ME 417 - Homework #4

Control of Mechanical Systems - Fall 2020

Homework Due: Fri, 06 Mar 2020 23:59

Complete the following problems and submit a hard copy of your solutions. You are encouraged to work together to discuss the problems but submitted work **MUST** be your own. This is an **individually** submitted assignment.

Problem 1

Derive State Space from Transfer Function (20pts)

For each of the following systems, derive an equivalent state-space representation. Then determine if the system is **controllable**.

a. $G(s) = \dfrac{10.0}{s^2 + 4.0s + 20.0}$

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b. $G(s) = \dfrac{40.0s + 80.0}{s^2 + 4.0s + 20.0}$

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c. $G(s) = \dfrac{20.0s^2 + 100.0s + 1.0 \cdot 10^3}{(s + 4.0)\,(s^2 + 4.0s + 20.0)}$

Solution:

a. The state space form of the transfer function is

$$\dot{\mathbf{x}} = \begin{bmatrix} 0.0 & 1.0 \\ -20.0 & -4.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 10.0 & 0.0 \end{bmatrix} \mathbf{x}$$

To determine the controllability of the system, we can evaluate the rank of the controllability matrix.

The controllability matrix is defined as

$$C_m = \begin{bmatrix} B & AB & \cdots & A^n \cdot B \end{bmatrix} = \begin{bmatrix} 0.0 & 1.0 \\ 1.0 & -4.0 \end{bmatrix}$$

If the rank of the controllability matrix is n, then the system is controllable. A sufficient test is to check if the determinant of the controllability matrix is $det(C_m) \neq 0$

Calculating the determinant of the controllability matrix we get. $det(C_m) = -1.0 \neq 0$

And thus the system is controllable.

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b. The state space form of the transfer function is

$$\dot{\mathbf{x}} = \begin{bmatrix} 0.0 & 1.0 \\ -20.0 & -4.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 80.0 & 40.0 \end{bmatrix} \mathbf{x}$$

To determine the controllability of the system, we can evaluate the rank of the controllability matrix.

The controllability matrix is defined as

$$C_m = \begin{bmatrix} B & AB & \cdots & A^n \cdot B \end{bmatrix} = \begin{bmatrix} 0.0 & 1.0 \\ 1.0 & -4.0 \end{bmatrix}$$

If the rank of the controllability matrix is n, then the system is controllable. A sufficient test is to check if the determinant of the controllability matrix is $det(C_m) \neq 0$

Calculating the determinant of the controllability matrix we get. $det(C_m) = -1.0 \neq 0$

And thus the system is controllable.

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c. The state space form of the transfer function is

$$\dot{\mathbf{x}} = \begin{bmatrix} 0.0 & 1.0 \\ -20.0 & -4.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 80.0 & 40.0 \end{bmatrix} \mathbf{x}$$

To determine the controllability of the system, we can evaluate the rank of the controllability matrix.

The controllability matrix is defined as

$$C_m = \begin{bmatrix} B & AB & \cdots & A^n \cdot B \end{bmatrix} = \begin{bmatrix} 0.0 & 1.0 \\ 1.0 & -4.0 \end{bmatrix}$$

If the rank of the controllability matrix is n, then the system is controllable. A sufficient test is to check if the determinant of the controllability matrix is $det(C_m) \neq 0$

Calculating the determinant of the controllability matrix we get. $det(C_m) = -1.0 \neq 0$

And thus the system is controllable.

Problem 2

Derive Transfer Function from StateSpace (20pts)

Given the following system, derive an equivalent transfer function, then compute the steady-state error for a step input $u = 12$

a.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0.0 & 1.0 \\ -2.0 & -5.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0.0 \\ 4.0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 2.0 & -0.5 \end{bmatrix} \mathbf{x}$$

b.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0.0 & 1.0 & 1.0 \\ 0.0 & 1.0 & 0.0 \\ -2.0 & -4.0 & -6.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0.0 \\ 1.0 \\ 0.0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1.0 & -0.2 & 0.0 \end{bmatrix} \mathbf{x}$$

c.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0.0 & 1.0 \\ 5.0 & -1.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 2.0 \\ -0.8 \end{bmatrix} u$$

$$y = \begin{bmatrix} 3.0 & 0.0 \end{bmatrix} \mathbf{x}$$

Solution:

a.

