Kuwait University
College of Engineering and Petroleum

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ME319 MECHATRONICS

PART IV: THE MUSCLES – ACTUATORS

LECTURE 2: PRACTICAL FEEDBACK CONTROL 1

Spring 2021

Ali AlSaibie

Objectives

- Understand the concept of feedback control
- Become familiar with practical aspects of feedback control
- Become familiar with PID Control
- Differentiate between each of the PID Control elements

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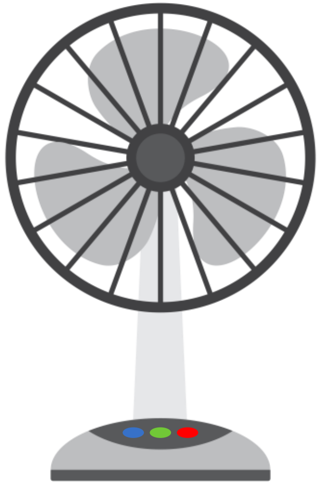
- Picture a desk fan
  - 1P induction motor typically
- Speed is a function of AC line frequency
- Fixed speed settings give a nominal voltage (via regulator)
- But load affects the speed too
  - Bigger fan blades will rotate slower
- This is an "open-loop" system
  - Can't control speed precisely
  - The actual speed is not measured
  - The actual speed is not used in regulating the input signal



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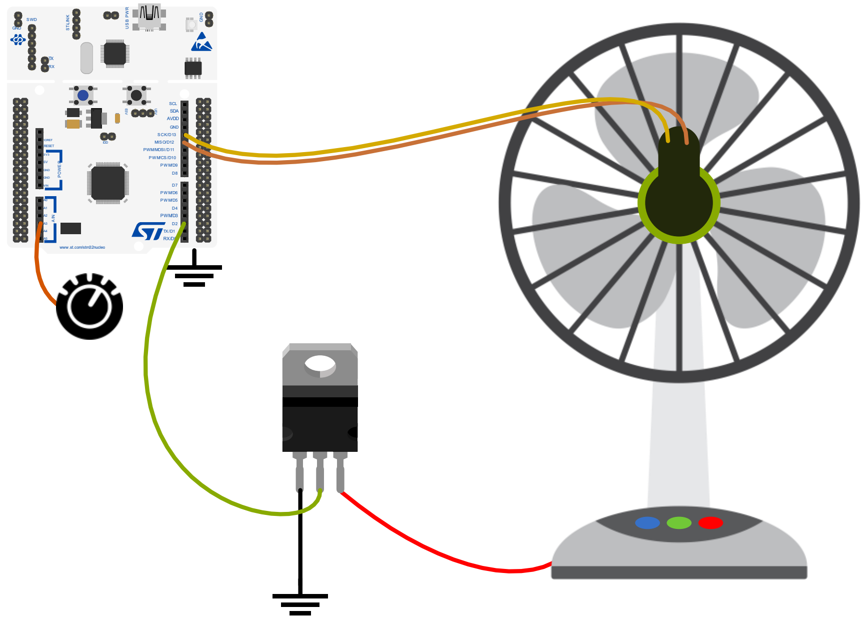
- Now you add an encoder / hall effect sensor to measure fan-motor speed
- You connect the sensor to an MCU
- In the MCU, you read the fan-motor speed
- User speed setting given to MCU
- MCU compares actual vs desired speed
- And adjust the input to fan continuously
  - Induction motor speed controller via
    - Variable frequency device
- To achieve a specific speed of rotation
- This is feedback control
  - In some context: regulation



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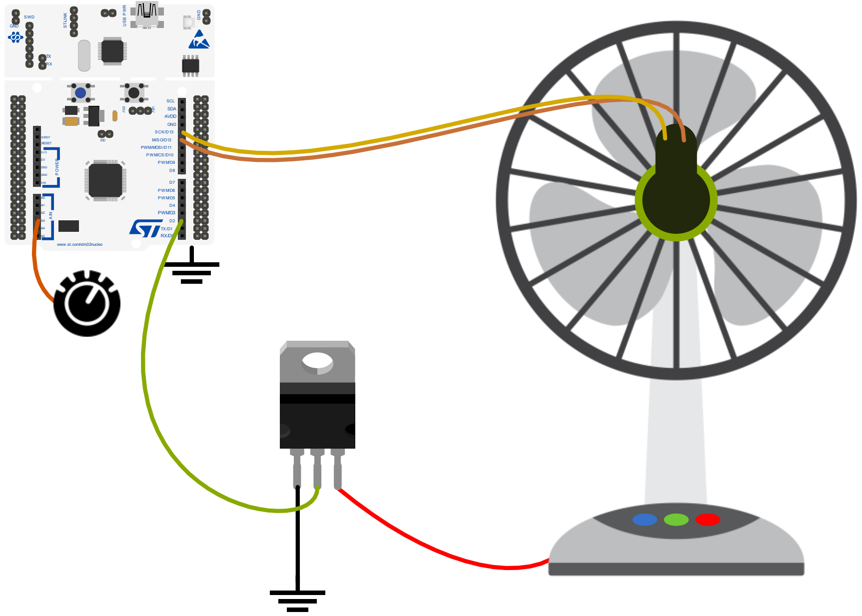
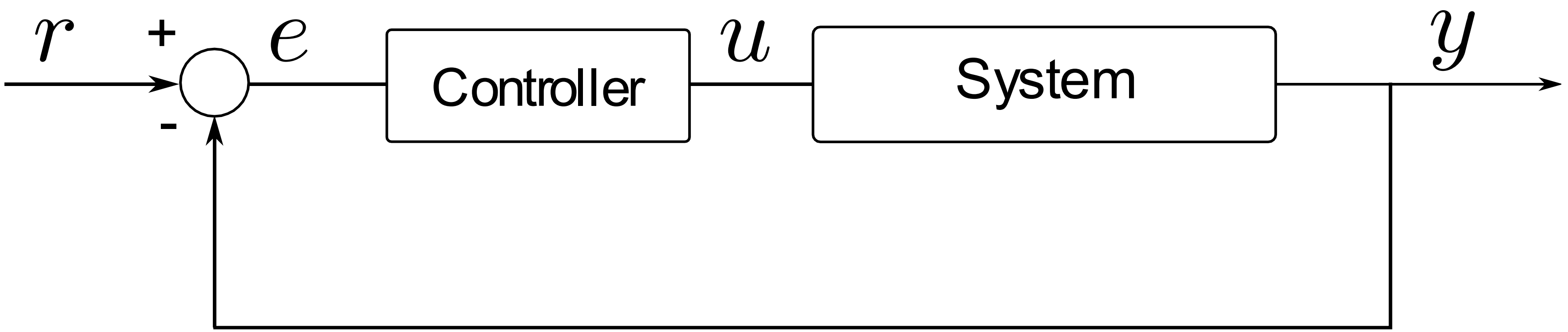
Feedback Control

- Think about the controller as a function: $\boldsymbol{u} = \boldsymbol{f}(\boldsymbol{e})$
- Where the error: $\boldsymbol{e} = \boldsymbol{r} - \boldsymbol{y}$
- r is the reference, a.k.a desired output (user speed setting).
- y is the system output (actual speed of motor-fan).
- The output y, is fed back into the input through the summation block, to calculate e. Hence the name, feedback control.





- A very practical method for designing the controller, is by constructing it as a PID controller.
  - The controller function (commonly called control law):
  - $u = f(e) = u_P + u_I + u_D$
  - P - ***Proportional*** to the error: $u_p = K_P e$
  - I - Proportional to the ***Integral*** of the error: $u_i = K_D \int e\, dt$
  - D – Proportional to the ***Derivative*** of the error: $u_d = K_D \frac{de}{dt}$

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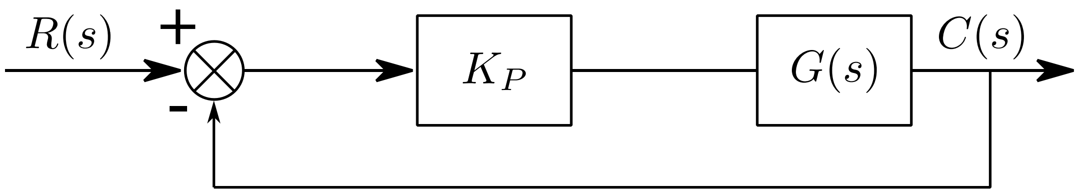
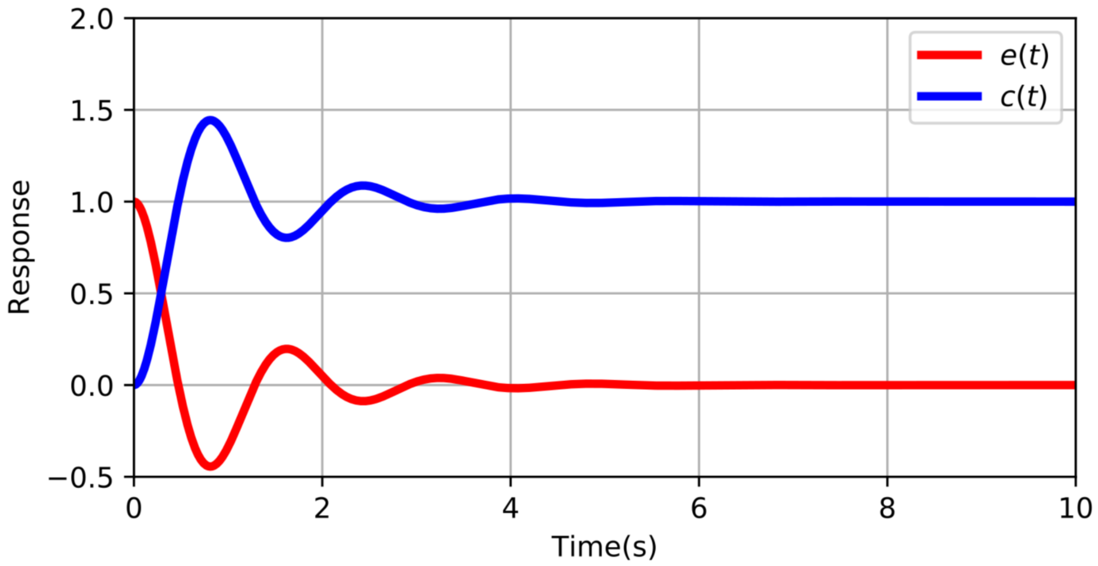
- If $Kp \neq 0$ and $K_I = K_D$. We call our controller a proportional controller (P Controller)
  - $u = u_P = K_P e$
- If $Kp \neq 0$ and $K_I \neq 0$, but $K_D = 0$. We have a PI Controller
  - $u = u_P + u_I = K_P e + K_I \int e\, dt$
- If $Kp \neq 0$ and $K_D \neq 0$, but $K_I = 0$. We have a PD Controller
  - $u = u_P + u_D = K_P e + K_D \dfrac{de}{dt}$
- And if all constants are not equal to zero, we have a full PID Controller

- The error: $e(t) = r(t) - c(t)$, in unity feedback
- Since the proportional controller is proportional to the error, we expect the control output u_p to follow the error dynamics proportionally

Error response to a unit-step input in a feedback control system with $G_{ol} = \frac{2}{s(s+2)}$, with a **P** Controller.

