ME 417 - Homework #3

Control of Mechanical Systems - Fall 2020

Homework Due: Sun, 14 Feb 2021 23:59

Complete the following problems and submit a hard copy of your solutions. You are encouraged to work together to discuss the problems but submitted work **MUST** be your own. This is an **individually** submitted assignment.

Problem 1

Root Locus Sketching (20pts)

For each of the following transfer functions, sketch a general shape root-locus, and include, as applicable, asymptote intercepts and angles

a. $G(s) = \dfrac{s+10}{s^2+3s+15}$

b. $G(s) = \dfrac{s(s-5)}{(s+4)(2s^2+4s+4)}$

c. $G(s) = \dfrac{s^2+8s+40}{s(s-8)}$

d. $G(s) = \dfrac{(s-10)(s+8)}{s(s^2+2s+60)}$

Problem 2

Root Locus Sketching (20pts)

For the following open-loop transfer functions, sketch a refined root locus, compute any applicable break-away and break-in points as well as imaginary axis crossing. Highlight the range of K for which the system is stable.

a. $G(s) = \dfrac{2s+6}{s^2+10s+61}$

b. $G(s) = \dfrac{s-8}{s(s+5)(s+8)}$

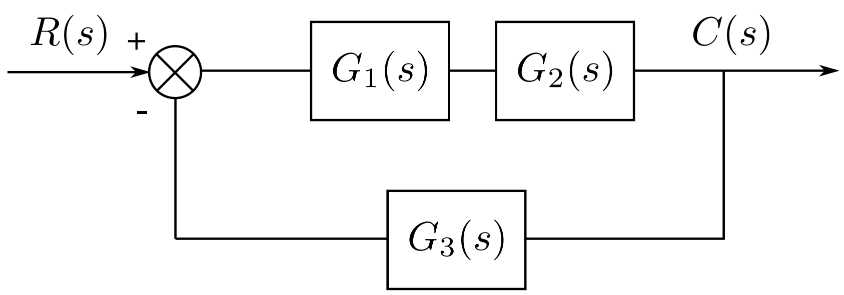
c. $G(s) = \dfrac{(s+20)(s+40)}{(s-20)(s-10)}$

d. $G(s) = \dfrac{(s-15)(s+10)}{s^2-8s+45}$

Problem 3

Root Locus Sketching (20pts)

Given the following feedback system



With $G_1 = s + z, G_2 = \frac{100}{s^2 + 4s + 20}, G_3 = \frac{20}{s + 10}$

a. Derive the characteristic polynomial of the system in the form $1 + zG(s) = 0$

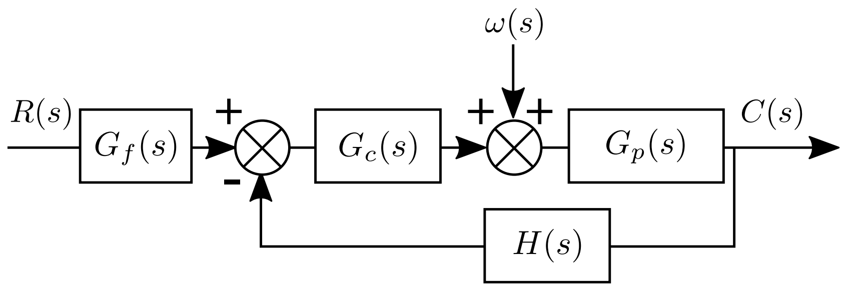
b. Sketch the root-locus of the system for varying values of the zero location z

c. Find the value of z that makes the closed-loop system's damped frequency $\omega_d = 6.283185307179586 rad/s$

Problem 4

Root Locus Sketching (20pts)

A closed-loop system with input disturbance is shown.



With $G_p = \frac{2s + 4}{(s - 3)(s + 6)}, H = \frac{3}{s + 1}, G_f = 5$

a. Design a controller that results in a stable response with

- $T_p = \frac{\pi}{4} s$
- Zero Steady-State error

Is a second-order approximation valid? Justify. Hint: Choose a convenient constraint for ζ or T_s

to simplify your calculations.

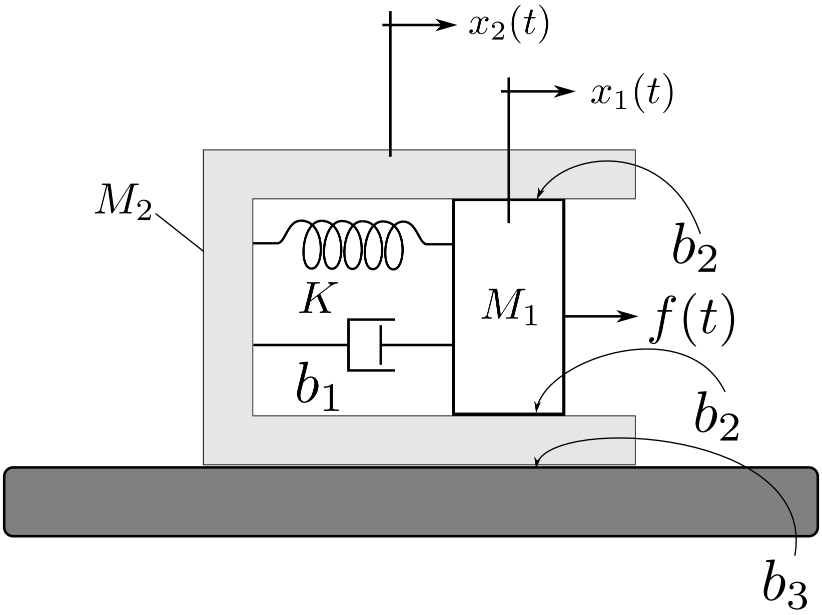
b. Show that a steady-state error of the closed loop system, to a step input, is zero.

c. Given your designed controller, derive the transfer function that relates the input $r(t)$ to the controller output $u(t)$

Problem 5

Root Locus Sketching (20pts)

Given the mechanical system shown on the figure. You can use MATLAB to aid in long calculations and verify your work.



With $M_1 = 2kg, M_2 = 2kg, b_1 = 7N \cdot s/m, b_2 = 4N \cdot s/m, b_3 = 2N \cdot s/m, K = 15N/m$

a. Derive the equations of motion for the system

b. Find the transfer function relating the input $f(t)$ to $x_2(t)$, $G_2(s) = \dfrac{X_2(s)}{f(s)}$

c. Analyze the stability of the system $G_2(s)$

d. Design a feedback controller, using root-locus technique, around G_2 to achieve

- Zero Steady-State error

- $T_s = 0.5s$

- $\zeta = 0.5$

Justify if the system can be approximated as second order.

e. Derive the transfer function relating the reference $r(t)$ to $x_2(t)$

f. Derive the transfer function relating the reference $r(t)$ to $x_1(t)$, with the feedback system derived above.