Modèle stochastique pour la détection d'essaims de véhicules aériens militaires

Stanislas de Charentenay[1], Alexandre Reiffers-Masson [1] and Gilles Coppin[1]

[1] *IMT Atlantique, LAB-STICC laboratory Brest, France. email: firstname.lastname@imt-atlantique.fr*

Cet article introduit un nouveau modèle de détection d'un essaim de véhicules aériens militaires autonomes par un radar sur des trajectoires prédéfinies. Ce modèle prend en compte l'impact des interactions entre membres de l'essaim sur la détection. Sur un scénario classique de pénétration, on montre qu'un modèle simplifié permet de calculer facilement une borne supérieure sur les performances du modèle avec défaillances.

Mots-clefs : UCAV Swarm, Performance Evaluation, Stochastic Model

1 Introduction

In the context of Unmanned Combat Aerial Vehicle (UCAV) swarm under radar-guided surface-to-air missile threats, it is frequent to look for optimal trajectories that minimize the probability to be detected before reaching a target. In such path-planning problems, a good detection model is needed as a first step to evaluate the trajectories. Although several models exist in the literature, few of them acknowledge the impact of internal swarm interactions on detection.

Literature on UCAV path planning can be divided into two categories. The first category is focused on stealth penetration problems for a single UCAV while the second addresses the problem in the case of UCAV swarms. The authors in the following works [KMZ06, ZWDH20, ZJWZ22] use complex detection models for a single UCAV that combine several characteristics of the radar, aircraft or enemy missiles. Although these models are more realistic, the coupling makes it difficult to adapt them to a swarm penetration problem. When part of a swarm, the probability for an UCAV to be detected is also affected by the presence of other swarm members, which is one of the advantages of swarm-based strategies. An example of such interaction is shown in papers [BGR17, BGR19] that minimize threat exposure of swarms with a stealth policy based on the effects of internal communication on radar detection. This shows the importance of internal swarm interactions in stealth problems but is not generalized to other types of interactions.

In this paper, we formulate a general model involving internal interactions between swarm members to calculate the probability for each UCAV of a swarm to be detected by a single fixed radar along predefined trajectories. Our model is based on the assumption that the detection rate of a swarm member is only depending on its distance to the radar and an internal swarm interaction term. We give an example of such a term representing effect of radar signal confusion for closely-spaced UCAVs. We extend the model by incorporating the failure of detected UCAVs. In a classic scenario of cooperative stealth penetration, we also prove that the first simple model provides a simple upper bound on swarm penetration success for this second model. Using numerical simulations, we study the impact of several model parameters in stealth penetration scenarios for straight trajectories of different swarm densities.

2 Model

2.1 Problem formulation

For every $i \in \{1, \cdots, n\}$ and $t \in \mathbb{R}_+$, we denote by $x_i(t) \in \mathbb{R}^3$ the position of UCAV i at instant t. The fixed position of the radar is denoted by $x_R \in \mathbb{R}^3$. For $i \in \{1, \cdots, n\}$, let us define the random variable $T_i \in \mathbb{R}_+$, the detection time of UCAV i by the radar. For every $i \in \{1, \cdots, n\}$ and $t \in \mathbb{R}_+$, we also define the random variable $D_i(t) \in \{0,1\}$ which indicates whether or not UCAV i is detected by the radar at time t. More precisely, $D_i(t) = 0$ when UCAV i is detected and $D_i(t) = 1$ when it is not, i.e, $D_i(t) = I\{T_i > t\}$. We now model the probability to be undetected at time t, denoted by $P_i(t) := \mathbb{P}(T_i > t)$. We introduce $\lambda_i(t)$ as the hazard rate at time t, so that for $h > 0$, we have the expected state transitions:

$$\mathbb{E}[D_i(t+h) \mid D_i(t)] = D_i(t)\big(1 - \lambda_i(t)h\big) + o(h). \tag{1}$$

The previous equation implies that :

$$P_i(t+h) = \mathbb{E}_{D_i(t)}\left[\mathbb{E}[D_i(t+h) \mid D_i(t)]\right]. \tag{2}$$