$K_p = 8$

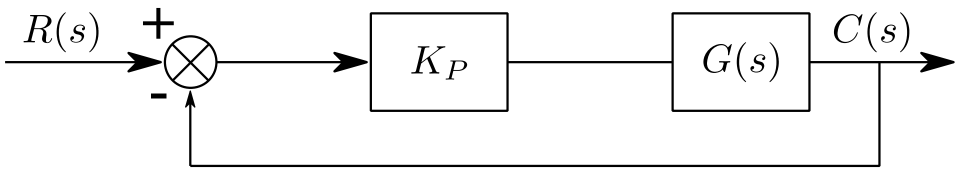


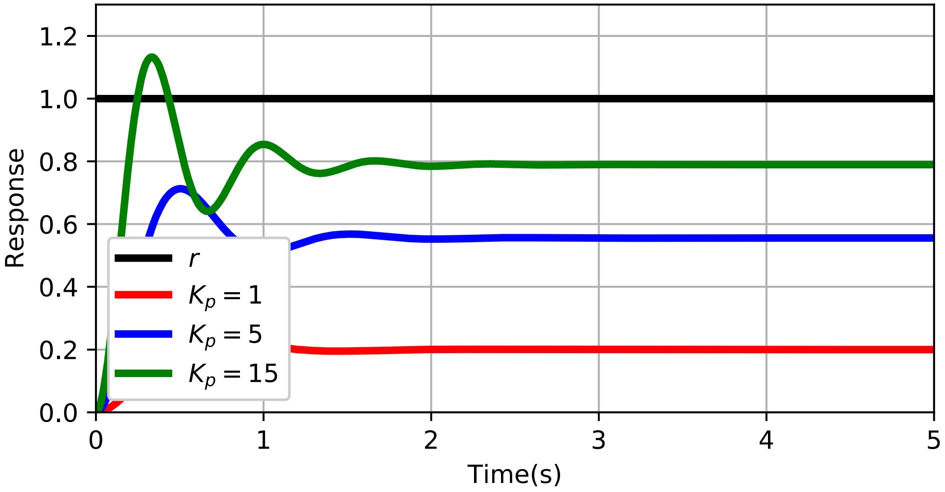


The P Controller

- In terms of its affect on the response, increasing the proportional gain:
  - (+) Decreases the steady-state error
  - (+) For 2nd order and higher systems, increases %OS.
  - (-) Increases overshoot
  - (-) Increases actuator effort
    - *In a real-world application, there is a limit to power/force/input to a system, we can't increase* K_p *without a limit.*

$$G(s) = \frac{5}{s^2+5s+20}$$





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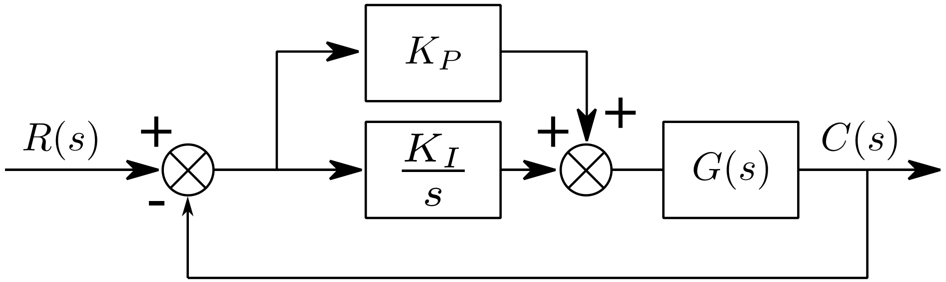
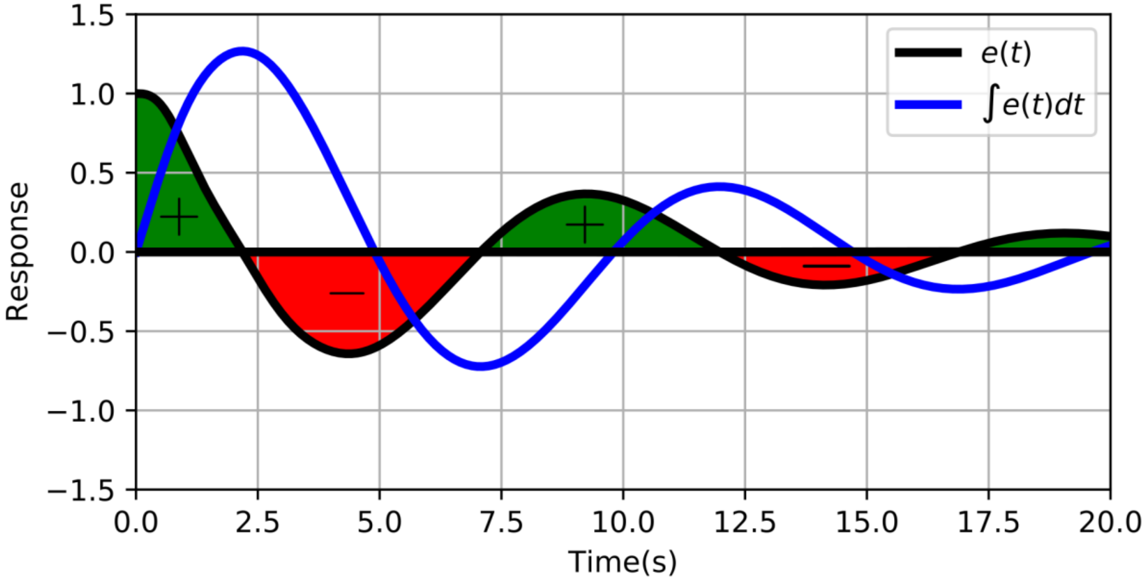
- The integral of the error is the area under the error curve
- The PI Controller: $u_{PI}(t) = K_P e(t) + K_I \int e(t)dt$
- Conceptually, the integral component of the controller tries to reduce the steady-state error by balancing between the positive and negative areas.
- If Green area > Red Area -> $u_I = K_I \int e(t) > 0$, and vice versa
- If Green area = Red Area -> $u_I = 0$

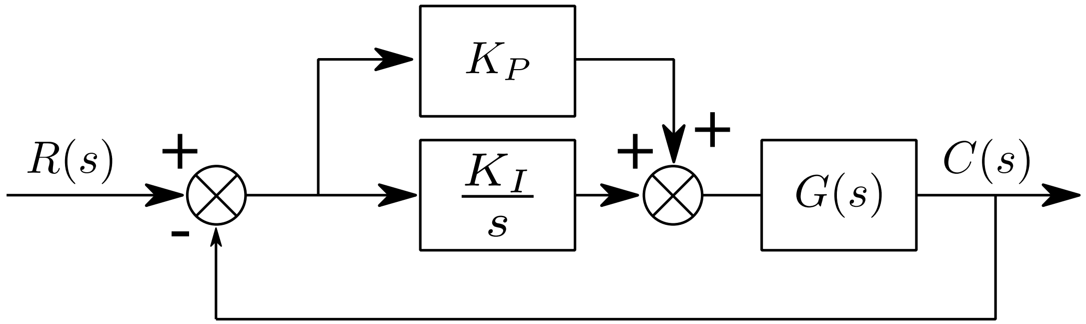
Error response to a unit step input in a feedback control system with $G_{ol} = \frac{2}{s(s^2+2s+15)}$, with a **PI** Controller. $K_p = 2, K_I = 3$

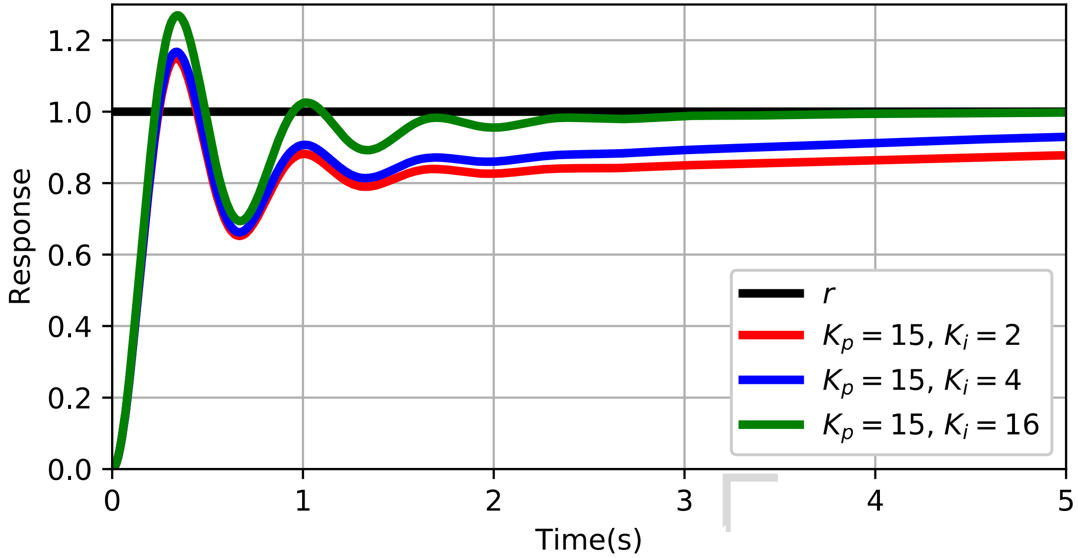




- In terms of its affect on the response, increasing the integral gain:
  - (+) Reduces the steady-state error
  - (+) Reduces the need for aggressive K_p gain
  - (-) Increases oscillation
  - (-) Increases settling time T_s
  - (-) Increases overshoot

