# 5
# Neural Networks

## Contents

Machine Learning (ML) is a field of research which studies algorithms which can learn to make predictions from data, i.e. predict a set of output variables, given a set of input variables. The algorithms typically take the form of a multivariate function, which is used to predict the output [83].

ML algorithms are often classified into four groups based on two distinctions: regression or classification, and supervised or unsupervised. The first distinction is based on the nature of the output distribution expected from the network; regression algorithms are designed to predict the outputs of a continuous function, whereas classification algorithms aim to separate data into groups. The distinction between supervised and unsupervised algorithms is based on the outputs used during training; in a supervised algorithm the true output is known, and the network's goal is to predict the true output, while in an unsupervised algorithm the output is unknown, and the networks goal is to extract meaningful structure from the data.

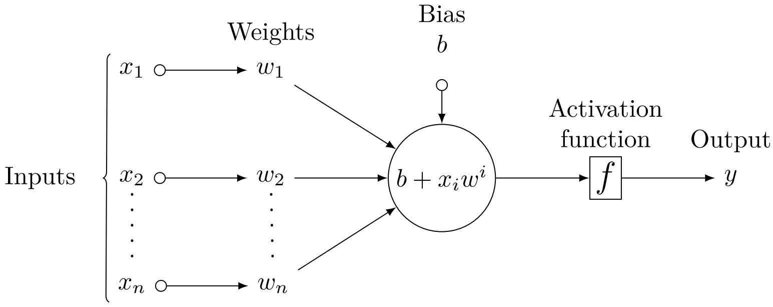

**Figure 5.1:** Graphical representation of an individual neuron in an artificial neural network.

Chapters 6 and 7 of this thesis describe two examples of the application of Neural Networks (NN) for event reconstruction in LArTPC data. These algorithms are classification algorithms based on the supervised learning approach. This chapter will not provide a full survey of the available ML techniques, instead it will briefly describe the theory behind NNs, as well as providing details of the techniques used in the subsequent chapters.

## 5.1 Artificial Neural Networks

Artificial neural networks (ANN) are a class of ML algorithm that draw inspiration from biological neurons. An ANN consists of a set of nodes, along with a set of connections between those nodes. The set of nodes and connections is often referred to as a graph or architecture. The nodes in the graph take the form of artificial neurons, which pass a number of inputs through an activation function to produce a single output, as depicted in Figure 5.1. The output of each neuron is either distributed to subsequent neurons in the network, or it is part of the output of the network.

One of the most widely used ANNs is the multi–layer perceptron (MLP)[83]; this class of network consists of at least three layers of nodes: an input layer, one or more hidden layers, and an output layer. The layers are connected in a feedforward configuration, such that the graph of nodes contains no cycles. The layers of an MLP are often fully connected or dense, meaning that the output of every node is

