ME 417 - Homework #4

Control of Mechanical Systems - Summer 2020

Homework Due: Thu, 12 Nov 2020 18:00

Complete the following problems and submit a hard copy of your solutions. You are encouraged to work together to discuss the problems but submitted work **MUST** be your own. This is an **individually** submitted assignment.

Problem 1

Derive State Space from Transfer Function (20pts)

For each of the following systems, derive an equivalent state-space representation. Then determine if the system is **controllable**.

a. $G(s) = \dfrac{1}{s^2 + 2s + 4}$

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b. $G(s) = \dfrac{s + 5}{s^2 + 2s + 4}$

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c. $G(s) = \dfrac{s^2 + 10s + 100}{(s + 2)\left(s^2 + 2s + 4\right)}$

Problem 2

Derive Transfer Function from StateSpace (20pts)

Given the following system, derive an equivalent transfer function

a.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0.0 & 1.0 \\ 1.0 & -2.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0.0 & 1.0 \end{bmatrix} \mathbf{x}$$

b.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0.0 & 1.0 & 1.0 \\ 0.0 & 1.0 & 0.0 \\ -1.0 & -2.0 & -3.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1.0 & 0.0 & 0.0 \end{bmatrix} \mathbf{x}$$

Problem 3

Derive Transfer Function from StateSpace (1pts)

Given the following system, derive an equivalent transfer function

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 5 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}$$

a. Derive the closed-loop form, given the input $u = \mathbf{Kx}$

b. Derive the closed-loop form, given the input $u = K_p y$

Problem 4

Full State Feedback Inverted Pendulum (20pts)

A very simple and linearized model of an inverted pendulum on a cart is given.

$m_c = 5kg, m_r = 0.25kg, l = 30.0cm$

a. Determine the stability of the closed-loop system

b. Show that the system is controllable

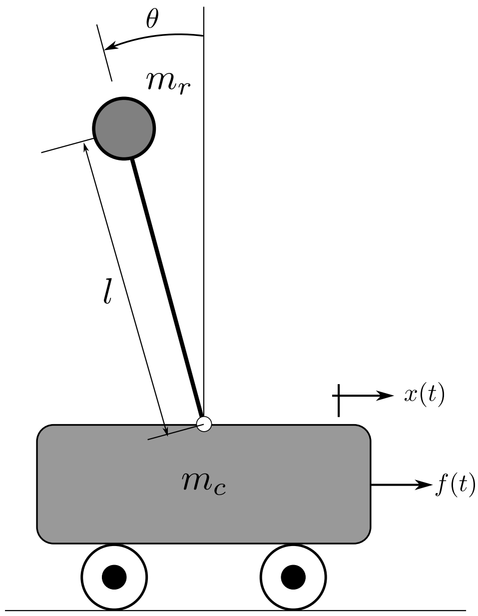
c. Design a full-state feedback controller to achieve $T_s = 2s$ and $\zeta = 0.5$

d. Justify your choice of additional poles' locations

You can use MATLAB symbolic to aid in the calculation of the determinants and adjugate, but the rest of the solution should be carried by hand.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.491 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 32.7 & 0.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0.0 \\ 0.2 \\ 0.0 \\ -0.667 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}$$



Problem 5

Design Integral Controller Given State Space Model) (20pts)

Given the following open-loop system

a. For what values of a is the system controllable

b. Show that applying full-state feedback, for a step input of $r(t) = 5$, the system yields a finite steady-state error. Show the value of the steady-state error.

c. Show that adding an integral controller to the closed-loop system in state-space, results in the elimination of the steady-state error.

d. What is the order of the system with just a state feedback controller? What is the order of the system when you include the integrator?

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & -5.0 \\ a & -1.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}$$