Kuwait University
College of Engineering and Petroleum

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ME417 CONTROL OF MECHANICAL SYSTEMS

PART I: INTRODUCTION TO FEEDBACK CONTROL

LECTURE 8: INTRODUCTION TO STABILITY AND FEEDBACK CONTROL

Summer 2020

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Lecture Plan

- Objectives:
  - *Overview of Block Diagram Reduction*
  - *Discuss the principles of Feedback Systems analysis and design*
  - *Discuss the conceptual fundamentals of stability*
- Reading:
  - *Nise: 5.1-5.3, 6.1-6.2*
- Practice Problems Included

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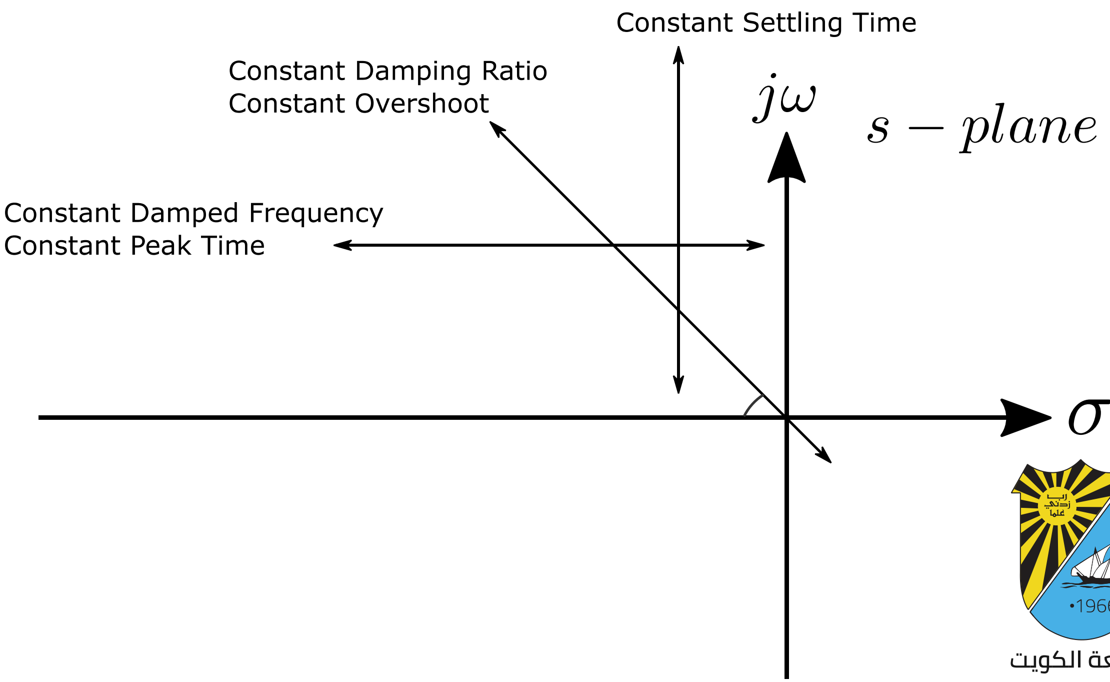
- Reviewed the performance specifications for the general second-order system

$$C(s) = \frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

- Discussed the effect of pole location on the step response



- Discussed the effect of additional poles on the step response of a system

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \times \frac{a}{\underbrace{s + a}_{Additional\ Pole}}$$

$$c(t) = \underbrace{K_1 u(t)}_{Forced\ Resp.} + \underbrace{e^{-\zeta\omega_n t}(K_2 cos\omega_d t + K_3 sin\omega_d t)}_{Nat.Resp.:\ Complex\ Poles} + \underbrace{K_4 e^{-at}}_{Nat.Resp.:\ Additional\ Pole}$$

- Discussed the effect of additional zeros on the step response of a system.

$$C'(s) = \underbrace{(s + a)}_{zero}\ C(s) = \underbrace{sC(s)}_{Derivative\ of\ Original\ Response} + \underbrace{aC(s)}_{Scaled\ Original\ Response}$$