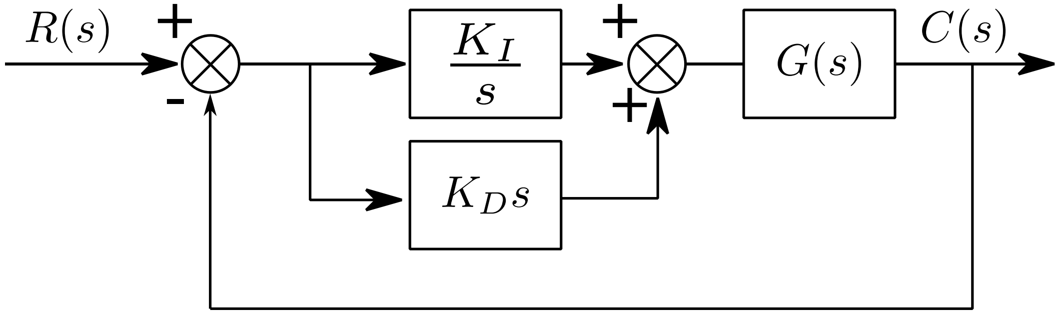
$$G(s) = \frac{5}{s^2+5s+20}$$

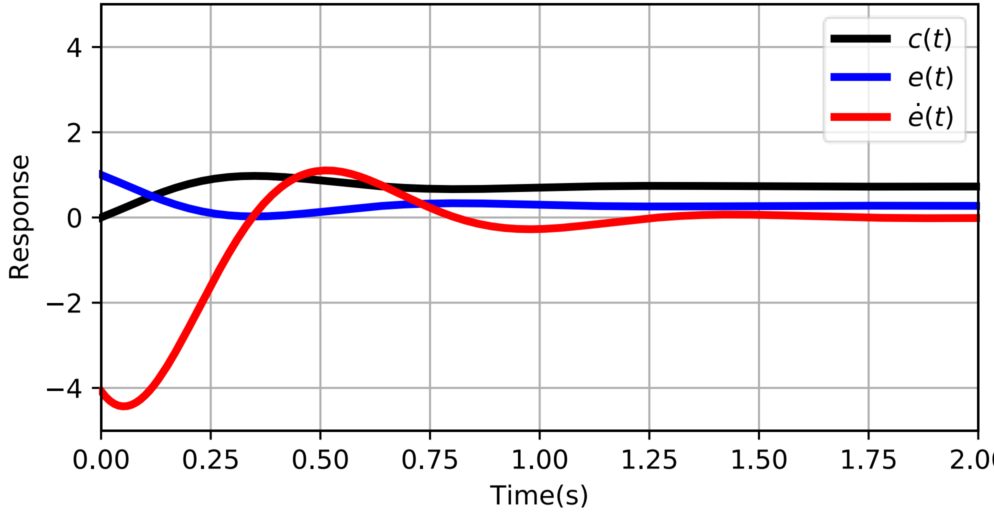




- The error derivative $\dot{e}(t)$ responds to the rate of change of the error.
- In a way, it predicts the future by looking at the rate of change. The rate of change is an indication of how quick the desired reference is moving away from the system state (output).
    - *If you're in your car, following a friend's car, it is one thing to speed up/down to maintain a fixed gap, but if your friend suddenly speeds up or slams on the brakes, your response would be more aggressive in anticipation of a big error change.*
    - *Because you **know** the error **will** change*

Error response to a unit step input in a feedback control system with $G_{ol} = \frac{2}{(s^2+2s+15)}$, with a **PD** Controller. $K_p = 20, K_D = 2$





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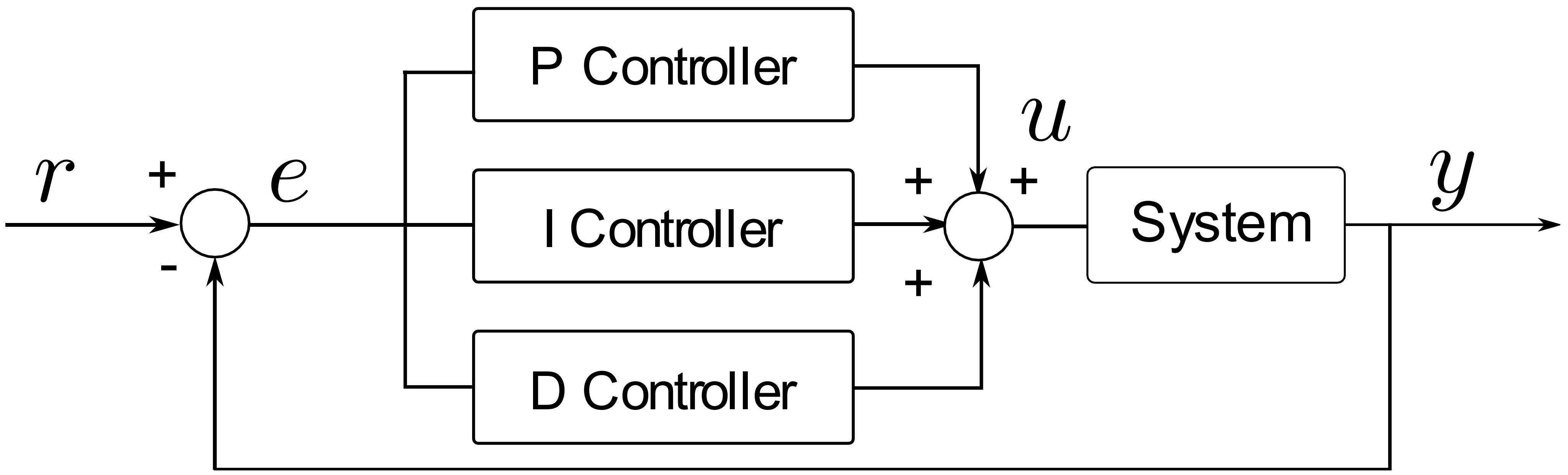
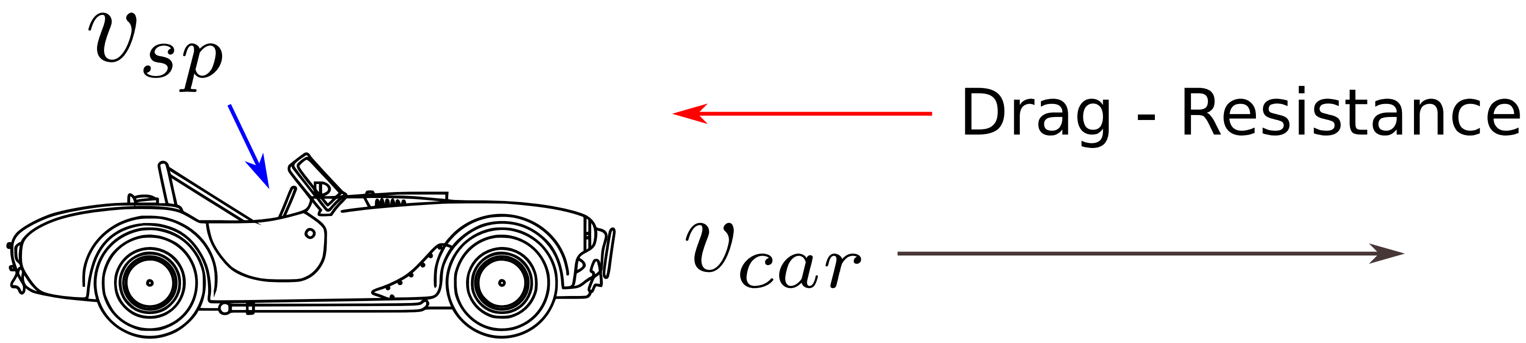
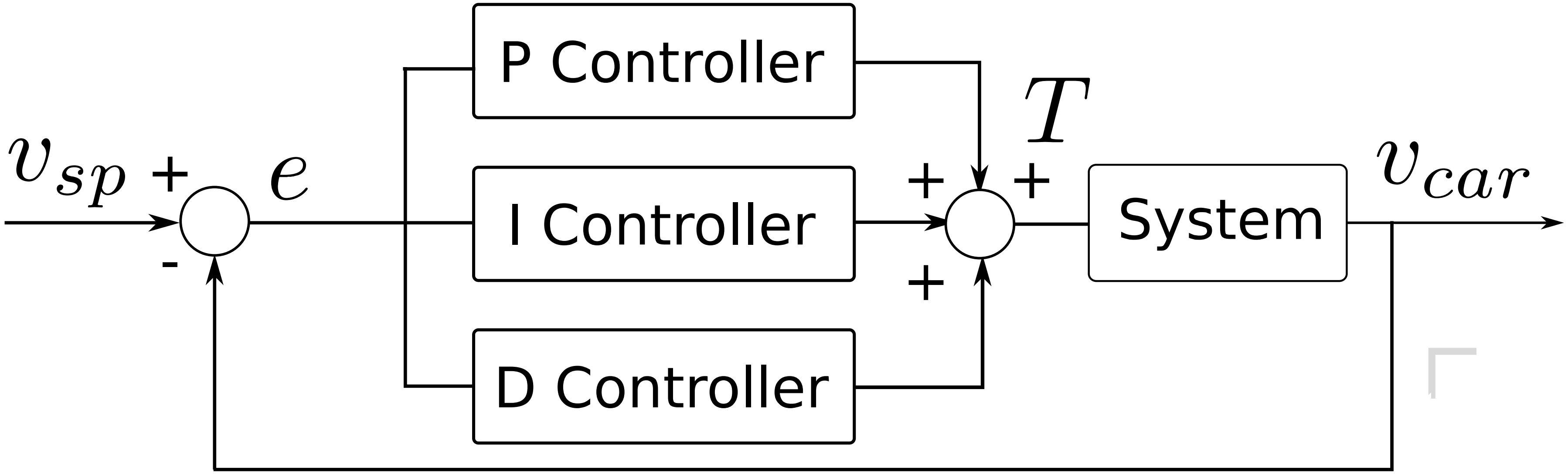
PID Control

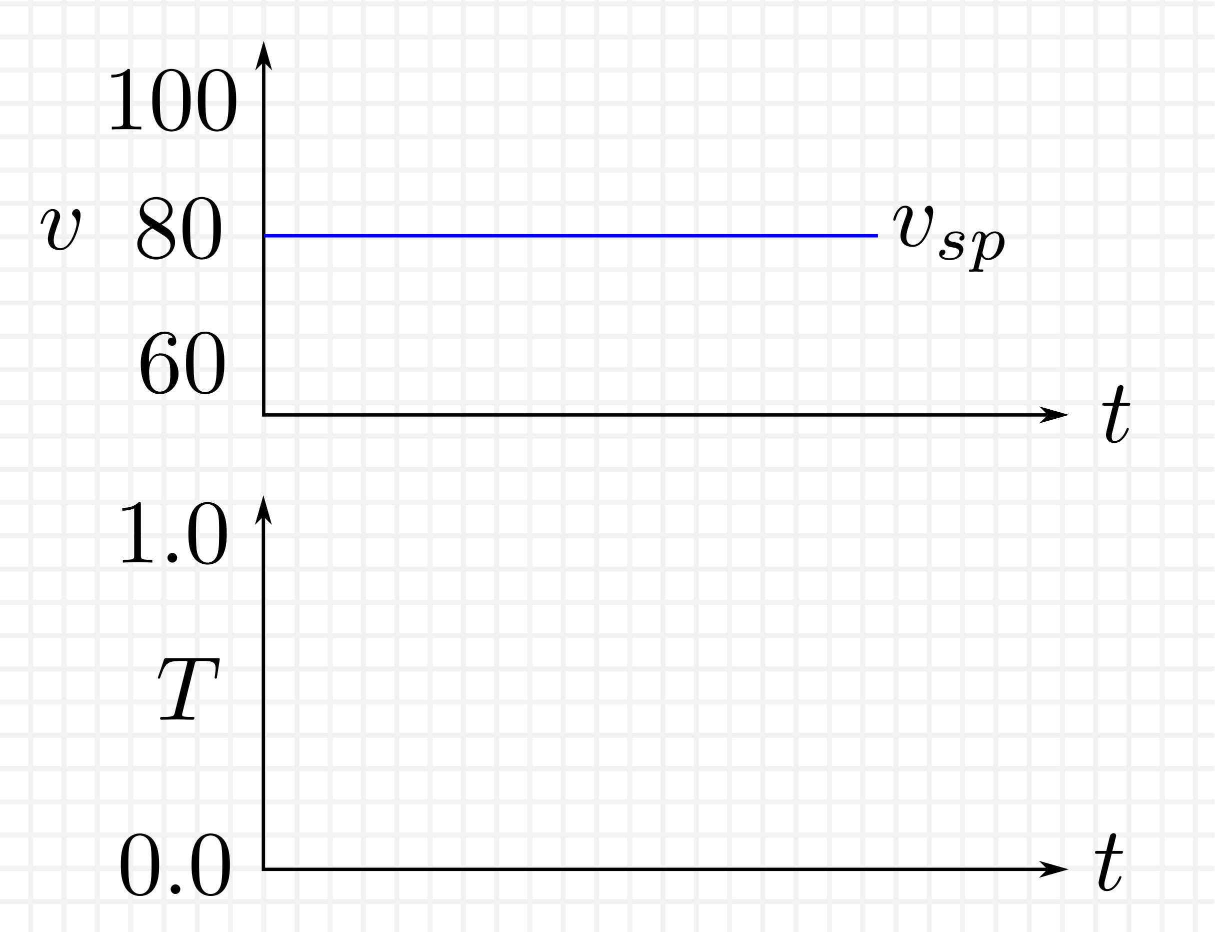


- T: Throttle (0% to 100%)





