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College of Engineering and Petroleum

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ME417 CONTROL OF MECHANICAL SYSTEMS

Part II: Controller Design Using Root-Locus

Lecture 4: Improving Steady-State Response

Summer 2020

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- Objectives:
  - Move beyond a proportional gain controller
  - Explore the use of ideal integral compensators to eliminate steady-state error
- Reading:
  - *Nise: 9.1-9.2*
- Practice problems included

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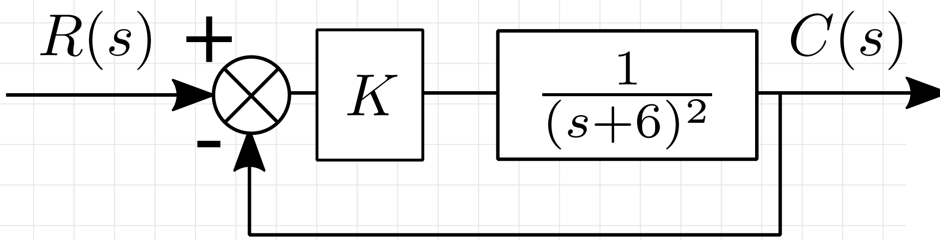
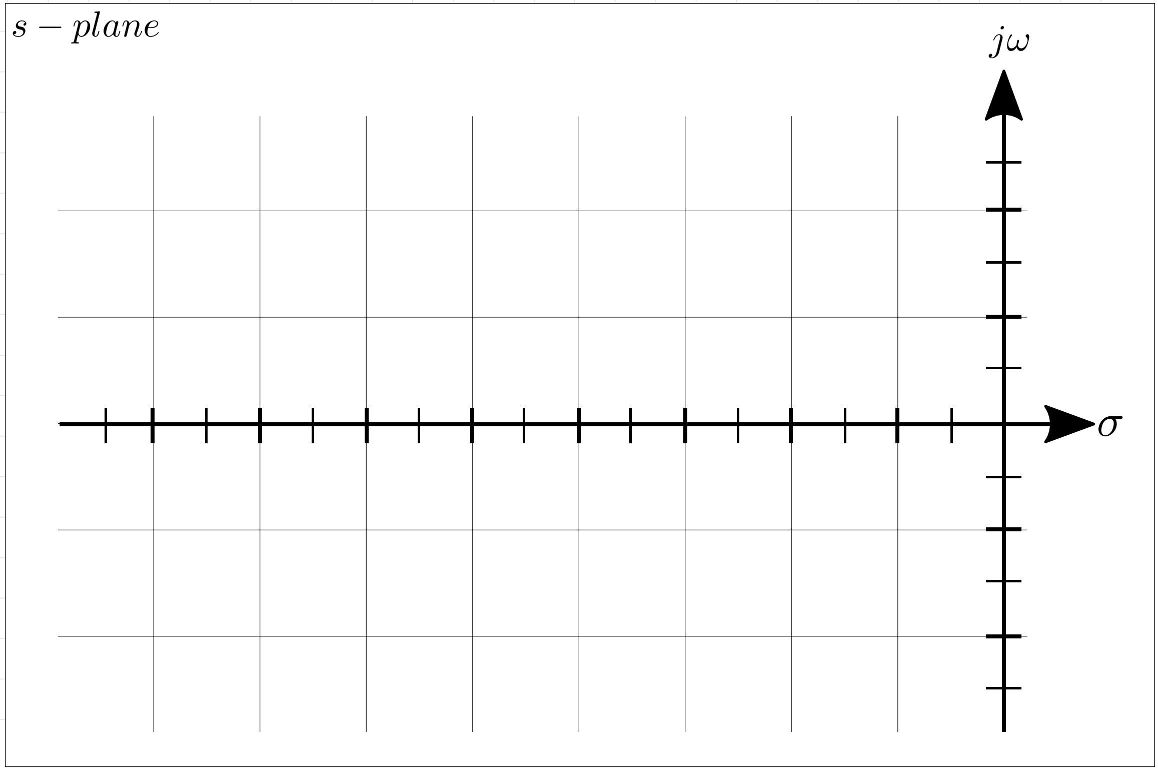
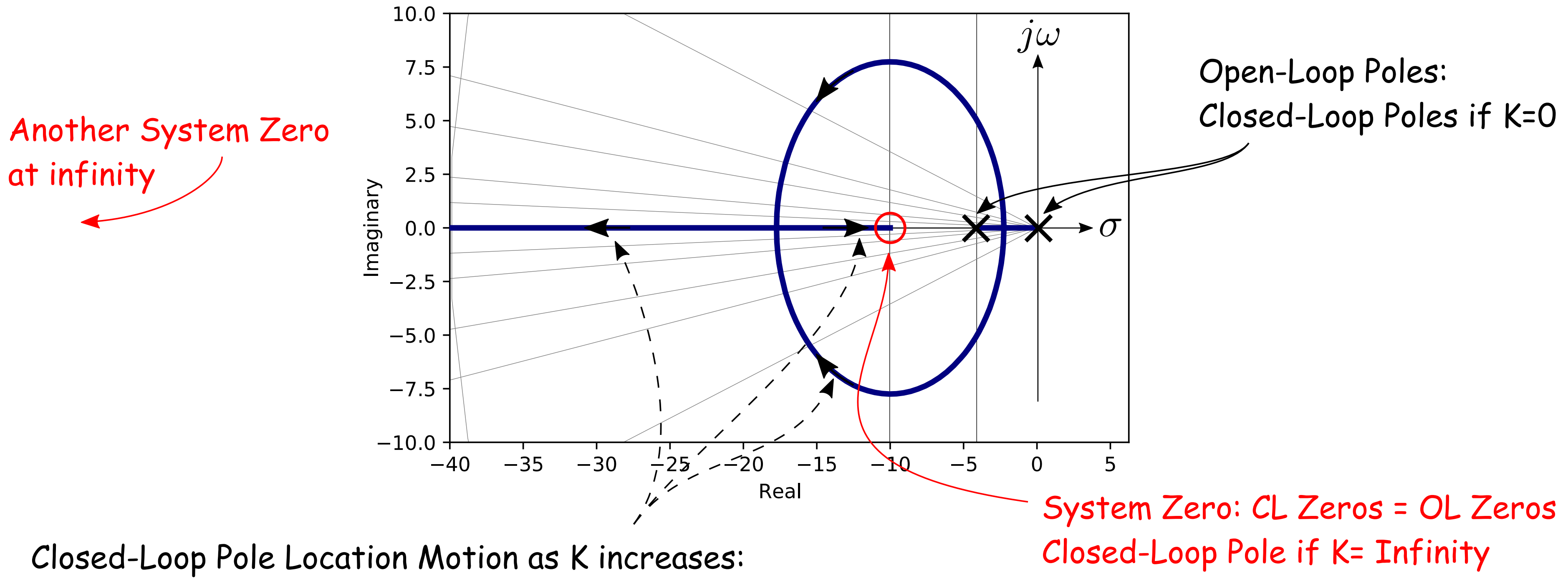
Design a proportional controller to obtain a settling time of $T_s < 2s$, for the feedback system shown. Hint: Is it possible?





