MATLAB #6 LOCAL REGULATOR

- domain of attraction
- separation principle does not hold in N.L.

suppose a linear system:

$$\begin{cases} \dot{x} = f(x,u) \\ u = \gamma(x) \end{cases} \Longrightarrow f_\gamma(x)$$

We then linearize the system:

$$A = \left.\frac{\partial f}{\partial x}\right|_{\substack{x = x^* \\ u = u^*}} \Longrightarrow \dot{x} = Ax + h(x) \quad x^n \to \infty$$

$$V(x) = \frac{1}{2} x^T P x$$

$$[A^T P + PA \leq -I_k]$$

$$\|h(x)\| \leq \varepsilon$$

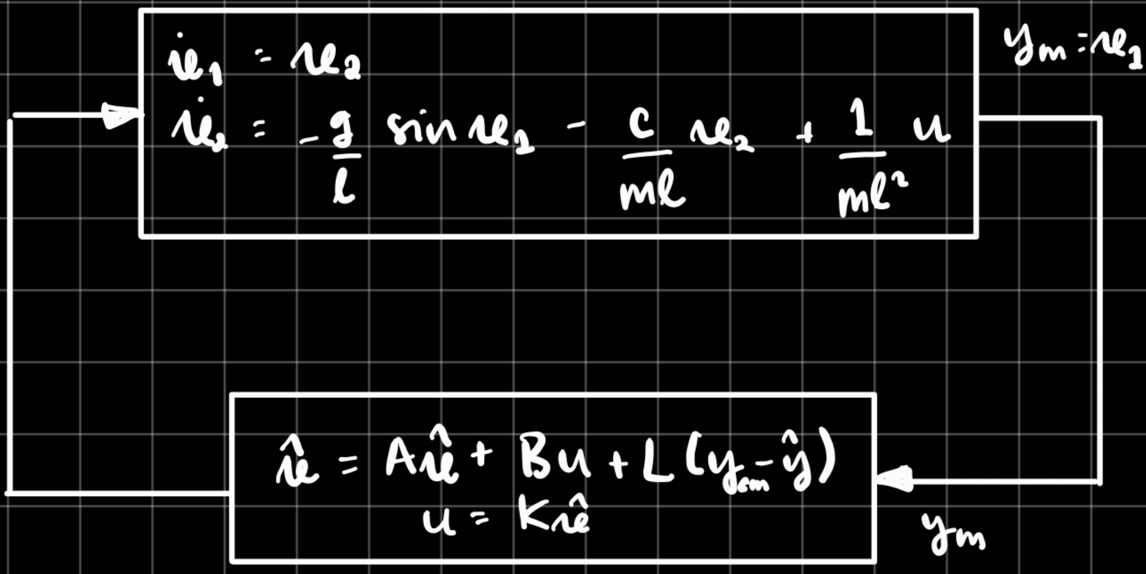
$$\dot{V}(x) = -x^T x + K \cdot h(x)$$

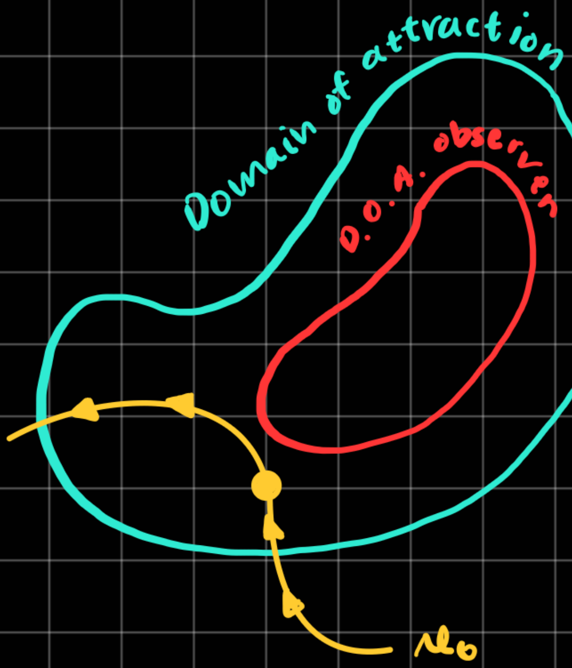
How the separation does not hold in N.L.?