- Let's apply a P Controller with $K_p = 0.05$
  - $u = K_P\, e$
  - Assume drag force: $D = 0.1v$, Traction force: $F = 10T$



	T_0	T_1	T_2
v_{sp}			
v_{actual}			
e			
T			
F			
D			
$\sum F$			

- Let's apply a P Controller with $K_p = 0.05$
  - $u = K_P\, e$
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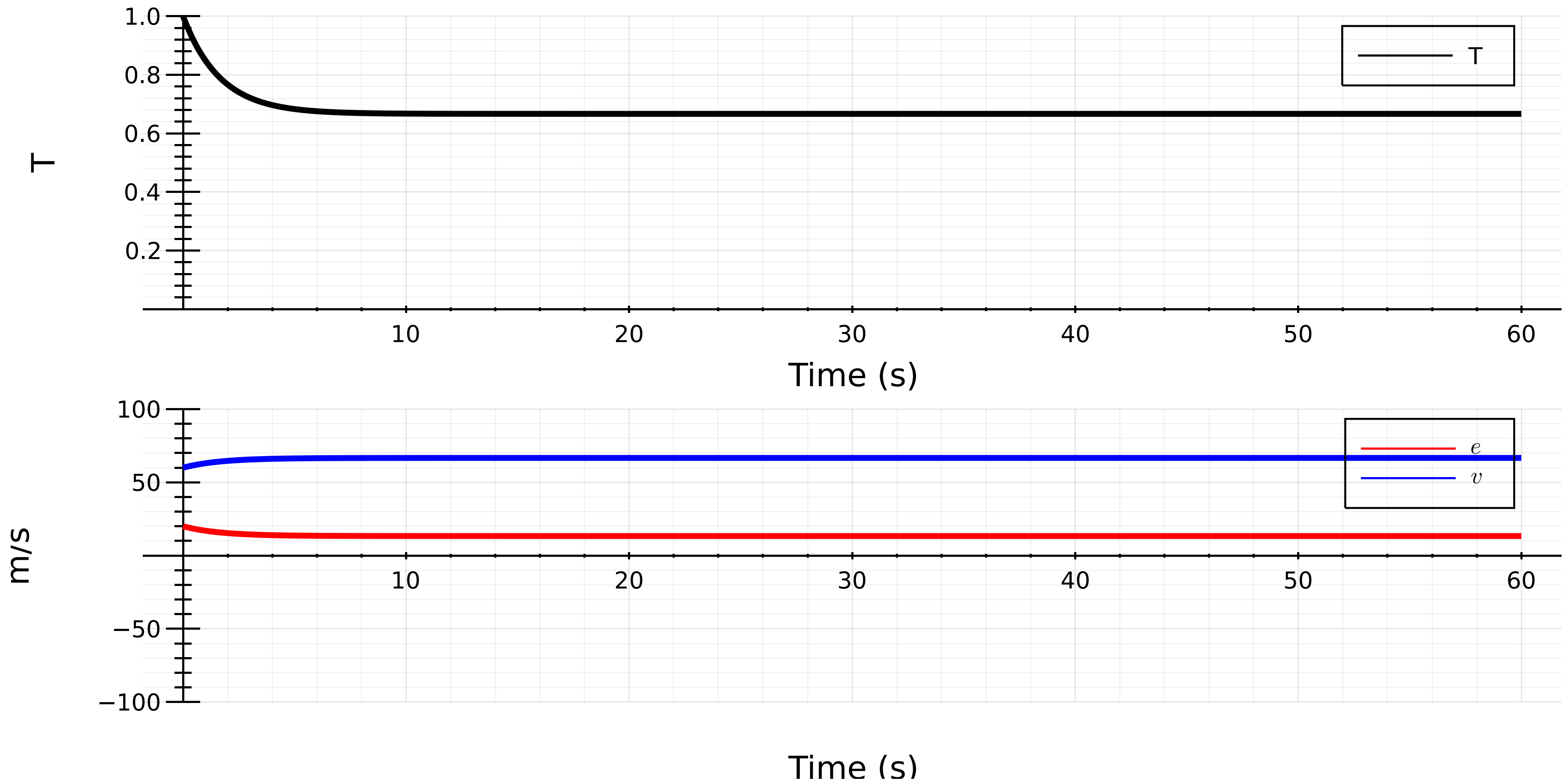

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- Now let's apply a P Controller with $K_p = 0.5$
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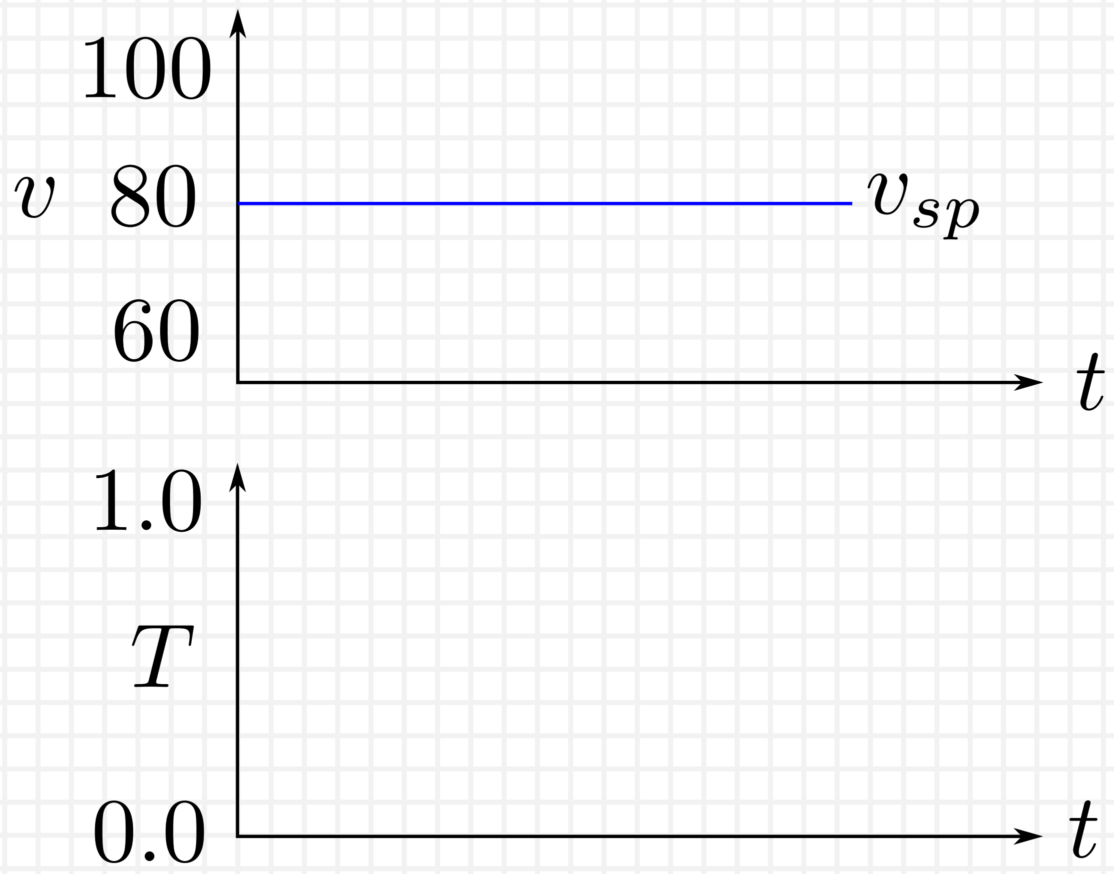

