ME 417 Control of Mechanical Systems

Fall 2019 Homework #1, Due February 20th, 2020

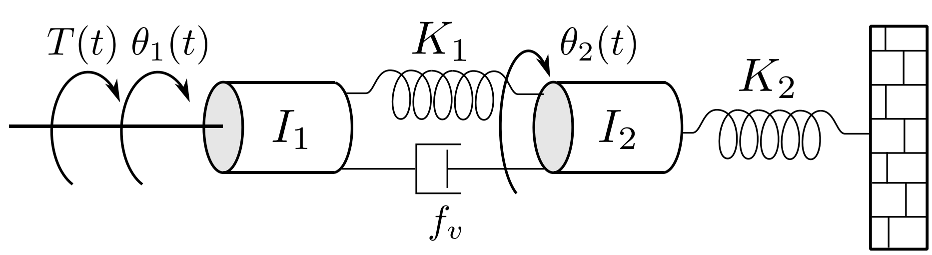
Complete the following problems and submit a hard copy of your solutions. You are encouraged to work together to discuss the problems but submitted work **MUST** be your own. This is an **individually** submitted assignment.

Problem 1 (20 Pts)

Given the mechanical system shown on the figure.

a) Derive the equations of motion for the system
b) Find the transfer function relating the input $f(t)$ to $x_1(t)$, $G_1(s) = \frac{\theta_1(s)}{T(s)}$
c) Analyze the stability of the system. Use the functions *pole()* and *zero()* in MATLAB to find the poles and zeros of the transfer function. See example at the end
d) Find the form of the response $\theta_1(t)$ to a step input $T(t) = 5Nm$, and **sketch** the time response to this input.
e) Find the steady state output for $\theta_1(t)$ to a step input $T(t) = 5Nm$

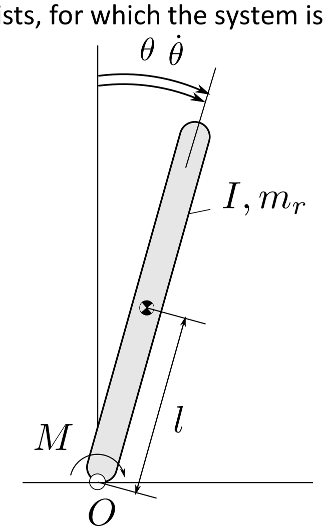
$$I_1 = 100kg \cdot m^2, I_2 = 10kg \cdot m^2, K_1 = 1000\ Nm/rad\ ,\ K_2 = 50\ Nm/rad\ , f_v = 300Nm \cdot s/rad$$



Problem 2 (20 Pts)

Given the following mechanical system. Where $I_o = \frac{4}{3}ml^2$ (l is half the length of the slender rod)

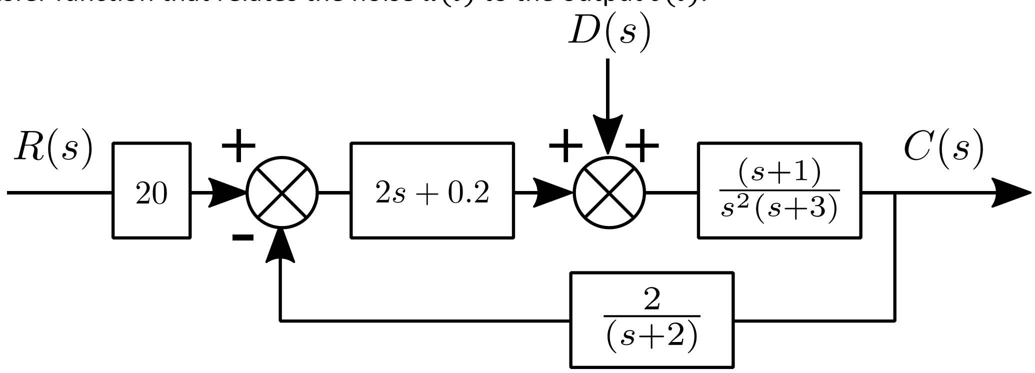
a) Find the transfer function $G(s) = \frac{\Theta(s)}{M(s)}$
b) Analyze the stability of the open-loop system.
c) Put the system into a unity feedback loop, draw the block diagram of the closed loop system. Highlight the reference input, error, system input, and output and the units for each of the terms.
d) Find the range of controller gain K, if it exists, for which the system is stable.



Problem 3 (20 Pts)

Given the following control system.

