

The robot's state consists of its position and velocity for each joint. The position of the robot is represented as  $\mathbf{q}_{err}$ , as the position error from the goal position.

$$\mathbf{x} = \begin{bmatrix} \mathbf{q}_{err} \\ \mathbf{q}_{vel} \end{bmatrix} \quad (1)$$

$$\dot{\mathbf{x}} = \begin{bmatrix} \mathbf{q}_{vel} \\ \mathbf{q}_{acc} \end{bmatrix} = \begin{bmatrix} \mathbf{q}_{vel} \\ \mathbf{M}^{-1}(\boldsymbol{\tau} - \mathbf{C}) \end{bmatrix} \quad (2)$$

where

$\mathbf{M}$  : inertia matrix

$\boldsymbol{\tau}$  : torque applied by joints

$\mathbf{C}$  : gravity, friction, and other external effects

$\mathbf{C}$  is calculated by Pinocchio library, it is completely separated from QP problem.

For the QP formulation we need to construct a different state. To start, here is an example QP formulation:

$$\begin{aligned} & \text{minimize} \quad \frac{1}{2} \mathbf{x}^T \mathbf{P} \mathbf{x} + \mathbf{q}^T \mathbf{x} \\ & \text{subject to} \quad \mathbf{l} \leq \mathbf{A} \mathbf{x} \leq \mathbf{u} \end{aligned}$$

The new state for QP will be:

$$\mathbf{x} = \begin{bmatrix} \mathbf{q}_{err}^{t+1} \\ \mathbf{q}_{vel}^{t+1} \\ \mathbf{u} \end{bmatrix} \quad (3)$$

Robot dynamics in the QP problem will be very simple:

$$\begin{aligned} \mathbf{q}_{err}^{t+1} &= \mathbf{q}_{err}^t + \mathbf{q}_{vel}^{t+1} \cdot T \\ \mathbf{q}_{vel}^{t+1} &= \mathbf{q}_{vel}^t + \mathbf{M}^{-1} \cdot \boldsymbol{\tau} \cdot T \end{aligned} \quad (4)$$

where

$T$  : time step between each controller iteration

: the superscripts mean time step  $t$  and  $t + 1$

We can write the dynamics as linear constraints as follows:

$$\mathbf{l} = \mathbf{u} = \mathbf{A} \mathbf{x} \quad (5)$$

$$\mathbf{l} = \mathbf{u} = \begin{bmatrix} \mathbf{q}_{err}^t \\ \mathbf{q}_{vel}^t \\ \mathbf{u}_{lim} \end{bmatrix} = \begin{bmatrix} 1 & -T & 0 \\ 0 & 1 & -\mathbf{M}^{-1} \cdot T \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{q}_{err}^{t+1} \\ \mathbf{q}_{vel}^{t+1} \\ \mathbf{u} \end{bmatrix} \quad (6)$$

The cost will be:

$$\mathbf{P} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \quad (7)$$

$$\mathbf{q} = \mathbf{0} \quad (8)$$