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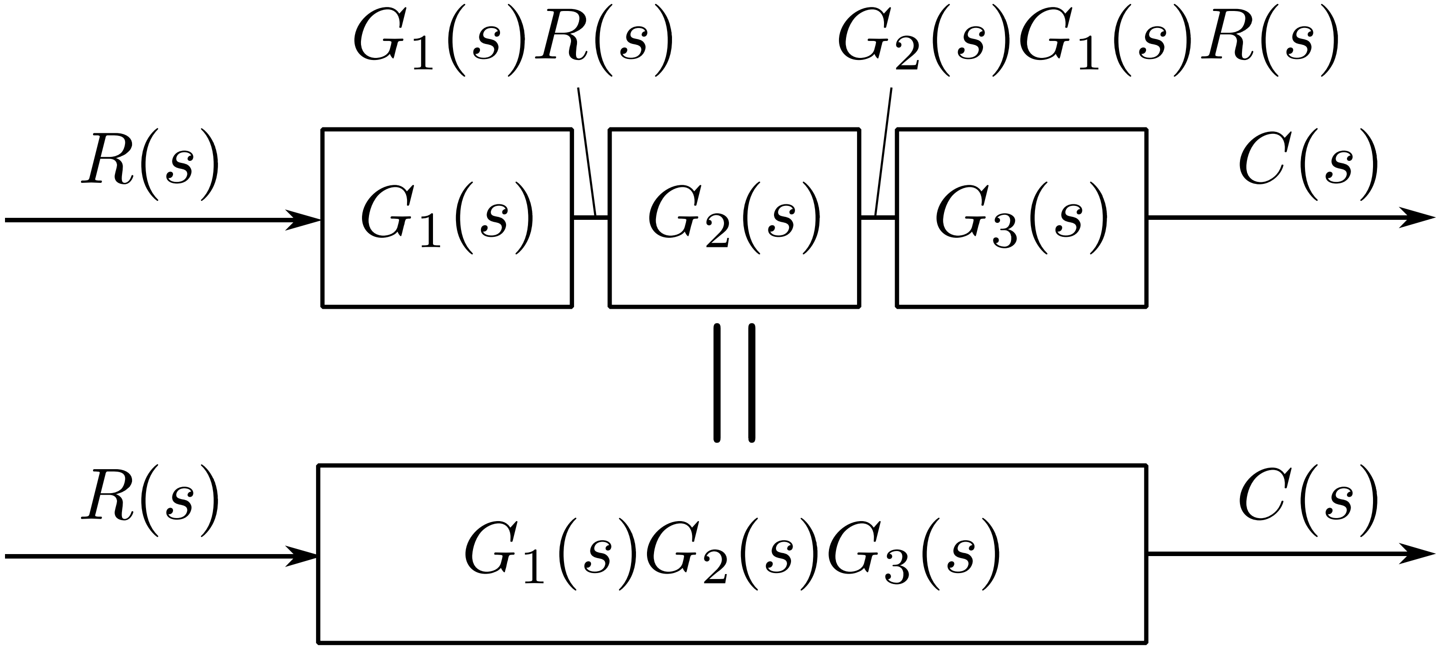
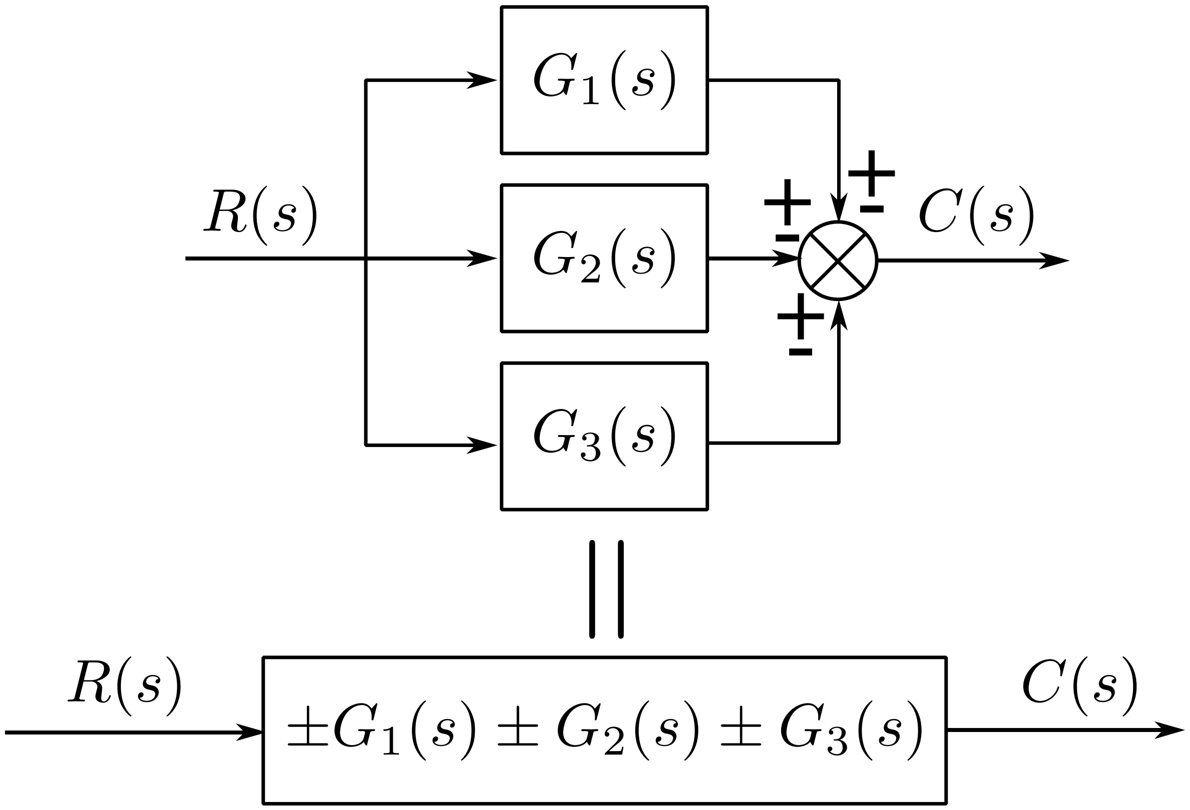
- When subsystem blocks are placed in series (cascade) form, the equivalent system is the product of the transfer functions.