	T_0	T_1	T_2
v_{sp}			
v_{actual}			
e			
T			
F			
D			
$\sum F$			

- Now let's apply a P Controller with $K_p = 0.5$
    - $u = K_P\, e$
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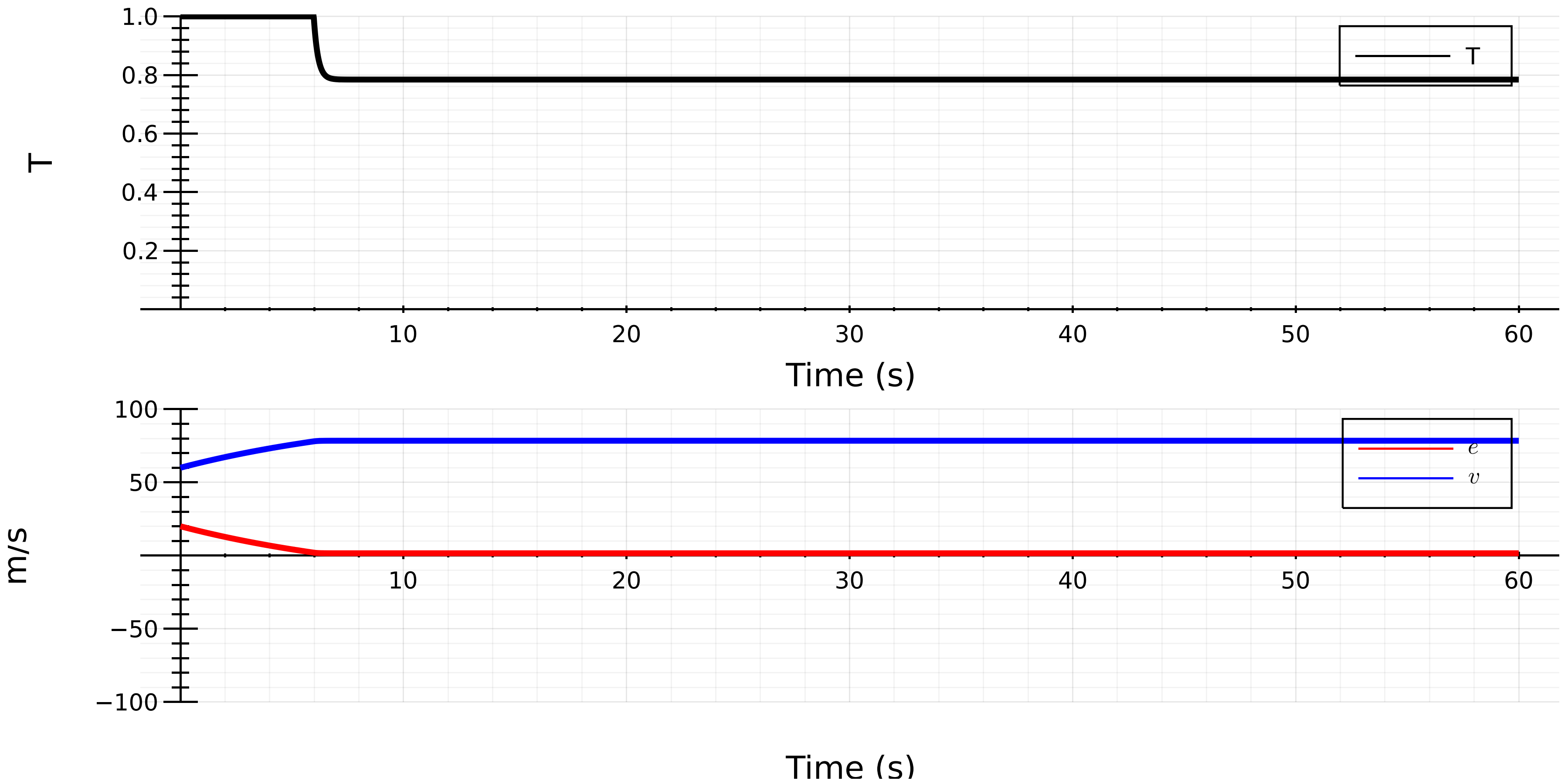
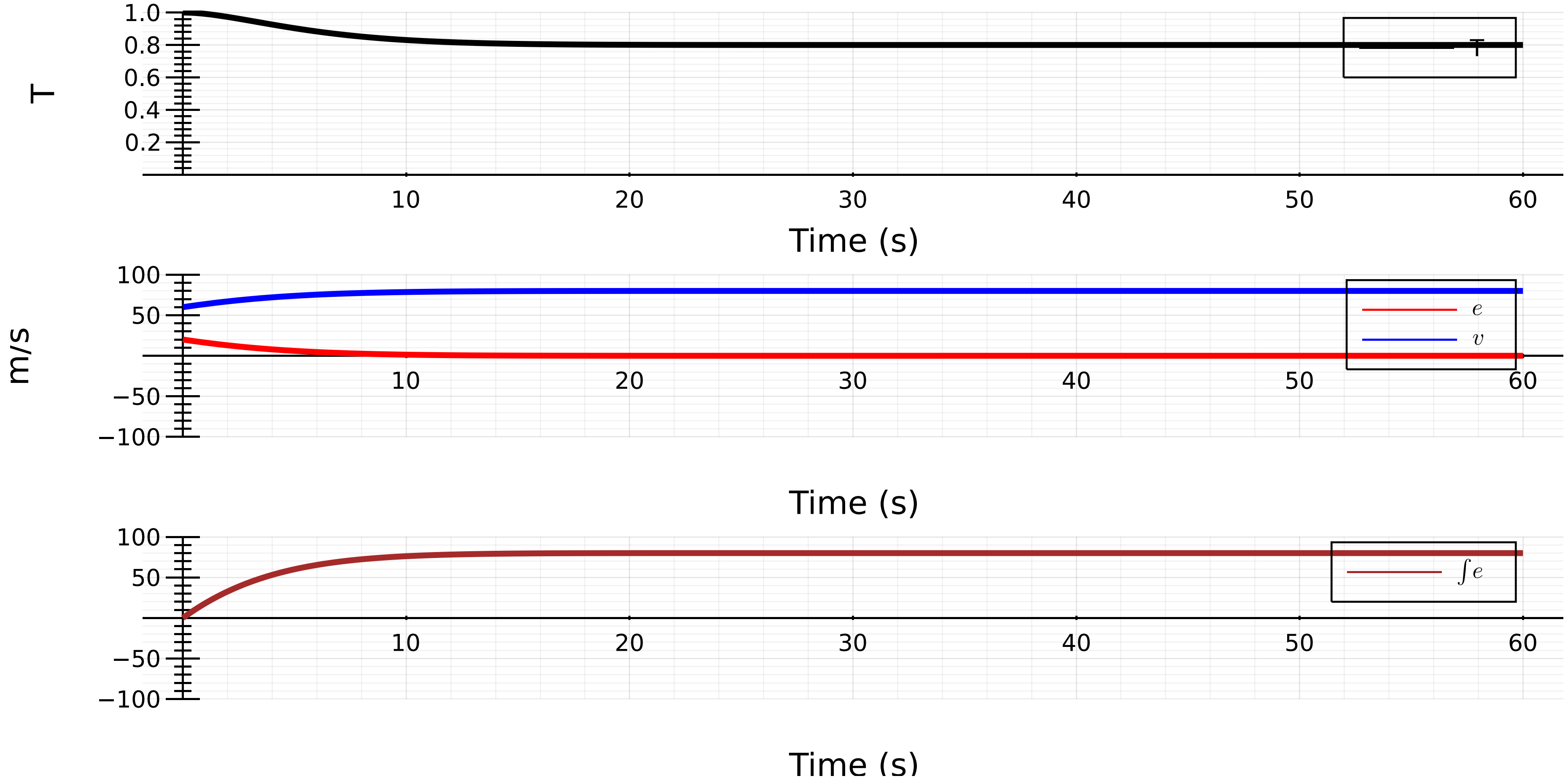


- How do we reduce the need to increase the P gain, and reduce the steady state error e_{ss}
- We can introduce the ***Integral*** Controller to make a PI Controller
  - $u = u_P + u_I = K_P e + K_I \int e\, dt$

- Using a PI Controller, with $K_P = 0.05$, and $K_I = 0.01$
- Assume drag force: $D = 0.1v$, Traction force: $F = 10T$



PI Controller

- $u = u_P + u_I = K_P e + K_I \int e\,dt$
  - The P and I term work together to produce a better output u
  - Near $e = 0$, the P term vanishes, but the I term converges to zero when the ***integral of the error*** is zero: when the area under the curve of error e was equally positive and negative.
  - Let's see the effect of changing the P and I gains on the response of a second order system
    - $G(s)_{plant} = \frac{1}{ms^2+bs+k}$
    - The PID Controller transfer function is represented as
      - $C_{PID}(s) = \frac{K_D s^2+K_P s+K_I}{s}$