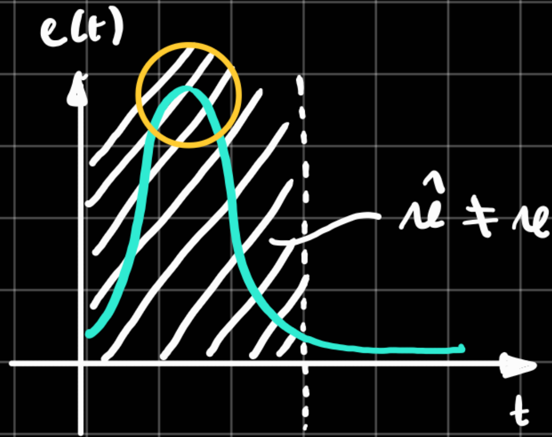
Suppose a pendulum

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\frac{g}{l} \sin x_1 - \frac{c}{ml} x_2 + \frac{1}{ml^2} u \end{cases}$$

$$y_m = x = \begin{bmatrix} x_1 \\ \cancel{x_2} \end{bmatrix} = x_1$$



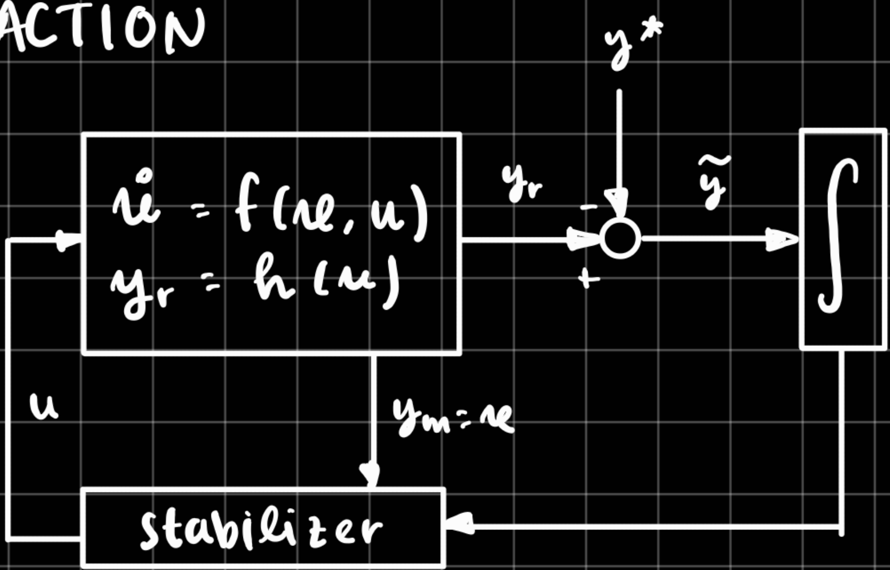




INTEGRAL ACTION

$u = u^* + K\tilde{x}$