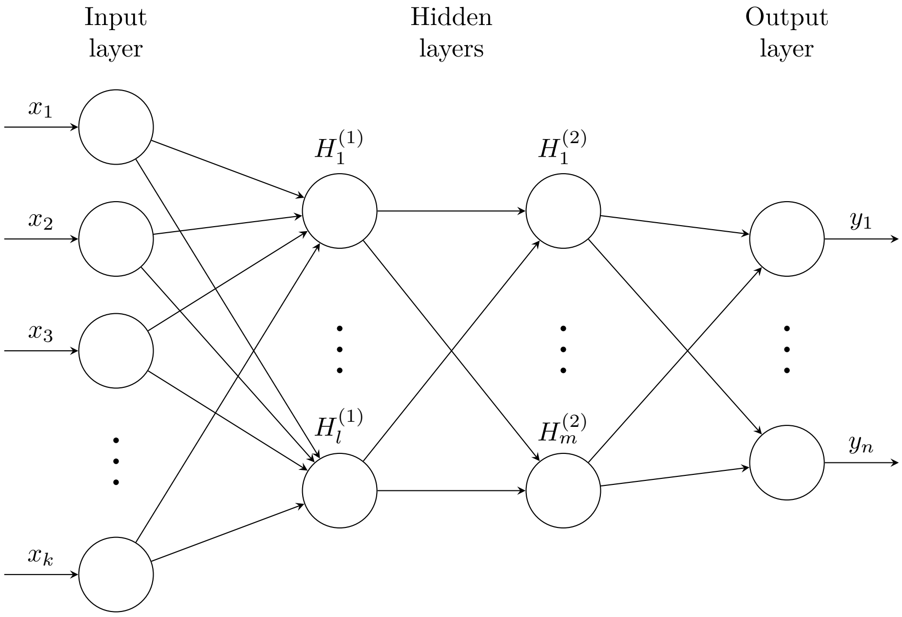

**Figure 5.2:** A graphical representation of a multi–layer perceptron.

connected to the input of every node in the next layer of the network. An example of a fully connected MLP, with two hidden layers, is shown in Figure 5.2.

Each node receives input from nodes in the previous layer, and uses the inputs to calculate a response function. For the $i^{th}$ node in the $j^{th}$ layer of a network,

$$z^{i,j} = \mathbf{w}^{i,j} \cdot \mathbf{x}^{j-1} + b^{i,j}$$

where $z^{i,j}$ is the response function, $\mathbf{w}^{i,j}$ is the weights vector, $b^{i,j}$ is the bias, and $\mathbf{x}^{j-1}$ is the input vector from the previous layer. This response function is usually passed through a nonlinear activation function, $f$, to produce the output from the node, $y$, which is part of the input for the next layer,

$$\left(\mathbf{x}^{j}\right)_i = y^{i,j} = f\left(z^{i,j}\right).$$

Some common choices for the activation function are the sigmoid function, the hyperbolic tangent, the rectified linear unit (ReLU), and the softmax function [84–86].

$$
\begin{aligned}
f(x) &= \frac{1}{1+e^{x}} && \text{(Sigmoid)}\\
f(x) &= \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}} && \text{(Tanh)}\\
f(x) &= \max(0,x) && \text{(ReLU)}\\
f(x) &= \frac{e^{x_i}}{\sum_j e^{x_j}} && \text{(Softmax)}
\end{aligned}
\tag{5.1}
$$

In the softmax function, the index $i$ indicates the current node, and the index $j$ includes all nodes in the current layer. This construction ensures that the outputs from a softmax layer sum to one, and as such this unit is commonly used in categorisation tasks when the output must belong to one of a given set of categories. The benefits and drawbacks of these common activation functions will be discussed later in this chapter, when we discuss modifying weights with the backpropagation algorithm.

The weights and biases of the nodes in an MLP can be adjusted to make accurate predictions of data. For a classification task, there are typically as many output nodes as there are classes, with output values in the range zero to one. The output value of a classification network, quantifies how well the image represents each class, based on the networks prediction. In principle, MLPs are able to approximate any function to arbitrary precision with a single hidden layer [87]; however, there is no limit on the number of nodes required in order to achieve a good approximation. In practice, networks with additional hidden layers can reach the required precision with fewer nodes than a network with a single hidden layer, but there are diminishing returns with more than a few hidden layers [83, 84].

### 5.1.1 Convolutional Neural Networks

An extension of the MLP with considerable success, particularly in image classification tasks, is the convolutional neural network (CNN) [86, 88, 89], which addresses some of the drawbacks of the traditional MLP. In particular, when evaluating data with a high dimensional input, having a fully connected network architecture leads to a large number of neurons and high computational cost. In addition, for spatially correlated data, such an architecture does not take into account the local spatial structure of the data, instead focussing on all the data at once. A CNN attempts to resolve these issues by exploiting the local spatial structure of the data.