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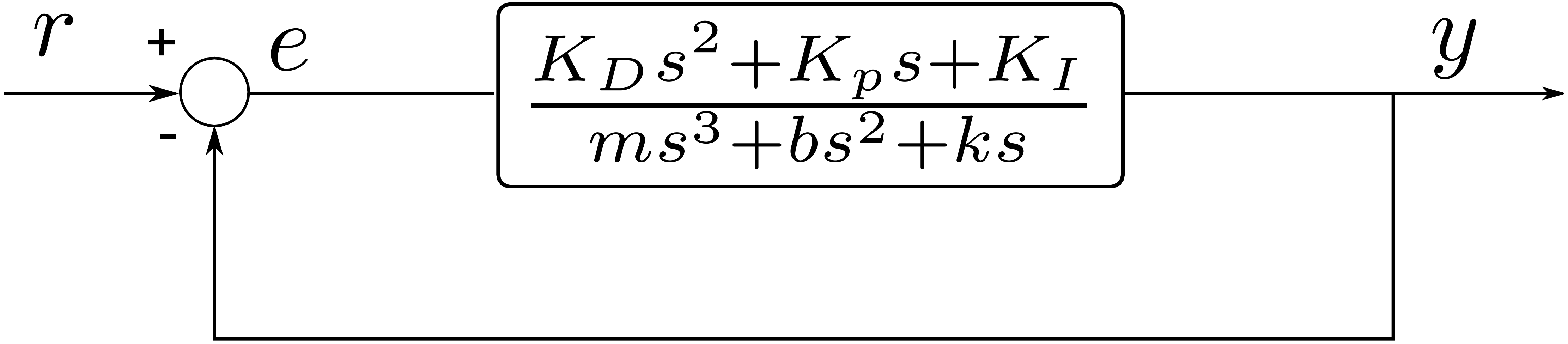
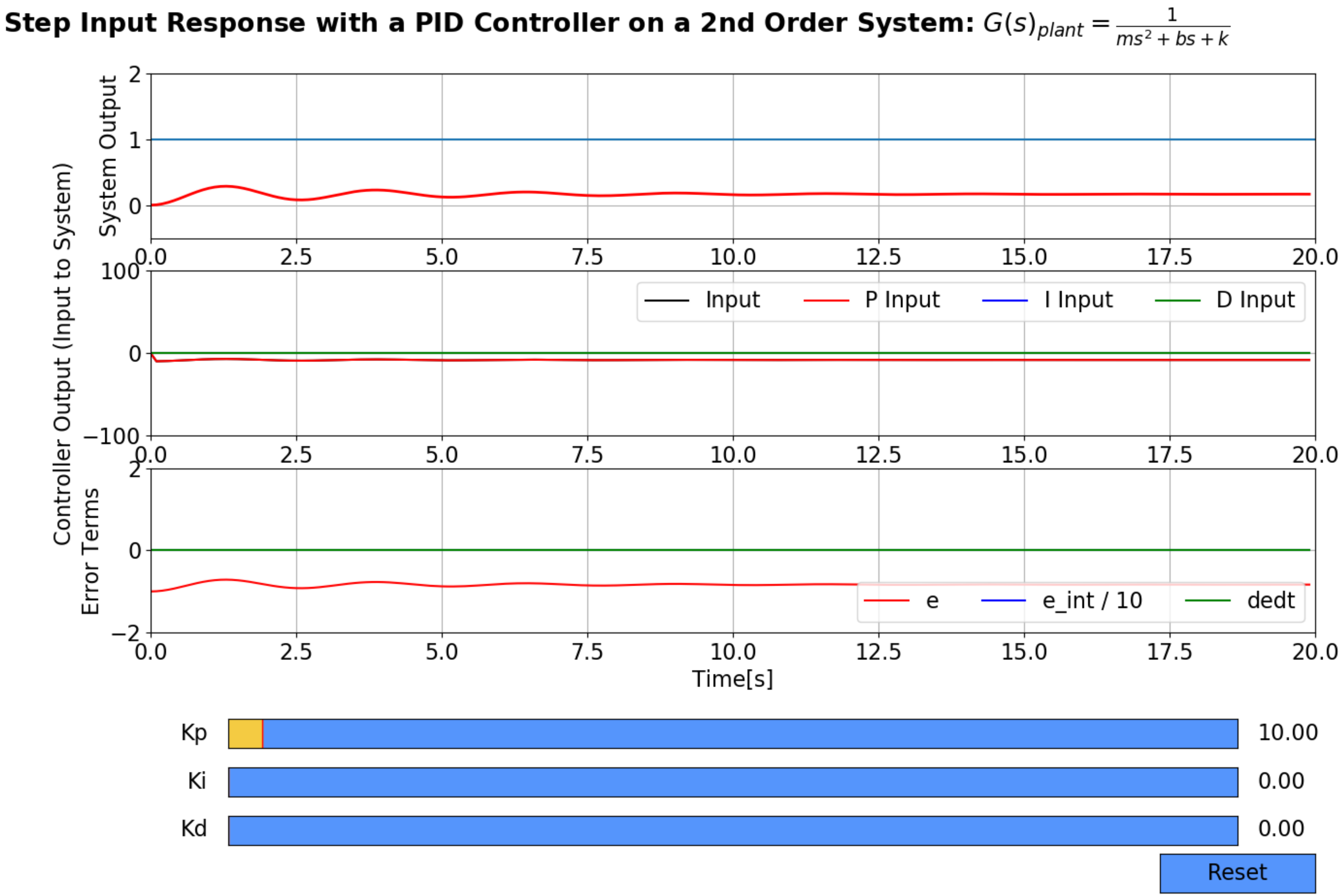
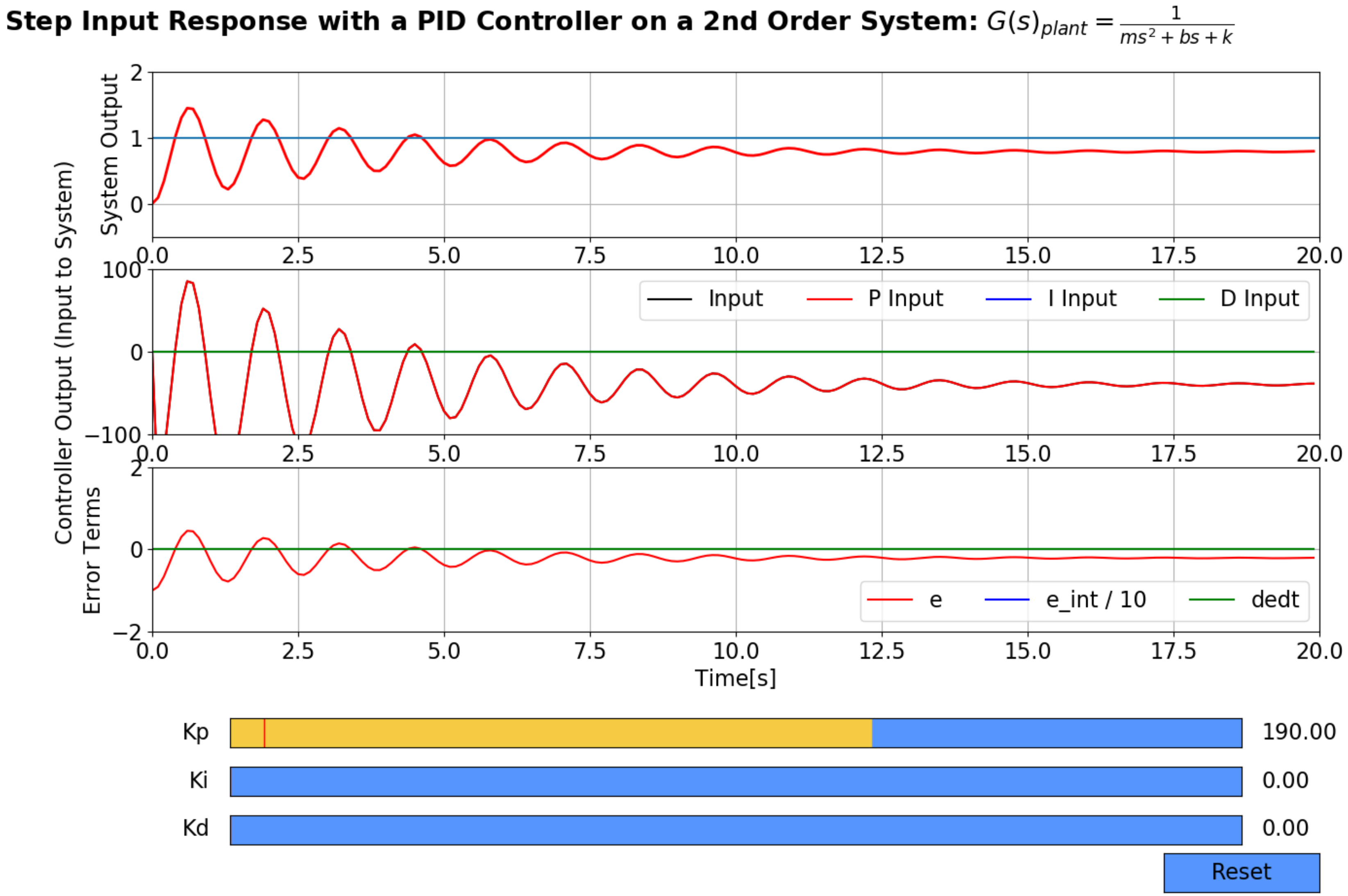

