Physics 410: Project 2

Arnold Choa – 32038144

12 November, 2018

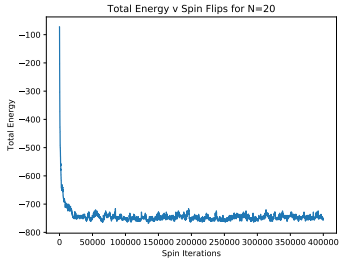
Please see `ising2D.py` in the code folder for the base 2d Ising model code. It returns `Elist` and `Mlist` instead of the $< E >$ and $< M >$ just so that the rest of the project has a common denominator of results to work with.

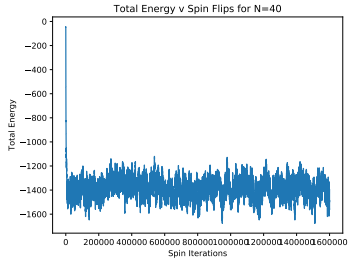
Firstly, to interpolate our 1D Ising model into a 2D Ising model, we need to make come changes:

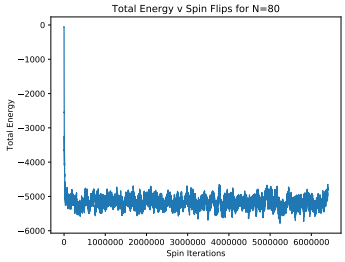
- $t \sim N^2$, since the number of steps should be proportional to the number of atoms in our lattice
- $E(\{\sigma\}) = -J\left(\sum_{j=1}^{N}\sum_{i=1}^{N}\sigma_{i,j}\sigma_{i,j+1}\right) - J\left(\sum_{j=1}^{N}\sum_{i=1}^{N}\sigma_{i,j}\sigma_{i+1,j}\right)$. This accounts for both horizontal and vertical coupling. This form also allows for a further generalization if vertical and horizontal coupling are different.
- Instead of `trials`, we have `trialsh` and `trialsv` to allow us to choose uniformly on a lattice.
- Instead of just left and right, we now also have up and down neighbors. We will not be taking into account the diagonal neighbors since those would be second-nearest neighbors.
- $dE = 2J\sigma_{i,j}\left(\sigma_{i,j-1} + \sigma_{i,j+1}\right) + 2J\sigma_{i,j}\left(\sigma_{i-1,j} + \sigma_{i+1,j}\right)$ to now account the 2nd dimension.

Relevant code for the next sections should be found in the `main.py` in the code folder.

Comparing the average number of spin flips needed for lattices of $N = 20$ and $N = 40$ ($J = 1$ and $T = 3$) to thermalize, we can be seen that the smaller lattice ($N = 20$) thermalizes faster than the $N = 40$. The $N = 20$ lattice takes around $4,000$ spin flips to thermalize while the $N = 40$ lattice takes around $12,000$ spin flips to thermalize. Also calculating for a $N = 80$ lattice, we can see that that configuration take around $60,000$ spin flips to thermalize. We can say thermalization time seems to increase linearly with lattice size (number of atoms).