In a CNN the singular input neutrons of a traditional MLP are replaced by convolutional kernels. A convolutional kernel is a matrix containing a set of weights, which are multiplied pixel–by–pixel with small regions of the input image. The convolution operator is defined as,

$$(x * y)_i = \sum_{j=-\infty}^{\infty} x_j \, y_{i-j}, \tag{5.2}$$

where x and y are discrete sequences. In the machine learning context, y represents the image, and x the convolutional kernel, which is defined to be zero beyond the finite size of the filter. The convolution operation is applied to all pixels in the input image, which are replaced by the resulting convolution in the next layer of the network. Many convolutional kernels are used in each layer of the network, each producing an output image which is passed to the next layer.

Convolutional kernels are sometimes referred to as feature detectors, which emphasises the fact that each kernel identifies a given feature in the data, based on the weights in the kernel matrix. The output image from each kernel is known as a feature map, reflecting the fact that they represent the spatial distribution of the features learned by a given feature detector.

### Pooling

The use of images as network input typically drastically increases the number of input parameters for a network, when combined with a large number of convolutional filters this can lead to a dramatic increase in the computational cost of training a network. Pooling [89] is a downsampling technique designed to reduce the number of parameters in the network, and therefore reduce the computational cost of making predictions. In a pooling algorithm the input data from each $m \times n$

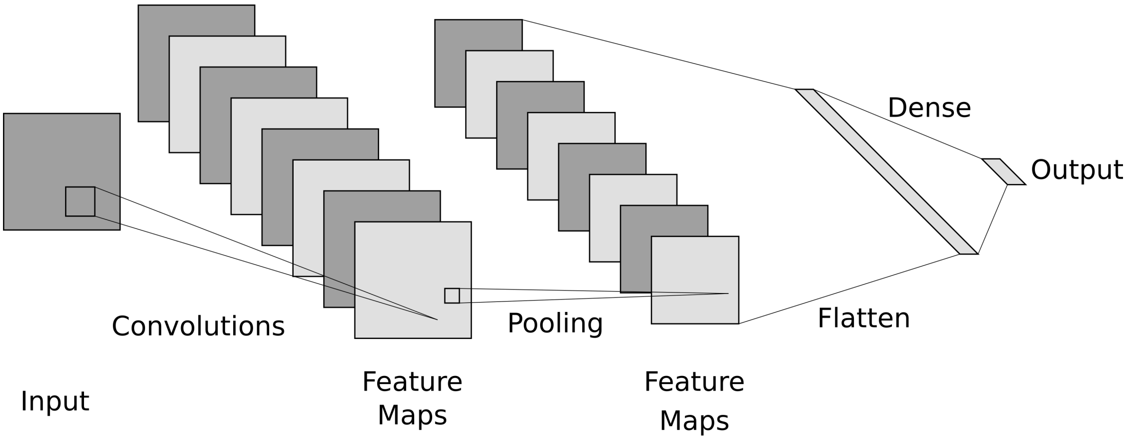

**Figure 5.3:** A graphical representation of a convolutional neural network. Figure generated with [90].

region of the input data is downsampled to a single value, the downsampled image is used as the input for the next layer of the network. Two common pooling algorithms are max pooling and average pooling; in max pooling the maximum value from within the downsampling region is used, whereas average pooling uses the average value from the region.

While convolutions are able to extract features from the data, they typically are not able to make predictions in the desired format for classification tasks. Therefore, after convolutions, data is typically flattened and passed through one or more dense layers in order to produce the classification prediction. A graphical representation of a typical CNN architecture is shown in Figure 5.3, it includes convolutional, pooling, and dense layers.