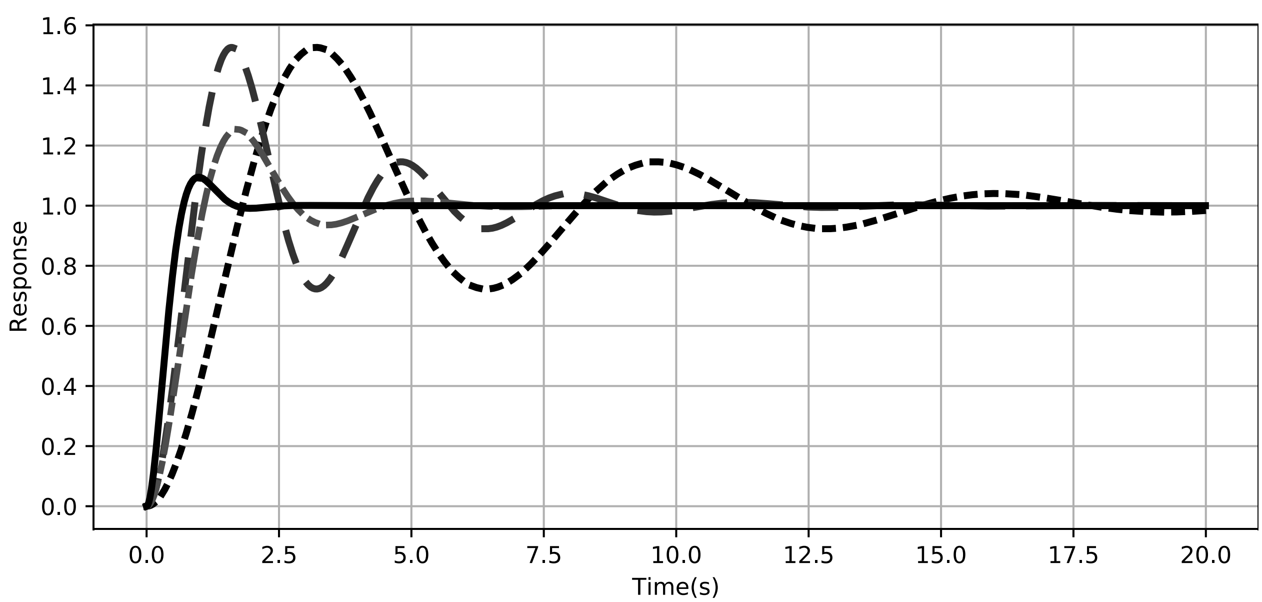
a) Is the open-loop system stable (the plant)?
b) Find the transfer function that relates the reference $r(t)$ to the output $c(t)$.
c) Analyze the stability of the closed-loop system. Use the functions *pole()* and *zero()* in MATLAB to find the poles and zeros of the transfer function. See example at the end
d) Find the steady-state error to
    a. A unit-step input
    b. A unit-ramp input
e) Find the transfer function that relates the noise $d(t)$ to the output $c(t)$.

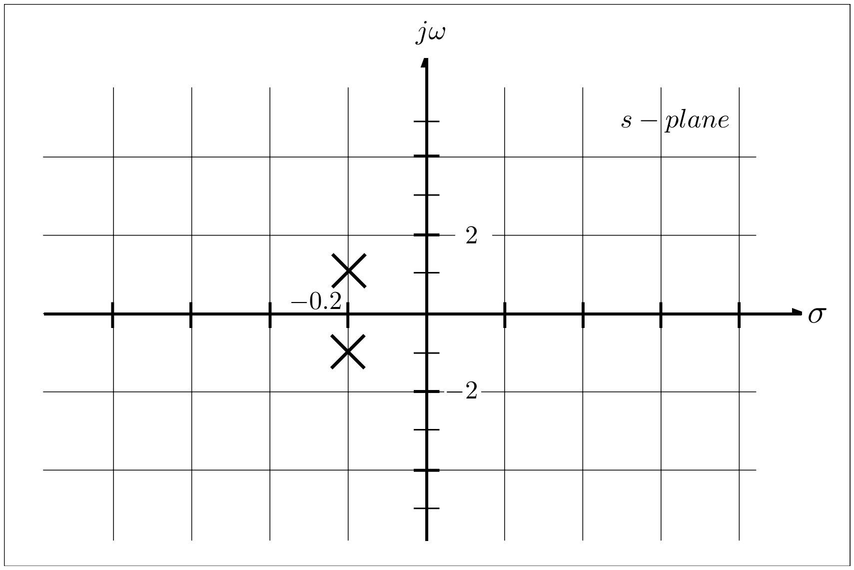


Problem 4 (10 Pts)

Given the following four responses for a general second-order systems to a step input.

a) Identify the damped frequency of the system to which the poles are already placed.
b) Place, qualitatively, the remaining poles of the systems on the s-plane, highlight which poles belong to which response.
c) Derive (approximate) the transfer function for each of the four systems.





Problem 5 (10 Pts)

For each of the following systems, determine its stability classification, justify your answer

a. $G(s) = \frac{s-5}{s^2+3s+9}$
b. $G(s) = \frac{s+1}{s^2-2s+1}$
c. $G(s) = \frac{s^2+4s+4}{(s+20)(s+10)(s^2+2s+8)}$
d. $G(s) = \frac{1}{(s^4+25)}$

Problem 6 (10 Pts)

For the following response functions, determine if pole-zero cancellation can be approximated. If it can, find percent overshoot, settling time, rise time, and peak time.

a. $C(s) = \frac{(s+3)}{s(s+2)(s^2+3s+10)}$

b. $C(s) = \frac{(s+2.5)}{s(s+2)(s^2+4s+20)}$

c. $C(s) = \frac{(s+2.1)}{s(s+2)(s^2+4s+20)}$

d. $C(s) = \frac{(s+2.01)}{s(s+2)(s^2+5s+20)}$

Problem 7 (10 Pts)

For each of the following systems, justify whether a second-order approximation is valid or not for a step response. State your assumptions.

a. $G(s) = \frac{1}{(s+4)(s^2+s+10)}$

b. $G(s) = \frac{100}{(s+4)(s^2+s+10)}$

c. $G(s) = \frac{300}{(s+10)^2(s^2+s+4)}$

d. $G(s) = \frac{300}{(s^2+16s+84)(s^2+2s+10)}$

EXAMPLE: Using MATLAB to find poles and zeros of a transfer function.

Given the following transfer function.

$$G(s) = \frac{s^2 + 4s + 4}{s^5 + 5s^2 + 20}$$

Here is an example code on how to find the poles and zeros of the transfer function.

```
s = tf('s')
G = (s^2 + 4 * s + 4) / (s^5 + 5 * s^2 + 20);
pole(G)
zero(G)
```

```
OUTPUT:
ans =
  -2.1156 + 0.0000i
   1.3837 + 1.3641i
   1.3837 - 1.3641i
  -0.3259 + 1.5485i
  -0.3259 - 1.5485i

ans =
    -2
    -2
```