

Lecture #1 — Introduction

Aim of the courses:

- Formulation of a theory of probability:
 - Rigorous (arithmetic)
 - Application-oriented
 - Basics of future classes
- Focus on standard models
- Introduction to stochastic process and non-linear optimization

KOLMOGOROV'S
HANDBOOK
1953

In discrete cases:

S Sample space

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

$$E_1 = \{1, 3, 5\}$$

$$E_3 = \{1, 2, 3\}$$

$$E_2 = \{2, 4, 6\}$$

Event set, a possible subset from Ω

Atomic event \rightarrow a single outcome of a possible subset from Ω

$$E_6 = \{6\}$$

discrete
example

For the finite case:

$$\Omega = [0, +\infty[$$

\rightarrow boundaries

PROBABILITY THEORIES exist when we only want to see the outcome without knowing the causations behind the results.

The probability function of an event is expressed

$P(E) = P$ ← modelling of the problem

STANDARD MODELS are tools to approximate the actual problem, and we only pick an interval or a set of points (or event a single point) as the event. For examples:

$$E = [10, +\infty[$$

$$E = [0, 5]$$

$$E = [2, 3]$$

$$E = \{1, 3\}$$

STOCHASTIC PROCESSES are approaches using random variables / function

X

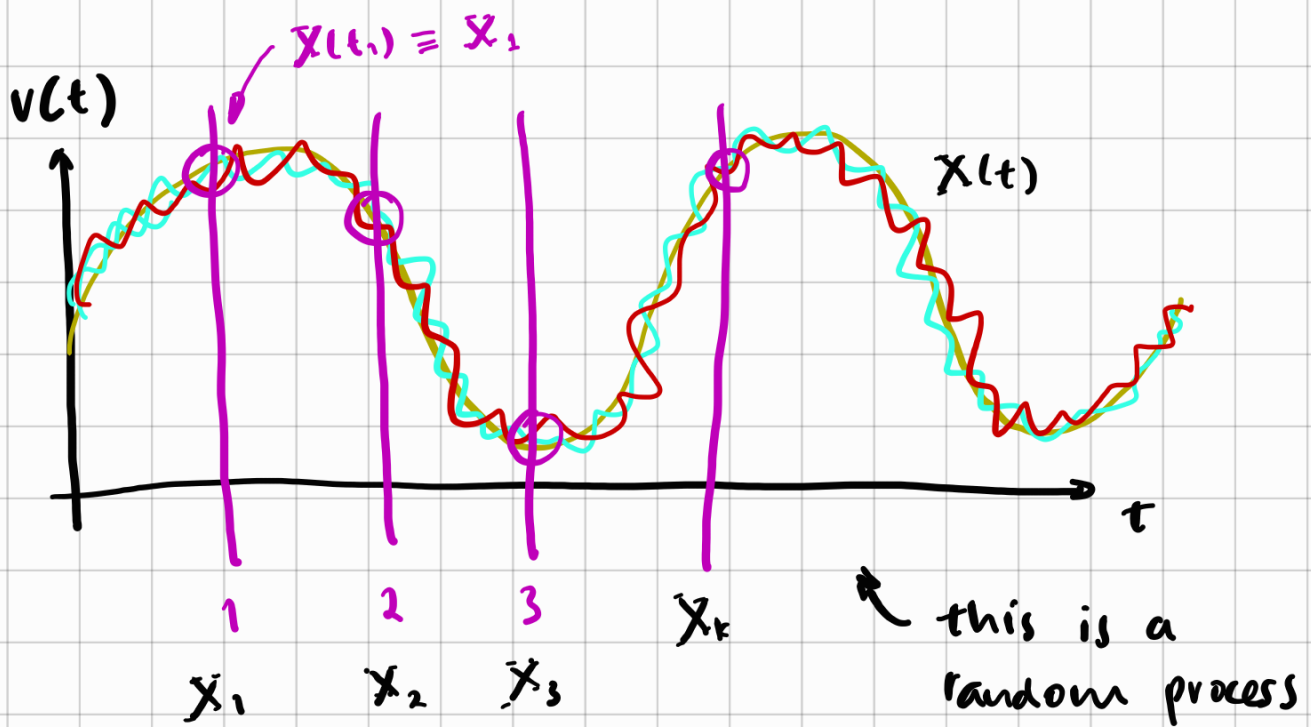
re

↘ random variable

↘ actual value of a variable

The function in engineering cases usually are time-based (t)

$X(t)$



In stochastic problem, the variables are random and independent.

NON LINEAR OPTIMIZATION. Examples:

weight (%)

$\begin{matrix} \mu_1 \\ \sigma_1 \end{matrix} \nearrow A_1 \rightarrow w_1$
 $\begin{matrix} \mu_2 \\ \sigma_2 \end{matrix} \nearrow A_2 \rightarrow w_2$

constraint

$w_1 + w_2 = 1$

we should determine the weight to produce the optimal outcome

return

volatility

$S = \frac{\mu}{\sigma}$

$$= \frac{w_1 \mu_1 + w_2 \mu_2}{\sqrt{\mu \cdot \sigma \cdot \mu}}$$

EIGENVALUES
EIGENVECTORS