$\dot{x}_1 = x_2$

$\dot{x}_2 = -x_1$

$\Rightarrow \dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x, \in \mathbb{R}$

$\boxed{y(t) = (\cos(t), \sin(t))}$

$\partial \rightarrow \dot{y}(t) = f(y(t))$

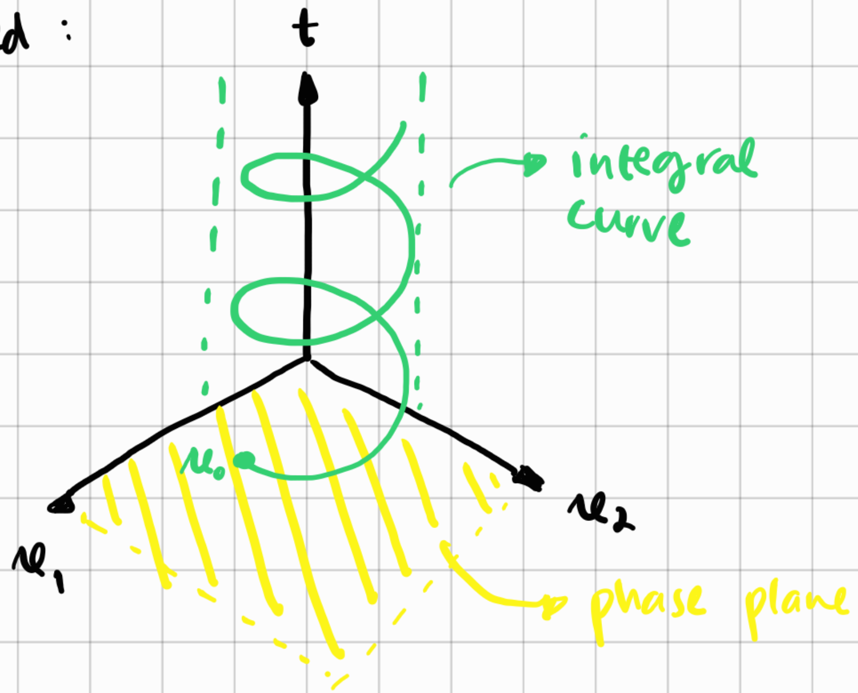
$f = \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$

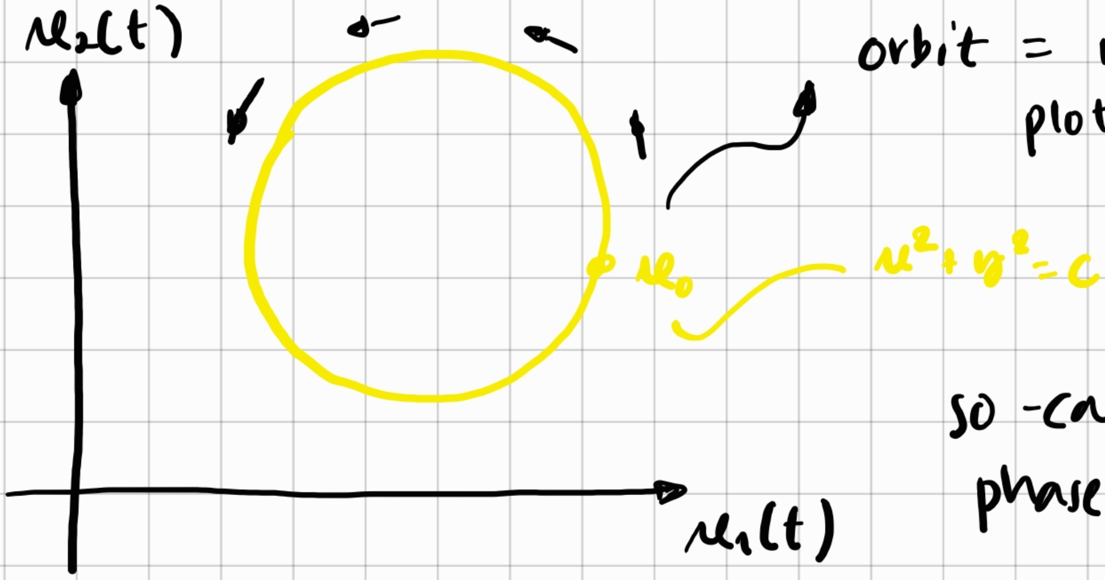
$x(0) = x_0$

$\uparrow$

$t_0 = \emptyset$

Plotted :