- Using the root-locus technique, we can visualize all the possible locations of the closed-loop poles of a system for varying values of the gain.



- But what if the controller produces a satisfactory transient response but falls short on the steady-state response? <- ***The topic of this lecture***
- What if the performance we seek requires the dominant closed-loop poles to be placed outside of the root-locus achieved with just a proportional controller? ***Next Lecture***

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Compensation

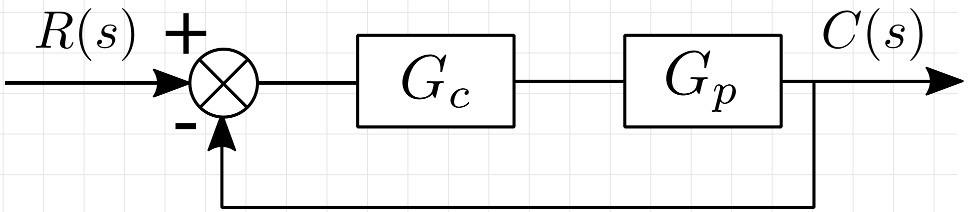
- We can design a controller beyond a simple proportional controller. A controller that places additional poles and/or zeros to the system.
- Such controller can affect the **shape** of the root-locus.
  - *Changing the possible closed-loop pole locations of the feedback system*
- Such controller can help improve the steady-state response
  - *By either reducing the steady-state error or eliminating the steady state error by increasing system type.*
- Such controller can help improve the transient response
  - *By allowing for the closed-loop poles to be located in a "better" region.*
- Such controller can help stabilize an unstable system
  - *By "pulling" the root locus into the LHP*
- We call a controller that adds poles and/or zeros to the feedback system:
  - *A **Compensator***

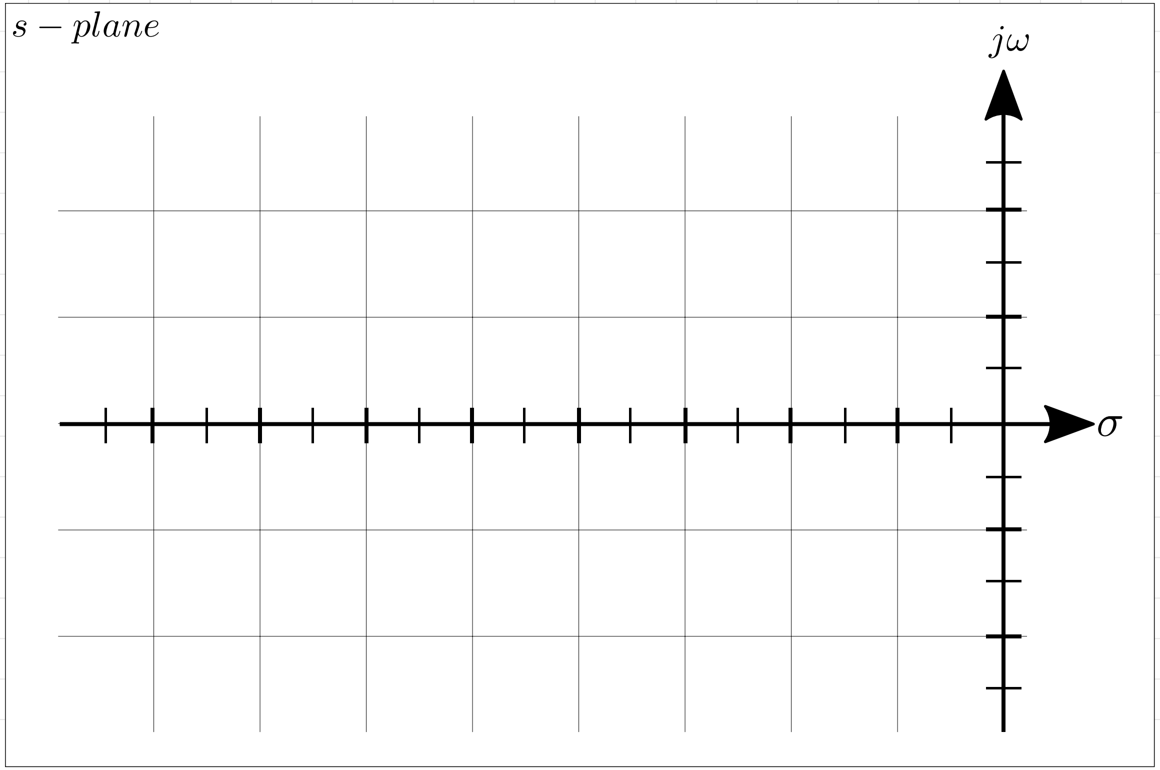
Example

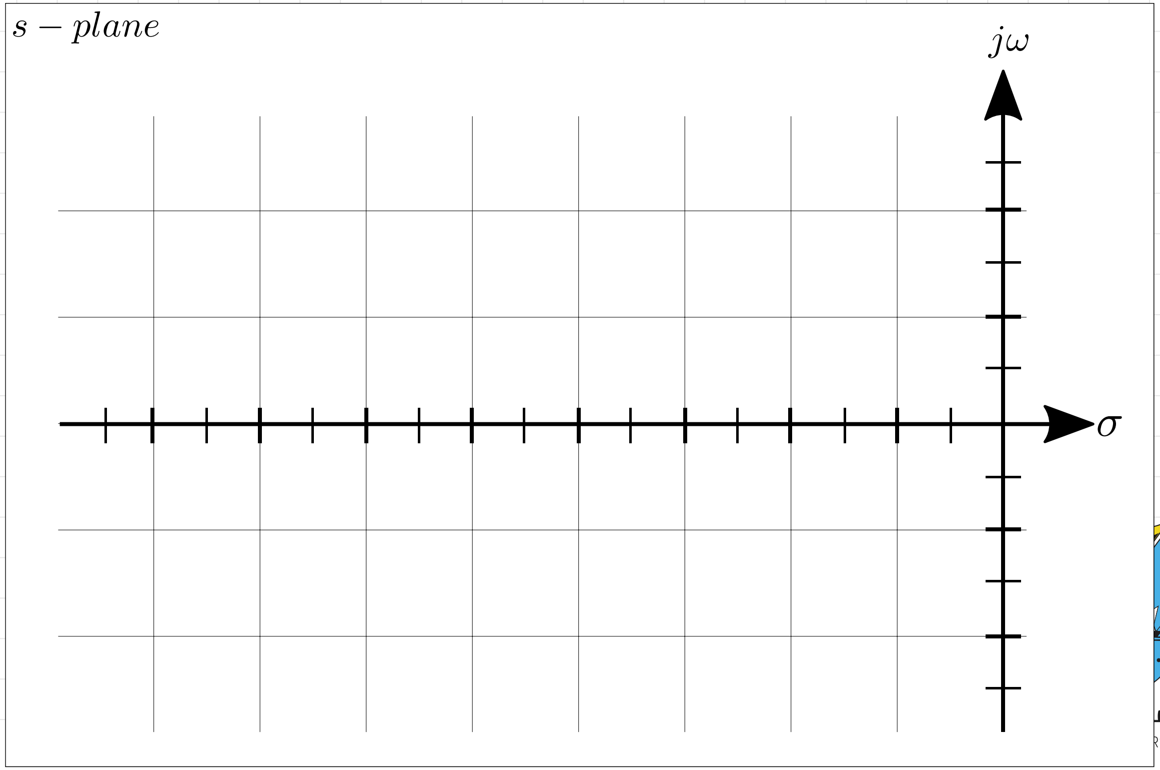
Sketch the root locus for the two unity feedback systems.