$$G_e(s) = G_1(s)G_2(s)G_3(s)$$
$$C(s) = G_e(s)R(s)$$

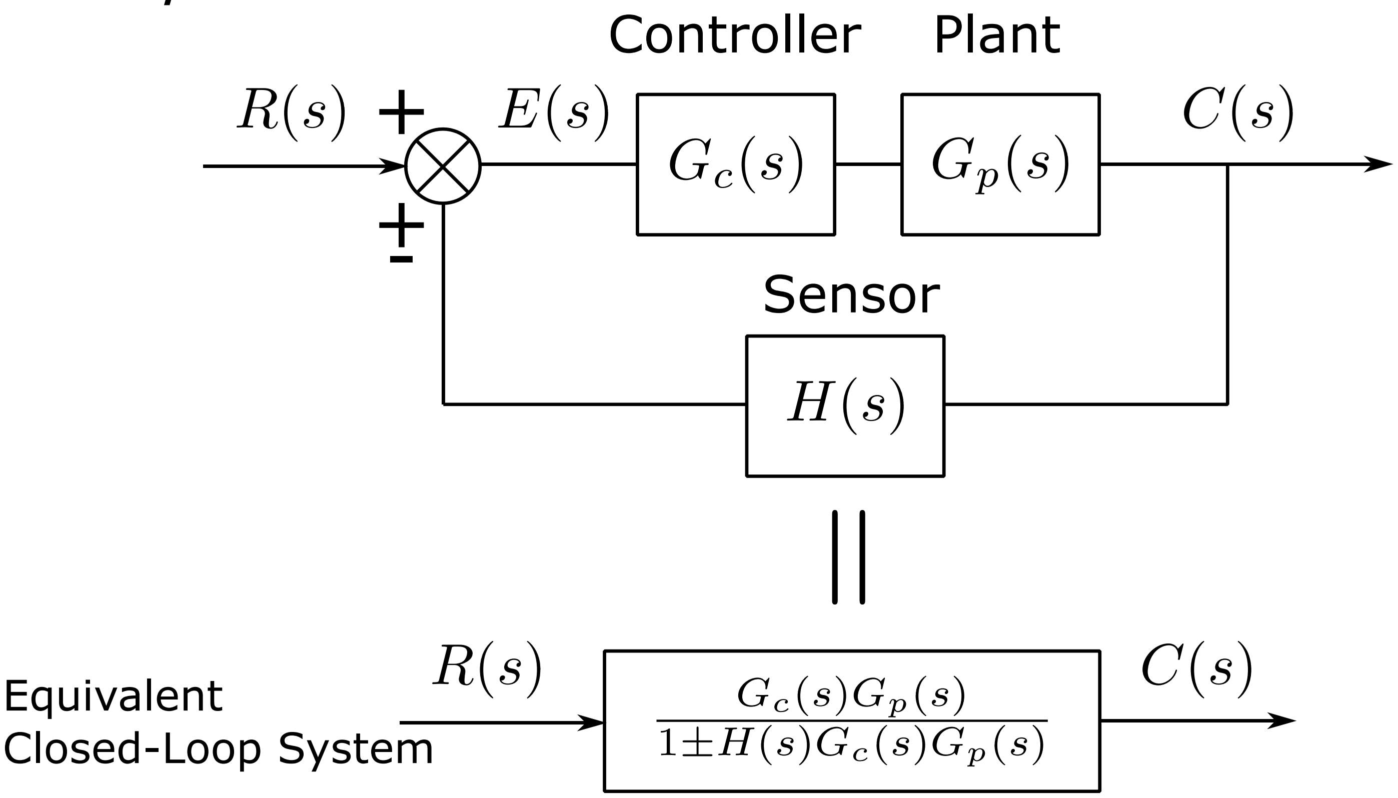


- When subsystem blocks are placed in parallel form, the equivalent system is the sum of the transfer functions.

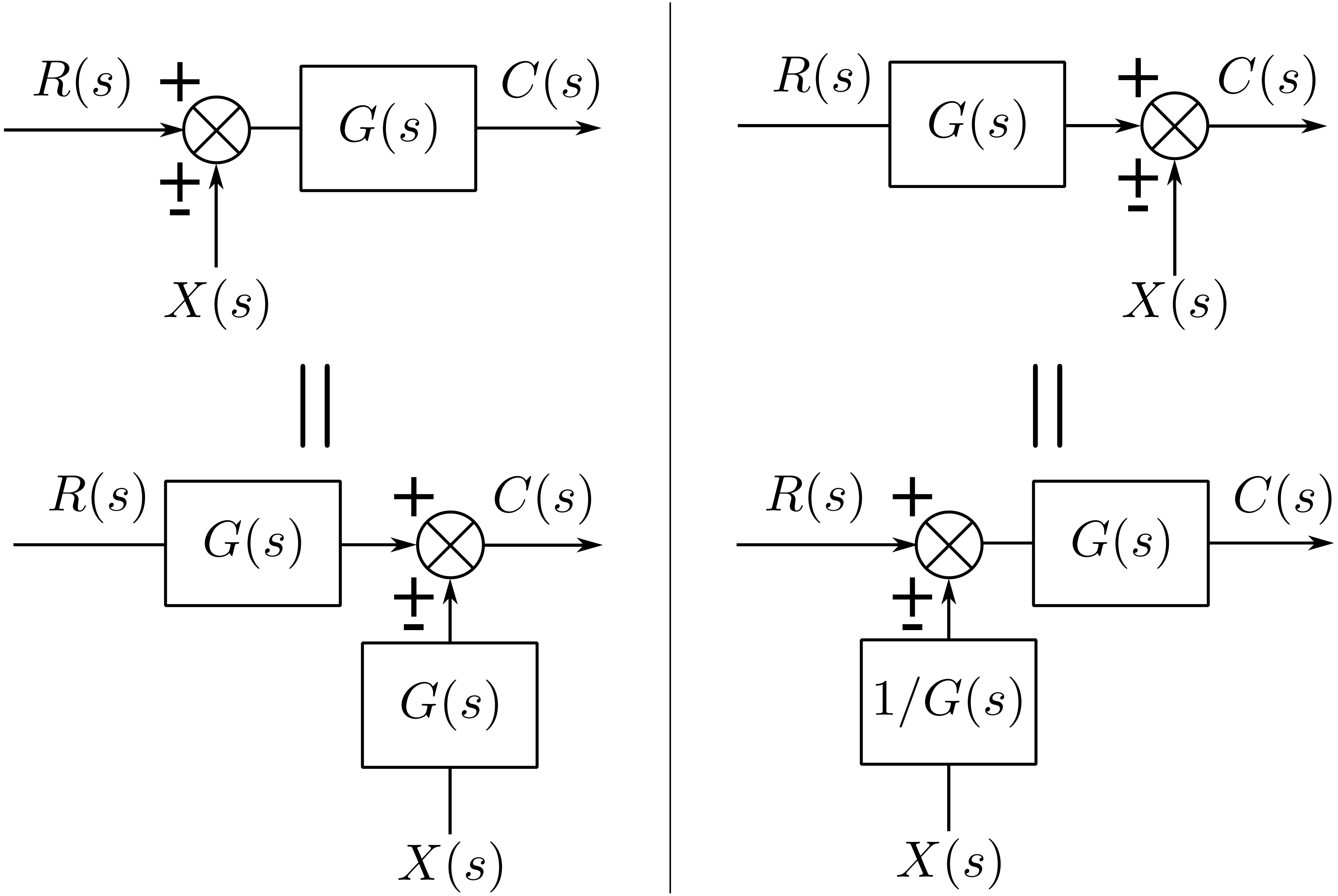
$$G_e(s) = \pm G_1(s) \pm G_2(s) \pm G_3(s)$$
$$C(s) = G_e(s)R(s)$$