### 5.1.2 Residual Neural Networks

Adding more layers to a neural network usually improves the performance, the popular intuition for this is that with each subsequent layer the network is able to learn more complex features of the image with which to make it's classification. In fact adding more layer should never decrease the performance of a network, because the new layers could be set to be the identity, which would result in the same output from the network. However, it has been demonstrated that adding

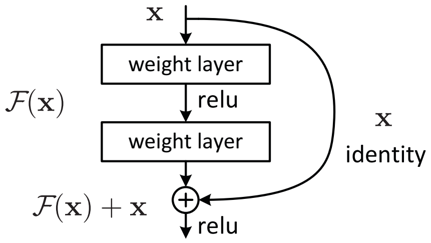
x
weight layer
$\mathcal{F}(\mathbf{x})$
relu
weight layer
x
identity
$\mathcal{F}(\mathbf{x}) + \mathbf{x}$
relu

**Figure 5.4:** An example of a shortcut connection in a convolutional neural network. Figure from [91].

too many layers to a simple neural network will reduce performance, suggesting that the optimisation of larger networks leads to degradation[91].

One method which has been shown to counter the degradation of deep networks, is the use of residual neural networks (ResNet). In a ResNet, the input to each layer is combined with the results of the convolutions as part of the input to a future layer; in effect, the identity transformation has been added to the set of filters in the layer. An example of these shortcut connections is shown in Figure 5.4[91].

## 5.2 Supervised Learning

Supervised learning is the process of teaching a neural network to accurately predict the correct output, the ground truth, for a given input. The network is optimised by minimising the loss function, which quantifies the difference between the network output and the ground truth. The choice of loss function is dependent on the problem being considered, specific examples will discussed in subsequent chapters, which will detail the CNN algorithms developed as part of this thesis.

The output of a neural network, g(x), is the result of a chain of function evaluations and matrix multiplication; for a network with $n$ layers,

$$g(x) = f^n(W^n f^{n-1}(W^{n-1} \dots f^1(W^1 x) \dots).$$

where $g$ is the output of the network, $x$ is the input, $f$ are the activation functions for each layer, and $W$ are the weights for each layer. The loss of the network is given by the loss function, L, evaluated on the output of the network and the ground truth, y,

$$L(y, g(x)).$$

To make accurate predictions with a NN, quantified by the loss function, the weights and biases of the network need to be optimised. The backpropagation algorithm[92] is currently the most widely used algorithm for minimising the loss function by adjusting the weights and biases in the network.

Consider the weight for the $j^{th}$ node in the $l^{th}$ layer of a network for incoming node $i$ from the previous layer, $W^l_{ij}$, the loss function varies with respect to these weights as $\partial L/\partial W^l_{ij}$, by adjusting the weights according to this gradient we can minimise $L$. If we adjust the weights with,

$$\Delta W^l_{ij} = -\eta \frac{\partial L}{\partial W^l_{ij}}, \tag{5.3}$$

for positive learning rates, $\eta > 0$, we are guaranteed to reduce the loss. The backpropagation algorithm provides an algorithm for calculating the derivatives required to adjust the weights in each layer.

Applying the chain rule to the right hand side of Equation 5.3,

$$\frac{\partial L}{\partial W^l_{ij}} = \frac{\partial L}{\partial a^l_j} \frac{\partial a^l_j}{\partial W^l_{ij}}, \tag{5.4}$$

where $a^l_j$ is the activation function. The derivative of the activation function with respect to the weights, is just the input to the layer from the previous layer,

$$\frac{\partial a^l_j}{\partial W^l_{ij}} = \frac{\partial}{\partial W^l_{ij}} \left( \sum_{k=1}^{n_l} W^l_{kj} x^{l-1}_k \right) = x^{l-1}_i, \tag{5.5}$$

where $x$ are the outputs from each layer, and $n_l$ is the number of nodes in layer $l$. The other term, which is often denoted as $\delta^l_j$, is known as the error; this term will be discussed shortly. Combining Equations 5.4 and 5.5 gives,

$$\frac{\partial L}{\partial W^l_{ij}} = \delta^l_j \, x^{l-1}_i.$$

The value of the error depends on the loss function being used in each particular model. The error can also be shown to have a recursive relationship via the chain rule,