a. $G_p = \frac{1}{s(s+2)^2}, G_c = K$

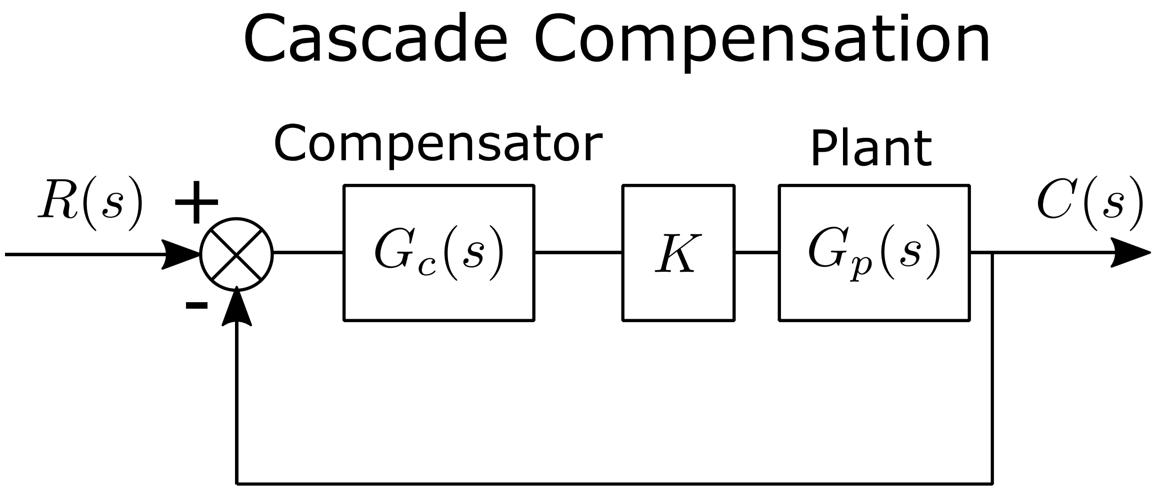
b. $G_p = \frac{1}{s(s+2)^2}, G_c = K(s+4)$ <- This is a compensator, note what it does.

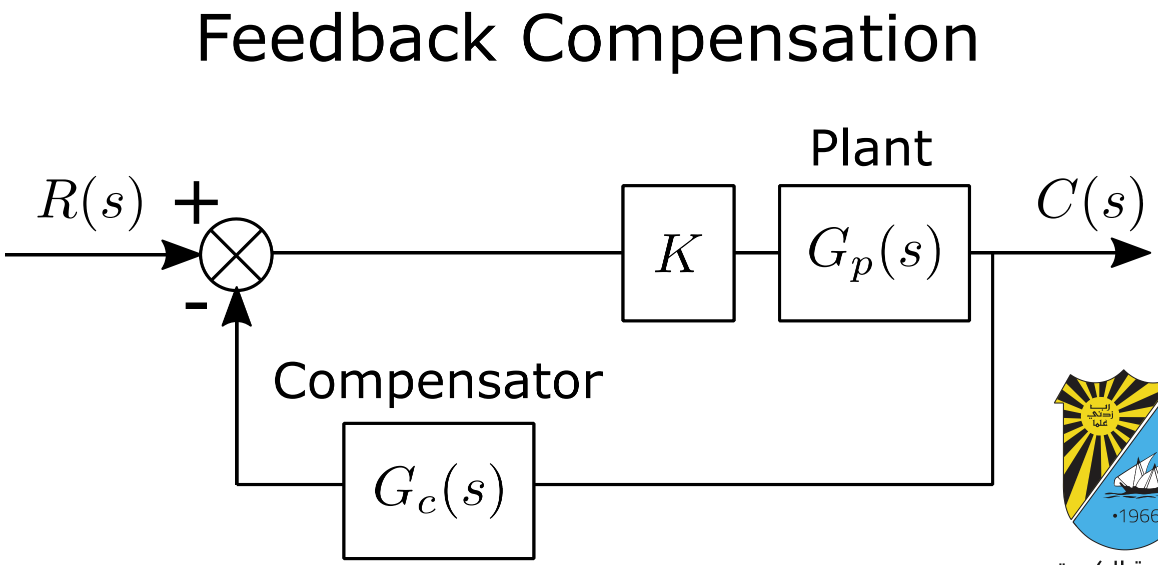






- If the compensator is placed in the forward path, we call it a **cascade compensator**.
  - If it is placed in the feedback path, we call it a **feedback compensator**.
- In practice, the names are often used interchangeably. A block diagram is always helpful in eliminating confusion and ambiguity.





- The ideal integral compensator is also known as the **Proportional-Integral Controller**. It seeks to:
  - *Eliminate the steady-state error (if it was already finite)*
  - *Increase system Type by introducing a pure/ideal integrator* $\frac{1}{s}$
    - Places a pole at the origin
  - *It places a matching zero to the left of the pole*
  - *Requires active components to implement*
- It's of the form

$$G_c = K_p \frac{s+K_i/K_p}{s},$$

where $-K_i/K_p$ is the zero location, K_p is the proportional error gain and K_i is the error integral gain.

- With the ideal integral compensator, the key idea is to **introduce an integrator to the system** $\frac{1}{s}$, that will increase the system type.
- The **ideal integral compensator** is also known as a **PI** (Proportional-Integral) controller.
- Consider the system with $G(s)_{OL} = \frac{(s+6)}{(s+2)(s+1)}$
    - The steady-state error to a step is given by $e_{ss} = \frac{1}{1+K_p}$, where the static position error constant $K_{position} = \lim_{s\to 0} G(s)_{OL} = 2$, making $e_{ss} = \frac{1}{3}$
- If we introduce the ideal dynamic compensator $G_c = K_p \frac{(s+K_i/K_p)}{s}$, then
    - $G(s)_{OL\ Compensated} = G_c G_{OL} = K_p \frac{(s+K_i/K_p)}{s} \frac{(s+6)}{(s+2)(s+1)}$
    - Which is a Type 1 system that has a $CL\ e_{ss} = 0$ to a step input
- *Beware that both the static position error constant and the proportional gain are labeled K_p. They are two different constants.*