By using the fact that $\lambda_i(t)$ is deterministic and h converges to 0, we obtain the following linear differential equation for the evolution of $P_i(t)$:

$$\dot{P}_i(t) = -\lambda_i(t)P_i(t), \quad P_i(0) = 0. \tag{3}$$

We make the assumption that our hazard rate is only dependent on the position of the swarm members at each instant t. More precisely, by defining vector $\mathbf{d}(t) = [d_i(t)]_{1 \le i \le n}$ where $d_i(t) = ||x_R - x_i(t)||_2$ is the distance of UCAV i to the radar, we can say $\lambda_i(t)$ depends on $d_i(t)$ and an interaction term $z_i(t)$. We define function $\lambda : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ so that: $\lambda_i(t) := \lambda\left(d_i(t), z_i(t)\right)$. The interaction function z_i is the accumulation of binary interactions of all swarm members. As such we define $z_i(t) = \sum_{j=1}^{n} a_{ij}(t)$, with $\mathbf{A}(t) = [[a_{ij}(t)]]_{i,j}$ a positive and symmetrical adjacency matrix whose diagonal is null. In this paper, we study models with function λ of the form:

$$\lambda_i(t) = \lambda\left(d_i(t), z_i(t)\right) = \frac{\alpha}{\varepsilon + d_i(t)^{\beta_1}\left(1 + z_i(t)\right)^{\beta_2}}, \tag{4}$$

with $\alpha > 0$ a positive factor and $\beta_1, \beta_2 > 0$ positive exponents to be tuned depending on the application. The term $\varepsilon > 0$ is a very small constant to assure λ_i is bounded on $\mathbb{R}_+$. This specific form of λ_i is coming from the assumption that the power of the radar signal reflecting on UCAV i is proportionate to $\frac{1}{d_i(t)^4}$. In the extreme case where the interaction is high and the term z_i tends to $+\infty$, the rate of detection decreases. However, when z_i tends to 0, the rate of detection depends solely on the distance to the radar. In the following, we give an example of z_i representing the effect of radar signal confusion for closely-spaced UCAVs.

As radars have limited resolution, we know that closely-spaced targets can induce errors due to the merging of their radar signal. To model this phenomenon in our function z_i, we take adjacency matrix $\mathbf{A}(t)$ such that $a_{ij}(t) = R||x_i(t) - x_j(t)||_2^{-2}$, for all $i, j \in \{1, \ldots, n\}^2$. We name coefficient $R \ge 0$ the hiding radius, which depends on the radar resolution and represents how close UCAVs have to be to confuse the radar. This leads to the extended formula:

$$\lambda_i(t) = \frac{\alpha}{\varepsilon + d_i(t)^{\beta_1}\left(1 + \sum_j \frac{R}{||x_i(t) - x_j(t)||_2^2}\right)^{\beta_2}}. \tag{5}$$

This formula is pertinent in extreme cases as when the distance between UCAV i and its neighbors is small, the interaction term is high and the detection rate decreases. However when UCAV i is isolated, then $z_i(t)$ is small and to a model with no interactions among UCAVs. When $R = 0$, we obtain $\lambda_i(t) = \dfrac{\alpha}{\varepsilon + d_i(t)^{\beta_1}}$ which can be linked to the instantaneous probability model defined in [ZWDH20].

2.2 Extending the model with UCAV failure

We now extend our model to a more realistic one where the UCAVs are attacked, when detected. Here, we suppose that UCAVs are instantly disabled when detected. This implies that detected UCAVs will not have any effect on the radar so they should not be involved in the detection rate of others. In this scenario, we denote by $\tilde{D}_i(t)$ (resp. $\tilde{T}_i$) the detection state (resp. detection time) of UCAV i at time t. Both variables are random. We also define a new interaction function $\tilde{z}_i$ that considers the detection state by removing interactions with the detected UCAVs. At instant t, for all i and for $\mathbf{s}$ a possible value of $\tilde{D}(t)$ with $\mathbf{s} := [s_i]_{1\leq i\leq n}$ $s_i \in \{0,1\}^n$, we define:

$$\tilde{z}_i\big(t,\mathbf{s}\big) = \sum_{j=1}^{n} s_j a_{ij}(t). \tag{6}$$

We can now introduce $\tilde{\lambda}_i(t,\mathbf{s}) = \lambda\left(d_i(t),\tilde{z}_i(t,\mathbf{s})\right)$ as the detection rate conditional to the detection state. For $i \in \{1,\ldots,n\}$, let $\tau_i > 0$ a realisation of T_i. Then for $\tau = [\tau_i]_i$, we call $\delta : \mathbb{R}_+ \to \{0,1\}^n$ the detection state evolution corresponding to event $\{\bigcap_i \{T_i = \tau_i\}\}$ with $\delta_\tau(t) = \sum_i I(t < \tau_i)e_i$. We have for UCAV i, at instant t and for $h > 0$:

$$\mathbb{E}[\tilde{D}_i(t+h) \mid \tilde{D}_i(t) \cap \{\tilde{D}_i(u) = \delta_\tau(u)\}_{u\in[0,t[}] = \tilde{D}_i(t)\big(1-\tilde{\lambda}_i(t,\delta_t)h\big) + o(h).$$

We introduce $\tilde{P}_i(t,\tau) := \mathbb{P}\big(\tilde{D}_i(t) = 1 \mid \{\tilde{D}(u) = \delta_\tau(u)\}_{\tau\in[0,t[}\big)$ the probability to stay undetected at instant t in our new scenario, conditional to detection state evolution d. In the same way as for (2), we get the transition equation $\tilde{P}_i(t+h,\tau) = \tilde{P}_i(t,\tau)\big(1-\tilde{\lambda}_i\big(t,\delta_\tau(t)\big)h\big)$, leading to the differential equation:

$$\frac{\partial \tilde{P}_i}{\partial t}(t,\tau) = -\tilde{\lambda}_i\big(t,\delta_\tau(t)\big)\tilde{P}_i(t), \quad \tilde{P}_i(0,\tau) = 1 \tag{7}$$

which has solution

$$\tilde{P}_i(t,\tau) = \exp\left(-\int_0^t \tilde{\lambda}_i(u,\delta_\tau(u))du\right). \tag{8}$$

2.3 Calculating the performance in a penetration problem

We want to study the use of our extended detection model in a basic penetration scenario. Here, a swarm of UCAVs aims to reach a target zone represented by a disk of center x_T and radius r while avoiding detection from a single radar. We suppose that the trajectory of each UCAV ends at the final instant t_{max} in the target zone with the condition on x_i for $i \in \{1,\ldots,n\}$, $||x_i(t_{max}) - x_T||_2 \leq r$. This ensures existence of $t_i^* = \min\{t \in \mathbb{R} \mid ||x_i(t) - x_T||_2 \leq r\}$ the instant that UCAV i reaches the target. We want to evaluate the performance of given trajectories with a metric we call success probability U, the probability that at least one UCAV reached the target zone before detection. We define the metric as follow:

$$U = 1 - \mathbb{P}\big(\bigcap_i \{\tilde{T}_i < t_i^*\}\big).$$

For the model with failures, this success probability will be computationally difficult to calculate when the size of the swarm increases. Instead, we show in appendix [SdCC23] that we can derive an upper bound on U from the simpler model without UCAV failure.

Theorem 1 *The following inequality is true:*

$$U \leq 1 - \prod_i \big(1 - P_i(t_i^*)\big)$$

3 Numerical studies

To illustrate the previous section, we use simulations to grasp how parameters α and R of the detection rate have an influence on the success probability U of trajectories of a swarm of 6 UCAVs. Here the radar is placed at the center of the target zone. We set other parameters with $\varepsilon = 0.01$, $\beta_1 = 2$ and $\beta_2 = 1$. To simplify the simulations, we focus on 2D trajectories $x : [0,t_{max}] \to \mathbb{R}^2$ during a finite time t_{max}. More precisely, trajectories generated for the simulation are uniform rectilinear motions that reach target x_T at time t_{max}. The only random parameter of the generation is the constant speed $v_i \in \mathbb{R}^2$ of each UCAV. As generated trajectory of UCAV i, $x_i : [0,t_{max}] \to \mathbb{R}^2$, has condition $x_i(t_{max}) = x_T$, we get for instant t the position $x_i(t) = v_i(t_{max} - t) + x_T$. To generate v_i, we have equation $v_i = C_i\left[\cos(\theta_i)\ \sin(\theta_i)\right]$, assuming that $C_i \sim U([C_{min}, C_{max}])$ and $\theta_i \sim U([0,\theta_{max}])$. θ_{max} can be used to control the angular proximity of the