↑ dependent from system param

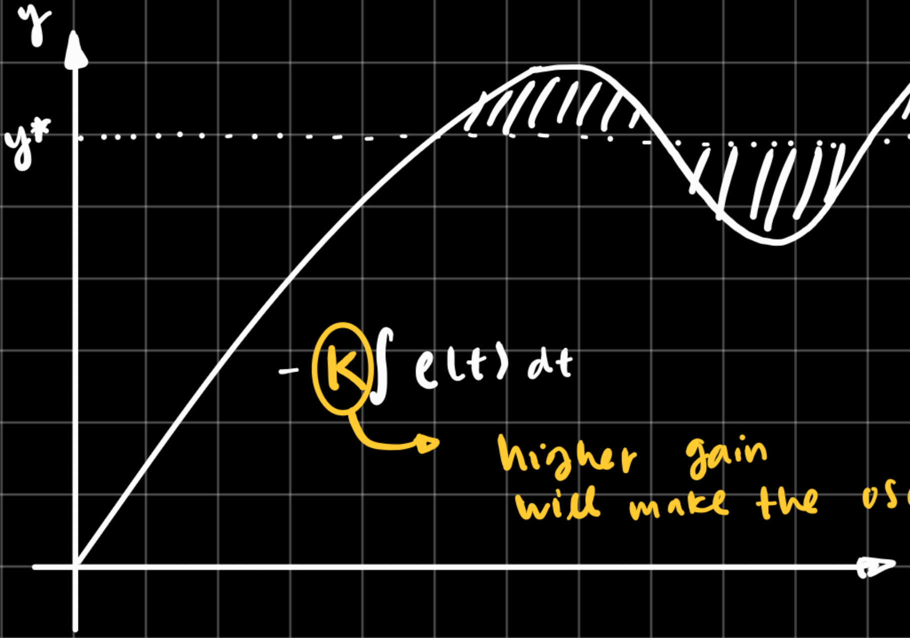


-) Static if $y_m \equiv x$
-) Static + observer (dynamic)

What the integral action does:

$u = K_P x_2 + K_D x_a + K_I \sigma_a$ (P.I.D. controller)

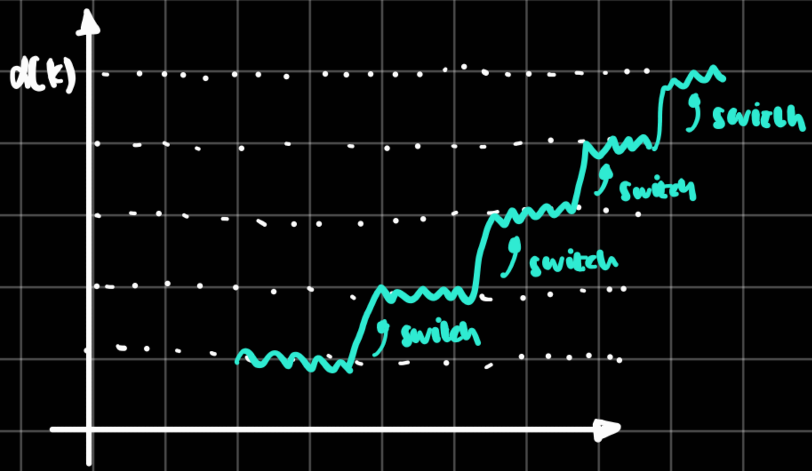
When we set the integral gain too high



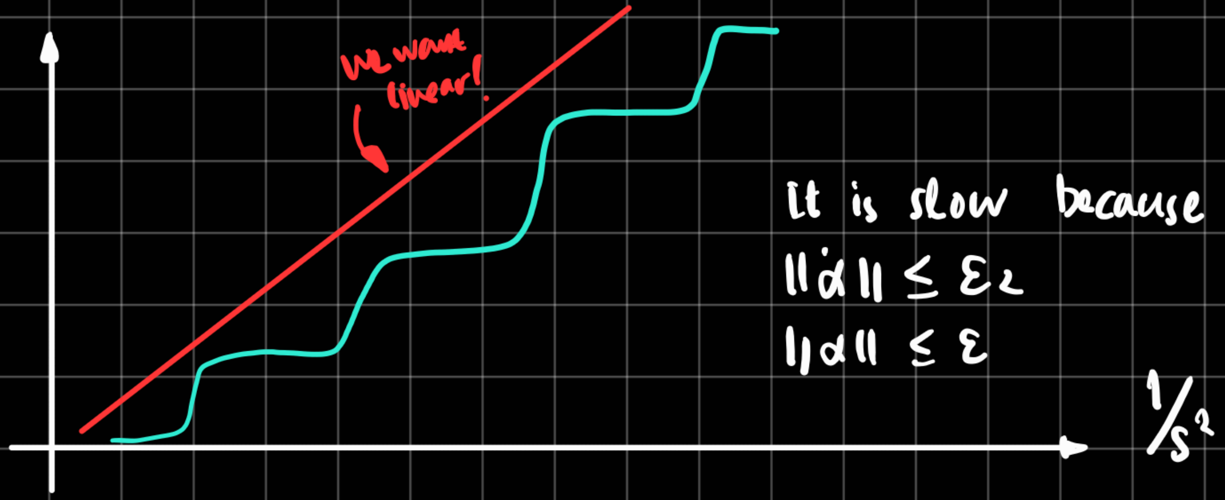
The solution:

