Stochastic model for the detection of UCAV swarm

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Cet article introduit un nouveau modèle de détection d'un essaim de véhicules aériens militaires autonomes par un radar sur des trajectoires prédéfinies. Ce modéle prends en compte l'impact des interactions entre membres de l'essaim sur la detection. Sur un scénario classique de penetration, on montre qu'un modéle simplifié permet de calculer facilement une borne supérieure sur les performances du modèle avec défaillances.

Mots-clefs : Swarm UCAV, Performance Evaluation, Stochastic Model

1 Introduction

In the context of Unmanned Combat Aerial Vehicle (UCAV) swarm under radar-guided surface-to-air missile threats, it is frequent to look for optimal trajectories that minimize the probability to be detected before reaching a target. In such path-planning problems, a good detection model is needed. Although several models exist in the literature, few of them acknowledge the impact of internal swarm interactions on detection.

Literature on UCAV path planning can be divided into two categories. The first category is focused on stealth penetration problems for a single UCAV while the second addresses the problem in the case of UCAV swarms. The authors in the following works [KMZ06, ZWDH20, ZJWZ22] use complex detection models for a single UCAV that combine several characteristics of the radar, aircraft or enemy missiles. Although these models are more realistic, the coupling makes it difficult to adapt them to a swarm penetration problem. When part of a swarm, the probability for an UCAV to be detected is also affected by the presence of other swarm members, which is one of the advantages of swarm-based strategies. An example of such interaction is shown in papers [BGR17, BGR19] that minimize threat exposure of swarms with a stealth policy based on the effects of internal communication on radar detection. This shows the importance of internal swarm interactions in stealth problems but is not generalized to other types of interactions.

In this paper, we formulate a general model involving internal interactions between swarm members to calculate the probability for each UCAV of a swarm to be detected by a single fixed radar along predefined trajectories. Our model is based on the assumption that the detection rate of a swarm member is only depending on its distance to the radar and an internal swarm interaction term. We give an example of such a term representing effect of radar signal confusion for closely-spaced UCAVs. We extend the model by incorporating the failure of detected UCAVs. In a classic scenario of cooperative stealth penetration, we also prove that the first simple model provides a simple upper bound on swarm penetration success for this second model. Using numerical simulations, we study the impact of several model parameters in stealth penetration scenarios for straight trajectories of different flocking densities.

2 Model

2.1 *Problem formulation*

For every $i \in \{1, \cdots, n\}$ and $t \in \mathbb{R}_+$, we denote by $x_i(t) \in \mathbb{R}^3$ the position of UCAV i at instant t. The fixed position of the radar is denoted by $x_R \in \mathbb{R}^3$. For $i \in \{1, \cdots, n\}$, let us define the random variable $T_i \in \mathbb{R}_+$, the detection time of UCAV i by the radar. For every $i \in \{1, \cdots, n\}$ and $t \in \mathbb{R}_+$, we also define the random variable $D_i(t) \in \{0, 1\}$ which indicates whether or not UCAV i is detected by the radar at time t. More precisely, $D_i(t) = 0$ when UCAV i is detected and $D_i(t) = 1$ when it is not, i.e, $D_i(t) = I\{T_i > t\}$. We now model the cumulative distribution of T_i, denoted by $P_i(t) := \mathbb{P}(T_i > t)$. We introduce $\lambda_i(t)$ as the hazard rate at time t, so that for $h > 0$, we have the expected state transitions:

$$\mathbb{E}[D_i(t+h) \mid D_i(t)] = D_i(t)\big(1 - \lambda_i(t)h\big) + o(h). \tag{1}$$

The previous equation implies that :

$$P_i(t+h) = \mathbb{E}_{D_i(t)}\big[\mathbb{E}[D_i(t+h) \mid D_i(t)]\big]. \tag{2}$$

By using the fact that $\lambda_i(t)$ is deterministic and h converges to 0, we obtain the following linear differential equation for the evolution of $P_i(t)$:

$$\dot{P}_i(t) = -\lambda_i(t)P_i(t), \quad P_i(0) = 0. \tag{3}$$

We make the assumption that our hazard rate is only dependent on the position of the swarm members at each instant t. More precisely, by defining vector $\mathbf{d}(t) = [d_i(t)]_{1 \le i \le n}$ where $d_i(t) = ||x_R - x_i(t)||_2$ is the distance of UCAV i to the radar, we can say $\lambda_i(t)$ depends on $d_i(t)$ and an interaction term $z_i(t)$. We define function $\lambda : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ so that: $\lambda_i(t) := \lambda\big(d_i(t), z_i(t)\big)$. The interaction function z_i is the accumulation of binary interactions of all swarm members. As such we define $z_i(t) = \sum_{j=1}^n a_{ij}(t)$, with $\mathbf{A}(t) = [[a_{ij}(t)]]_{i,j}$ a positive and symmetrical adjacency matrix whose diagonal is null. In this paper, we study models with function λ of the form:

$$\lambda_i(t) = \lambda\big(d_i(t), z_i(t)\big) = \frac{\alpha}{\varepsilon + d_i(t)^{\beta_1}\big(1 + z_i(t)\big)^{\beta_2}}, \tag{4}$$

with $\alpha > 0$ a positive factor and $\beta_1, \beta_2 > 0$ positive exponents to be tuned depending on the application. The term $\varepsilon > 0$ is a very small constant to assure λ_i is bounded on $\mathbb{R}_+$. This specific form of λ_i is coming from the assumption that the power of the radar signal reflecting on UCAV i is proportionate to $\frac{1}{d_i(t)^4}$. In the extreme case where the interaction is high and the term z_i tends to $+\infty$, the rate of detection decreases. However, when z_i tends to 0, the rate of detection depends solely on the distance to the radar. In the following, we give an example of z_i representing effect of radar signal confusion for closely-spaced UCAVs.

As radars have limited resolution, we know that closely-spaced targets can induce errors due to the merging of their radar signal. To model this phenomenon in our function z_i, we take adjacency matrix $\mathbf{A}(t)$ such that $a_{ij}(t) = R||x_i(t) - x_j(t)||_2^{-2}$, for all $i, j \in \{1, \ldots, n\}^2$. We name coefficient $R \ge 0$ the hiding radius, which depends on the radar resolution and represents how close UCAVs have to be to confuse the radar. This leads to the extended formula:

$$\lambda_i(t) = \frac{\alpha}{\varepsilon + d_i(t)^{\beta_1}\big(1 + \sum_j \frac{R}{||x_i(t) - x_j(t)||_2^2}\big)^{\beta_2}}. \tag{5}$$

This formula is pertinent in extreme cases as when the distance between UCAV i and its neighbors is small, the interaction term is high and the detection rate decreases. However when UCAV i is isolated, then $z_i(t)$ is small and to a model with no interactions among UCAVs. When $R = 0$, we obtain $\lambda_i(t) = \dfrac{\alpha}{\varepsilon + d_i(t)^{\beta_1}}$ which can be linked to the instantaneous probability model defined in [ZWDH20].

sometimes you talk about first model or second model. It is not defined anymore. So you need to check carefully.

2.2 Extending the model with UCAV failure

We now extend our model to a more realistic one where the UCAVs are attacked, when detected. Here, we suppose that UCAVs are instantly disabled when detected. This implies that detected UCAVs will not have any effect on the radar so they should not be involved in the detection rate of others. In this scenario, we denote by $\tilde{D}_i(t)$ (resp. $\tilde{T}_i$) the detection state (resp. detection time) of UCAV i at time t. Both variables are random. We also define a new interaction function $\tilde{z}_i$ that takes into account the detection state by removing interactions with the detected UCAVs. At instant t, for all i and for $\mathbf{s}$ a possible value of $\tilde{D}(t)$ with $\mathbf{s} := [s_i]_{1\leq i\leq n}$ $s_i \in \{0,1\}^n$, we define:

$$\tilde{z}_i\big(t,\mathbf{s}\big) = \sum_{j=1}^{n} s_j a_{ij}(t). \tag{6}$$

vector are denoted like that: $[x_i]_i$ and a sequence is denoted like that: $\{x_i\}_i$. You need to change that everywhere. We can now introduce $\tilde{\lambda}_i(t,\mathbf{s}) = \lambda\left(d_i(t), \tilde{z}_i(t,\mathbf{s})\right)$ as the detection rate conditional to the detection state. For $i \in \{1,\ldots,n\}$, let $\tau_i > 0$ a realisation of T_i. Then for $\tau = [\tau_i]_i$, we call $\delta : \mathbb{R}_+ \to \{0,1\}^n$ the detection state evolution corresponding to event $\{\bigcap_i \{T_i = \tau_i\}\}$ with $\delta_\tau(t) = \sum_i I(t < \tau_i) e_i$. We have for UCAV i, at instant t and for $h > 0$:

$$\mathbb{E}[\tilde{D}_i(t+h) \mid \tilde{D}_i(t) \cap \{\tilde{D}_i(u) = \delta_\tau(u)\}_{u\in[0,t[}] = \tilde{D}_i(t)\big(1 - \tilde{\lambda}_i(t,\delta_t)h\big) + o(h).$$

pb conditionnement pas le meme partout+besoin explication We introduce $\tilde{P}_i(t,\tau) := \mathbb{P}\big(\tilde{D}_i(t) = 1 \mid \{\tilde{D}(u) = \delta_\tau(u)\}_{\tau\in[0,t[}\big)$ the probability to stay undetected at instant t in our new scenario, conditional to detection state evolution d. We get the transition equation $\tilde{P}_i(t+h,\tau) = \tilde{P}_i(t,\tau)\big(1 - \tilde{\lambda}_i\big(t,\delta_\tau(t)\big)h\big)$, leading to the differential equation:

$$\dot{\tilde{P}}_i(t,\tau) = -\tilde{\lambda}_i\big(t,\delta_\tau(t)\big)\tilde{P}_i(t), \quad \tilde{P}_i(0,\tau) = 1 \tag{7}$$

which has solution

$$\tilde{P}_i(t,\tau) = \exp\left(-\int_0^t \tilde{\lambda}_i(u,\delta_\tau(u))du\right). \tag{8}$$