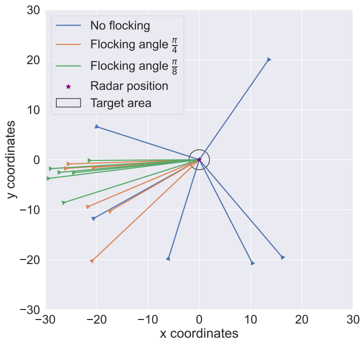

Figure 1: Example of trajectories in different batches

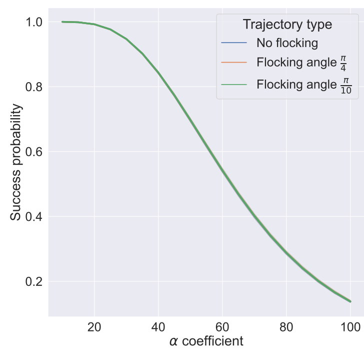

Figure 2: Impact of α on success probability

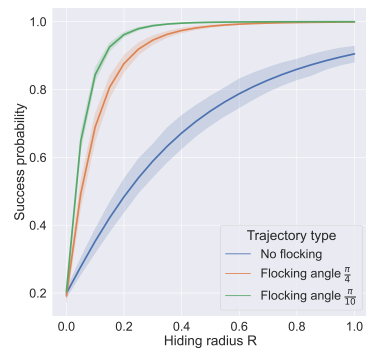

Figure 3: Impact of R on success probability

generated trajectories and induce a swarm formation. Our experiments are made with $C_{min} = 2$, $C_{max} = 3$ and $\theta_{max} := \{2\pi, \frac{\pi}{4}, \frac{\pi}{10}\}$, as illustrated in figure 1. Trajectories in first batch (with $\theta_{max} = 2\pi$) have completely random speed angles and no swarm formation.

Impact of α: The first experiment, illustrated in figure 2, studies the impact of detection rate coefficient α, capturing the efficiency of the radar, on the success probability of a swarm. Here we set the hiding radius R to 0 in order to neglect UCAV interactions, leading to similar results in all batches. As expected, the results show that a higher α coefficient leads to a lower success probability.

Impact of R: The hiding radius parameter R represents how much the proximity of UCAVs confuses the radar and prevents detection. In figure 3 illustrating the influence of this parameter on success probability, we see a decrease in success probability as R increases. The impact is much bigger for the closely-spaced swarms, as trajectories generated with $\theta_{max} = \frac{\pi}{10}$ have a 202% higher mean success probability to succeed for $R = 0.2$ compared to the trajectories with a flocking angle of $\theta_{max} = 2\pi$. The experiment shows the importance of swarm formation when such interactions are taken into account.

4 Conclusion

In this paper, we introduce two detection models for UCAV swarms that take internal swarm interactions into account. The model with failures, and therefore more realistic, is highly coupled and computationally costly to evaluate its performance. Therefore, we show that a simpler model can provide an upper bound on the performance of the model with failure. Such bound is easier to evaluate. Future work will focus on finding other more accurate approximations and use them to find optimal trajectories to solve the path planning problem.

References

[BGR17] Nícolas Pereira Borges, Cinara Guellner Ghedini, and Carlos HC Ribeiro. A local communication model for improving stealth in collaborative uav networks. *Anais do Computer on the Beach*, pages 249–258, 2017.

[BGR19] NÃcolas Pereira Borges, Cinara Guellner Ghedini, and Carlos Henrique Costa Ribeiro. Sun - stealth uav networks: A cooperative local model to mitigate uavs exposure to threats. In *2019 IEEE International Conference on Systems, Man and Cybernetics (SMC)*, pages 4316–4323, 2019.