- The standard feedback form shows a cascade controller $G_c(s)$ with the plant $G_p(s)$, a sensor subsystem $H(s)$ in the feedback loop, the error $E(s)$ computes a difference between the input and output.
- Using the previous properties allows us to obtain an equivalent closed-loop form for a feedback system.

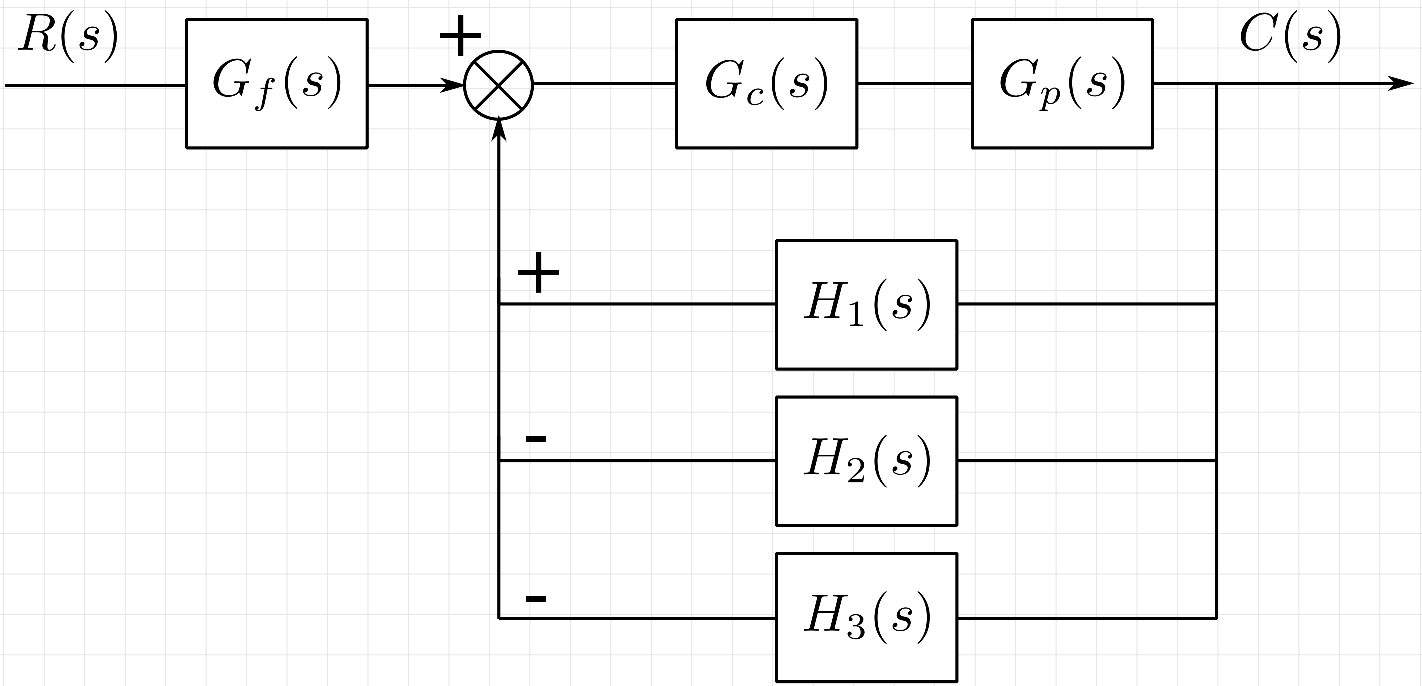


- It is often desired to manipulate the block diagram to produce a familiar form



Example 1

Reduce the block diagram shown to a single transfer function.