r
+
-
e
$\frac{K_D s^2 + K_p s + K_I}{m s^3 + b s^2 + k s}$
y

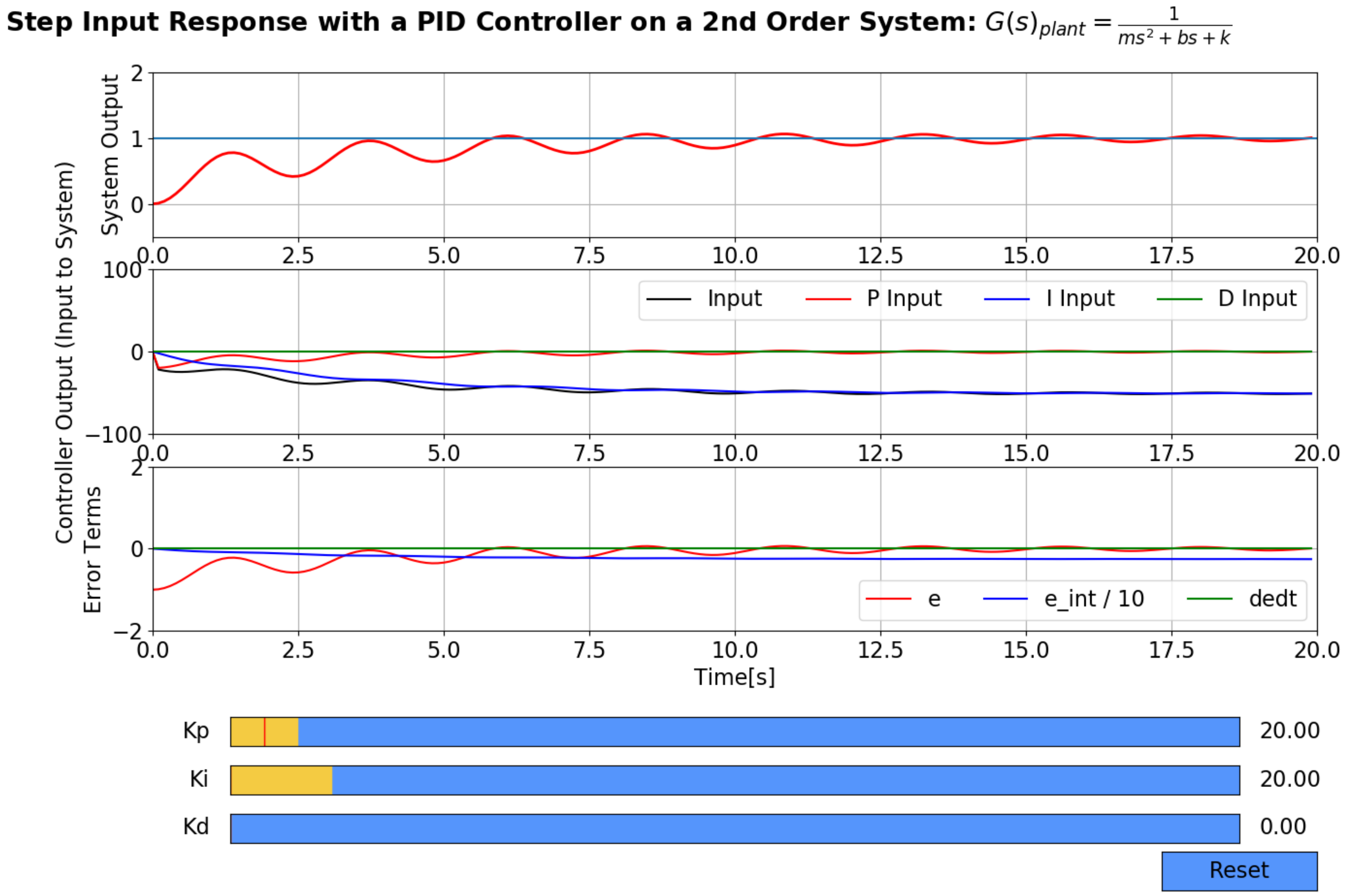
PID Control of 2nd Order System



- Increase K_P gain

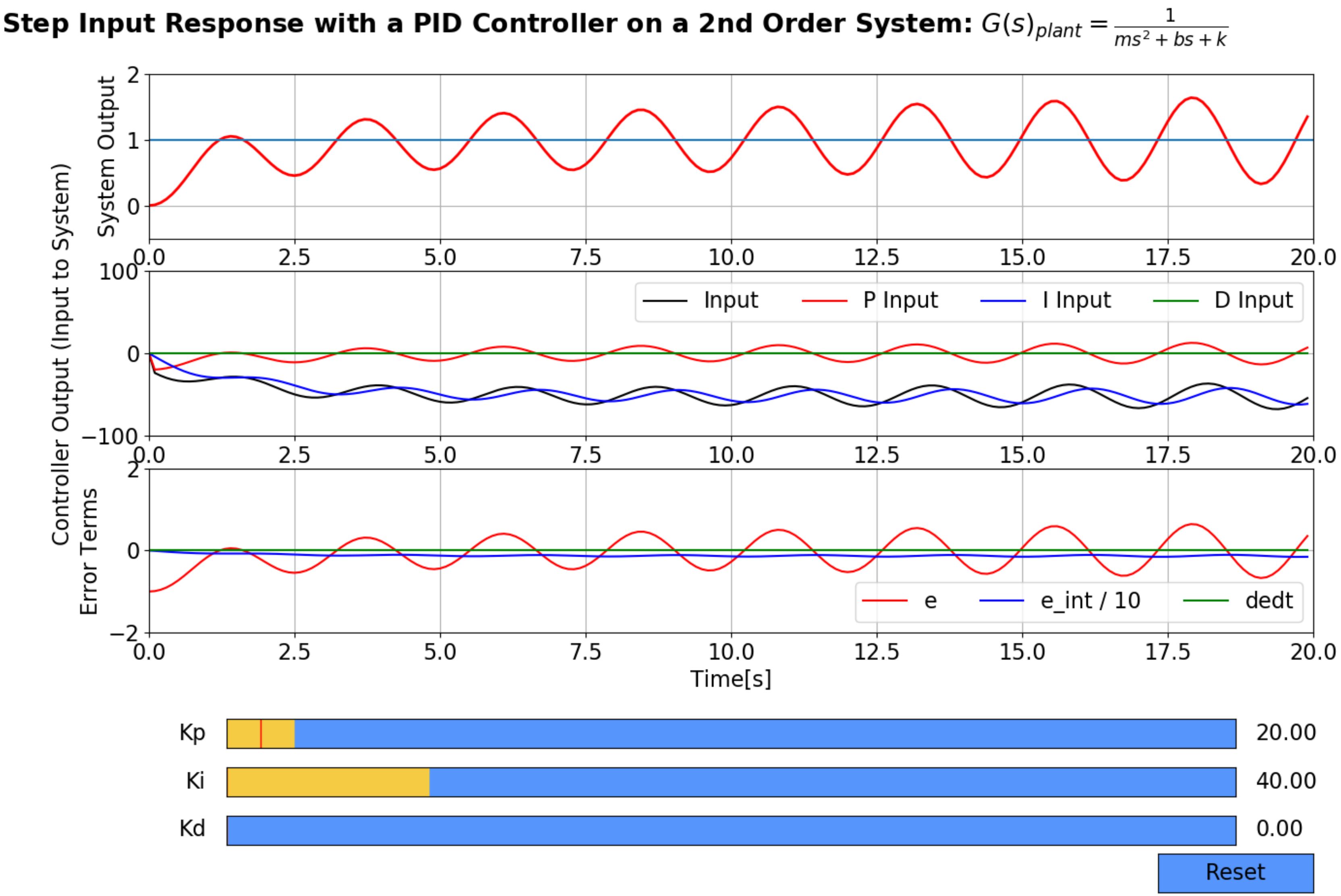


- Introduce K_I gain

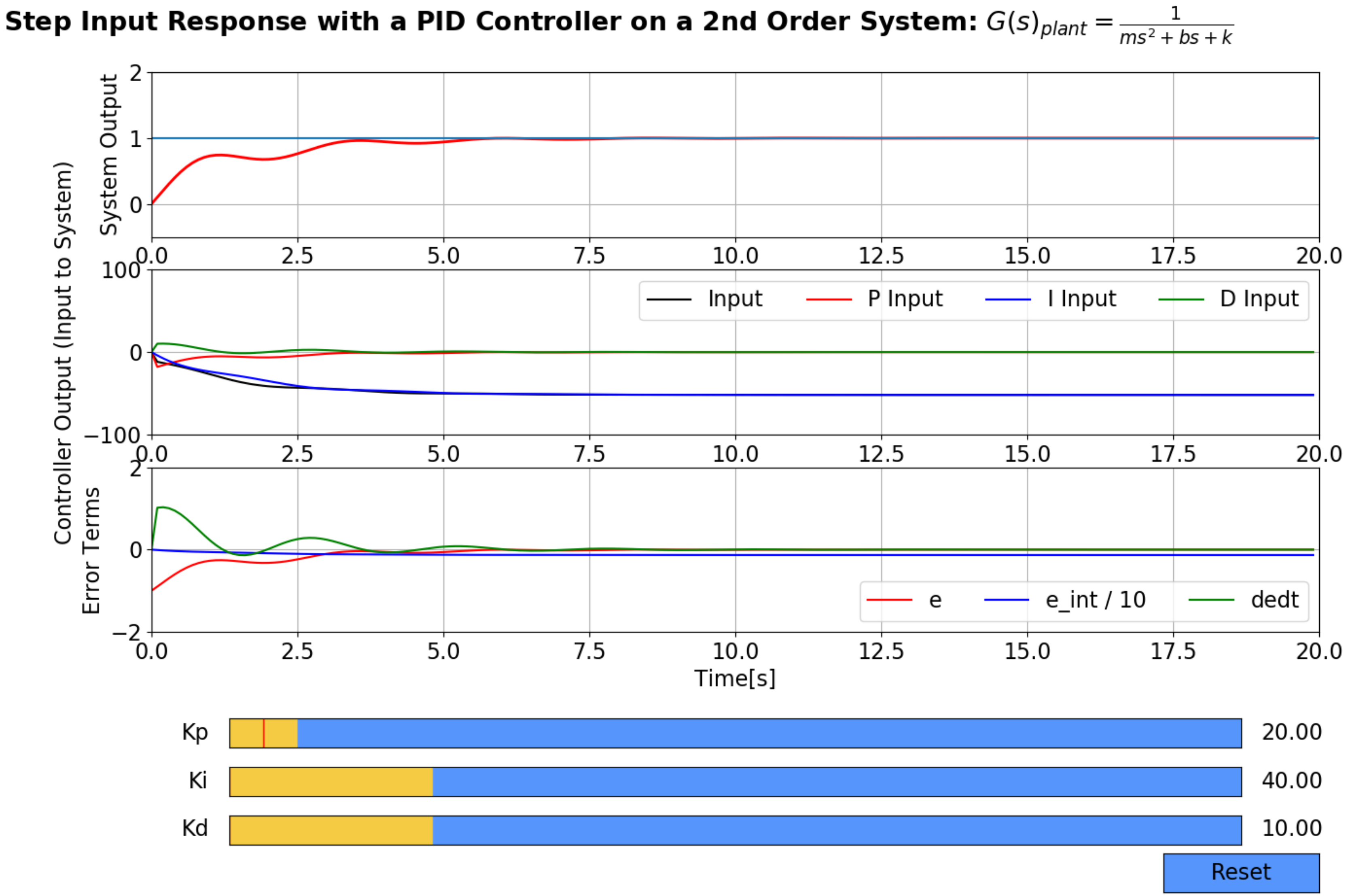


PID Control of 2nd Order System

- Increase K_I gain



- Introduce K_D gain



Effects of P, I, and D Gains

Gain	Rise Time	Overshoot	Settling Time	e_{ss}
K_P	Decrease	Increase	Small Change	Decrease
K_I	Decrease	Increase	Increase	Decrease
K_D	Small Change	Decrease	Decrease	No Change

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Integrator Wind-up

- Actuators have a finite amplitude
  - In the CCS example, the throttle input can only take values between 0.0 and 1.0.
  - What happens when the controller output value exceeds these limits, while we have an integrator term?
  - Take a look at the CCS example with P Controller, $K_p = 0.1$ and let's just observe what happens to the I term if $K_I = 0.1$

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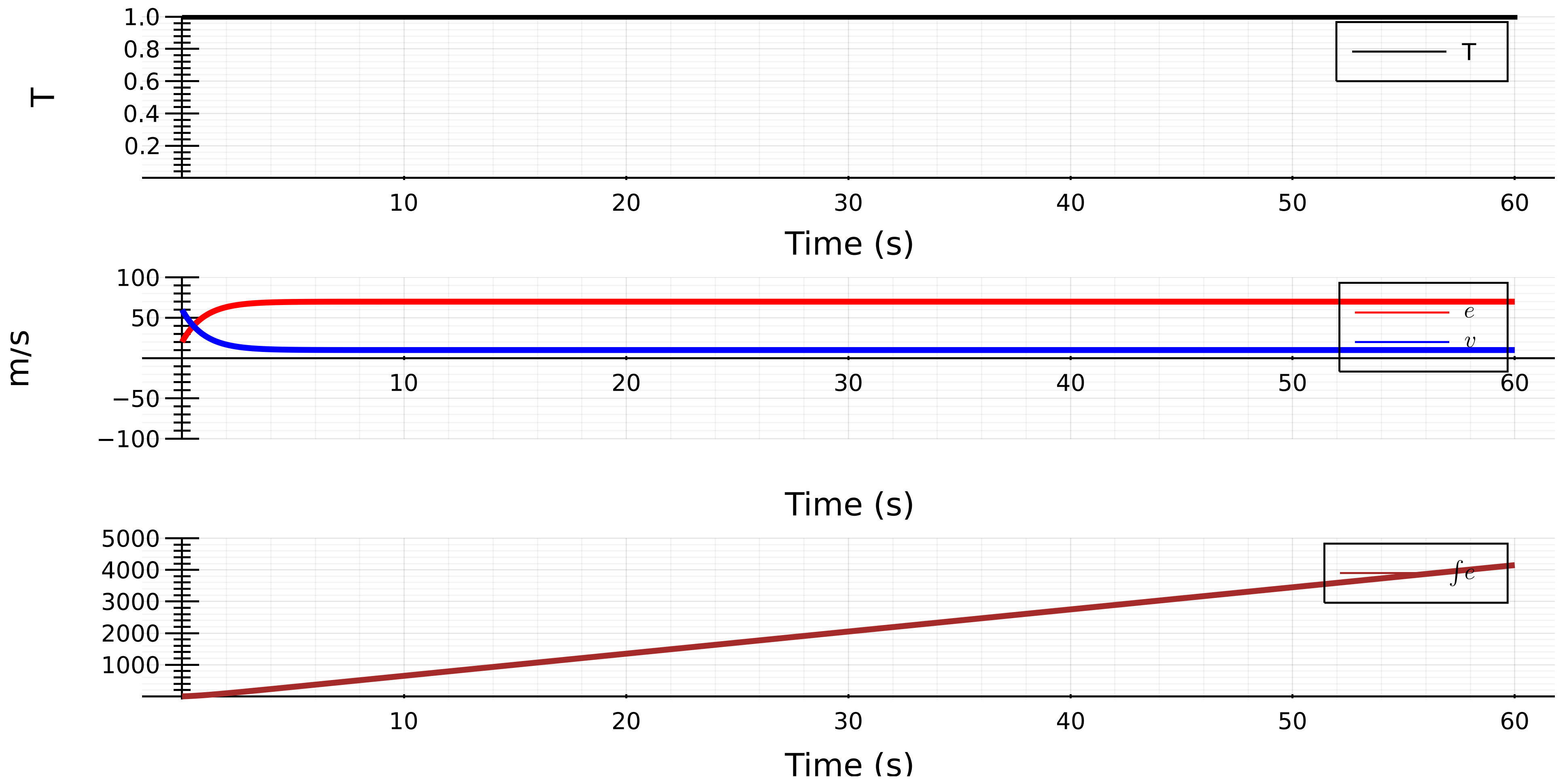
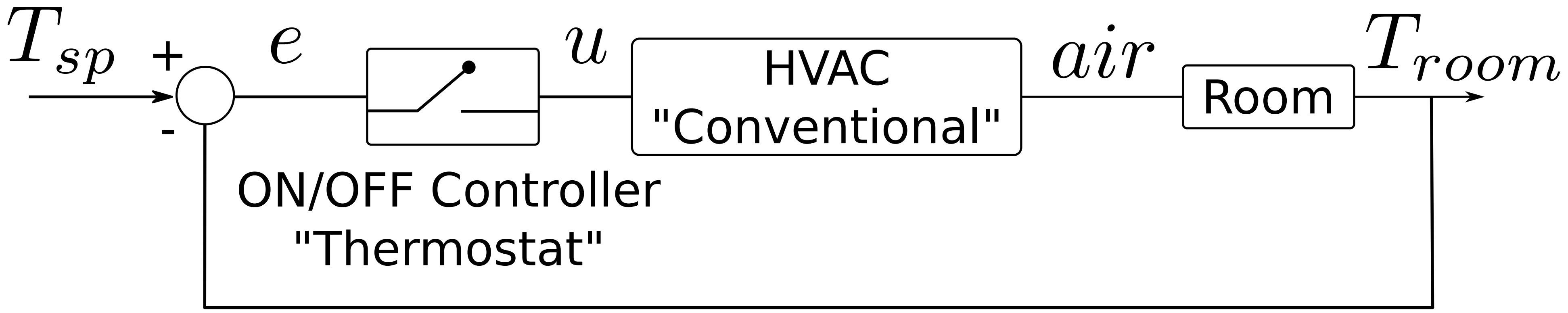
- What happens to $\int e\,dt$ when we increase $D = 1v$?