[KMZ06] Pierre T. Kabamba, Semyon M. Meerkov, and Frederick H. Zeitz. Optimal path planning for unmanned combat aerial vehicles to defeat radar tracking. *Journal of Guidance Control and Dynamics*, 29:279–288, 2006.

[SdCC23] Alexandre Reiffers-Masson Stanislas de Charentenay and Gilles Coppin. Extended version - a stochastic model for the detection of ucav swarm. 2023. `https://github.com/alreiff/alreiff.github.io/blob/master/Submission_for_AlgoTel_with_appendix.pdf`.

[ZJWZ22] Zhe Zhang, Ju Jiang, Jian Wu, and Xiaozhou Zhu. Efficient and optimal penetration path planning for stealth unmanned aerial vehicle using minimal radar cross-section tactics and modified a-star algorithm. *ISA transactions*, 2022.

[ZWDH20] Zhe Zhang, Jian Wu, Jiyang Dai, and Cheng He. A novel real-time penetration path planning algorithm for stealth uav in 3d complex dynamic environment. *IEEE Access*, 8:122757–122771, 2020.

5 Appendix

Proof of Theorem 1: We want to study the use of our extended detection model on a basic penetration scenario. Here, a swarm of UCAVs aims to reach a target zone represented by a disk of center x_T and radius r while avoiding detection from a single radar. We suppose that the trajectories of each UCAV ends at final instant t_{max} in the target zone with condition on x_i for $i \in \{1,\dots,n\}$: $||x_i(t_{max}) - x_T||_2 \leq r$. This ensures existence of $t_i^* = \min\{t \in \mathbb{R} \mid ||x_i(t) - x_T||_2 \leq r\}$ the instant that UCAV i reaches the target. We want to evaluate the performance of given trajectories with a metric we call success probability U, the probability that at least one UCAV reached the target zone before detection. As such,

$$U = 1 - \mathbb{P}\big(\bigcap_i \{\tilde{T}_i < t_i^*\}\big).$$

From the previous definition, we get

$$U \quad = \quad 1 - \prod_i \mathbb{P}\big(\tilde{T}_i < t_i^* \mid \bigcap_{j=1}^{i-1} \{\tilde{T}_j < t_j^*\}\big) \tag{9}$$

From equation (6) we have inequality $\tilde{z}_i(u, \delta_\tau(u)) \leq z_i(u)$ for any $u \geq 0$ and $\tau \in \mathbb{R}_+^n$. This implies that $\tilde{\lambda}_i(u, \delta_\tau(u)) \geq \lambda_i(u)$. We can conclude from the previous observations and from (8) that for any $t \geq 0$,

$$\tilde{P}_i(t, \tau) \leq P_i(t). \tag{10}$$

Let $i \in \{1,\dots,n\}$, $t > 0$. For clarity purposes, we define event $C_i := \bigcap_{j=1}^{i-1} \{\tilde{T}_j < t_j^*\}$. As $\mathbb{P}\big(\tilde{T}_i \geq t_i^* \mid C_i\big) = \mathbb{E}\big(\tilde{D}_i(t_i*) \mid C_i\big)$ is defined we can apply the total expectation theorem to the random variable $\{\tilde{D}(u)\}_{u \in [0, t_i^*[}$:

$$\mathbb{P}\big(\tilde{T}_i \geq t_i^* \mid C_i\big) = \mathbb{E}_{\{\tilde{D}(u)\}_{\tau \in [0, t_i^*[}}\big[\mathbb{E}[\tilde{D}_i(t_i^*) \mid \{\tilde{D}(u)\}_{\tau \in [0, t_i^*[}] \mid C_i\big] \tag{11}$$

As $\mathbb{E}[\tilde{D}_i(t_i^*) \mid \{\tilde{D}(u)\}_{\tau \in [0, t_i^*[}] = \tilde{P}_i(t, \tau)$ we have using inequality (10) that $\mathbb{P}\big(\tilde{T}_i \geq t_i^* \mid C_i\big) \leq P_i(t_i^*)$. Looking back on equation (9) we find upper bound on U:

$$U \quad \leq \quad 1 - \prod_i \big(1 - P_i(t_i^*)\big) \tag{12}$$