To convert the state space respresentation to a transfer function, we use the following equation

$$G(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} = \mathbf{C}\frac{adj(s\mathbf{I} - \mathbf{A})}{det(s\mathbf{I} - \mathbf{A})}\mathbf{B} + \mathbf{D}$$

$$\rightarrow G(s) = \begin{bmatrix} 2.0 & -0.5 \end{bmatrix} (\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0.0 & 1.0 \\ -2.0 & -5.0 \end{bmatrix})^{-1} \begin{bmatrix} 0.0 \\ 4.0 \end{bmatrix}$$

$$\rightarrow G(s) = \begin{bmatrix} 2.0 & -0.5 \end{bmatrix} \frac{adj(\begin{bmatrix} s & -1.0 \\ 2.0 & s + 5.0 \end{bmatrix})}{det(\begin{bmatrix} s & -1.0 \\ 2.0 & s + 5.0 \end{bmatrix})} \begin{bmatrix} 0.0 \\ 4.0 \end{bmatrix}$$

$$\rightarrow G(s) = \frac{8.0 - 2.0s}{1.0s^2 + 5.0s + 2.0}$$

To find the steady-state error to a step input $u = 12$, given a state-space representation, we use the derived property

$$e_{ss} = (1 + \mathbf{C}\mathbf{A}^{-1}\mathbf{B})u = (1 + \mathbf{C}\frac{adj(\mathbf{A})}{det(\mathbf{A})}\mathbf{B})u$$

$$\rightarrow e_{ss} = (1 + \begin{bmatrix} 2.0 & -0.5 \end{bmatrix} \frac{adj(\begin{bmatrix} 0.0 & 1.0 \\ -2.0 & -5.0 \end{bmatrix})}{2.0} \begin{bmatrix} 0.0 \\ 4.0 \end{bmatrix})12 = -60.0$$

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b.

To convert the state space respresentation to a transfer function, we use the following equation
$$G(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} = \mathbf{C}\frac{adj(s\mathbf{I} - \mathbf{A})}{det(s\mathbf{I} - \mathbf{A})}\mathbf{B} + \mathbf{D}$$

$$\rightarrow G(s) = \begin{bmatrix} 1.0 & -0.2 & 0.0 \end{bmatrix} (\begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0.0 & 1.0 & 1.0 \\ 0.0 & 1.0 & 0.0 \\ -2.0 & -4.0 & -6.0 \end{bmatrix})^{-1} \begin{bmatrix} 0.0 \\ 1.0 \\ 0.0 \end{bmatrix}$$

$$\rightarrow G(s) = \begin{bmatrix} 1.0 & -0.2 & 0.0 \end{bmatrix} \frac{adj(\begin{bmatrix} s & -1.0 & -1.0 \\ 0.0 & s - 1.0 & 0.0 \\ 2.0 & 4.0 & s + 6.0 \end{bmatrix})}{det(\begin{bmatrix} s & -1.0 & -1.0 \\ 0.0 & s - 1.0 & 0.0 \\ 2.0 & 4.0 & s + 6.0 \end{bmatrix})} \begin{bmatrix} 0.0 \\ 1.0 \\ 0.0 \end{bmatrix}$$

$$\rightarrow G(s) = \frac{-0.25s^2 - 0.25s + 1.5}{1.0s^3 + 5.0s^2 - 4.0s - 2.0}$$

To find the steady-state error to a step input $u = 12$, given a state-space representation, we use the derived property

$$e_{ss} = (1 + \mathbf{C}\mathbf{A}^{-1}\mathbf{B})u = (1 + \mathbf{C}\frac{adj(\mathbf{A})}{det(\mathbf{A})}\mathbf{B})u$$

$$\rightarrow e_{ss} = (1 + \begin{bmatrix} 1.0 & -0.2 & 0.0 \end{bmatrix} \frac{adj(\begin{bmatrix} 0.0 & 1.0 & 1.0 \\ 0.0 & 1.0 & 0.0 \\ -2.0 & -4.0 & -6.0 \end{bmatrix})}{2.0} \begin{bmatrix} 0.0 \\ 1.0 \\ 0.0 \end{bmatrix})12 = 9.600000000000001$$

c.