orbit = not plotting time

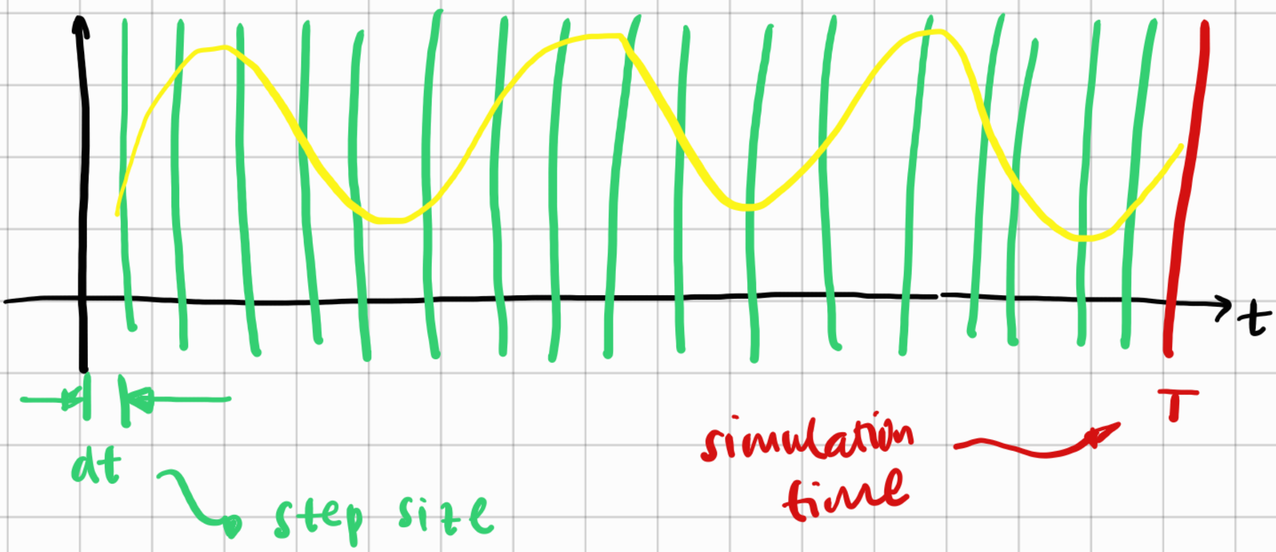
so-called phase space

cycle $\Leftrightarrow$ periodic solution,
$x(t) = x(t+T)$

limit cycle <u>does not</u> imply periodic solution as $t \to \infty$

Exercise 1

- Harmonic Oscillator



Using naive-approach (Euler), it can be defined:

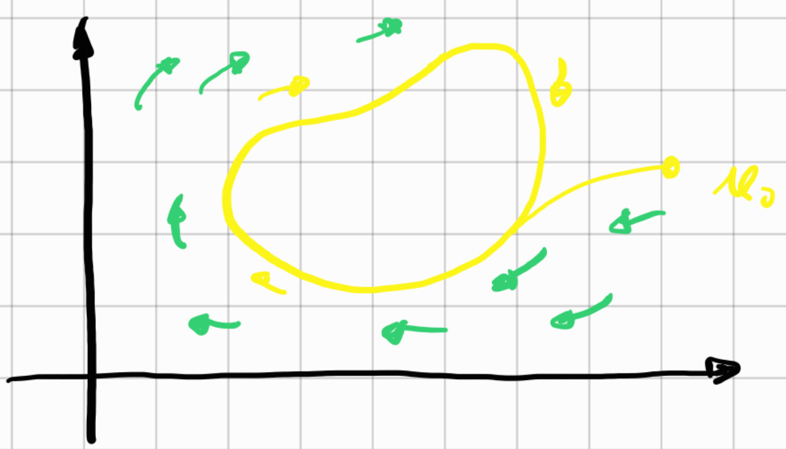
$$\dot{x} = f(x)$$
$$= \left(\frac{x_{k+1} - x_k}{dt}\right)$$

