ME 417 - Homework #2

Control of Mechanical Systems - Summer 2020

Homework Due: Sun, 25 Oct 2020 23:59

Complete the following problems and submit a hard copy of your solutions. You are encouraged to work together to discuss the problems but submitted work **MUST** be your own. This is an **individually** submitted assignment.

Problem 1

Stability Analysis (20pts)

For each of the following systems, find the poles of the system and determine the system's stability classification. Justify your answer.

a. $G(s) = \dfrac{s+19}{s^2+5s+9}$

b. $G(s) = \dfrac{s-5}{s^2+3s+100}$

c. $G(s) = \dfrac{s^2+20}{(s+3)\,(s^2+2s+20)}$

d. $G(s) = \dfrac{s\,(s-19)}{s^2-5s+20}$

Solution:

---

a.

The poles of the system are $\begin{bmatrix} -2.5-1.625i \\ -2.5+1.625i \end{bmatrix}$

The system is stable.

There are no poles in the RHP plane nor on the imaginary axis

---

b.

The poles of the system are $\begin{bmatrix} -1.5-9.875i \\ -1.5+9.875i \end{bmatrix}$

The system is stable.

There are no poles in the RHP plane nor on the imaginary axis

---

c.

The poles of the system are $\begin{bmatrix} -3.0 \\ -1.0 - 4.375i \\ -1.0 + 4.375i \end{bmatrix}$

The system is stable.

There are no poles in the RHP plane nor on the imaginary axis

---

d.

The poles of the system are $\begin{bmatrix} 2.5 - 3.75i \\ 2.5 + 3.75i \end{bmatrix}$

The system is unstable.

There are poles in the RHP plane @ $\begin{bmatrix} 2.5 - 3.75i \\ 2.5 + 3.75i \end{bmatrix}$

Problem 2

Second Order Approximation (20pts)

For each of the following systems, determine if a 2nd-order approximation is valid. Justify your answer.

a. $G(s) = \dfrac{s+2}{(s+3)\left(s^2+2s+10\right)}$

b. $G(s) = \dfrac{s+1.99}{(s+1.9)\left(s^2+3s+10\right)}$

c. $G(s) = \dfrac{s+45}{(s+50)\left(s^2+5s+40\right)}$

d. $G(s) = \dfrac{s+1}{(s+3)\left(s^2+10s+200\right)}$

Hint: 5 times rule of thumb for higher order poles, or if zeros are present, compare the magnitude of the higher order term.

Solution:

---

a.

First, finding the partial fraction expansion form. $G(s) = \dfrac{0.077\,(s+12.0)}{s^2+2.0s+10.0} - \dfrac{0.077}{s+3.0}$

The poles are close and the resiudes have similar magnitudes, 2nd order approximation can not me made.

---

b.

First, finding the partial fraction expansion form. $G(s) = -\dfrac{0.099\,(0.012s-1.0)}{0.1s^2+0.3s+1.0} + \dfrac{0.006}{0.53s+1.0}$

While the zero and pole at 1.99 and 1.9 are close, the other two complex poles are also close to them, it is not clear whether 2nd order approximation can be made.

---

c.

First, finding the partial fraction expansion form. $G(s) = \dfrac{0.0022\left(s+4.1\cdot 10^2\right)}{s^2+5.0s+40.0} - \dfrac{0.0022}{s+50.0}$

Both the zero and third pole are more than 5 times further into the LHP, and they are relatively close to each other and can be cancelled. A second order approximation can be made.

d.

First, finding the partial fraction expansion form. $G(s) = \frac{0.0056\left(2.0s + 1.9 \cdot 10^2\right)}{s^2 + 10.0s + 2.0 \cdot 10^2} - \frac{0.011}{s + 3.0}$

A second order approximation can not me made, as the third pole and zero are close to other poles, in fact the pole at -3 is the dominant pole.