2.3 Calculating the performance in a penetration problem

We want to study the use of our extended detection model on a basic penetration scenario. Here, a swarm of UCAVs aims to reach a target zone represented by a disk of center x_T and radius r while avoiding detection from a single radar. We suppose that the trajectories of each UCAV ends at final instant t_{max} in the target zone with condition on x_i for $i \in \{1,\ldots,n\}$: $||x_i(t_{max}) - x_T||_2 \leq r$. This ensures existence of $t_i^* = \min\{t \in \mathbb{R} \mid ||x_i(t) - x_T||_2 \leq r\}$ the instant that UCAV i reaches the target. We want to evaluate the performance of given trajectories with a metric we call success probability U, the probability that at least one UCAV reached the target zone before detection. We define the netric as follow:

$$U = 1 - \mathbb{P}\Big(\bigcap_i \{\tilde{T}_i < t_i^*\}\Big).$$

This success probability will be computationally difficult to calculate when the size of the swarm increases. Instead, we show in appendix annex ref that we can derive an upper bound on U from the simpler model without UCAV failure.

Theorem 1 *The following inequality is true:*

$$U \leq 1 - \prod_i \big(1 - P_i(t_i^*)\big)$$

what are the implications?

3 Numerical studies

To illustrate the previous section, we use simulations to grasp how parameters α and R of the detection rate have an influence on success probability U of trajectories of a swarm of 6 UCAVs. Here the radar is placed at the center of the target zone. We set other parameters with $\varepsilon = 0.01$, $\beta_1 = 2$ and $\beta_2 = 1$. To simplify the simulations, we focus on 2D trajectories $x : [0,t_{max}] \to \mathbb{R}^2$ during a finite time t_{max}. More precisely, trajectories generated for the simulation are uniform rectilinear motions that reach target x_T at time t_{max}. The only random parameter of the generation is the constant speed $v_i \in \mathbb{R}^2$ of each UCAV. As generated trajectory of UCAV i, $x_i : [0, t_{max}] \to \mathbb{R}^2$, has condition $x_i(t_{max}) = x_T$, we get for instant t the position $x_i(t) = v_i(t_{max} - t) + x_T$. To generate v_i, we have equation $v_i = C_i \left[\cos(\theta_i)\ \sin(\theta_i)\right]$, assuming that $C_i \sim U([C_{min}, C_{max}])$ and $\theta_i \sim U([0, \theta_{max}])$. θ_{max} can be used to control angular proximity of the generated trajectories. Our experiments are made with $C_{min} = 2$, $C_{max} = 3$ and $\theta_{max} := \{2\pi, \frac{\pi}{4}, \frac{\pi}{10}\}$, as illustrated in figure 1. The first batch (with $\theta_{max} = 2\pi$) can be seen as a control experiment as the speed angles are completely random. The other experiments are used to study the impact of the density in flocked trajectories.

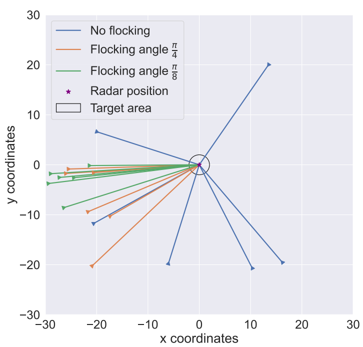

Figure 1: Example of trajectories in different batches

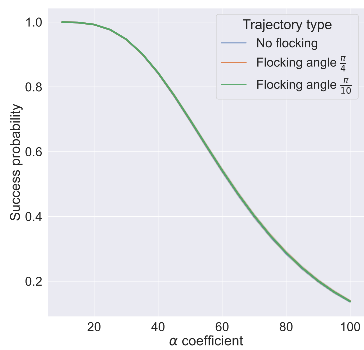

Figure 2: Impact of α on success probability

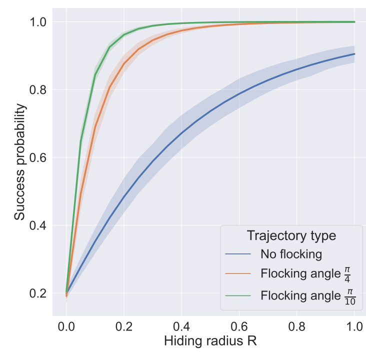

Figure 3: Impact of R on success probability

Impact of α: The first experiment, illustrated in figure 2, studies the impact of detection rate coefficient α, capturing the efficiency of the radar, on the success probability of a swarm. Here we set hiding radius R to 0 in order to neglect UCAV interactions, leading to similar results in all batches. As expected, the results show that a higher α coefficient leads to a lower success probability.