To convert the state space respresentation to a transfer function, we use the following equation

$$G(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} = \mathbf{C}\frac{adj(s\mathbf{I} - \mathbf{A})}{det(s\mathbf{I} - \mathbf{A})}\mathbf{B} + \mathbf{D}$$

$$\rightarrow G(s) = \begin{bmatrix} 3.0 & 0.0 \end{bmatrix} \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0.0 & 1.0 \\ 5.0 & -1.0 \end{bmatrix} \right)^{-1} \begin{bmatrix} 2.0 \\ -0.8 \end{bmatrix}$$

$$\rightarrow G(s) = \begin{bmatrix} 3.0 & 0.0 \end{bmatrix} \frac{adj\left(\begin{bmatrix} s & -1.0 \\ -5.0 & s + 1.0 \end{bmatrix} \right)}{det\left(\begin{bmatrix} s & -1.0 \\ -5.0 & s + 1.0 \end{bmatrix} \right)} \begin{bmatrix} 2.0 \\ -0.8 \end{bmatrix}$$

$$\rightarrow G(s) = \frac{6.0s + 3.5}{1.0s^2 + 1.0s - 5.0}$$

To find the steady-state error to a step input $u = 12$, given a state-space representation, we use the derived property

$$e_{ss} = (1 + \mathbf{C}\mathbf{A}^{-1}\mathbf{B})u = (1 + \mathbf{C}\frac{adj(\mathbf{A})}{det(\mathbf{A})}\mathbf{B})u$$

$$\rightarrow e_{ss} = (1 + \begin{bmatrix} 3.0 & 0.0 \end{bmatrix} \frac{adj\left(\begin{bmatrix} 0.0 & 1.0 \\ 5.0 & -1.0 \end{bmatrix} \right)}{-5.0} \begin{bmatrix} 2.0 \\ -0.8 \end{bmatrix})12 = -132.0$$

Problem 3

Derive Transfer Function from StateSpace (1pts)

Given the following system, derive an equivalent transfer function

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 5 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}$$

a. Derive the closed-loop form, given the input $u = \mathbf{Kx}$

b. Derive the closed-loop form, given the input $u = K_p y$

Solution:

a.

Substituting into u we get $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}(r - \mathbf{Kx}) = (\mathbf{A} - \mathbf{BK})\mathbf{x} + \mathbf{B}r$

Where $K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}$

$$\dot{\mathbf{x}} = \begin{bmatrix} -1.0K_1 & 1.0 - 1.0K_2 \\ 5.0 & -1.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1.0 \\ 0 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1.0 & 0 \end{bmatrix} \mathbf{x}$$

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b.

Substituting into u we get $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}K_p(r - y) = \mathbf{Ax} + \mathbf{B}K_p(r - \mathbf{Cx}) = (\mathbf{A} - \mathbf{B}K_p\mathbf{C})\mathbf{x} + \mathbf{B}K_p r$

$$\dot{\mathbf{x}} = \begin{bmatrix} -K_p & 1 \\ 5 & -1 \end{bmatrix} \mathbf{x} + \begin{bmatrix} K_p \\ 0 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}$$

Problem 4

Full State Feedback Inverted Pendulum (20pts)

A very simple and linearized model of an inverted pendulum on a cart is given.

$m_c = 3kg, m_r = 0.4kg, l = 25.0cm$

a. Determine the stability of the closed-loop system

b. Show that the system is controllable

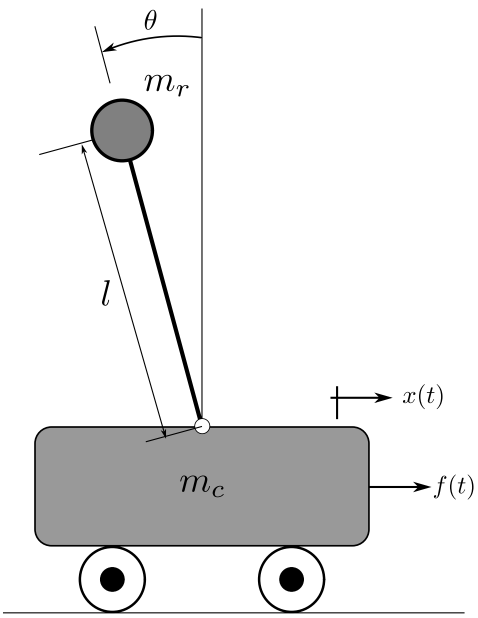
c. Design a full-state feedback controller to achieve $\omega_d = 5\,rad/s$ and $\zeta = 0.7$

d. Justify your choice of additional poles' locations

You can use MATLAB symbolic to aid in the calculation of the determinants and adjugate, but the rest of the solution should be carried by hand.

$$\dot{\mathbf{x}} = \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.308 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 39.24 & 0.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0.0 \\ 0.3333333333333333 \\ 0.0 \\ -1.3333333333333333 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \end{bmatrix} \mathbf{x}$$