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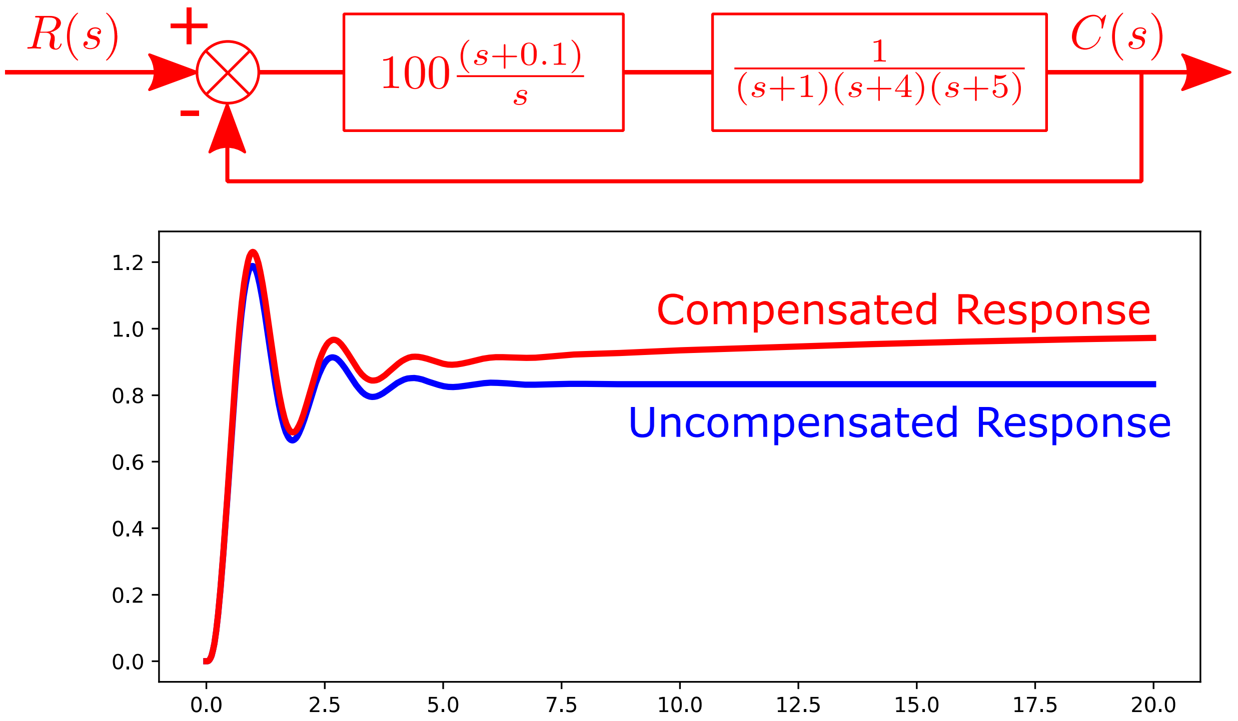
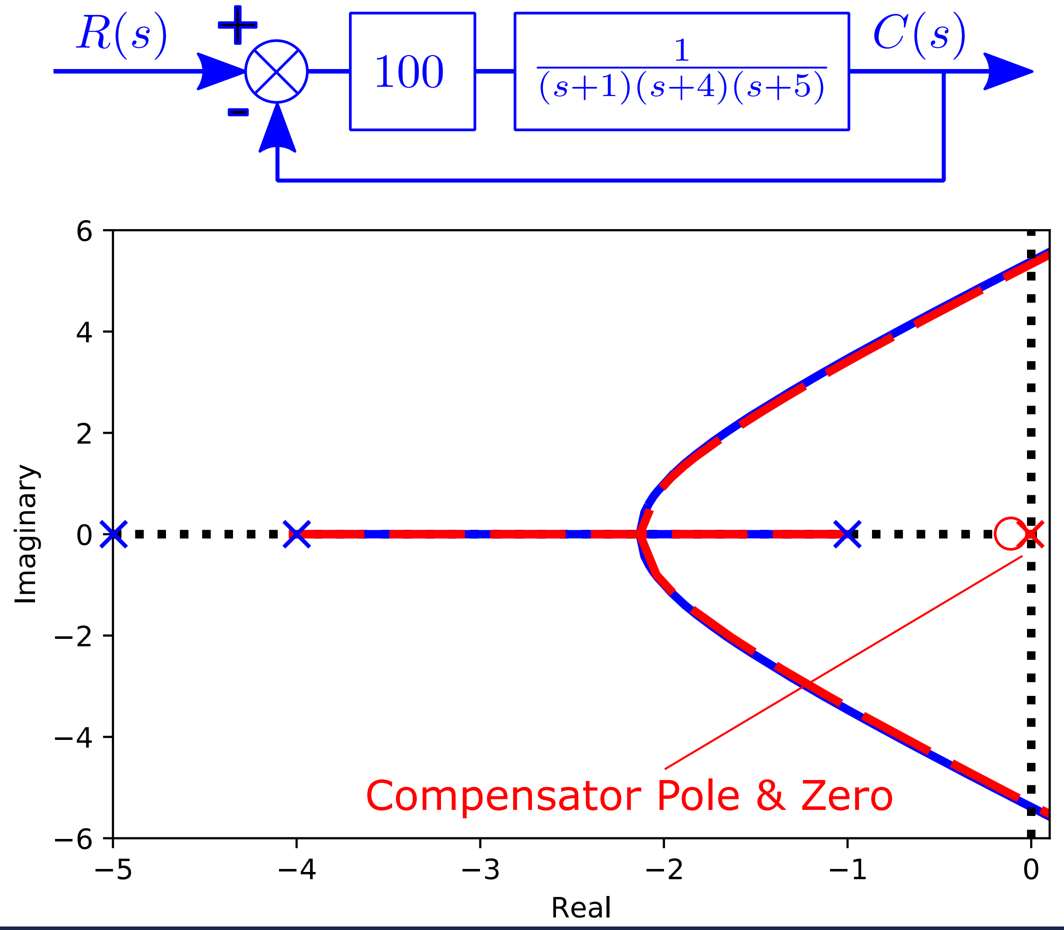
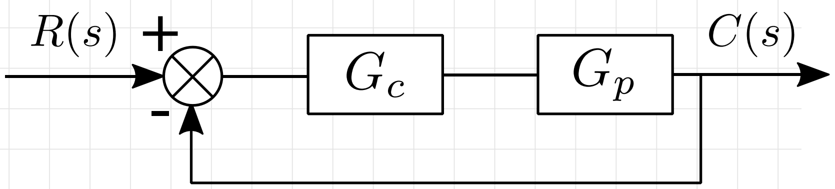
- Note how adding a pole and zero pair, both near the origin, does not affect the shape of the root-locus significantly (does not effect the transient response behavior). This is due to the compensator's pole-zero cancellation.

