Math 2001 Homework 2

Due date: 1 February by 1pm

1. (**LaP 2.5.2**) Here's a logic puzzle by George J. Summers, called "Murder in the Family." Murder occurred one evening in the home of a father and mother and their son and daughter. One member of the family murdered another member, the third member witnessed the crime, and the fourth member was an accessory after the fact.

   (a) The accessory and the witness were of opposite sex.
   (b) The oldest member and the witness were of opposite sex.
   (c) The youngest member and the victim were of opposite sex.
   (d) The accessory was older than the victim.
   (e) The father was the oldest member.
   (f) The murderer was not the youngest member.

   Which of the four—father, mother, son, or daughter—was the murderer? Solve this puzzle, and write a clear argument to establish that your answer is correct. Use grammatically correct English sentences (not a natural induction derivation tree).

2. (**LaP 15.5.1**) Let f be any function from X to Y, and let g be any function from Y to Z.
   a. Show that if $g \circ f$ is injective, then f is injective.
   b. Give an example of functions f and g as above, such that that $g \circ f$ is injective, but g is not injective.
   c. Show that if $g \circ f$ is injective and f is surjective, then g is injective.

3. (**LaP 3.6.2**) Give a natural deduction proof of $(A \to C) \land (B \to \neg C) \to \neg(A \land B)$.

4. (**LaP 3.6.4**) Take another look at Exercise 2.5.3. Using propositional variables A, B, and C for "Alan likes kangaroos," "Betty likes frogs" and "Carl likes hamsters," respectively, express the three hypotheses in the previous problem as symbolic formulas, and then derive a contradiction from them in natural deduction.

5. (**LaP 3.6.6**) Give a natural deduction proof of $\neg A \land \neg B \to \neg(A \lor B)$.