ME 417 - Homework #1

Control of Mechanical Systems - Spring 2021

Homework Due: Thu, 22 Apr 2021 23:59

Complete the following problems and submit a hard copy of your solutions. You are encouraged to work together to discuss the problems but submitted work **MUST** be your own. This is an **individually** submitted assignment.

Problem 1

System Modeling (25pts)

An inverted pendulum on a rotating disk is shown. Where θ is the pendulum angle measured from the vertical and ψ is the disk angle.

The equations of motion for the system are given as:

$$l^2 m\ddot{\theta} + lmr\cos(\theta)\ddot{\psi} = b_1\dot{\theta} + glm\sin(\theta)$$
$$lmr\cos(\theta)\ddot{\theta} + \left(J + mr^2\right)\ddot{\psi} = b_2\dot{\psi} + lmr\sin(\theta)\dot{\theta}^2 + \tau$$

a. Linearize the equations of motion (small angle approximation)

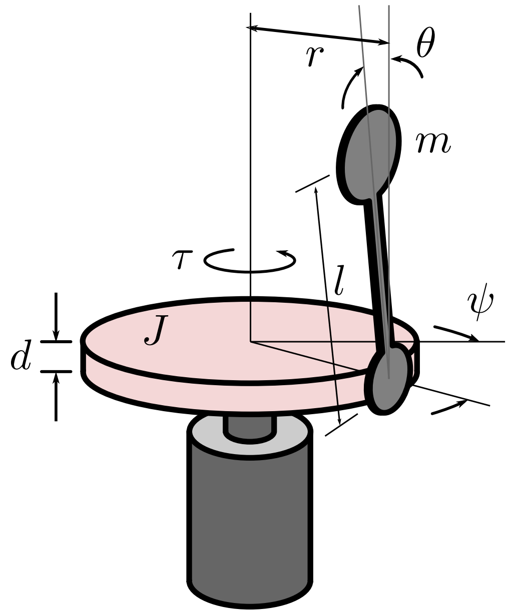
b. Find the transfer function that relates τ to θ and τ to $\dot{\theta}$

c. Draw the block diagram if feedback control is applied to control the pendulum angle θ

c. Find the pole locations of the transfer function derived in part (b)

Given: $r = 12.0cm$, $m_r = 0.3kg$, $J = 15kg \cdot m^2$, $l = 20.0cm$

Neglect friction in the system.



Problem 2

System Modeling (25pts)

Given the following system

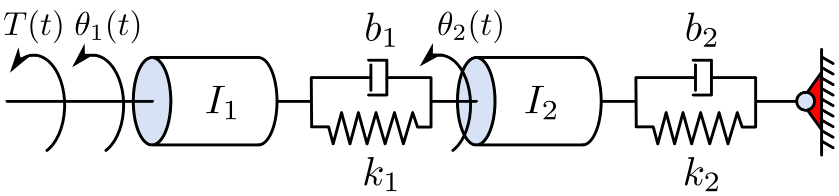
a. Derive the equations of motion for the system

b. Find the transfer function that relates T to θ_2

c. Find the steady state value of θ_2 given a step-input $T(t) = 20$

d. Draw a feedback block diagram if you wanted to control θ_2, show how the output signal θ_1 will be represented.

Given: $I_1 = 0.3kg \cdot m^2$, $I_2 = 0.25kg \cdot m^2$, $k_1 = 280N/m$, $k_2 = 380N/m$, $b_1 = 45N \cdot s/m$, $b_2 = 35N \cdot s/m$



Problem 3

Time Response (25pts)

Given the following transfer function relating force to position

$$\frac{X}{F} = \frac{40}{s\,(s+5)}$$

Derive the partial fraction expansion form for the output, sketch (by hand) the time response for position and velocity on the same figure, and find the steady-state output value for position for each of the following inputs.

a. $u_a(t) = 2$

b. $u_b(t) = 6t + 3$

c. $u_c(t) = 0.2e^{-2t}$

d. $u_d(t) = 2te^{-4t}$

Problem 4

Transfer Function Components (25pts)

For each of the following 3rd order systems, perform a partial fraction expansion, then cancel the third pole term if it is real magnitude is five times or higher than the real magnitude of the other two poles

a. $G(s) = \dfrac{10}{(s+10)\,(s^2+2s+20)}$

b. $G(s) = \dfrac{4}{(s+4)\,(s+5)\,(s+30)^2}$

c. $G(s) = \dfrac{10}{(s+5)\,(s^2+2s+8)}$

d. $G(s) = \dfrac{1}{(s+18)\,(s^2+6s+100)}$

e. $G(s) = \dfrac{5}{(s+5)\,(s^2+4s+20)}$