Lecture #1 – Introduction

ST&AC

EXAMINATION

- **ONLY ORAL** and single exam although it consists of two modules.
- Some simple exercise (presented during the course) delivered during the exam.
- Some simple MATLAB implementation

TEACHING METHODS – Slides are only the final computation. The process almost is on the blackboard.

Laplace $\rightarrow$ STATE SPACE DOMAIN

SYSTEM THEORY as a fundamental tool for the systematic study of structural properties of complex linear system.

CLASSICAL vs MODERN CONTROL

BODE & NYQUIST

KALMAN

more complex system

Control theory aims to model a complex system mathematically. It connects many <u>sub-systems</u> and interacts with the environment through <u>input/output channels</u>.

COURSE CONTENTS

Module 1:

- Dynamic systems representation
- Stability of linear and non-linear systems
- Reachability and controllability subspace and matrix
- observability and reconstructability

Module 2:

- "Advanced" control
  - Linearization
  - Gain scheduling
  - Robust control via integral action
  - Non-Linear sys control via linearization
  - State-feedback stabilization
  - High-gain nonlinear observers

Dynamic Systems

To understand the system, we need to describe its behavior by means of a <u>mathematical model</u>.

The system <u>interats</u> with the environment through $\boxed{\text{INPUT}}$ and $\boxed{\text{OUTPUT}}$

DYNAMIC

TIME - DOMAIN

STATE - SPACE REPRESENTATION — to describe the dynamic system by means of ODEs for continuous - time systems an DEs for discrete - time systems.

Continuous - time systems $t \in \mathbb{R}$

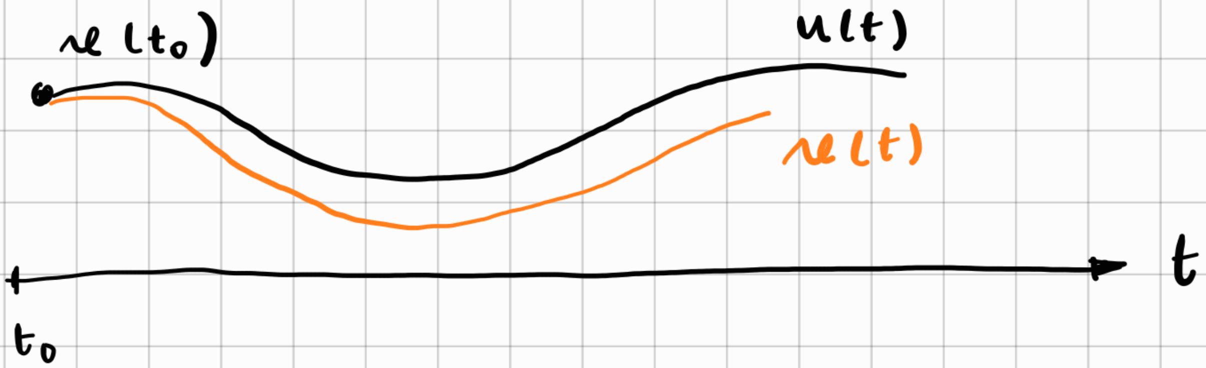
$$\dot{x}(t) = f(x(t), u(t), t) \qquad x \in \mathbb{R}^n, \; u \in \mathbb{R}^m$$

$$y(t) = h(x(t), u(t), t) \qquad y \in \mathbb{R}^p$$

typially assigned ← $x(t_0) \in \mathbb{R}$

n = n. input m = n. ext. input p = n. ourput

x = input u = ext. input y = output



Discrete-time systems $t \in \mathbb{Z}$

$$x(t+1) = f(x(t), u(t), t) \qquad x \in \mathbb{R}^n, \; u \in \mathbb{R}^m$$

$$y(t) = h(x(t), u(t), t) \qquad y \in \mathbb{R}^p$$

typically assigned ← $x(t_0) \in \mathbb{R}$

- STATIONARY systems if the vector fields $f(\cdot)$ and $h(\cdot)$ do not depends on t ($t_0 = 0$ without loss of generality)

$$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \qquad f(x,u) = \begin{pmatrix} f_1(x,u) \\ \vdots \\ f_n(x,u) \end{pmatrix}$$

LINEAR SYSTEMS

$$\left.\begin{array}{ll} \text{ODEs} & \dot{x}(t) \\ \text{DEs} & x(t+1) \end{array}\right\} = A(t)\,x(t) + B(t)\,u(t)$$

$$y(t) = C(t)\,x(t) + D(t)\,u(t)$$

$x \in \mathbb{R}^n$

$u \in \mathbb{R}^m$

$y \in \mathbb{R}^p$

where:

$$A = \begin{pmatrix} & \\ & \end{pmatrix}_{n \times n} \qquad B = \begin{pmatrix} & \\ & \end{pmatrix}_{n \times m}$$

$$C = \begin{pmatrix} & \\ & \end{pmatrix}_{p \times n} \qquad D = \begin{pmatrix} & \\ & \end{pmatrix}_{p \times n}$$

Stationary linear systems

$$\left.\begin{array}{l} \dot{x}(t) \\ x(t+1) \end{array}\right\} = A\,x(t) + B\,u(t)$$

$$y(t) = C\,x(t) + D\,u(t)$$

Proper linear systems

$$\left.\begin{array}{l} \dot{x}(t) \\ x(t+1) \end{array}\right\} = A\,x(t) + B\,u(t)$$

$$y(t) = C\,x(t)$$

The input $u(t)$ doesn't affect the output

Autonomous linear system

$$\left.\begin{array}{l}\dot{x}(t)\\ x(t+1)\end{array}\right\} = Ax(t)$$

$$y(t) = Cx(t)$$

Doesn't require input $u(t)$