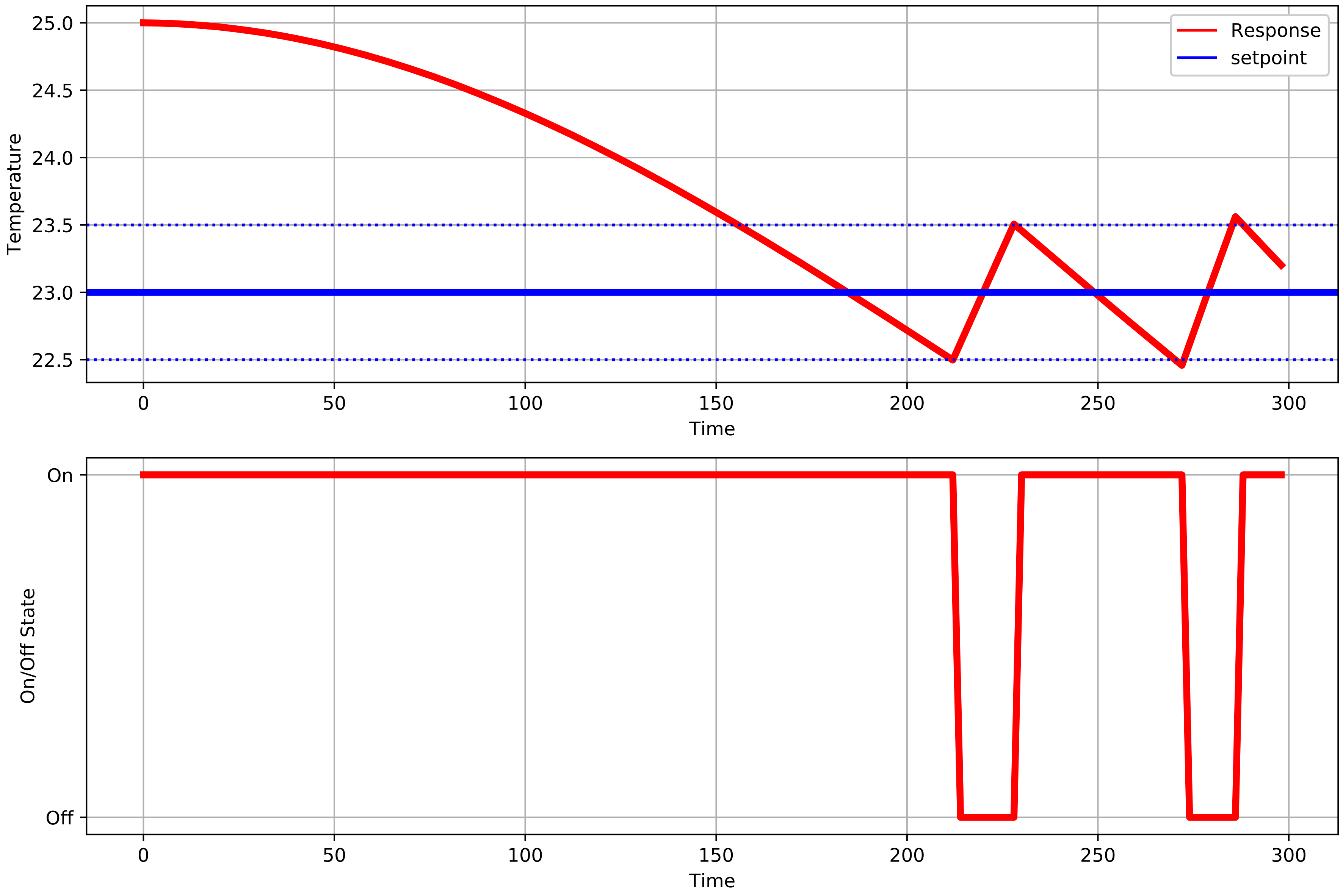


Another example: Feedback Control System - Conventional HVAC

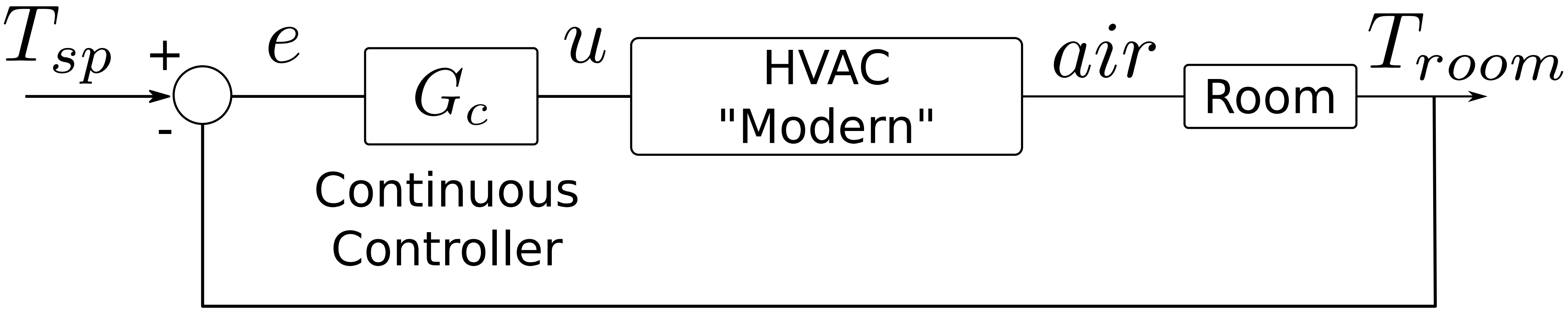
- Controller is a simple On/Off controller
  - If $e \geq 0.5 \quad and\ AC\_Is\_On$, then turn a/c off: $u = 0$
  - If $e \leq -0.5 \quad and\ AC\_Is\_Off$, then turn a/c on: $u = 1$
  - Else, keep u unchanged.



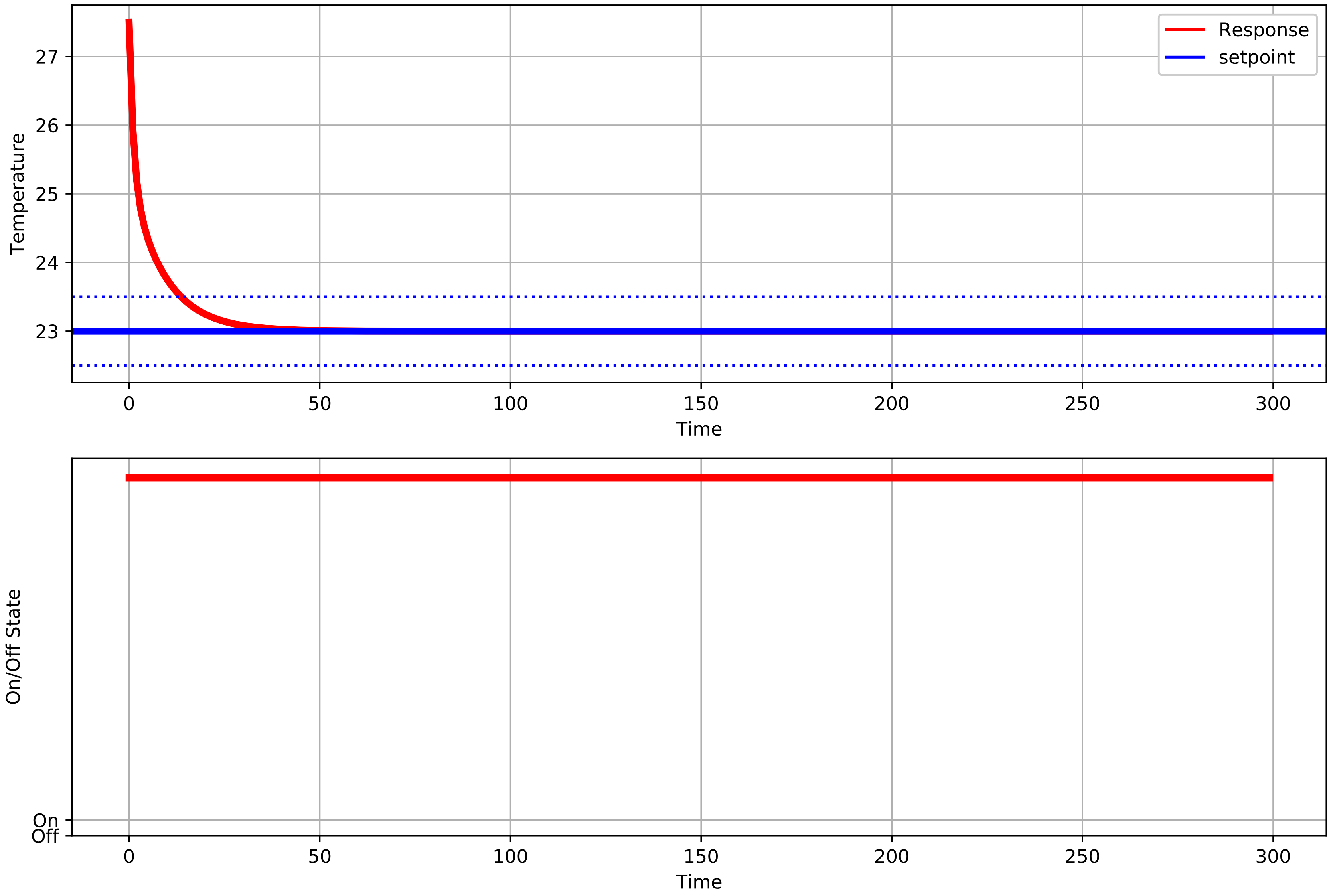
Conventional HVAC: Response



- What if we introduce a variable frequency drive (VFD) compressor?
  - The output of the controller will instead be a speed command to the compressor.



Modern HVAC: Response



- Assume is the HVAC system is On 50% (Duty Cycle is 50%)
  - Power: $P = \tau\omega$, and $\tau = K_T\omega^2$, so $P = K_T\omega^3$
  - Power is related to the speed of the compressor motor, cubed.
  - With 50% duty cycle, the power: $P_{conventional} = \frac{K_T\omega_c^3}{2}$

- Now, assume we install a VFD compressor on the same capacity HVAC, and would need to run it at 100% duty cycle, at 75% of the conventional setup speed
  - $\omega_{vfd} = \frac{3\omega_c}{4}$
  - Power would then be,

$$P_{VFD} = K_T\omega_{VFD}^3 = \frac{27}{2 \cdot 32} K_T\omega_c^3 = \frac{27}{32} P_{conventional}$$

  - That is 16% reduction in power consumption, plus accurate temperature control

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