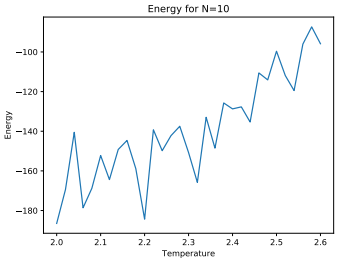
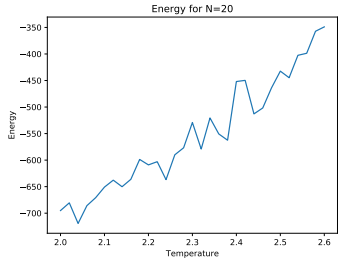
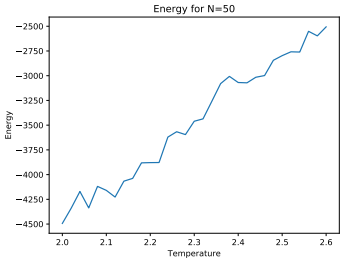
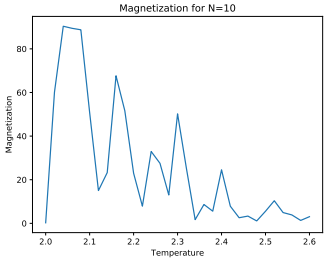
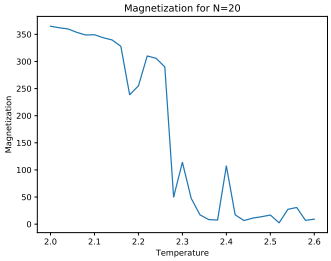
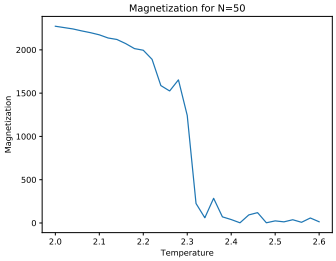
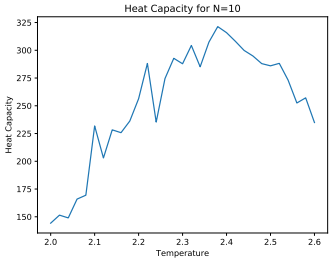
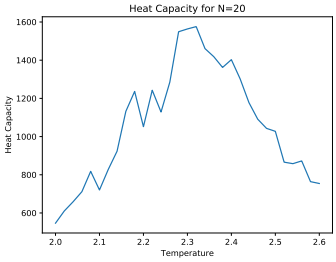
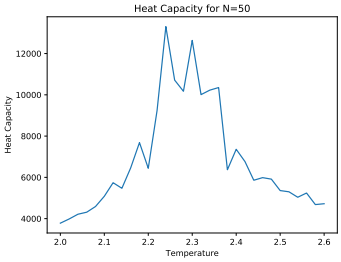
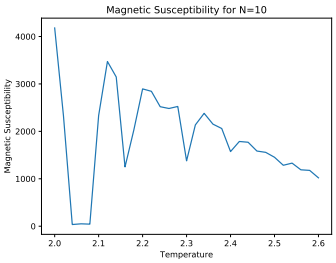
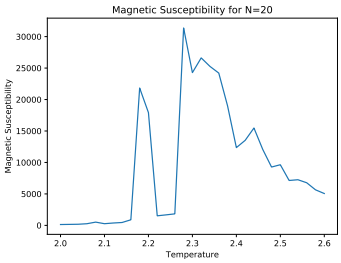
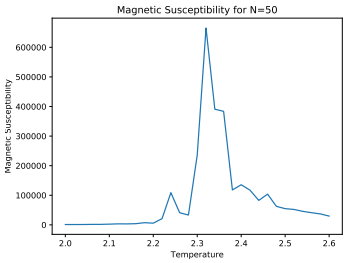






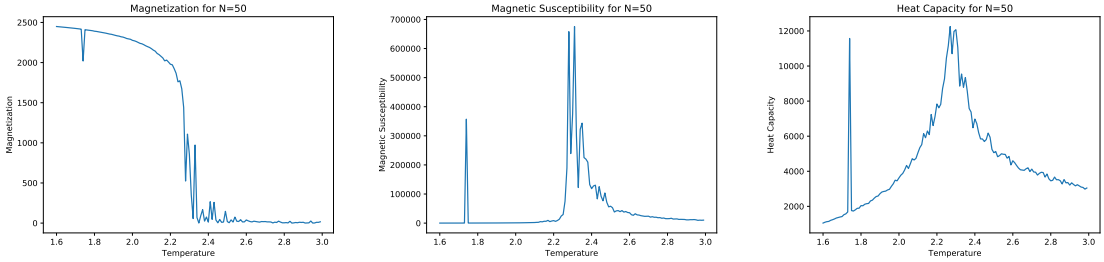
For the next couple of plots, I will assume that $J = 1$ for convenience and consistency with the last section.

I plot the quantities of total energy, magnetization, heat capacity, and magnetic susceptibility:

	N=10	N=20	N = 50
Total Energy			
Total Magnetization			
Heat Capacity			
Magnetic Susceptibility			

As we can see, accuracy of the approximations of the various quantities increase as the lattice size increases. This is mostly because we are getting more and more disordered in our calculations. When we were at $N = 10$, everytime we would flip 1 spin, we would be flipping every 1 out of a 100 spins, while when we were at $N = 50$, everytime we would flip 1 spin, we would be flipping every 1 out of a 2500 spins whicch is a better approximation of disorder. At the asymptotic limit, as $N \to \infty$, we would get the best approximation at each Temperature (though at that point, we would also have an enormously big runtime).

As for the approximate location of the phase transition, we take the $50x50$ lattice with a smaller step size to better approximation of calculation and resolution of the curve. Since the Curie Temperature is characterized to be the temperature where the Magnetization drastically drops to 0, we can use the magnetic susceptibility to see where the largest change in magnetization happens. We can also use the Heat capacity since a drastic change in Magnetization also makes a drastic change in Energy of the lattice.



We can see then that the Curie Temperature, at least as we can see from a $50x50$ lattice, is around $(2.28-2.31)\ \frac{J}{k_b}$. We can also see that the clearest signature of the transistion is at the magnetic susceptibility with the Heat capacity being a close second.