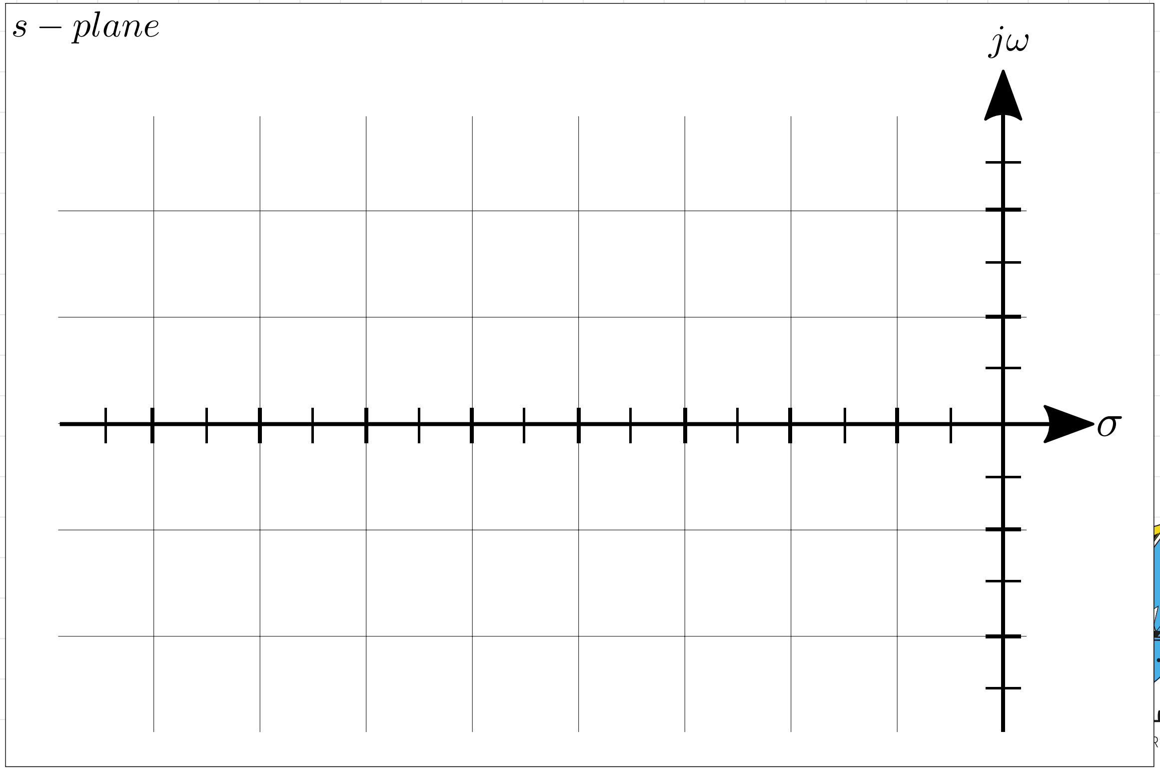




If it is desired to maintain a damping ratio $\zeta = 0.15$ to a step response for the feedback system shown. Design a controller that achieves this damping ratio in addition to having a zero steady-state error to a step response. For the following system