Solution:

Taking the determinant of $(s\mathbf{I} - \mathbf{A})$ we then derive the characteristic polynomial for the closed-loop system. Characteristic polynomial : $det(s\mathbf{I} - \mathbf{A}) = det(s \begin{bmatrix} 1.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} - \begin{bmatrix} 0.0 & 1.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 1.25 & 0.0 \\ 0.0 & 0.0 & 0.0 & 1.0 \\ 0.0 & 0.0 & 39.25 & 0.0 \end{bmatrix} =$

$1.0s^4 - 39.3s^2 = 0$

The poles of the system are $\begin{bmatrix} 0.0 & 0.0 & 6.25 & -6.25 \end{bmatrix}$

The system is unstable.

There are poles in the RHP plane @ $\begin{bmatrix} 6.25 \end{bmatrix}$

To determine the controllability of the system, we can evaluate the rank of the controllability matrix.

The controllability matrix is defined as

$$C_m = \begin{bmatrix} B & AB & \cdots & A^n \cdot B \end{bmatrix} = \begin{bmatrix} 0.0 & 0.25 & 0.0 & -1.75 \\ 0.25 & 0.0 & -1.75 & 0.0 \\ 0.0 & -1.25 & 0.0 & -52.25 \\ -1.25 & 0.0 & -52.25 & 0.0 \end{bmatrix}$$

If the rank of the controllability matrix is n, then the system is controllable. A sufficient test is to check if the determinant of the controllability matrix is $det(C_m) \neq 0$

Calculating the determinant of the controllability matrix we get. $det(C_m) = 232.56250000000003 \neq 0$

And thus the system is controllable.

First we put the system in to a closed-loop form

Substituting into u we get $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}(r - \mathbf{K}\mathbf{x}) = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} + \mathbf{B}r$

Where $K = \begin{bmatrix} K_1 & K_2 & K_3 & K_4 \end{bmatrix}$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1.0 & 0 & 0 \\ -0.333K_1 & -0.333K_2 & 1.31 - 0.333K_3 & -0.333K_4 \\ 0 & 0 & 0 & 1.0 \\ 1.33K_1 & 1.33K_2 & 1.33K_3 + 39.2 & 1.33K_4 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0.333 \\ 0 \\ -1.33 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1.0 & 0 & 0 & 0 \\ 0 & 0 & 1.0 & 0 \end{bmatrix} \mathbf{x}$$

Taking the determinant of $(s\mathbf{I} - \mathbf{A})$ we then derive the characteristic polynomial for the closed-loop system in terms of the full-state feedback controller gains.

$p(s) =$

$$\begin{aligned} &-4.93K_1^2 - 9.86K_1K_2s + 1.0s^6 + s^5\left(0.666K_2 - 1.33K_4\right) \\ &+ s^4\left(0.666K_1 + 0.111K_2^2 - 0.443K_2K_4 - 1.33K_3 - 39.3\right) \\ &+ s^3\left(0.222K_1K_2 - 0.443K_1K_4 - 0.443K_2K_3 - 27.9K_2\right) + s^2\left(0.111K_1^2 - 0.443K_1K_3 - 27.9K_1 - 4.93K_2^2\right) \end{aligned} \tag{1}$$

Given a desired damping ratio of $\zeta = 0.7$ and a damped frequency of $\omega_d = 5rad/s$, we can construct a second-order polynomial that captures this requirement

With $\sigma = -\frac{4}{T_s} = 4.900980294098034$

$p_d(s) = (s - \sigma + \omega_d i)(s - \sigma - \omega_d i) = s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 - 9.8s + 49.0$

We have a 4th-order polynomial, we need to choose where to place the additional pole/s

If we have a LHP zero, and we want the outcome to achieve an ideal second-order response, we can place a pole on the zero and cancel its effect. Generally, we can place the third and higher poles 5 times further away into the LHP relative to the real location of the dominant poles. The desired characteristic polynomial becomes

$p_d(s) = (s - 49.0)(s - 24.5)\left(s^2 - 9.8s + 49.0\right) = s^4 - 83.3s^3 + 1.97 \cdot 10^3 s^2 - 1.54 \cdot 10^4 s + 5.89 \cdot 10^4$

Matching the coefficients to find the values of the K matrix

$$\mathbf{K} = \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix} = \begin{bmatrix} -3976.0 \\ 1038.5 \\ -2506.75 \\ 322.75 \end{bmatrix}$$