$$\begin{aligned} \delta^l_j = \frac{\partial L}{\partial a^l_j} &= \sum_{k=1}^{n^{l+1}} \frac{\partial L}{\partial a^{l+1}_k} \frac{\partial a^{l+1}_k}{\partial a^l_j} \\ &= \sum_{k=1}^{n^{l+1}} \delta^{l+1}_k \frac{\partial a^{l+1}_k}{\partial a^l_j}, \end{aligned}$$

where the sum is over all nodes in layer $l+1$. We can used the definition of $a$ to simplify this further,

$$\begin{aligned}\frac{\partial a_k^{l+1}}{\partial a_j^l} &= \frac{\partial}{\partial a_j^l}\left(\sum_{p=1}^{n^l} W_{pk}^{l+1} f(a_p^l)\right)\\ &= W_{jk}^{l+1}\, f'(a_k^l)\end{aligned}$$

where $f$ is the activation function. The error is therefore,

$$\delta_j^l = \sum_{k=1}^{n^{l+1}} \delta_k^{l+1} W_{jk}^{l+1} f'(a_j^l),$$

which can be calculated recursively, if we go backwards through the network, based on the error in the previous layer.

Based on these manipulations with the chain rule, the backpropagation algorithm is as follows,

1. Calculate the forward pass, while storing the results for $g(x)$, $a_j^l$, and $x_j^l$.
2. Starting at the output layer, calculate the derivatives, going backwards through the network.
   - Store the results for $\delta_j^l$, to use when calculating subsequent layers.
3. Update the weights according to the update rule in Equation 5.3.

The rate of learning in the backpropagation algorithm is dependent on the derivative of the activation function, therefore, the choice of activation function has an impact on learning. Two important factors when considering activation functions are the computational cost of calculating the derivative, and the gradient of the function.

Traditional activation functions, such as sigmoid and tanh, are usually bounded and differentiable, however the presence of exponentials makes computing the gradients expensive. They also typically have vanishing gradients in the tails, which can lead to slow learning if the value of the activation is consistently in the tail of the distribution.

The ReLU activation function is much cheaper to compute computationally, and while the gradient is undefined at zero, in practice this is not a problem. One issue with ReLU units, is the vanishing gradient in the negative region. This issue can be resolved by implementing a small negative gradient below zero, which is known as a Leaky ReLU or Parametric ReLU [85]. In practice, ReLU units have been found to outperform traditional sigmoid units[93, 94].

### 5.2.1 Regularisation

Regularisation refers to a set of techniques which are used to prevent over–fitting during the training of a NN. This ensures that the trained network can generalise well beyond the training data. A number of regularisation techniques can be used, such as applying penalty terms to the loss function which are proportional to the square of the weights, ensuring that the weights stay relatively small. In this thesis we will make use of two regularisation techniques, dropout[95] and early stopping[96].

In each training iteration, the Dropout algorithm sets a subset of weights in each layer to zero. Each weight has a probability, $p$, of being set to zero for each iteration. The product of the remaining weights is scaled up by a factor of $1/r$, which ensures that the size of the input to the activation function remains reasonably consistent. This reduced network is then trained using the backpropagation algorithm, and the non–zero weights are updated. During the next iteration, the set of non–zero weights is reselected, and a new reduced network is trained. Finally, after training, the final weights of the network are multiplied by $p$ to give the so–called model average of the reduced networks. The dropout algorithm with $p = 0.5$, has proven to provide robust regularisation, and is a popular technique in the field[84].

"Early stopping (is) beautiful free lunch", according to Geoff Hinton[97]. It refers to the act of monitoring the loss on the validation set during training, in order to stop training before over–fitting has occurred. The work in this thesis implements the early stopping technique proposed by Lutz Prechelt[96], which suggests stopping training after the first checkpoint when the validation loss increases.