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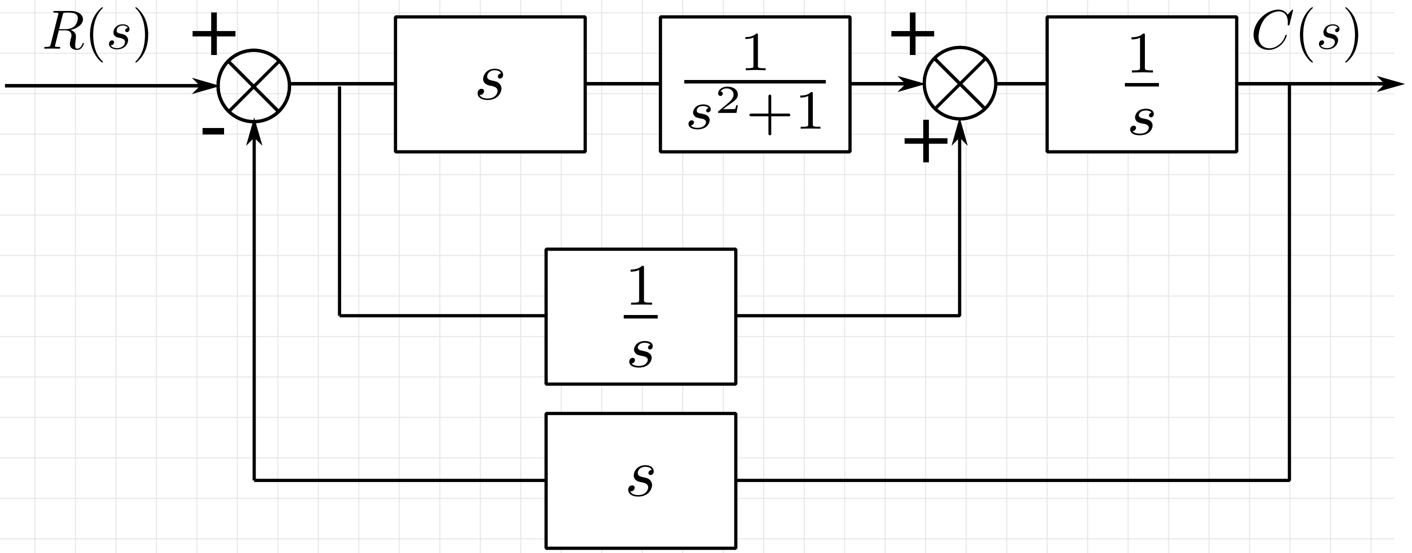
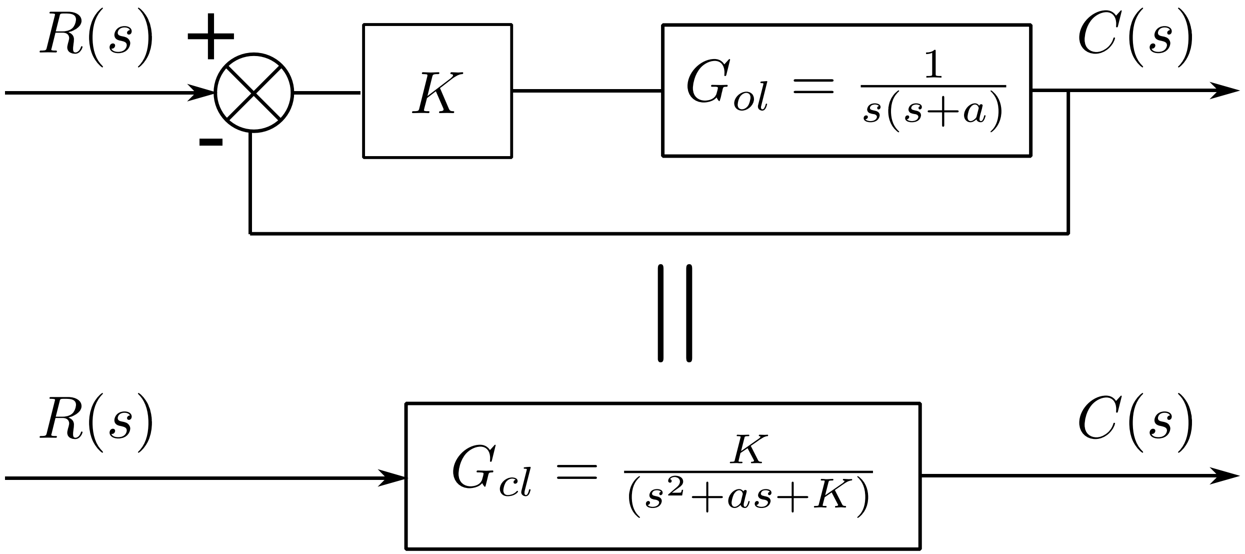
Reduce the block diagram shown to a single transfer function.

