Physics 410 - Homework 3

1. Write a code for a 7-point centred formula for differentiating a function at the point x^*, using evaluations of the function at x^* and $x^* \pm n * h$, for $n = 1, 2, 3$.

   - Derive the required weights using the idea of differentiating the Lagrange interpolant.

   - Derive the optimal values of h, which minimizes the combination of discretization and round off errors, and use it in your code.

   - Check your formula by differentiating the function $\sin(x^2)$ for $0 \leq x \leq 1$ and comparing to expectation.

2. Write a code for Romberg integration of a function $f(x)$ over an interval $[a, b]$, using the composite trapezoid formula (with error of order h^2, where $h = b - a$) applied to sub-intervals of length $h2^{-m}$. Eliminate all errors to order h^6.

   Apply the code to calculate $\int_0^1 \sin(x)dx$, and compare the result to expectations.