Problem 3

Stability and Feedback Form (20pts)

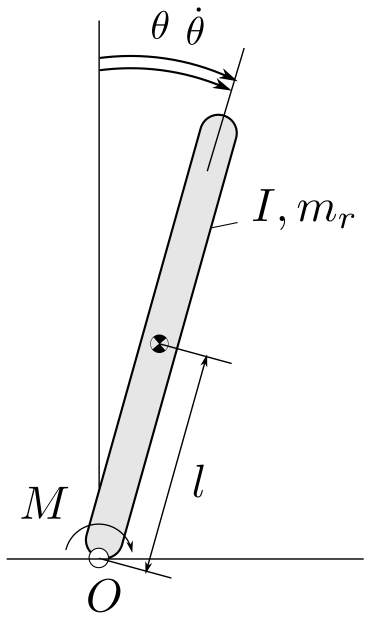
Given the following system representing a simple inverted pendulum with $m_r = 1.3kg, l = 0.3cm$. We wish to design a feedback controller to keep it balanced

a. Derive the equation of motion and the transfer function relating M to Θ

b. Analyze the stability of the plant

c. Draw a feedback diagram, highlighting the reference, error, plant input, plant output and the units of each signal. Include the sensor in the feedback block diagram. Assume the sensor is a first-order system with $\tau = 100\mu s$ with unity dc gain ($H(s \to 0) = 1$). What is the value of r, the reference?

d. If the controller chosen is $G_c = 30s + 60$, analyze the stability of the closed-loop system.



Solution:

a.

By summing the moment about O, we can derive the equation of motion

$\sum M_o = M + m_r g sin\theta \cdot l = I_o\ddot{\theta}, I_o = \frac{4}{3}m_r l^2$

$\frac{4}{3}m_r l^2\ddot{\theta} - m_r g sin\theta \cdot l = M$, linearizing with small angle approximation assumption: $sin\theta \approx \theta$, we get

$0.156\ddot{\theta} - 12.8\theta = M$

The transfer function is then $G(s) = \dfrac{1}{1.33l^2 m_r s^2 - 9.81 l m_r} = \dfrac{1}{0.15561 s^2 - 3.8259}$

---

b.

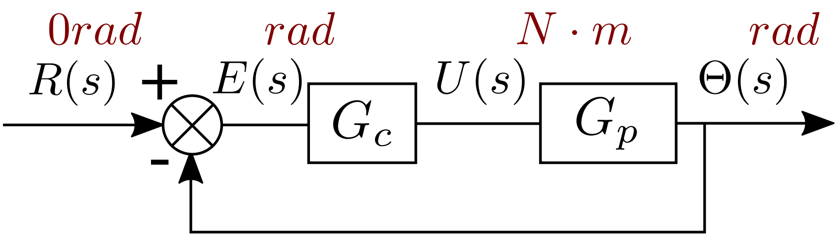
The poles of the system are $\begin{bmatrix} -5.0 \\ 5.0 \end{bmatrix}$

The system is unstable.

There are poles in the RHP plane @ $\begin{bmatrix} 5.0 \end{bmatrix}$

---

c.



---

d.

With the controller $G_c = 30s + 60$, the closed-loop system becomes $G_{cl} = \dfrac{30.0\,(s + 2.0)}{0.16s^2 + 30.0s + 56.0}$

The poles of the system are $\begin{bmatrix} -191.0 \\ -1.875 \end{bmatrix}$

The system is stable.

There are no poles in the RHP plane nor on the imaginary axis

Problem 4

Block Diagram Manipulation (20pts)

Given the following block diagram, with

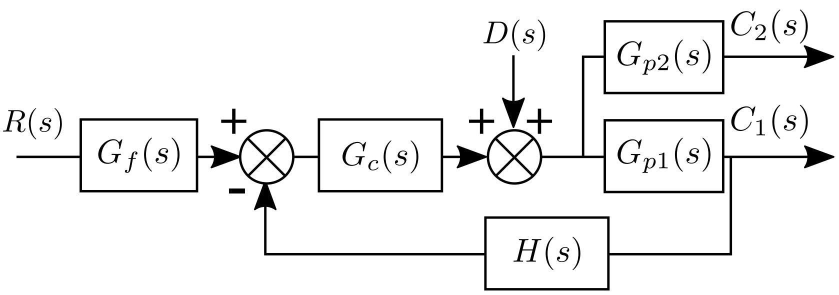
$G_f = 12,\ G_c = 3.0s + 16.5,\ G_{p1} = \frac{6.0}{1.0s^2 + 8.0s + 10.0},\ G_{p2} = \frac{1.0s + 1.0}{1.0s^2 + 8.0s + 20.0},\ H = \frac{1.0}{s}$

a. Derive the transfer function that relates the reference $R(s)$ to the output $C_1(s)$

b. Derive the transfer function that relates the reference $R(s)$ to the input to the plant $U(s)$

c. Derive the transfer function that relates the reference $R(s)$ to the output $C_2(s)$

d. Derive the transfer function that relates the disturbance (noise) $D(s)$ to the output $C_1(s)$



Solution:

---

a.