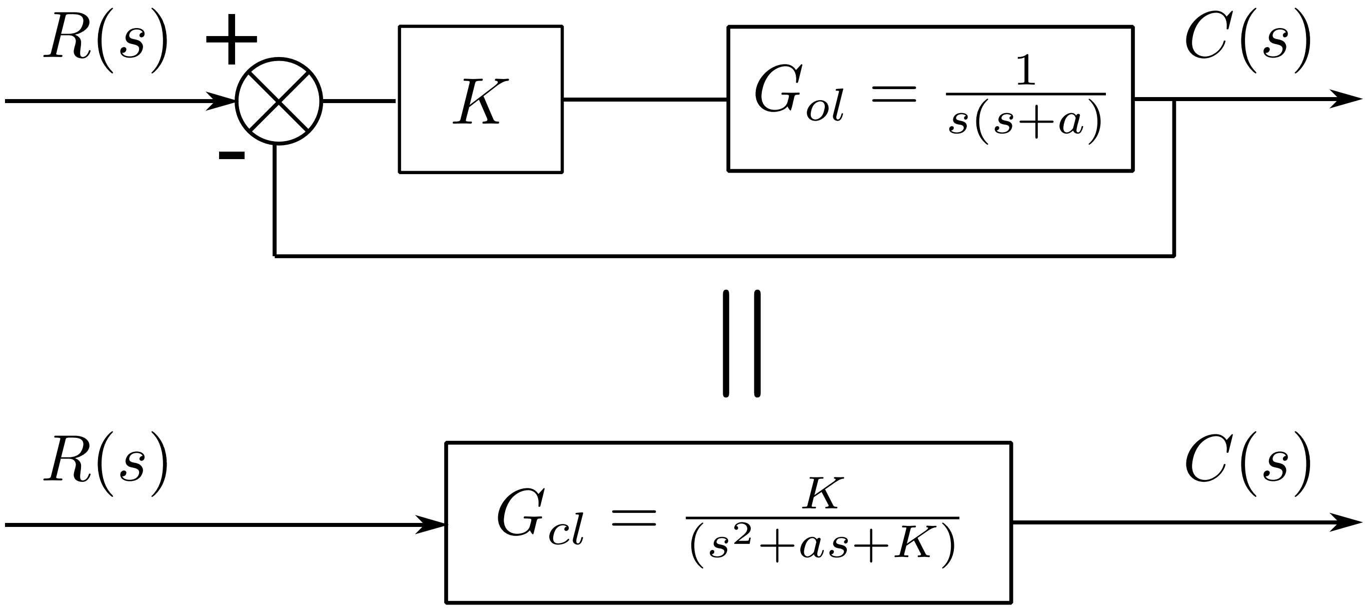
Example 2



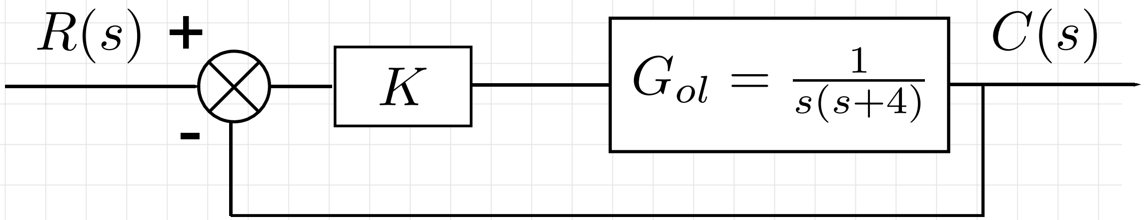
- *Design* (of a controller) indicates a presence of a variable that can be changed to produce a more desired design outcome.
- In designing a feedback controller, there is at least one variable that can be tweaked to produce the desired performance specifications.
- The simplest feedback controller is a simple gain.
    - Here, when we say: "Design a gain feedback controller", we mean: "find the suitable value for the gain K to achieve a desired performance"



- Note how in the above example, the open-loop system has an integrator $\frac{1}{s}$, that disappears in the closed loop form. And the closed loop can become a general second-order system.
- Note how the value of gain K in the characteristic polynomial of the closed-loop system, is a design variable that affects the second-order system performance specifications (T_s, T_r, T_p, $\%OS$)

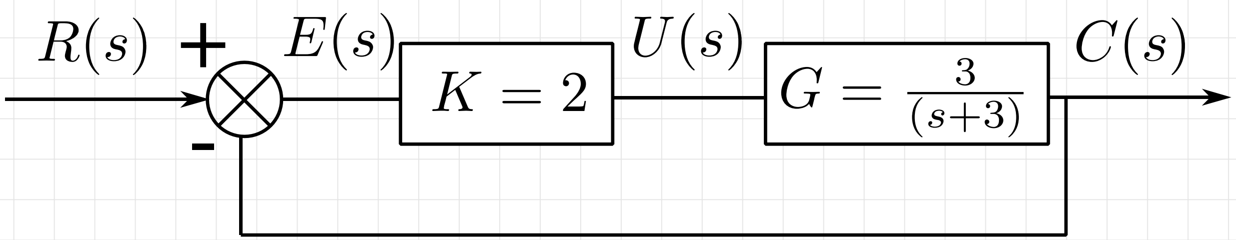


Design the value of gain, K, for the feedback control system shown. So that the system will have a damping ratio ζ of 0.25