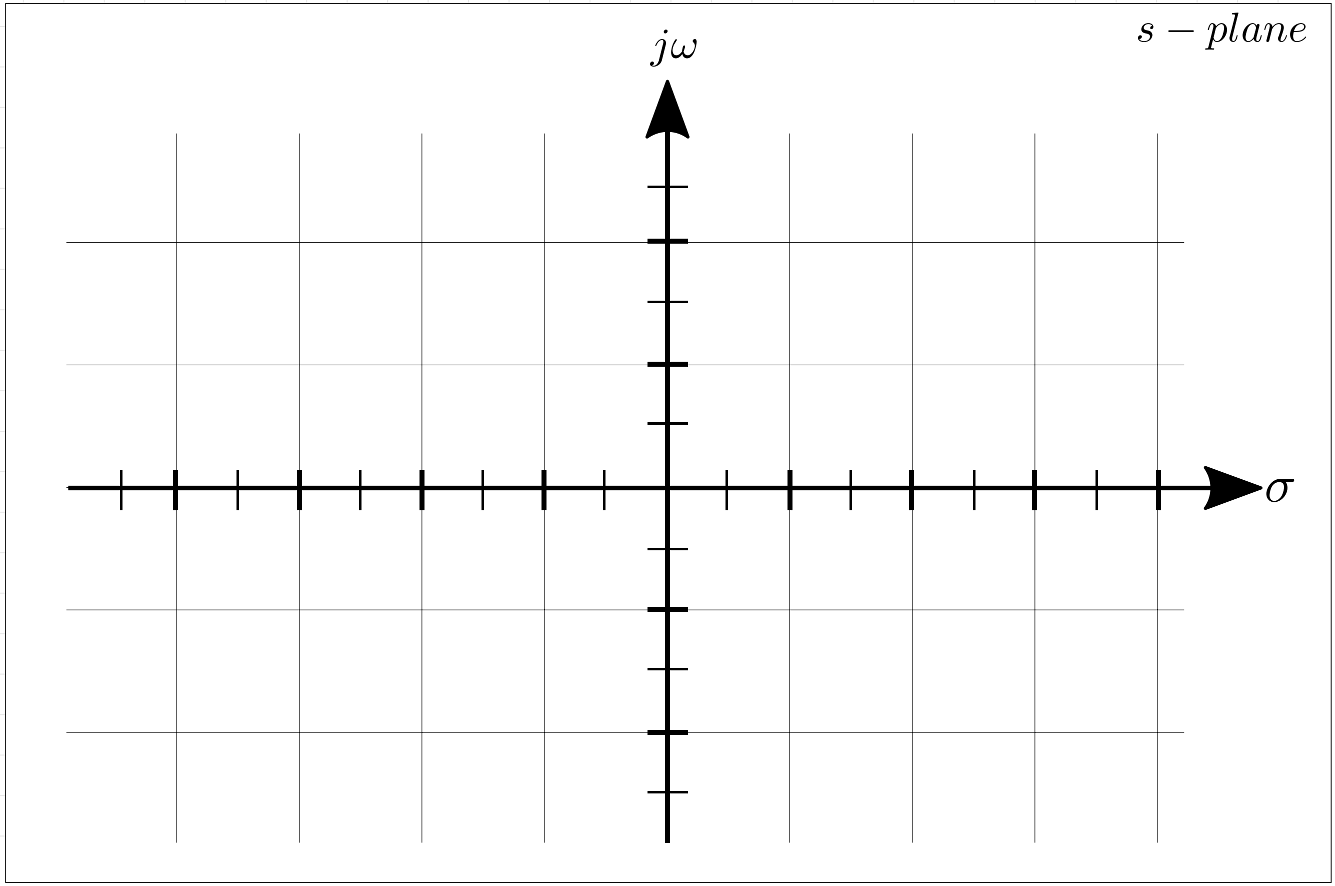
$$G_p(s) = \frac{19}{(s+1)(s+2)}$$



Example

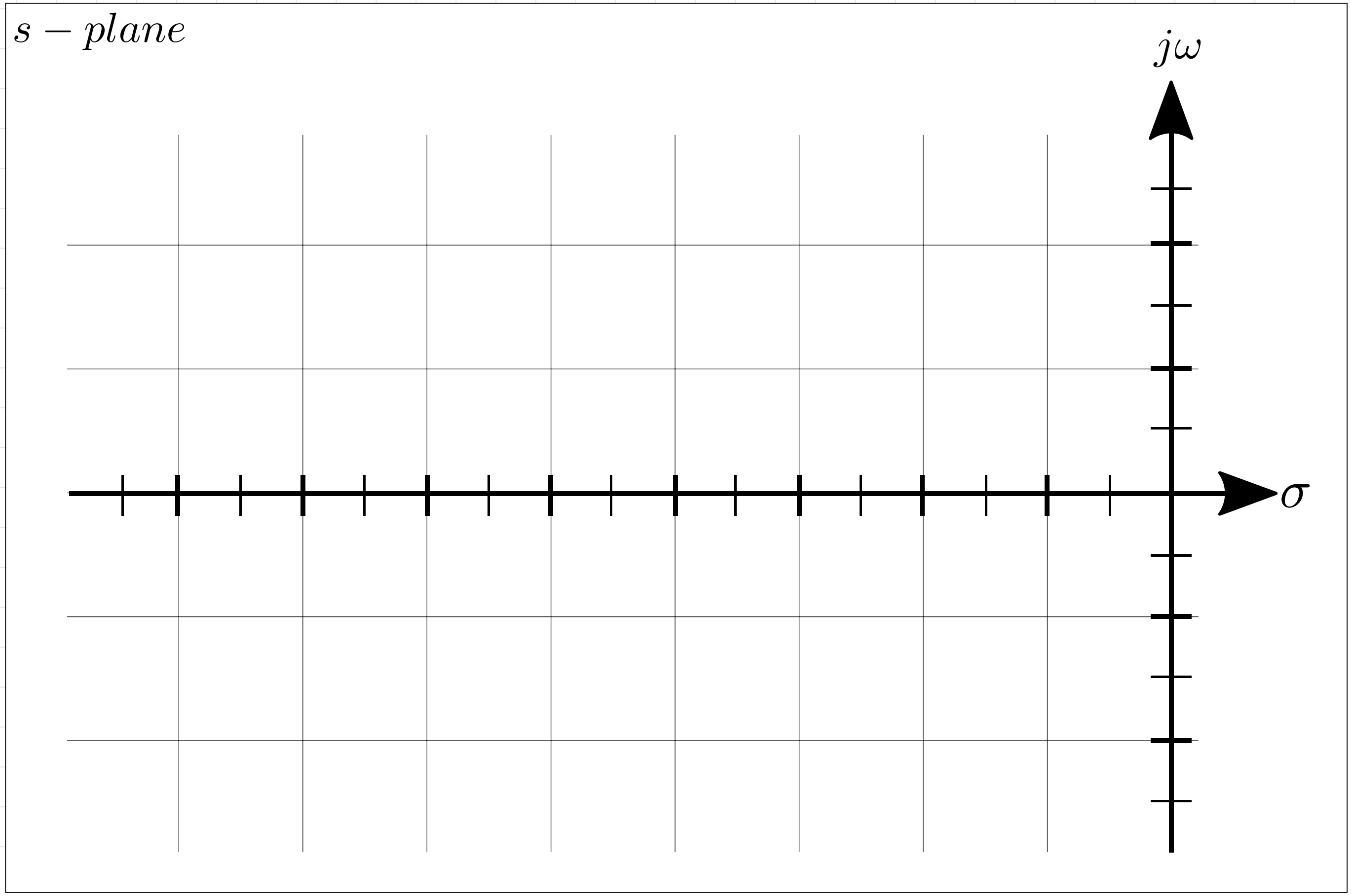
Show that for the system given, adding just an integral controller in a feedback loop, will result in an unstable response, and why we must include a proportional controller with the integral controller (Why we need to have a **PI controller**, and not just an **I Controller**)

$$G_p = \frac{1}{(s^2 + 2s + 10)}$$



Design a feedback system, using the root-locus technique, for the following plant to achieve a maximum overshoot of $OS\% = 4.3\%$, and zero steady-state error.

$$G_p = \frac{1}{(s+3)(s+5)}$$



- An ideal integral compensator (a.k.a **PI** Controller) requires an active integrator
  - In electronic systems, this would mean active circuits such as op-amps (additional supply of energy)
  - In mechanical systems, this would mean that mechanical actuation is required (motor/pump/hydraulic systems etc.)
- We can instead use a passive controller to **reduce** the steady-state error.
  - In electrical systems, this controller will be composed of passive elements (R,C,L)
  - In mechanical systems, this controller can be implemented using springs/mass/dampers (passive devices)
  - *What is a controller, but a system component that changes the total system behavior -> producing a "new" system.*
- For systems that are actuated by default and the controller is applied digitally (much of modern control applications), this distinction has little effect.

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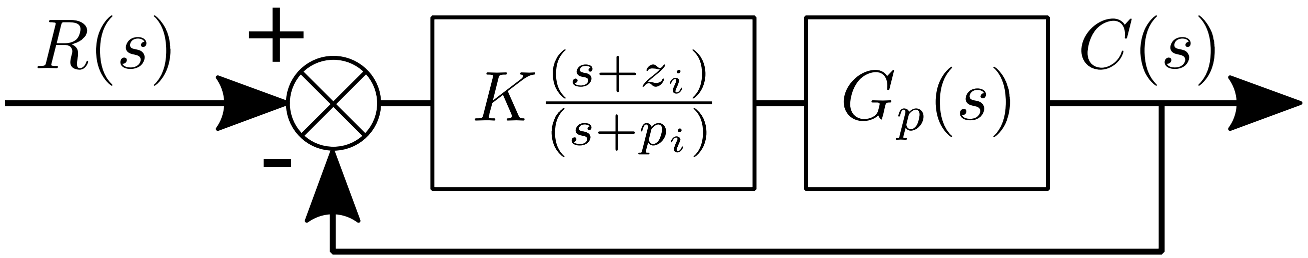
- The lag compensator is a type of controller that seeks to
  - Reduce the steady-state error (if it is already finite), but doesn't eliminate it
  - Without affecting the transient response behavior of the system
  - It places a pole in the LHP and near the origin
  - And a matching zero to the left of the pole
  - Doesn't require active components
- It's of the form

$$G_c = \frac{s+z}{s+p}, \text{ or } G_c = K\frac{s+z}{s+p}$$

where $-z$ is the zero location and $-p$ is the pole location.



Lag Compensation

- Instead of introducing an ideal integral term: $\frac{1}{s}$, the lag compensator **reduces**, but **doesn't eliminate**, the steady-state error by adding a pair of pole and zero near the origin.
- Which would not quite increase the system type, but rather, *increase the static error constant (reducing e_{ss})*
- Consider the system with $G(s)_{OL} = \frac{(s+6)}{(s+2)(s+1)}$
  - The steady-state error to a step is given by $e_{ss} = \frac{1}{1+K_p}$, where the static position error constant $K_{position} = \lim_{s\to 0} G(s)_{OL} = 2$, making $e_{ss} = \frac{1}{3}$
- If we introduce the lag compensator $G_c = \frac{(s+z_i)}{(s+p_i)}$, then
  - $G(s)_{OL\,Compensated} = G_c G_{OL} = \frac{(s+z_i)}{(s+p_i)} \frac{(s+6)}{(s+2)(s+1)}$
  - $K_{position} = \lim_{s\to 0} \frac{(s+z_i)(s+6)}{(s+p_i)(s+2)(s+1)} = \frac{z_i \cdot 2}{p_i \cdot 1}$
  - If $p_i < z_i$ the position static error constant is increased, reducing e_{ss}
- The PI Controller can be thought of as a Lag Compensator with the pole at the origin making:
  - $K_{position} = \frac{z_i}{0} = \infty$, $e_{ss} = 0$

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Effect of Lag Compensator on Transient Response