$$x_{k+1} = x_k + dt \cdot f(x)$$

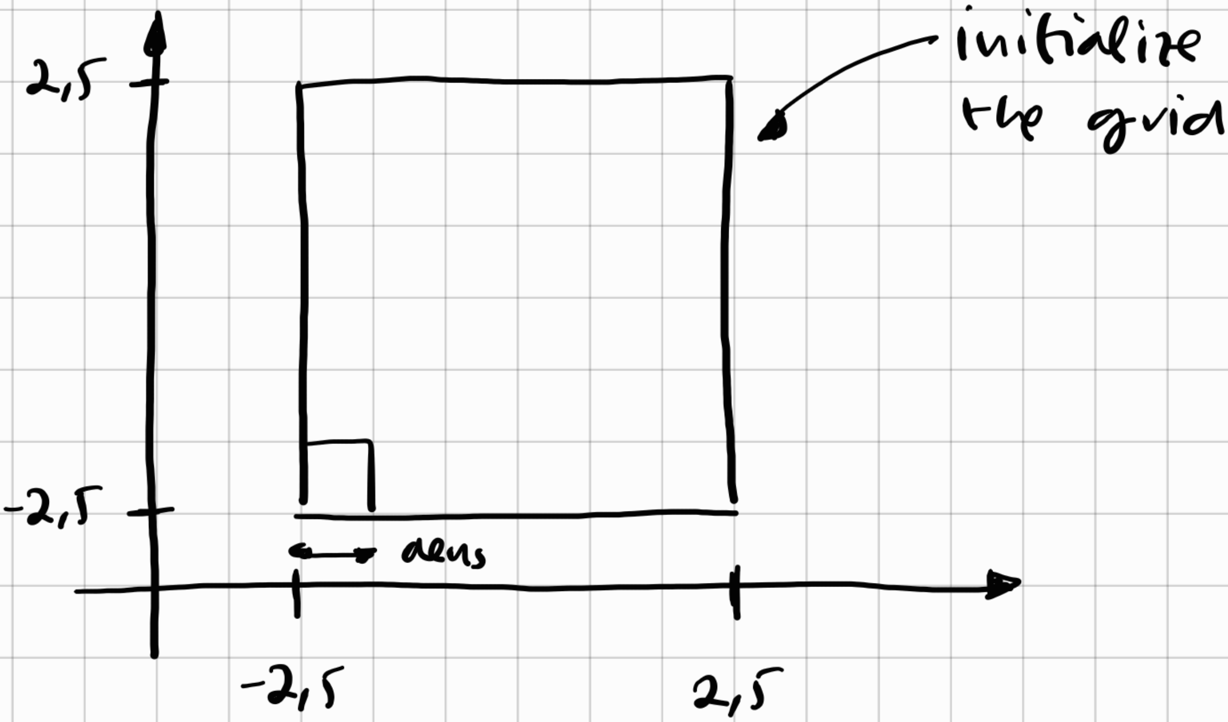
$\boxed{\text{ode 45}}$ $\longrightarrow$ a built-in solver

- Van der Pol Oscillator - Vector field plotting

It's an "expensive" computation since the process requires every solution in a plane



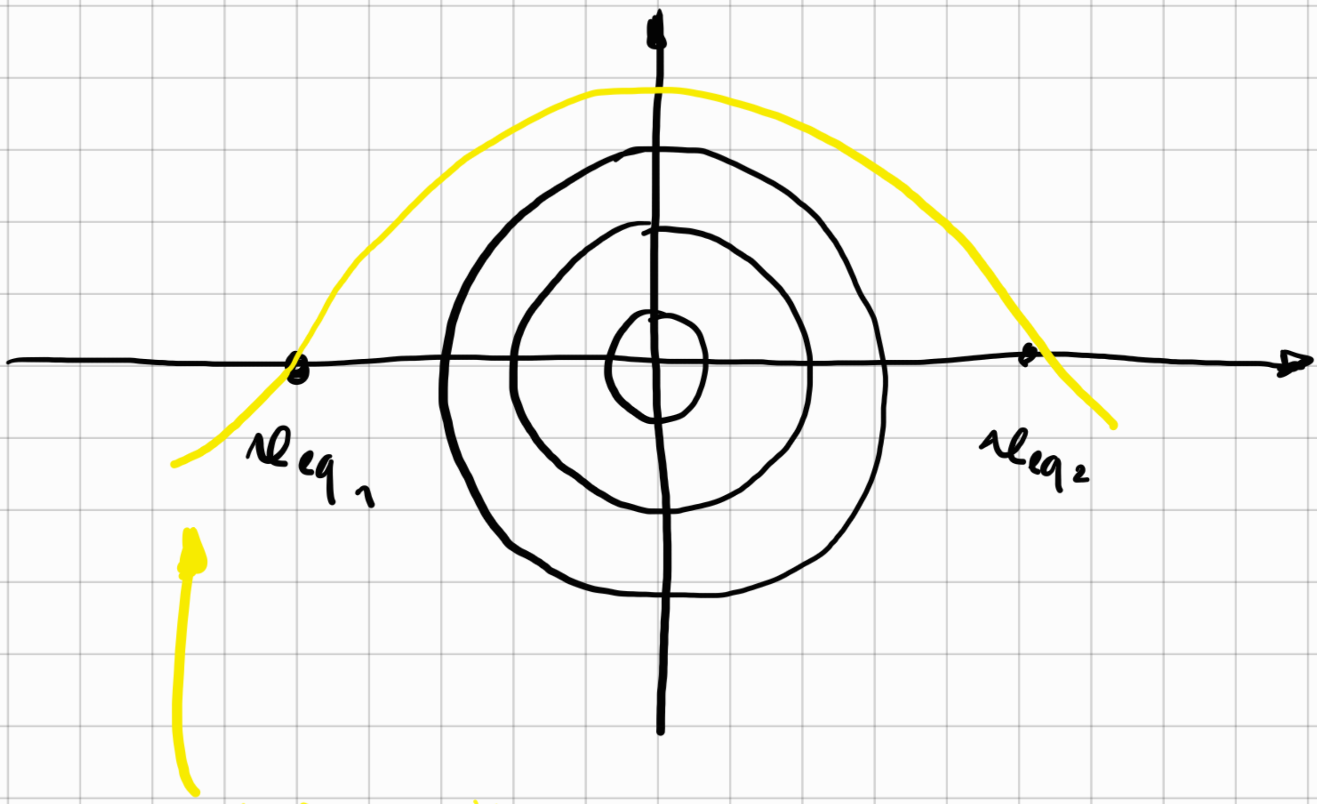
How to plot the vector field:



