ME 417 - Num Assignment #3

Control of Mechanical Systems - Summer 2020

Num Assignment Due: Thu, 05 Nov 2020 23:59

Complete the following problems and submit your work as a single MATLAB Livescript + a saved pdf version. Collaboration is only allowed within the group members. Provide response plots as relevant, ensure that you label the figures, the axes, title plots and legends. The design specifications should be met by observing the time response of the system.

Problem 1

Controller Design over a Transfer Function (25pts)

We wish to design a controller for the following system.

- $G(s) = \dfrac{s+1}{s^2 + 2s + 2}$

a. Using Control System Designer, design a continuos domain PID controller to achieve the following performance specifications:

- $\%OS < 1.0\%$
- $T_s < 0.25s$
- Zero Steady-State Error

Provide a screenshot of the Control System Designer setup.

b. Test your controller by simulating the closed loop response using step()

c. Discretize your controller and simulate the closed-loop system using basic numerical integration. Integrate using $\Delta t = 0.01s$

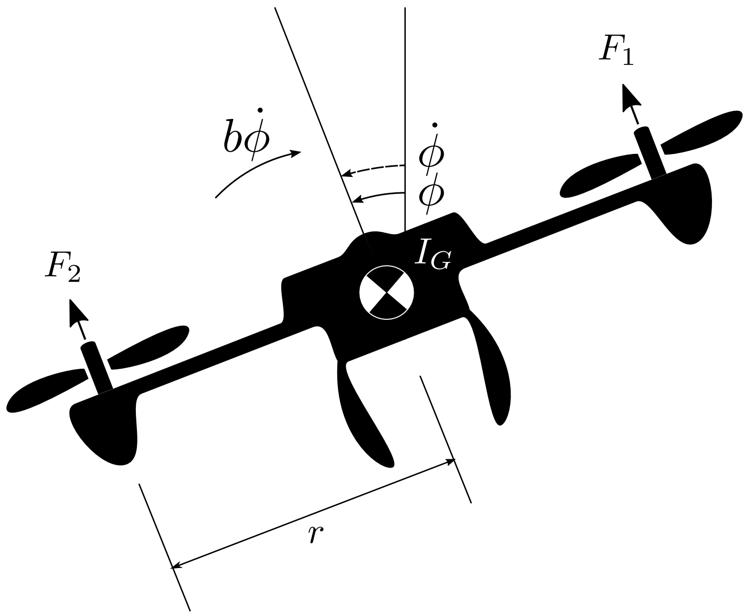
Note that you can retrieve the differential equation from the transfer function and can define a model (@xdot) from the differential equation.

Problem 2

Cascaded Control with Sensor (25pts)

Given the simplified roll attitude control system of a quadrotor. The thurst force is a function of the angular velocity of the propellers $F = k_T\omega^2$

With $k_T = 0.01, I_G = 0.14kg \cdot m^2, b = 0.11kg \cdot s \cdot m^2, r = 15.0cm$



We wish to design a cascaded controller. First we want to design the inner loop, which is a roll rate control system (angular velocity), then design the outer loop, which is to control the roll angle.

The roll rate (roll angular velocity) sensor is modeled as a first order system, with a DC gain of 1 and time constant $\tau = 1.0ms$

a. Derive the transfer function that relates $\Delta\omega^2$, the difference between the square of the propeller velocities, and the roll rate $\dot{\phi}$

b. Design a PI controller using Control System Designer, to achieve the following specifications to a step input:

- $T_p = 0.1s$
- $T_s = 0.2s$
- Max PI Gain $K_{max} = 5000$

- Zero Steady-State Error

Remember to include the sensor in the Control System Designer.

Simulate the closed-loop system response to the reference input

$$r_{\dot{\phi}} = \begin{cases} 0 & 0 < t \leq 0.5s \\ 6rad/s & 0.5s < t \leq 1.5s \\ -6rad/s & 1.5s < t \leq 2.5s \\ 0 & 2.5s < t \end{cases}$$

c. With the rate controller closed-loop system, design a roll attitude (angular position) P controller, to achieve the following performance specifications:

- $\%OS = 0$
- $T_s < 0.25s$

Simulate the closed-loop cascaded system to the following reference input

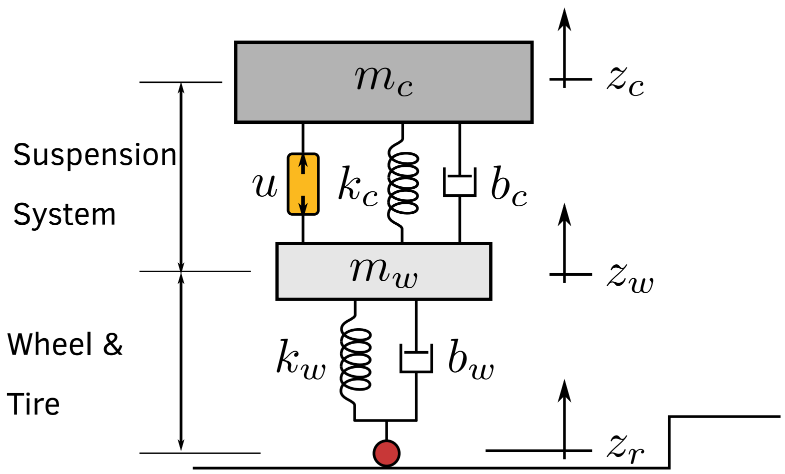
$$r_{\phi} = \begin{cases} 0 & 0 < t \leq 2s \\ 1.5rad & 1s < t \leq 2s \\ 0rad & 2s < t \leq 4s \\ -1.5rad & 4s < t \leq 5s \\ 0 & 5s < t \end{cases}$$

You can use lsim() to simulate the response, no numerical integration required in this problem.

Problem 3

Regulation (50pts)

Given the following car active quarter suspension model. Design a controller to smooth the response of the car level in response to a road bump. You can treat the bump z_r as an input.



The equations of motion of the system are:

$m_c\ddot{z}_c + b_c\dot{z}_c + k_c z_c - b_c\dot{z}_w - k_c z_w = u$

$m_w\ddot{z}_w + (b_w + b_c)\dot{z}_w + (k_w + k_c)z_w - b_c\dot{z}_c - k_c z_c = -u + b_w\dot{z}_r + k_w z_r$

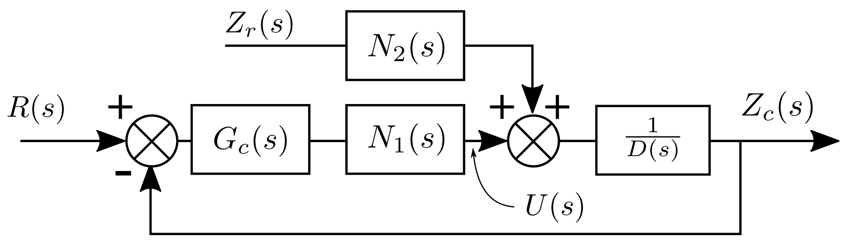
With $m_c = 1.2 MetricTons, m_w = 10kg, b_c = 100N \cdot s/m, b_w = 200N \cdot s/m, k_c = 150N/m, k_w = 800N/m$

Since you are treating the z_r as an input, you have two inputs to the system.

a. Symbolically solve for $\mathbf{x}$ in $\mathbf{Ax} = \mathbf{B}$ to find the four transfer functions that relate the inputs to outputs: $\frac{Z_c}{U}, \frac{Z_c}{Z_r}, \frac{Z_w}{U}, \frac{Z_w}{Z_r}$

Looking at the two transfer functions for Z_c, you will notice both have the same internal dynamics (poles), but different output relationship (zeros).

You can factor out the numerator for each of the transfer functions as $N_1(s)$ and $N_2(s)$ as shown in the feedback block diagram.



b. Using Control System Design, and remember to use the open-loop transfer function, design a controller $G_c(s)$ to achieve the following response characteristics to a step input.

- $\%OS < 15\%$
- $T_s < 0.25s$
- $K_{max} = 70000$

c. Simulate the response of the closed loop system to a step input $z_r(t) = 0.05m$, the closed-loop system here is $\frac{Z_c(s)}{Z_r(s)}$, in the context of the feedback loop designed.

d. Simulate the system response to the step input in the previous step, using basic numerical integration with the PID controller discretized. Simulate the system with $\Delta t = 0.01s$

Note that the input $z_r(t)$ needs to be differentiated, you can ignore the $\dot{z}_r(t)$ term in the numerical simulation.

- Plot the position $z_c(t)$, and acceleration $\ddot{z}_c(t)$ as well as the input $u(t)$.

e. Bonus (15pts), simulate the response to a bumpy road using the basic numerical integration setup, you can be creative in your definition of the $z_r(t)$ function, but consider how a road bump will be shaped at. You can assume a constant car velocity for simplicity. You can not however, ignore the differential term $\dot{z}_r(t)$, but you can predefine the function $\dot{z}_r(t)$ from your definition of $z_r(t)$. Hint: Use trigonometry.