- Similar to the case with the **PI** Controller, placing the pole and zero pair of the compensator near the origin allows us to:
  - Place them close to each other, such that the compensator pole cancels the compensator zero during the transient response
  - Still have the ratio $\frac{z_i}{p_i}$ large enough to influence the steady state error
    - If the pole and zero become relatively far from one another, the compensator will begin to influence the transient response.
- Remember, the static position error constant from the prior example:
  - $K_{position} = \lim_{s \to 0} \frac{(s+z_i)(s+6)}{(s+p_i)(s+2)(s+1)} = \frac{z_i \cdot 2}{p_i \cdot 1}$
  - For a unit step: $e_{ss} = \frac{1}{1+K_{position}} = \frac{1}{1+\frac{2z_i}{p_i}}$

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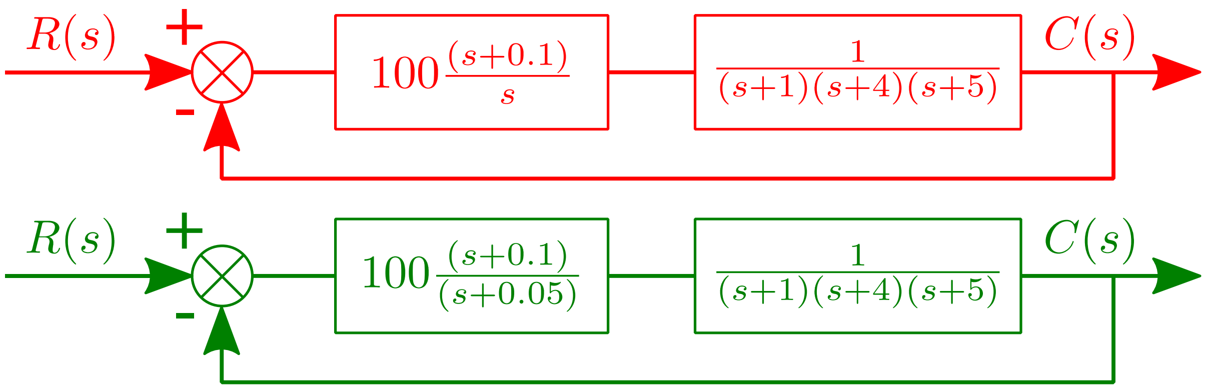
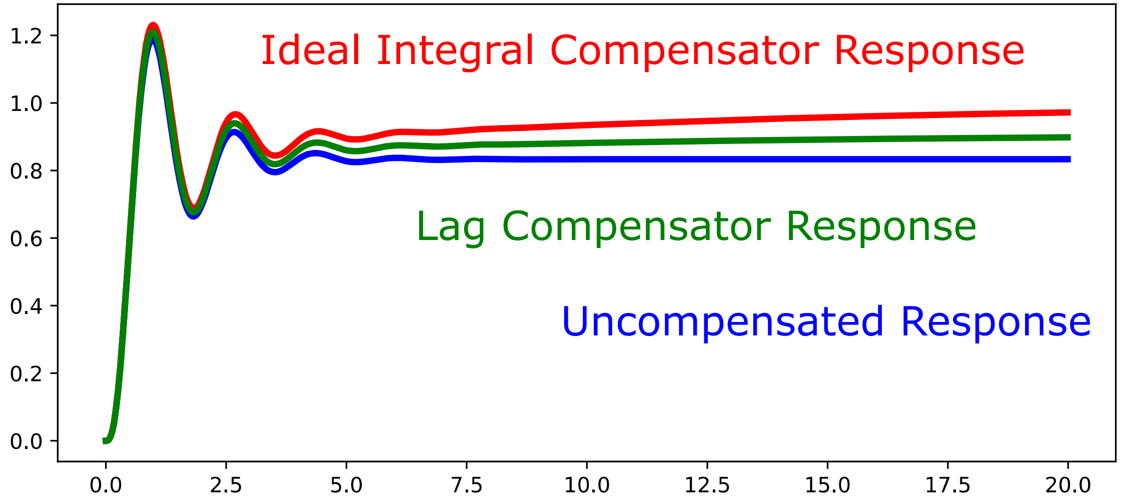
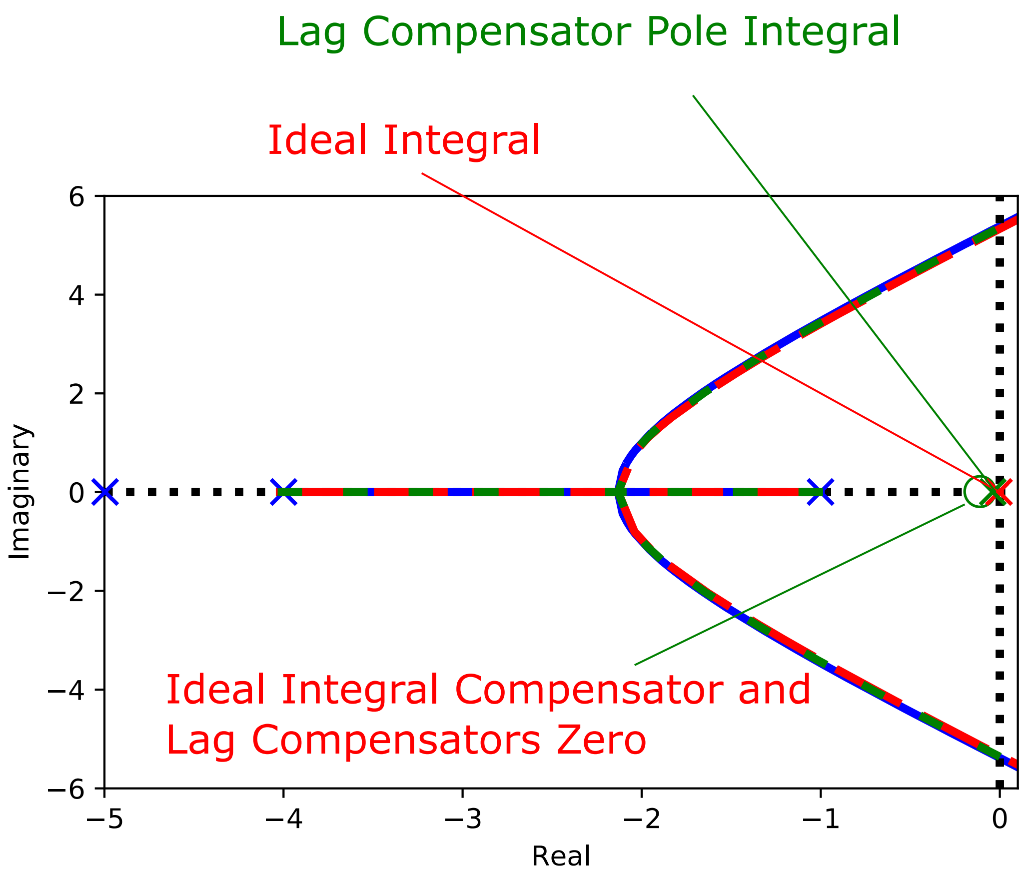
- Note how the lag compensator reduces but does not eliminate the steady-state error. And since the pole/zero pair are relatively close to each other, their influence on the transient response is negligible.