Impact of R: The hiding radius parameter R represents how much the proximity of UCAVs confuses the radar and prevent detection. In the second experiment, we studied the impact of different values for this parameter on the success probability of the trajectories. On figure 3, we see a decrease in success probability as R increases. The impact is much bigger for the closely-spaced swarms, as trajectories generated with $\theta_{max} = \frac{\pi}{10}$ have a 195% higher mean success probability to succeed for $R = 0.2$ compared to the trajectories without flocking. The trajectories corresponding to $\theta_{max} = \frac{\pi}{4}$ has a slightly less important increase of 190% in mean success probability. For $R \geq 1$, the success probability of trajectories with flocking converges rapidly to 1, which shows that R parameter is very sensitive. The experiment shows the importance of swarm formation when such interactions are taken into account.

4 Conclusion

In this paper, we introduce two detection models for UCAV swarms involving internal swarm interactions. to rephrase? Using the more realistic model to evaluate given trajectories in a penetration scenario is a highly coupled and computationally costly problem. We show that the simpler model can provide an upper bound on the performance of the model with failure. Such bound is easier to calculate. Future work will focus on finding simpler approximations that will enable us to solve the trajectory optimization problem.

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5 Appendix

Proof of Theorem 1: We want to study the use of our extended detection model on a basic penetration scenario. Here, a swarm of UCAVs aims to reach a target zone represented by a disk of center x_T and radius r while avoiding detection from a single radar. We suppose that the trajectories of each UCAV ends at final instant t_{max} in the target zone with condition on x_i for $i \in \{1,\dots,n\}$: $||x_i(t_{max}) - x_T||_2 \leq r$. This ensures existence of $t_i^* = \min\{t \in \mathbb{R} \mid ||x_i(t) - x_T||_2 \leq r\}$ the instant that UCAV i reaches the target. We want to evaluate the performance of given trajectories with a metric we call success probability U, the probability that at least one UCAV reached the target zone before detection. As such,

$$U = 1 - \mathbb{P}\big(\bigcap_i \{\tilde{T}_i < t_i^*\}\big).$$

From the previous definition, we get

$$U \quad = \quad 1 - \prod_i \mathbb{P}\big(\tilde{T}_i < t_i^* \mid \bigcap_{j=1}^{i-1} \{\tilde{T}_j < t_j^*\}\big) \tag{9}$$

From equation (6) we have inequality $\tilde{z}_i(u, \delta_\tau(u)) \leq z_i(u)$ for any $u \geq 0$ and $\tau \in \mathbb{R}_+^n$. This implies that $\tilde{\lambda}_i(u, \delta_\tau(u)) \geq \lambda_i(u)$. We can conclude from the previous observations and from (8) that for any $t \geq 0$,

$$\tilde{P}_i(t, \tau) \leq P_i(t). \tag{10}$$

Let $i \in \{1,\dots,n\}$, $t > 0$. For clarity purposes, we define event $C_i := \bigcap_{j=1}^{i-1} \{\tilde{T}_j < t_j^*\}$. As $\mathbb{P}\big(\tilde{T}_i \geq t_i^* \mid C_i\big) = \mathbb{E}\big(\tilde{D}_i(t_i*) \mid C_i\big)$ is defined we can apply the total expectation theorem to the random variable $\{\tilde{D}(u)\}_{u \in [0, t_i^*[}$:

$$\mathbb{P}\big(\tilde{T}_i \geq t_i^* \mid C_i\big) = \mathbb{E}_{\{\tilde{D}(u)\}_{\tau \in [0, t_i^*[}}\big[\mathbb{E}[\tilde{D}_i(t_i^*) \mid \{\tilde{D}(u)\}_{\tau \in [0, t_i^*[}] \mid C_i\big] \tag{11}$$

As $\mathbb{E}[\tilde{D}_i(t_i^*) \mid \{\tilde{D}(u)\}_{\tau \in [0, t_i^*[}] = \tilde{P}_i(t, \tau)$ we have using inequality (10) that $\mathbb{P}\big(\tilde{T}_i \geq t_i^* \mid C_i\big) \leq P_i(t_i^*)$. Looking back on equation (9) we find upper bound on U:

$$U \quad \leq \quad 1 - \prod_i \big(1 - P_i(t_i^*)\big) \tag{12}$$