-) Set (K_I, K_D), K_P
-) Anti-windup

GAIN SCHEDULER

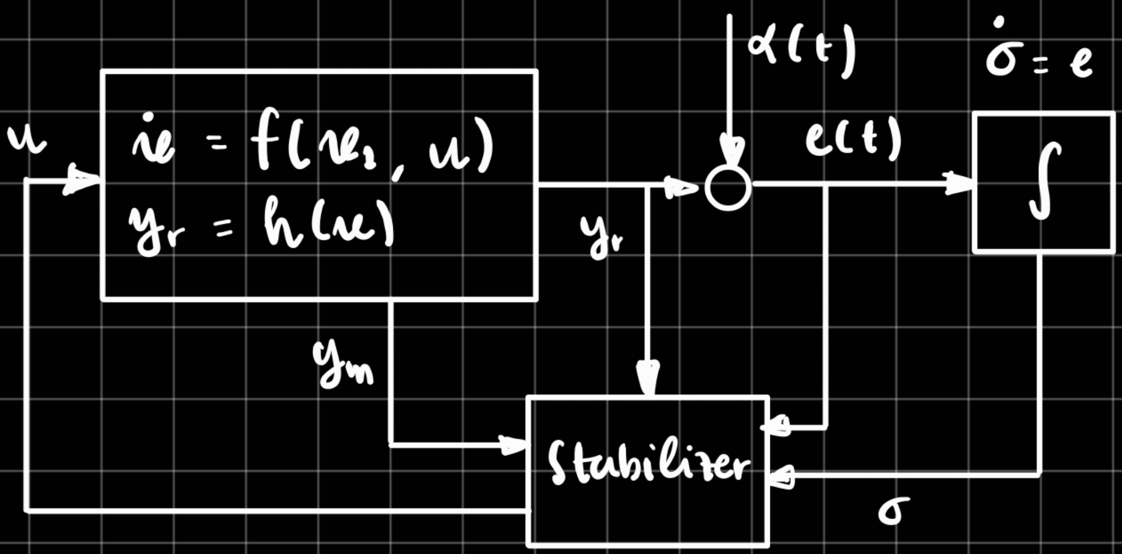


We will see the switching variable, expect this behavior



It is slow because
$\|\dot{\alpha}\| \leq \varepsilon_2$
$\|\alpha\| \leq \varepsilon$

The idea behind the scheduling:



$\dot{\sigma} = \underbrace{y_r - \alpha}_{e}$ $\longrightarrow$ α is "slow enough".
so we start "slow enough"
$\tilde{\alpha}(0) = \alpha(0) - x^*(\alpha(0))$

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{g}{l}\sin x_1 - \dfrac{c}{ml}x_2 + \dfrac{1}{ml^2}u \end{cases}$$

$$\begin{cases} 0 = x_2^* \\ 0 = -\dfrac{g}{l}\sin(\underbrace{x_1^*}_{\alpha}) - \cancel{\dfrac{c}{ml}x_2}^{\,0} + \dfrac{1}{ml^2}u^* \end{cases}$$

$$u(\alpha) = u^*(\alpha) + K\left(x - \begin{bmatrix}\alpha \\ 0\end{bmatrix}\right) \longrightarrow \text{Feed-forward + Local}$$