$\dot{x} = f(x)$ where $f : \mathbb{R}^2 \to \mathbb{R}^2$

$$f(x) = \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} \begin{array}{l} \to u \\ \to v \end{array}$$

PENDULUM WITHOUT FRICTION



this orbit
is not solvable
in current MATLAB
algorithm

CHAOTIC BEHAVIOR OF THE PENDULUM

$$\ddot{x} + \sin(x) + c x = u(t)$$

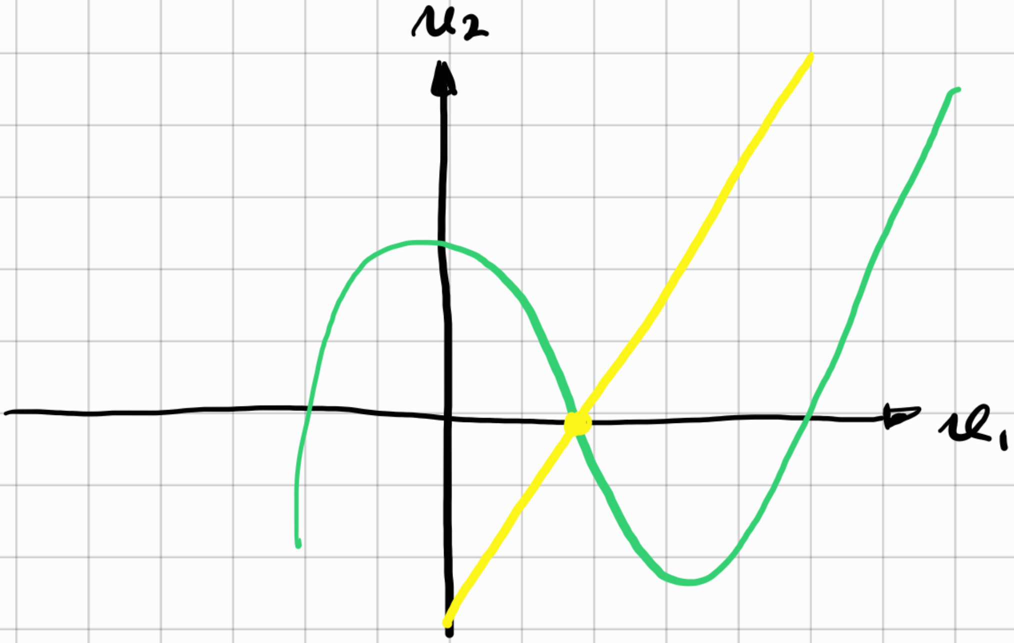
$c \ll 1$

input: $\sin(t)$

If we're using Euler solver (ode45), the trajectory does not make sense.

FITZ-HUGH-NAGUMO

- BIBO Stability
- Excitable



$$\dot{x} = \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix} \Rightarrow f(x) = 0$$

$f_1(x) = 0 \longrightarrow x_1 - \frac{1}{3}x_1^3 + I$

$f_2(x) = 0 \longrightarrow \frac{x_2 + a}{b}$