Preparation of Papers for IEEE Trans on Industrial Electronics (Apr. 2021)

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I. SSA Decompsition

Let $N > 2$. Consider a real-valued time series:

$$F = (f_0, f_1, \dots, f_{N-1}) \tag{1}$$

A. Embeddings

This step involve transforming the time series in a multi dimentioned lagged vectors. Let N be an integer values representing the window length, where $1 \leq L \leq N$ the embeddings precedure, such that $K = N - l - 1$ lagged vectors.

$$x_i = (f_{i-1}, \dots, f_{i+L-2})^T \quad 1 \leq i \leq k$$

Therefore the trajectory matrix can be defined as follows:

$$X = (x_{ij})_{i,j}^{L,K} = \begin{bmatrix} f_0 & f_1 & f_2 & \dots & f_{k-1} \\ f_1 & f_2 & f_3 & \dots & f_k \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ f_{L-1} & f_L & f_{L+1} & \dots & f_{N-1} \end{bmatrix} \tag{2}$$

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The terms $X_{(i,j)} = f_{i+j-2}$. Therefore, the matrix X is the Henkel matrix of the time series F.

B. SVD: Singular Value Decomposition

After creating the Henkel matrix, i.e. a multi dimention representation of the univariate time series, SSA employes singular value decomposition in order to break down the matrix X into smaller blocks. Thus decomposting the Henkel matrix X as follows:

$$X = U \Sigma V^T \tag{3}$$

where U is an $L \times L$ unitary ortonormal matrix, representing the set of left singular vectors, Σ is LxK rectangular matrix cintaining the L singular values of the matrix X, ordered in magnitude ($\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \dots \geq \lambda_N$), and V a $K \times K$ unitary matrix containing the ortonormal set of right singular vectors of X. The SVD of the singular value can be rewritten as:

$$X = \sum_{i=0}^{d-1} \sigma_i U_i V_i^T = \sum_{i=0}^{d-1} X_i \tag{4}$$

Eq.4 shows how the X_i is the i-th elementary block of the matrix X given by the eigentriple (U_i, σ_i, V_i). Where the values of σ determines the magnitude of the i-th contribution of the block X_i. d represents the dimensionality of the of the Henkel matrix. This formulation extend the well-know decomposition framework of a time series in trend, seasonality and noise. As results of this decomposition, the initial time series can be seen as the sum of multiple components ordered by magnitude (trend, multiple seasonality, exogenous and noise). If we sum up all the d sub-blocks, the matrix is entirely reconstructed, while considering only the first r-th sub.blocks would filter the time series.

C. Reconstruction

To extract a time series from the elementary matrices, we'll employ **diagonal averaging**, which defines the values of the reconstructed time series $F^{(j)}$ as averages of the corresponding anti-diagonals of the matrices $X^{(j)}$. Formally, this is represented by introducing the **Hankelisation** operator,

$\hat{H}$, that acts on the $L \times K$ matrix $X^{(j)}$ to give a Hankel matrix $\hat{X}^{(j)}$, that is,

$$\hat{X}^{(j)} = \hat{H} X^{(j)}$$

The element $\hat{x}_{m,n}$ in $\hat{X}$, for $s = m + n$, is given by

$$\hat{x}_{m,n} = \begin{cases} \frac{1}{s+1} \sum_{l=0}^{s} x_{l,s-l} & \text{for } 0 \le s \le L-1 \\ \frac{1}{L} \sum_{l=0}^{L-1} x_{l,s-l} & \text{for } L \le s \le K-1 \\ \frac{1}{K+L-s-1} \sum_{l=s-K+1}^{L} x_{l,s-l} & \text{for } K \le s \le K+L-2 \end{cases}$$

