# MATLAB #5 - Local regulation

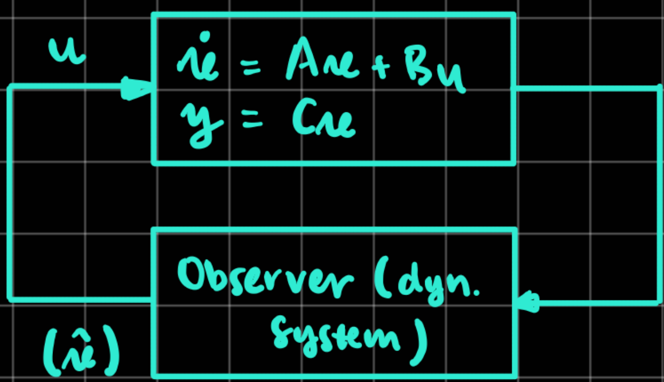
stabilization

Pendulum — Satellite

What we are going to observe:

1) Separation principle is no longer the case.
   Let a system:

$$\begin{cases} \dot{x} = Ax + Bu \\ u = Kx \end{cases} \longrightarrow (A, B) \longrightarrow \dot{x} = \underbrace{(A + BK)}_{\text{Hurwitz}}x$$



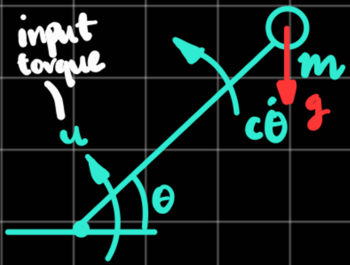

$$A_{cl} = \begin{bmatrix} A+BK & * \\ 0 & A+LC \end{bmatrix}$$

$$\sigma(A_{cl}) = \sigma(A+BK) \cup \sigma(A+LC)$$

2) Domain of attraction $\mathcal{A}$. With the LaSalle principle, using the "maximum" from: $\dot{V} \leq 0$
   $= 0$
   $< 0$

Lyapunov function

# PENDULUM EXAMPLE




Model equation:

$$ml^2\ddot{\theta} + cl\dot{\theta} + mgl\sin\theta = u$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \end{bmatrix} \Rightarrow \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{g}{l}\sin x_1 - \dfrac{c}{ml}x_2 \underbrace{+\dfrac{1}{ml^2}u}_{\tilde{u}} \end{cases}$$

1) Stability eq. (Dynamical Coherence)

2) Regulator eq. $u^*$

$$\rightarrow \quad u = u^* + K(x - x^*)$$

$$A = \frac{\partial f}{\partial x}\bigg|_{x^*} \qquad B = \frac{\partial f}{\partial x}\bigg|_{x = x^*}$$

$$A = \begin{bmatrix} \partial f_1/\partial x_1 & \partial f_1/\partial x_2 \\ \partial f_2/\partial x_1 & \partial f_2/\partial x_2 \end{bmatrix} \qquad B = \begin{bmatrix} \partial f_1/\partial u \\ \partial f_2/\partial u \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 \\ -g/l\cos(x_1) & -\dfrac{c}{ml} \end{bmatrix}_{x = x^*} \qquad B = \begin{bmatrix} 0 \\ 1/ml^2 \end{bmatrix}$$

From the expression above, we want to observe
$\dot{x} = Ax + Bu + h(x)$, where $\Theta \subset \mathbb{R}^n$

$$u = u^* + \overbrace{K\tilde{x}}^{\text{P.D. controller}}$$
$$= \underbrace{mgl\ \sin(x_1^*)}_{\text{feed forward action}} + K_1 x_1^* + K_2 x_2^*$$

$$\tilde{x} \xrightarrow[t\to\infty]{} 0 \quad \Rightarrow \quad x \xrightarrow[t\to\infty]{} x^*$$

$$\dot{\tilde{x}}_1 = \dot{x}_1 - \dot{x}_1^* = x_2 = \tilde{x}_2$$

$$\dot{\tilde{x}}_2 = \dot{x}_2 - 0 = -\frac{g}{l}\sin(x_1) - \frac{c}{ml}\tilde{x}_2 + \frac{1}{ml^2}u$$

$$\dot{\tilde{x}}_1 = \tilde{x}_2$$
$$\dot{\tilde{x}}_2 = -\frac{g}{l}\sin(\tilde{x}_1 + x_1^*) - \frac{c}{ml}\tilde{x}_2 + \frac{1}{ml^2}u$$
$$u = u^* + K\tilde{x}$$
$$= mgl\ \sin(x_1^*) + K\tilde{x} \longrightarrow \text{P. D. controller}$$

Applying to Lyapunov :

$$V(\tilde{x}) = \frac{1}{2}\tilde{x}_2^2 + \frac{g}{l}(1-\cos(\tilde{x}_1))$$

$$\dot{V}(\tilde{x}) = \frac{\partial V}{\partial \tilde{x}_1}\dot{\tilde{x}}_1 + \frac{\partial V}{\partial \tilde{x}_2}\dot{\tilde{x}}_2$$
$$= \left(\frac{g}{l}\sin(\tilde{x}_1)\right)\tilde{x}_2 + \tilde{x}_2\left(-\frac{g}{l}\sin(\tilde{x}_1 + x_1^*)\right.$$

$$-\frac{c}{ml}\tilde{x}_2 + \frac{1}{ml^2}u$$

$$= -\tilde{x}_2 \frac{g}{l}\left(\sin(\tilde{x}_2 + x_2^*) - \sin(\tilde{x}_2)\right) - \frac{c}{ml}\tilde{x}_2^2$$

$$+ \frac{1}{ml^2}u\tilde{x}_2$$

$$= \frac{1}{ml^2}\tilde{x}_2\left(mgl\sin(x_1^*) - K_1\tilde{x}_1 - K_2\tilde{x}_2\right)$$

$$= \frac{g}{l}\sin(x_1^*)\tilde{x}_2 = \frac{K_1}{ml^2}\tilde{x}_1\tilde{x}_2 - \frac{K_2}{ml^2}\tilde{x}_2^2$$

$$= -\frac{g}{l}\tilde{x}_2\left(\sin(\tilde{x}_1 + x_1^*) + \sin(x_1^*) - \sin(\tilde{x}_1)\right)$$

$$- \frac{K_1}{ml^2}\tilde{x}_1\tilde{x}_2 - \frac{K_2}{ml^2}\tilde{x}_2^2 - \frac{c}{ml}\tilde{x}_2^2$$

## PENDULUM ON MATLAB

For the solver, we can use Runge-Kutta, Euler, ODE45, or whatever. However, qualitative analysis does not require such much details, so it the solver does not matter that much.

provides more details