P.I.D

$$u = K_p x + K_D \dot{x} + K_I \int e(t)\,dt$$

$$A(\alpha)\Big|_{\substack{x = x^*(\alpha) \\ u = u^*(\alpha)}} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l}\cos(\underbrace{x_1}_{\alpha(t)}) & -\frac{c}{ml} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l}\cos(\alpha) & -\frac{c}{ml} \end{bmatrix}$$

$$B(\alpha) = B = \begin{bmatrix} 0 \\ 1/ml^2 \end{bmatrix} \Rightarrow \begin{cases} \dot{\sigma} = x_1 - \alpha \\ u = \underbrace{[K_P \ K_D]}_{K_\alpha} x + K_I \sigma \end{cases}$$

$$\begin{cases} \dot{x} = A(\alpha)x + Bu \\ \dot{\sigma} = x_1 - \alpha \end{cases}$$

$$A_{cl}(\alpha) = \begin{bmatrix} A(\alpha) + B(\alpha)K(\alpha) & BK_I \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -g/l\cos(\alpha) & -c/ml \end{bmatrix} + \begin{bmatrix} 0 \\ 1/ml^2 \end{bmatrix} \begin{bmatrix} K_P & K_D \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -g/l\cos(\alpha) + \frac{1}{ml^2}K_P & -\frac{c}{ml} + \frac{1}{ml^2}K_D \end{bmatrix}$$

$$BK_I = \frac{1}{ml^2} K_I$$

Using the change of coordinate:

$$A = \begin{bmatrix} 0 & 1 & & \\ & & \ddots & \\ & & & 1 \\ -\alpha_0 & \cdots & & -\alpha_{n-1} \end{bmatrix} \qquad T = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \theta \\ \varphi \end{bmatrix} = T \begin{bmatrix} \varphi \\ \sigma \end{bmatrix}$$

$$\underbrace{T A_{cl}(\alpha)}_{\tilde{A}_{cl}(\alpha)} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_0 & -\alpha_1 & -\alpha_2 \end{bmatrix}$$

$$\varphi(\hat{A}_{cl}) = \lambda^3 + \alpha_2 \lambda^2 + \alpha_1 \lambda + \alpha_0$$

$$\begin{cases} \alpha_0 = -K_I / ml^2 \\ \alpha_1 = g/l \cos(\alpha) - \dfrac{K_P}{ml^2} \\ \alpha_2 = \dfrac{c}{ml} - \dfrac{K_D}{ml^2} = \dfrac{cl - K_D}{ml^2} \end{cases}$$

Based on Ruth-Hurwitz criterion:

$$\begin{cases} \alpha_2 > 0 \\ \alpha_0 > 0 \\ \alpha_2 \cdot \alpha_1 > \alpha_0 \end{cases} \Rightarrow \quad \alpha_2 > 0 \longrightarrow \frac{cl - K_D}{ml^2} > 0$$

$$K_D < cl$$

$$\alpha_0 > 0 \longrightarrow K_I < 0$$

$$\left(\frac{cl - K_D}{ml^2}\right)\left(\frac{g}{l}\cos(\alpha) - \frac{K_P}{ml^2}\right) > \frac{K_I}{ml^2}$$

$$\frac{g}{l}\cos(\alpha) - \frac{K_P}{ml^2} > -\frac{K_I}{\cancel{ml^2}} \cdot \frac{\cancel{ml^2}}{cl - K_D}$$

$$-K_P > -mgl\cos(\alpha) - \frac{ml^2 K_I}{cl - K_D}$$

$$K_P < mgl\cos(\alpha) + \frac{ml^2 K_I}{cl - K_D} < 0$$

$$\left.\begin{array}{l} K_P < mgl\Big|_{+1} \\ \\ K_P < -mgl\Big|_{-1} \end{array}\right\} + \frac{ml K_I}{cl - K_D}$$