This is the closed loop transfer function of the system

$G_{cl} = \frac{C_1}{R} = G_f \frac{G_c G_{p1}}{1 + G_c G_{p1} H} = \frac{216.0s^2 + 1188.0s}{1.0s^3 + 8.0s^2 + 28.0s + 99.0}$

---

b.

With $G_{cl}\frac{C_1}{R}$, we observe that $\frac{U}{R} = G_{cl}\frac{1}{G_{p1}} = G_f \frac{G_c}{1 + G_c G_{p1} H} = \frac{36.0s^4 + 486.0s^3 + 1944.0s^2 + 1980.0s}{1.0s^3 + 8.0s^2 + 28.0s + 99.0}$

---

c.

Note that this is note the closed-loop system, this transfer function relates R to the output C_2, through U, the input to G_{p1} in the feedback system.

Also note that $\frac{U}{R} = G_{cl}\frac{1}{G_{p1}} = G_f \frac{G_c}{1 + G_c G_{p1} H}$, and since

$\frac{C_2}{U} = G_{p2}$, we get $\frac{C_2}{R} = G_f \frac{G_c G_{p2}}{1 + G_c G_{p1} H} = \frac{36.0s^5 + 522.0s^4 + 2430.0s^3 + 3924.0s^2 + 1980.0s}{1.0s^5 + 16.0s^4 + 112.0s^3 + 483.0s^2 + 1352.0s + 1980.0}$

d.

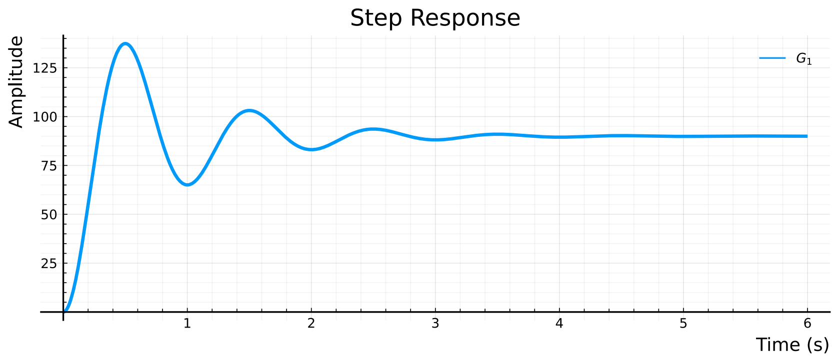
The disturbance signal forward path is only through G_{p1}. The feedback path is the same as before

$$\frac{C_1}{D} = \frac{G_{p1}}{1 + G_c G_{p1} H} = \frac{6.0s}{1.0s^3 + 8.0s^2 + 28.0s + 99.0}$$

Problem 5

Derive System from Response (20pts)

The following is the response of a second-order system to a step input $u = 9$



Derive, approximately, the transfer function of the system

Solution:

Peak time can be approximated from the response and this gives ω_d. $T_p = \dfrac{\pi}{\omega_d} = 0.5 \rightarrow \omega_d = 6.28$

The percent overshoot can give us the damping coefficient ζ.

$$\%OS = 0.527 \rightarrow \zeta = \frac{-ln(\%OS)}{\sqrt{\pi^2 + ln(\%OS)^2}} = 0.2$$

We know from F.V.T that $c(\infty) = |u|K\omega_n^2/\omega_n^2$, where K is the system gain s.t. $G(s) = K\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$K = 10$

And so the transfer function of the system is $G(s) = 10\dfrac{6.41^2}{s^2 + 2 \cdot 0.2 \cdot 6.41s + 6.41^2} = \dfrac{410.875}{1.0s^2 + 2.625s + 41.125}$