ME 417 - Homework #3

Control of Mechanical Systems - Summer 2020

Homework Due: Sun, 08 Nov 2020 23:59

Complete the following problems and submit a hard copy of your solutions. You are encouraged to work together to discuss the problems but submitted work **MUST** be your own. This is an **individually** submitted assignment.

Problem 1

Root Locus Sketching (20pts)

For each of the following transfer functions, sketch a general shape root-locus, and include, as applicable, asymptote intercepts and angles

a. $G(s) = \dfrac{s+5}{s^2+2s+6}$

b. $G(s) = \dfrac{(s-2)(s+10)}{s(s^2+4s+16)}$

c. $G(s) = \dfrac{s^2+4s+25}{s(s-8)(s-5)}$

d. $G(s) = \dfrac{(s-20)(s+4)}{s^2(s^2+4s+60)}$

Problem 2

Root Locus Sketching (20pts)

For the following open-loop transfer functions, sketch a refined root locus, compute any applicable break-away and break-in points as well as imaginary axis crossing. Highlight the range of K for which the system is stable.

a. $G(s) = \dfrac{s+5}{s^2+10s+61}$

b. $G(s) = \dfrac{(s-6)(s-1)}{(s+3)(s+8)}$

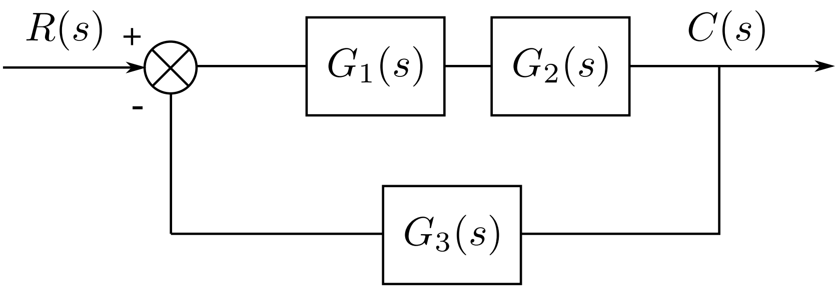
c. $G(s) = \dfrac{(s+10)(s+20)}{(s-8)(s-5)}$

d. $G(s) = \dfrac{(s-19)(s+20)}{s^2-4s+25}$

Problem 3

Root Locus Sketching (20pts)

Given the following feedback system



With $G_1 = s + z, G_2 = \dfrac{10}{s^2 + 4s + 10}, G_3 = \dfrac{10}{s + 100}$

a. Derive the characteristic polynomial of the system in the form $1 + zG(s) = 0$

b. Sketch the root-locus of the system for varying values of the zero location z

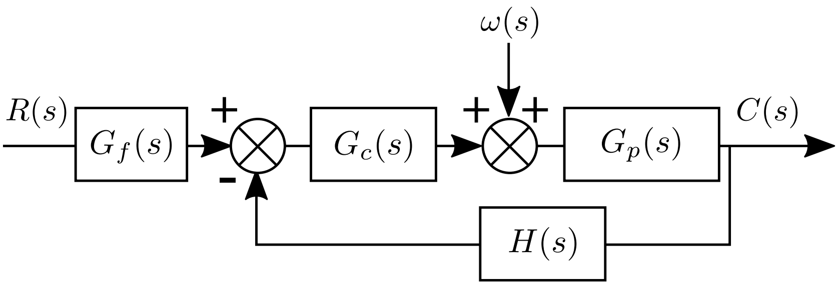
c. Find the value of z that makes the closed-loop system's damped frequency $\omega_d = \pi rad/s$

Note that this is almost a PD controller design problem, but rather than try to find the zero location from the proportional gain root-locus, the characteristic polynomial is rearranged in a way to make the zero act like the system gain.

Problem 4

Root Locus Sketching (20pts)

A closed-loop system with input disturbance is shown.



With $G_p = \dfrac{2s + 2}{(s-2)(s+5)}, H = \dfrac{3}{s+1}, G_f = 5$

a. Design a controller that results in a stable response with

- $T_p = \dfrac{\pi}{2} s$
- Zero Steady-State error

Is a second-order approximation valid? Justify. Hint: Choose a convenient constraint for ζ or T_s to simplify your calculations.

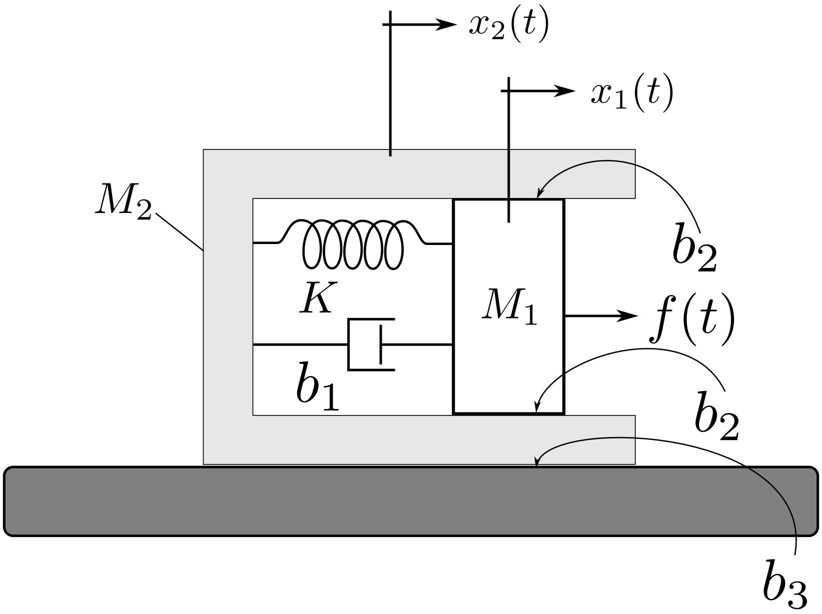
b. Show that a steady-state error of the closed loop system, to a step input, is zero.

c. Given your designed controller, derive the transfer function that relates the input $r(t)$ to the controller output $u(t)$

Problem 5

Root Locus Sketching (20pts)

Given the mechanical system shown on the figure. You can use MATLAB to aid in long calculations and verify your work.



With $M_1 = 5kg, M_2 = 3kg, b_1 = 5N \cdot s/m, b_2 = 5N \cdot s/m, b_3 = 5N \cdot s/m, K = 10N/m$

a. Derive the equations of motion for the system

b. Find the transfer function relating the input $f(t)$ to $x_2(t)$, $G_2(s) = \dfrac{X_2(s)}{f(s)}$

c. Analyze the stability of the system $G_2(s)$

d. Design a feedback controller, using root-locus technique, around G_2 to achieve

- Zero Steady-State error

- $T_s = 1s$
- $\zeta = 0.866$

Justify if the system can be approximated as second order.

e. Derive the transfer function relating the reference $r(t)$ to $x_2(t)$

f. Derive the transfer function relating the reference $r(t)$ to $x_1(t)$, with the feedback system derived above.