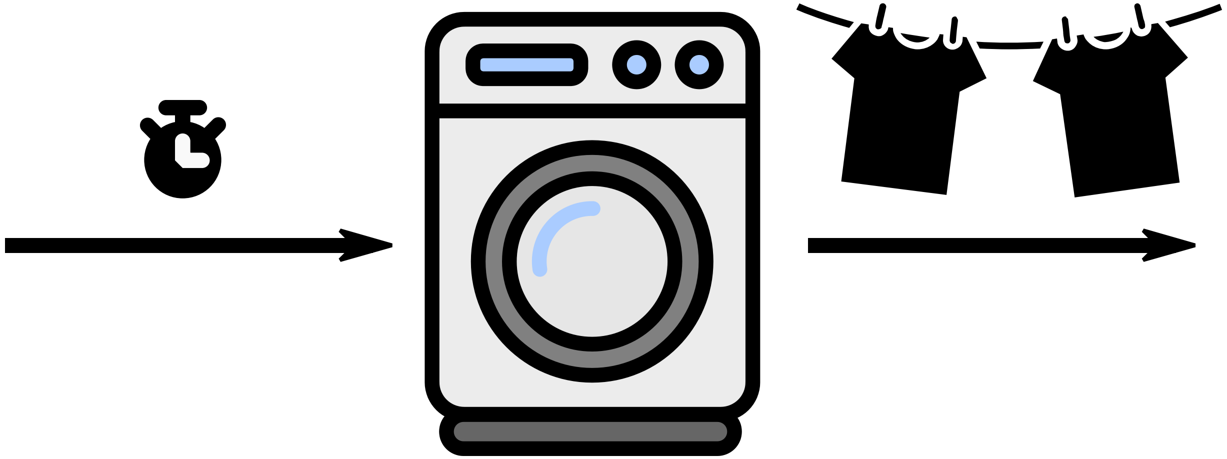
Example 4

Calculate the value of the input signal $\boldsymbol{u}$ to the plant G for a unit-step input **r** to the control system, at times $t = 0, 2, 4s,$ & $t = \infty$, for the system shown on the figure.



Open-Loop Systems

- An Open-Loop Control System is one in which the input to the system is **not** a function of the system. It is blind to the system's state and output.
- An example of an open-loop control system is a clothes washing machine. Where the control variables are time / water temperature / water volume.
    - For a given input, the outcome (level of cleanliness), varies from the desired outcome (clean, undamaged clothes).
    - The outcome depends on the system states (how dirty and delicate the clothes were)