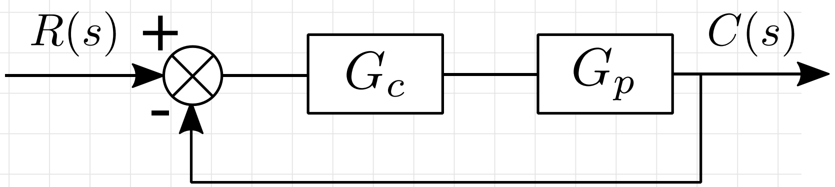






Example-Solved

Design a lag compensator to reduce the steady state error by a factor of 10 for the system shown, and achieve a damped frequency of $10rad/s$



$$G_p(s) = \frac{10}{(s+20)(s+25)(s+30)}$$

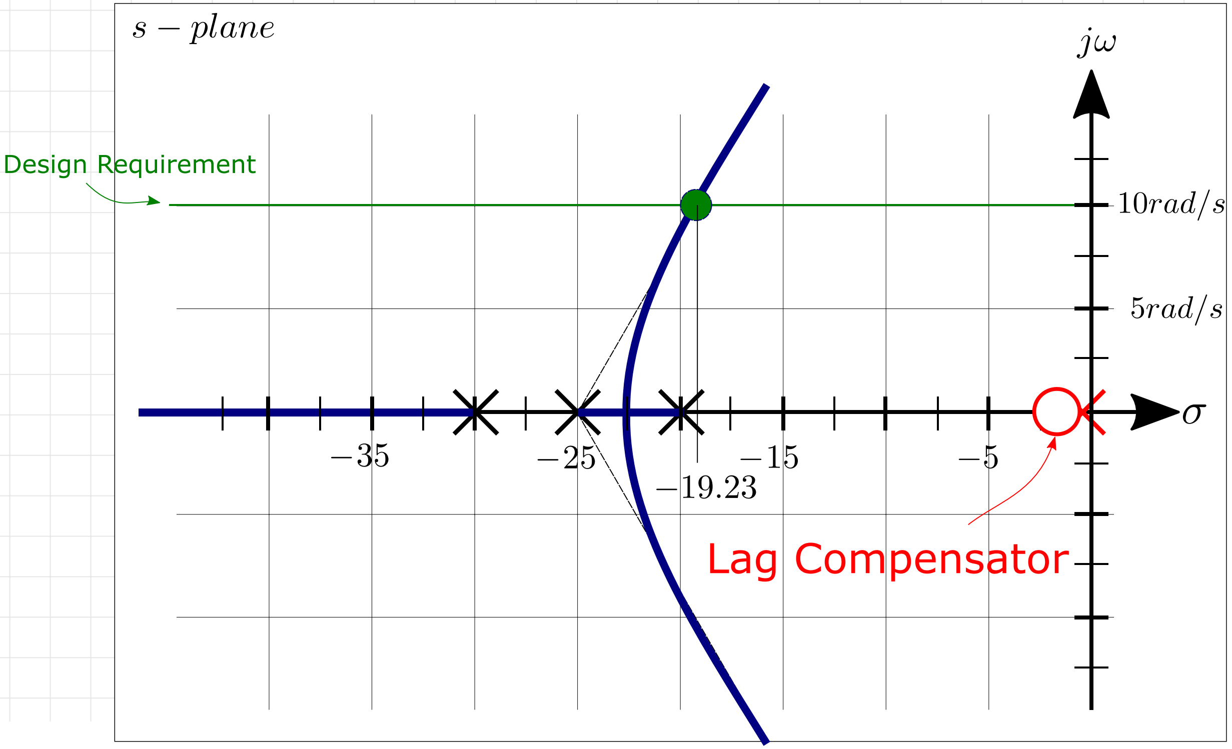
In this problem, we are required to achieve a specific transient response (10rad/s) and specific reduction in steady-state error.

Generally, we first seek to achieve the required transient response then the required steady-state response.

Start by specifying the transient response region or location. In this problem, the transient requirement is $\omega_d = 10rad/s$. This requirement is represented by the green line

Draw the root-locus without compensation and see whether it intersects the design region. In this problem we assume the root locus joins the asymptote at $\omega_d = 10$. The root-locus does intersect the design line, so all we need is to find the value of K that places the CL poles at the design point.

We already have the imaginary location of the desired closed-loop pole locations, next: we need to find the real axis value of this intersection



To find the point of intersection, note that the asymptote angle is 60^o, and since $\tan(60^o) = \frac{\omega_d}{\Delta\zeta\omega_n}$, then the real axis location of the intersection point is: $\sigma_a - \Delta\zeta\omega_n = \frac{-20-25-35}{3} - \frac{10}{\tan(60^o)} = -19.23$ (The intercept minus the x-component of the asymptote)

So the desired closed-loop pole location is: $s = -19.23 + 10j$

To find the gain we use the magnitude condition: $K = \frac{1}{|G(s)|} = \frac{Length\ Poles}{Length\ Zeros} = \frac{\sqrt{10^2+(30-19.23)^2}\sqrt{10^2+(25-19.23)^2}\sqrt{10^2+(20-19.23)^2}}{1} = 1702$

So, to achieve the desired damped frequency, we can use a proportional controller $10K = 1702$. Note that we ignore the contribution from the lag compensator. $K = 170.2$

Verifying with MATLAB, we find that the value of $K = 170.2$ actually places the closed loop poles at $s = -18.7 + 9.75i$. This discrepancy is primarily due to our assumption that the root-locus joins the asymptote before the design line.

So far we haven't included the **lag compensator**, the lag compensator's role is to reduce steady-state error. The requirement is to reduce the steady-state error to a step by a factor of 10, this governs the ratio of the zero-to-pole location of the lag compensator.

$$e_{ss\ compensated} = \frac{1}{10} e_{ss\ uncompensated} = \frac{1}{10}\frac{1}{1 + K_{p\ uncompensated}} = \frac{1}{1 + K_{pos\ compensated}} \Rightarrow 9 + 10K_{pos\ uncompensated} = K_{pos\ compensated}$$

$$9 + 10\lim_{s\to 0} KG_p(s) = \lim_{s\to 0} KG_cG_p(s) \ : \textit{Don't forget to include the gain}$$

$$9 + 10\lim_{s\to 0}\frac{1702}{(s+25)(s+30)(s+35)} = \lim_{s\to 0}\frac{s+z}{s+p}\frac{1702}{(s+25)(s+30)(s+35)} \Rightarrow 9 + \frac{1702}{15000} = \frac{z}{p}\frac{1702}{15000}$$

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$$9 + \frac{1702}{15000} = \frac{z}{p}\frac{1702}{15000} \Rightarrow \frac{z}{p} = 1 + 9\frac{15000}{1702} = 80.3184$$

This means that the ratio of the zero location to the pole location is 80.3. There is no other requirement mentioned, and we should try to relatively place the pole and zero close to each other compared to the other poles in the system. So, if we place the pole of the lag compensator at $p = -0.01$, then placing $z = -0.81$ should satisfy the design requirement.

$$G_c = 170.2\frac{s + 0.81}{s + 0.01}$$

Testing the design in MATLAB, we observe that the steady-state error before compensation is around 0.9
While with the compensator the steady state error is 0.09, which is a 10 fold decrease as required.

Note: The gain calculated by the root-locus is the full open loop gain, not just the controller. Meaning, the gain of 10 coming from the plant is included in the calculation. So, in this example, the open loop gain with the controller is $10K$, 10 from the plant and the controller gain K.

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