ME 417 - Num Assignment #4

Control of Mechanical Systems - Summer 2020

Num Assignment Due: Tue, 17 Nov 2020 23:59

Complete the following problems and submit your work as a single MATLAB Livescript + a saved pdf version. Collaboration is only allowed within the group members. Provide response plots as relevant, ensure that you label the figures, the axes, title plots and legends. The design specifications should be met by observing the time response of the system.

Problem 1

Controlling a non-linear system using linear tools (50pts)

Given a 2-DOF system with the following nonlinear equations of motion

$$2x_1^2 + 3x_1 + 23\dot{x}_1\dot{x}_2 - 15\dot{x}_1 + 34\ddot{x}_1 = 2f_1 + 3f_2$$
$$2x_1 + 0.5x_2^2 + 18\dot{x}_2 + 14\ddot{x}_2 = f_1 + 5f_2$$

a. Put the nonlinear equations in a vector form $\dot{x} = f(x, t, u)$.

b. Simulate the natural response of the nonlinear system with the following initial conditions $x_0 = \begin{bmatrix} 0.0 & 0.1 & 0.1 & 0.0 \end{bmatrix}^T$ for $t = 0 : 15s$

c. Linearize the system (ignore the square and coupling terms) and put the system into the state space form $\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}u$

d. Design a full-state feedback controller using pole placement method, to stabilize the system. Test your controller on the linearized system first using lsim, then simulate the closed-loop system response using the linear and nonlinear model in a numerical integration setup. Assume you can control both inputs (What is the dimension of the gain matrix?)

Your controller is a regulator, so observe the closed-loop response with $r = 0$ and initial conditions $x_0 = \begin{bmatrix} 10.0 & 0.5 & 0.5 & 10.0 \end{bmatrix}^T$

Plot the response of all the 4 states, for both the linear and non-linear system in one subplot.

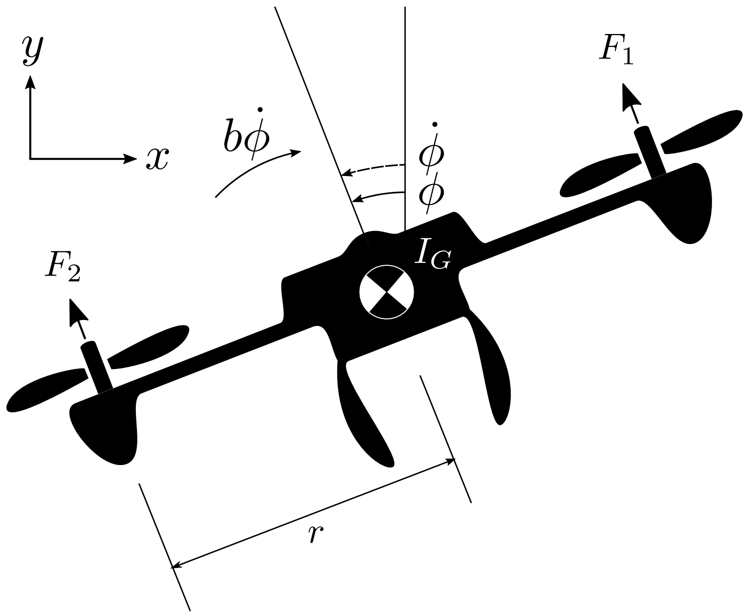
Can you control the non-linear (actual) system with the same controller? Explain why or why not.

Problem 2

2-D Control of Multirotor - State-Space (50pts)

Given the simplified control system of a quad-rotor. The thrust force is a function of the angular velocity of the propellers $F = k_T\omega^2$

With $k_T = 0.1, I_G = 0.025kg \cdot m^2, b_x = 0.5kg \cdot s \cdot m^2, b_y = 0.8kg \cdot s \cdot m^2, b_\phi = 0.5N \cdot s \cdot m, r = 15.0cm, m = 1kg, g = 9.81m/s^2$,



We wish to design a full-state feedback controller in state-space.

a. Derive the equations of motion for the system in the x, y, ϕ directions, then put the system in nonlinear vector form $\dot{x} = f(x, t, u)$, include gravity.

b. Design a full-state feedback controller to control the position of the quad-rotor (or better called bi-rotor?), you will design the controller using the linear state-space model, but simulate it using both, the linear and non-linear model.

The model of the multi-rotor is non-linear and the states are coupled. We can linearize using small angle approximation, but even then we will have the term $\omega^2\phi$ which is a coupling between the input and the roll angle. One assumption we can make is to say that the multi-rotor is operating near hover thrust, meaning the total thrust is $\approx$ gravity force W, such that the contribution of the thrust in the x-direction is proportional to $W\phi$, where $W = mg$. The resulting linearized state-space model will be:

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{-b_x}{m} & 0 & 0 & \frac{-W}{m} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{-by}{m} & \frac{-W}{m} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{-b_\phi}{IG} \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{K_T}{m} & \frac{K_T}{m} \\ 0 & 0 \\ \frac{r \cdot K_T}{I_G} & \frac{(-r) \cdot K_T}{I_G} \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \end{bmatrix}$$

Simulate the response of the system with the controller, using basic numerical integration, with the following reference input.

$r = \begin{bmatrix} 2 \cdot m & \frac{0 \cdot m}{s} & 2 \cdot m & \frac{0 \cdot m}{s} & 0 \cdot rad & 0 \cdot rad \end{bmatrix}^T$, we are asking the drone to move to a specific position. To implement this, keep $r = 0$ in the simulation and change the initial state condition to be the negative of the desired r

Plot the x, y positions in one subplot.

Plot the ϕ response in one subplot.

Plot the $\dot{x}, \dot{y}$ velocities in one subplot.

Plot the $\dot{\phi}$ response in one subplot.

Hint: Don't be aggressive with your gains, keep the dominant poles at a reasonable range.