D. Forecast using SSA

II. System Identification

ARX Model

We have chosen to represent the system in *discrete time*,

$$y_t + a_1 y_{t-1} + \cdots + a_n y_{t-p} = b_1 u_{t-1} + \cdots + b_q u_{t-q} \quad (5)$$

$$\theta = \left[a_1, \ldots, a_p, b_1, \ldots, b_q\right]^T \quad (6)$$

$$\varphi(t) = \left[-y_{t-1} \cdots - y_{t-p}, u_{t-1} \cdots u_{t-q}\right]^T \quad (7)$$

$$y_t = \varphi_t^T \theta_t \quad \forall t \in (1, N) \quad (8)$$

For a given system we can collect inputs and outputs over a time interval t

$$Z^N = \left\{u(t_0), y(t_0), \cdots, u(N), y(n)\right\} \quad (9)$$

$$\hat{\theta} = \arg\min_x V_N(\theta) + \lambda(\theta - \theta^*) R(\theta - \theta^*) \quad (10)$$

The ARX(p, q) model is given by:

$$y_t = \sum_{i=1}^{p} \varphi_i y_{t-i} + \sum_{j=1}^{q} \beta_j x_{t-j} + \epsilon_t \quad (11)$$

A. Piecewise Linear function

Algorithm 1 Grouping Time Series Data by Bin Medians

Require: Time series data $y = \{y_1, y_2, \ldots, y_T\}$, excitation data $u = \{u_1, u_2, \ldots, u_T\}$, bin size n
Ensure: Grouped data by bin medians
1: Divide the time series y into bins of size n
2: **for** $i = 1$ to $\lceil \frac{T}{n} \rceil$ **do**
3: $B_i \leftarrow \{y_{(i-1)n+1}, y_{(i-1)n+2}, \ldots, y_{\min(in,T)}\}$
4: Compute the median m_i of the bin B_i
5: **end for**
6: Group the data in y by their corresponding bin medians m_i

III. Higher-Level MPC

$$\min_{\zeta,} \sum_{t=1}^{T} \Gamma \left\|E\right\|_t^2 + \Lambda_t^T \sigma_t \quad (12a)$$

$$\text{s.t.} \quad \forall \sigma_t \ge 0 \quad (12b)$$

$$\zeta_t = \sum_{j=1}^{3} \zeta_{t-1} \quad (12c)$$

$$h_{min} \le h_t \le h_{max} \quad (12d)$$

IV. Lower-Level MPC

$$\min_{\omega, E, P, Q_{\text{out}}} \sum_{k=1}^{h} \mathcal{Q} \left\|h_k - h_r\right\|^2 + \mathcal{R} \left\|\omega_k\right\|^2 + \Gamma \left\|E\right\|_k^2 + \Lambda_k^T \sigma_k \quad (13a)$$

$$\text{s.t.} \quad \forall \sigma_k \ge 0,\ l \ge 0 \quad (13b)$$

$$Q_{\text{out},k} = \sum_{j=1}^{3} Q_{out_{j,k-1}} \quad (13c)$$

$$E_k = \sum_{j=1}^{3} E_{k-l} \quad (13d)$$

$$P_k = \sum_{j=1}^{3} P_{k-l} \quad (13e)$$

$$Q_{in,k} = \tilde{Q}_{in,k-1} \quad (13f)$$

$$h_k = \frac{1}{A}\left(\tilde{Q}_{in,k} - Q_{out,k}\right) \quad (13g)$$

$$\omega_l - \sigma_\omega \le \omega_k \le \omega_u + \sigma_\omega \quad (13h)$$

$$P_l - \sigma_P \le P_k \le P_u + \sigma_P \quad (13i)$$

$$h_r - \sigma_{h_r} \le h_k \le h_l + \sigma_{h_r} \quad (13j)$$

A. References

This is the paper I am citing [1]

References

[1] A. Quattrociocchi, R. K. Subroto, W. M. Oppedijk, and T. Dragičević, "Energy efficiency optimization of a wastewater pumping station through iot and ai: A real-world application of digital twins," in *IECON 2023-49th Annual Conference of the IEEE Industrial Electronics Society*, pp. 1–6. IEEE, 2023.