And the controller now becomes $u = r - \begin{bmatrix} -3976.0 \\ 1038.5 \\ -2506.75 \\ 322.75 \end{bmatrix} \mathbf{x}$

Problem 5

Design Integral Controller Given State Space Model) (20pts)

Given the following open-loop system

a. For what values of a is the system controllable

b. Show that applying full-state feedback, for a step input of $r(t) = 10$, the system yields a finite steady-state error. Show the value of the steady-state error.

c. Show that adding an integral controller to the closed-loop system in state-space, results in the elimination of the steady-state error.

d. What is the order of the system with just a state feedback controller? What is the order of the system when you include the integrator?

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & -10.0 \\ a & -2.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1.0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1.0 & 0.2 \end{bmatrix} \mathbf{x}$$

Solution:

1. To determine the controllability of the system, we can evaluate the rank of the controllability matrix.

The controllability matrix is defined as

$$C_m = \begin{bmatrix} B & AB & \cdots & A^n \cdot B \end{bmatrix} = \begin{bmatrix} 0 & -10.0 \\ 1.0 & -2.0 \end{bmatrix}$$

If the rank of the controllability matrix is n, then the system is controllable. A sufficient test is to check if the determinant of the controllability matrix is $det(C_m) \neq 0$

Calculating the determinant of the controllability matrix we get. $det(C_m) = 10.0 \neq 0$

And thus the system is controllable.

2. Deriving the full-state feedback closed-loop form:

Substituting into u we get $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}(r - \mathbf{K}\mathbf{x}) = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} + \mathbf{B}r$

Where $K = \begin{bmatrix} K_1 & K_2 \end{bmatrix}$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & -10.0 \\ -1.0K_1 + a & -1.0K_2 - 2.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1.0 \end{bmatrix} r$$

$y = \begin{bmatrix} 1.0 & 0.2 \end{bmatrix} \mathbf{x}$

To find the steady-state error to a step input $u = 10$, given a state-space representation, we use the derived property

$$e_{ss} = (1 + \mathbf{C}\mathbf{A}^{-1}\mathbf{B})u = (1 + \mathbf{C}\frac{adj(\mathbf{A})}{det(\mathbf{A})}\mathbf{B})u$$

$$\rightarrow e_{ss} = (1 + \begin{bmatrix} 1.0 & 0.2 \end{bmatrix} \frac{adj(\begin{bmatrix} 0 & -10.0 \\ a & -2.0 \end{bmatrix})}{10.0a} \begin{bmatrix} 0 \\ 1.0 \end{bmatrix})10 = 10.0a + 6.0$$

The system will have a finite steady-state error, unless the following is true:

$a = -0.6$

3.

To add integral control in state-space, given a full-state feedback form, we are basically adding $u_i = K_I \int e$ to $u = \mathbf{K}\mathbf{x}$

This requires augmenting the state to include $\int e$, such that $\tilde{\mathbf{x}} = \begin{bmatrix} x_1 \\ x_2 \\ \int e \end{bmatrix}$

With the closed-loop integral form

$$\dot{\tilde{\mathbf{x}}} = \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{K} & \mathbf{B}K_I \\ -\mathbf{C} & 0 \end{bmatrix} \tilde{\mathbf{x}} + \begin{bmatrix} \mathbf{B} \\ 1 \end{bmatrix} r$$

$$y = \begin{bmatrix} \mathbf{C} & 0 \end{bmatrix} \tilde{x}$$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & -10.0 & 0 \\ a & -2.0 & K_I \\ -1.0 & -0.2 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1.0 \\ 1.0 \end{bmatrix} r$$

$$y = \begin{bmatrix} 1.0 & 0.2 & 0 \end{bmatrix} \mathbf{x}$$

To find the steady-state error to a step input $u = 10$, given a state-space representation, we use the derived property

$$e_{ss} = (1 + \mathbf{C}\mathbf{A}^{-1}\mathbf{B})u = (1 + \mathbf{C}\frac{adj(\mathbf{A})}{det(\mathbf{A})}\mathbf{B})u$$

$$\rightarrow e_{ss} = (1 + \begin{bmatrix} 1.0 & 0.2 \end{bmatrix} \frac{adj(\begin{bmatrix} 0 & -10.0 \\ a & -2.0 \end{bmatrix})}{10.0a} \begin{bmatrix} 0 \\ 1.0 \end{bmatrix})10 = 10.0a + 6.0$$