Phys 410 Project 3

This project discusses a driven damped pendulum motion, exemplifying chaotic motion.
The angle of the pendulum is θ, and it moves under the influence of gravity, friction, and potentially an external periodic force. The tangential component of the velocity of the pendulum is $v = l\frac{\mathrm{d}\theta}{\mathrm{d}t}$ and the angular acceleration is $\alpha = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$. The equation obeyed by $\theta(t)$ is then:

$$m\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \nu\frac{\mathrm{d}\theta}{\mathrm{d}t} + g\sin(\theta(t)) = A\sin(\omega t) \tag{0.1}$$

Henceforth we set the length of the pendulum to be $l = 1$ meter, the mass $m = 1$ kg, and $g = 1$ m/sec^2 . To solve the second order equation youd need to put it in a standard form, as pair of first order equations.

1. Using the provided code (with RK4, RFK, or Matlab's inbuilt ode45), study the unforced ($A = 0$) motion for ten cycles, for $\nu = 1, 5, 10$. Choose as initial conditions $\theta = 0.2$ and $v = 0$. Make plots of θ as function of time (recalling that θ it is a periodic variable), as well as phase portraits of θ versus v. Comment on the resulting motion, which case corresponds to damped, under-damped or over-damped motion?

2. Now we switch on the driving force, with the same constants and initial values as above, and $\nu = \frac{1}{2}$. Set $\omega = \frac{2}{3}$ and $A = 0.5, 1.2$. In both cases study the motion for at least 300 periods, making plots of $\theta(t)$ and phase portraits. Comments on features of the resulting motion.

3. You might need to choose your parameters (step size in RK4, tolerance and minimum step size in RFK, or the options such as `RelTol` in ode45) appropriately. Pay special attention to reproducing the correct long time behaviour of the solutions. Comment and explain the results for different amplitudes. Comment on the choices involved.

4. For the same parameters, except for $A = 1.35, 1.44$, and 1.465, run the simulation for at least 300 periods, plot $\theta(t)$ and phase portraits. Comment on your results.

5. Further analyze the motion in questions two and four by making phase space plots using only points for which $\omega t = 2\pi n$ for integer n. Plot $\theta(2\pi n/\omega)$ vs n. These are an examples of Poincare sections. Comment on your results, do the Poincare sections have a well defined limit as $n \to \infty$? What is the pendulum doing in each case?

6. Reexamine the large n behaviour of the Poincare plots for $0.5 \leq A \leq 1.2$ and $1.35 \leq A \leq 1.5$ by ploting the large n values of $\theta(2\pi n/\omega)$ vs A over those ranges. Comment on the results.