MATLAB

$A = [v_i \; \dots \; v_n] \in \mathbb{R}^{n \times n}$

$\mathcal{R}^+ = \text{image}(R) = \text{image}\{B \;\; AB \;\; \dots \;\; A^{n-1}B\}$

$\downarrow$

$\mathcal{R}^+(\infty)$

$= \text{span}\{w_1 \;\; \dots \;\; w_n\}$

$= \{x : x = Rv, \; \forall v \in \mathbb{R}^n\}$

Call operator $T(x, y, z) = (x, y, 0)$

$$T(\vec{x}) = A \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$\Downarrow$

image $(T(\vec{x}))$ in x-y plane $=$ $\{x : x = Rv, \; \forall v \in \mathbb{R}^n\}$

$\mathcal{R}_t^+ = \text{image } R_t$

$= \text{image}\{B \;\; AB \;\; \dots \;\; A^{t-1}B\}$

Suppose that $x_{t+1} = A x_t + B u_t$, $\alpha \in \mathbb{R}$

$A = \begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 \\ \alpha \end{bmatrix} \Rightarrow \quad R =$ (next page) cont...

$$R = \begin{bmatrix} 1 & 2-\alpha \\ \alpha & -4+2\alpha \end{bmatrix} = \text{rank}(R) = n$$

$$\det(R) = -4 + 2\alpha - \alpha(2-\alpha) = \alpha^2 - 4 = 0$$

$\alpha = +2$ $\qquad$ $\alpha = -2$

For $\alpha = -2$ $\qquad \mathcal{R}^+ = \text{image} \begin{bmatrix} 1 & 4 \\ -2 & -8 \end{bmatrix}$

$\qquad\qquad\qquad\qquad\qquad\quad v_1 \quad 4v_1$

$$\{ x : x = Rw, \forall w \in \mathbb{R}^2 \}$$

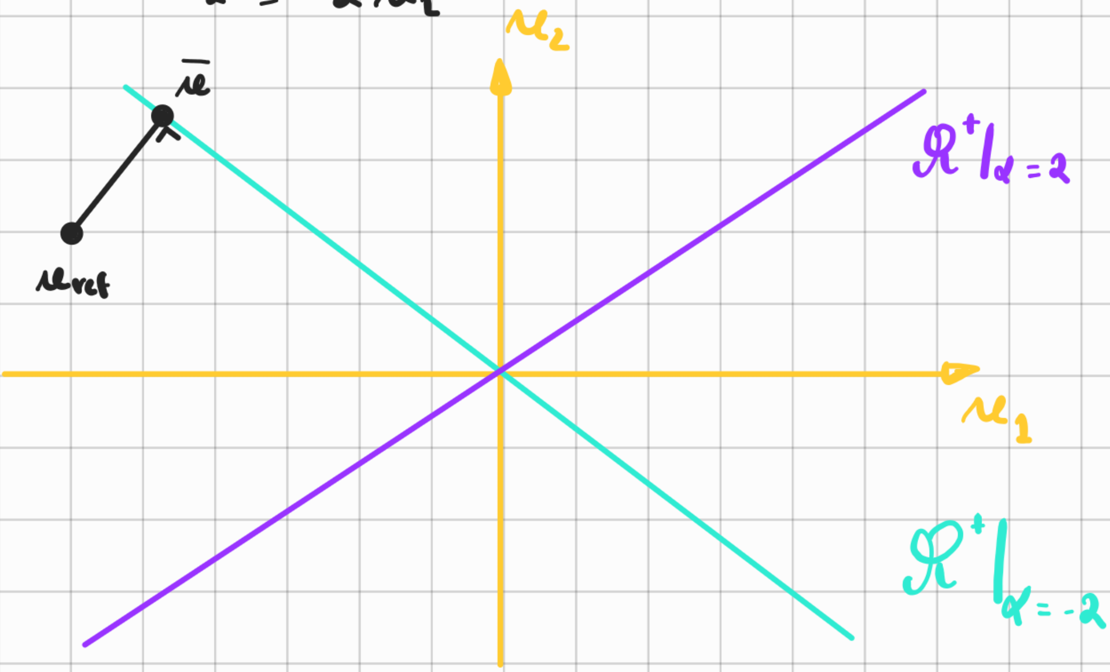
$$R \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} w_1 + 4w_2 \\ -2w_1 - 8w_2 \end{bmatrix} \rightarrow \begin{cases} x_1 = w_1 + 4w_2 \\ x_2 = -2w_1 - 8w_2 \end{cases}$$

$$= -2(w_1 + 4w_2)$$

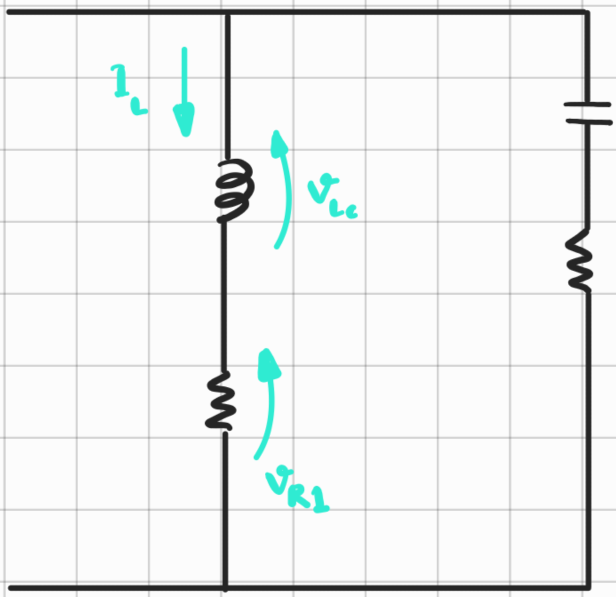
$$= -2x_1$$

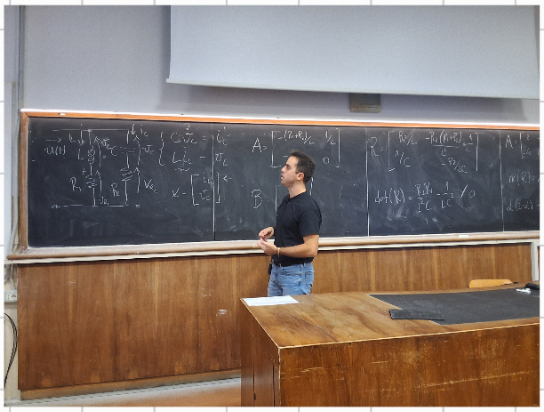
x_1 is whatever

$x_2 = -2x_1$



RLC Circuit





$$\det(R) = \frac{R_2 R_1}{L^2 C} - \frac{1}{L C^2}$$

$$\neq 0 \qquad R^2 = \frac{L}{C}$$