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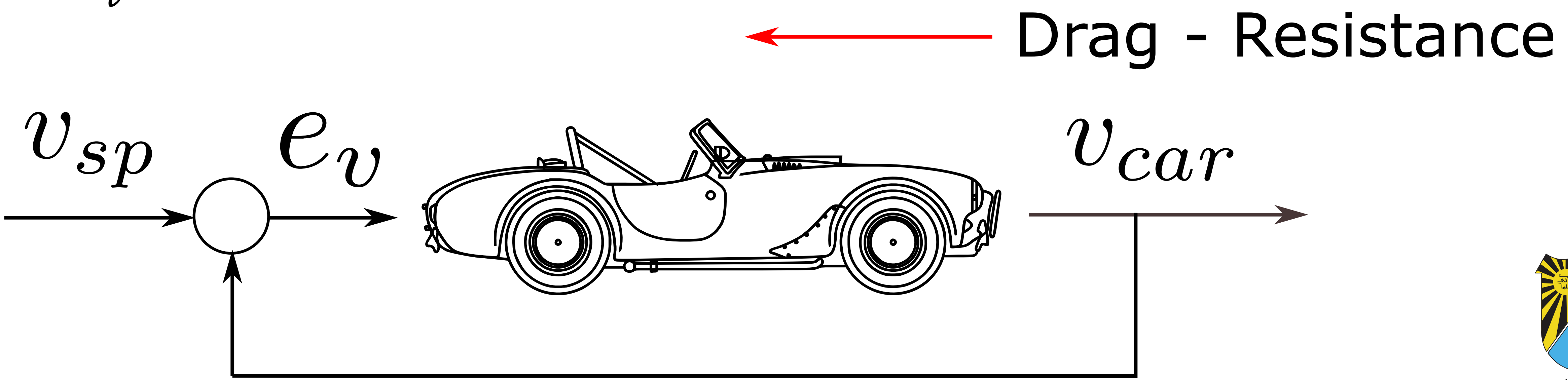
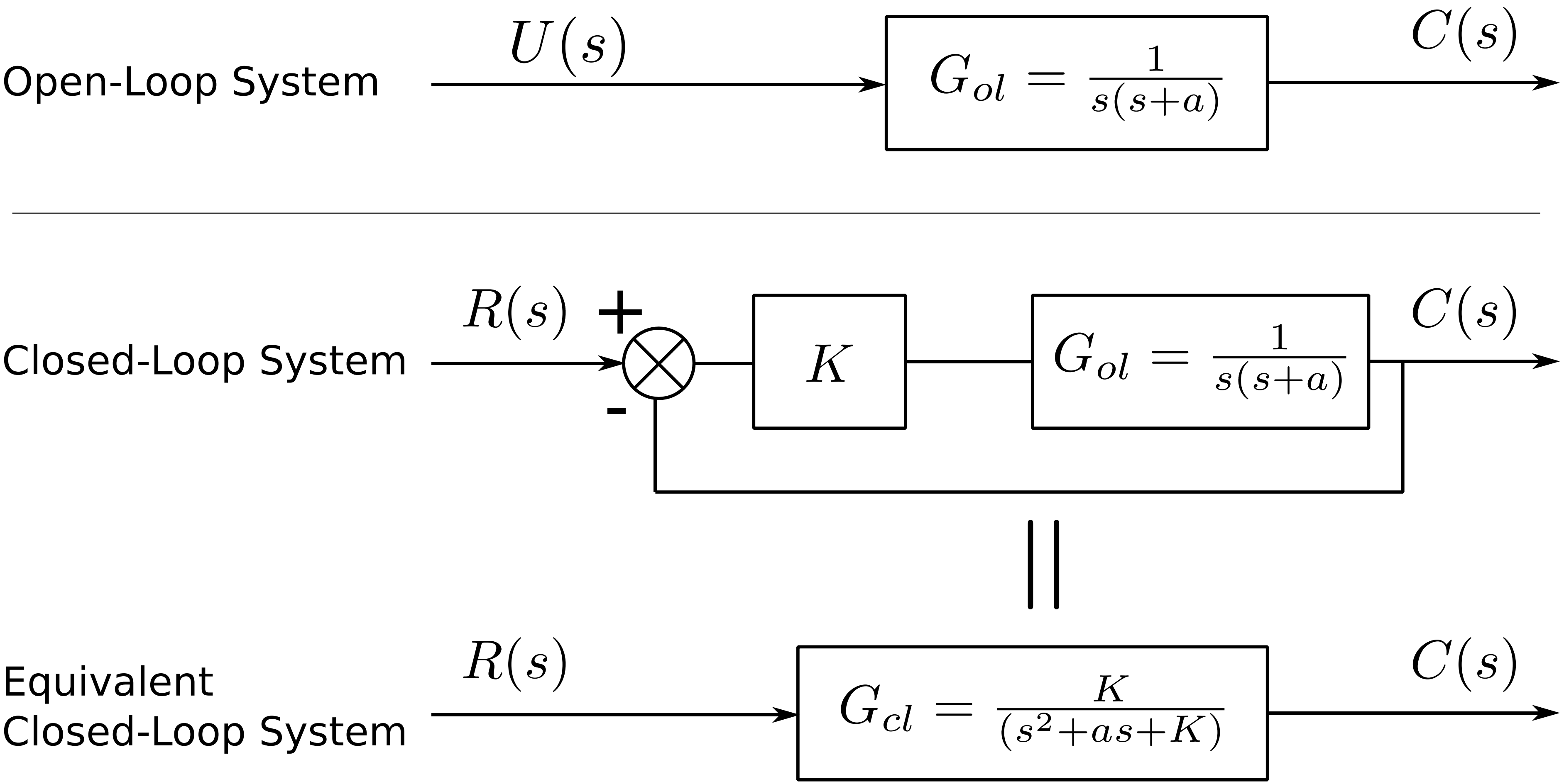
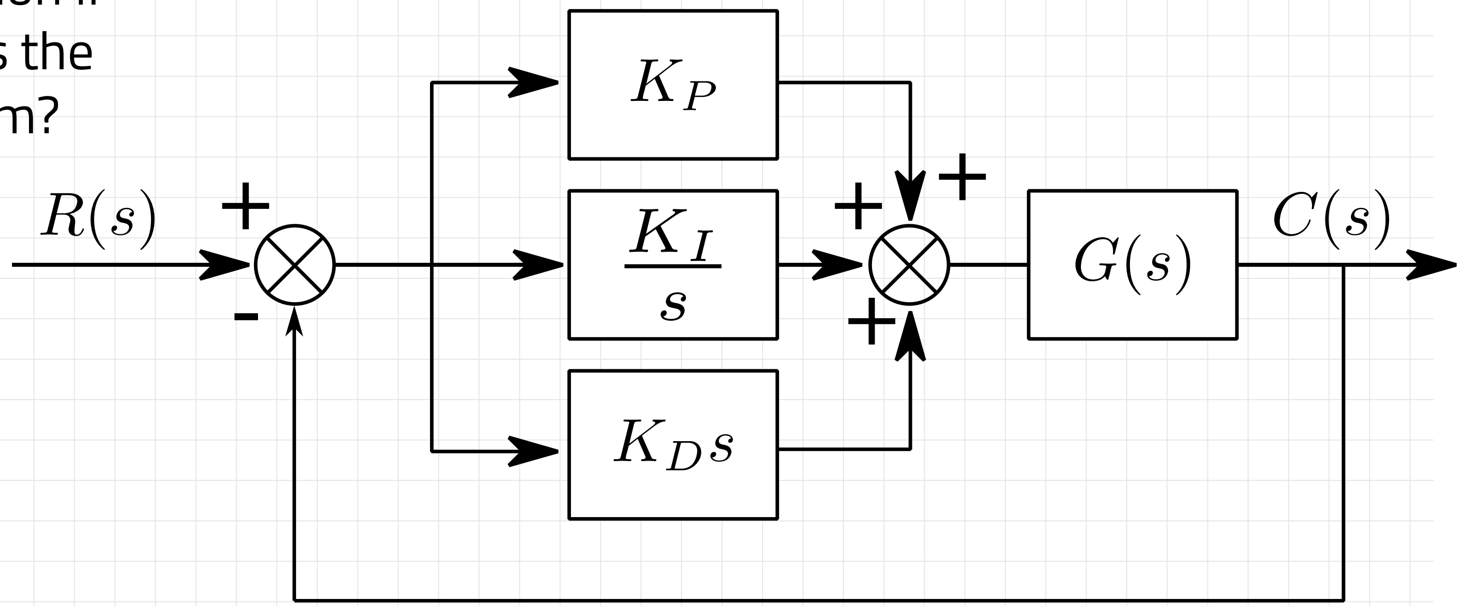
- Closed-Loop Control Systems are those that take into consideration, the output state of the system, in the control input value to the system.
- A car cruise control system, CCS, is an example of a closed-loop control system.
- The driver sets a desired velocity, the CCS reads the actual velocity of the car, computes the error, e_v, then increases/decreases gas throttle to bring the error e_v down to zero.



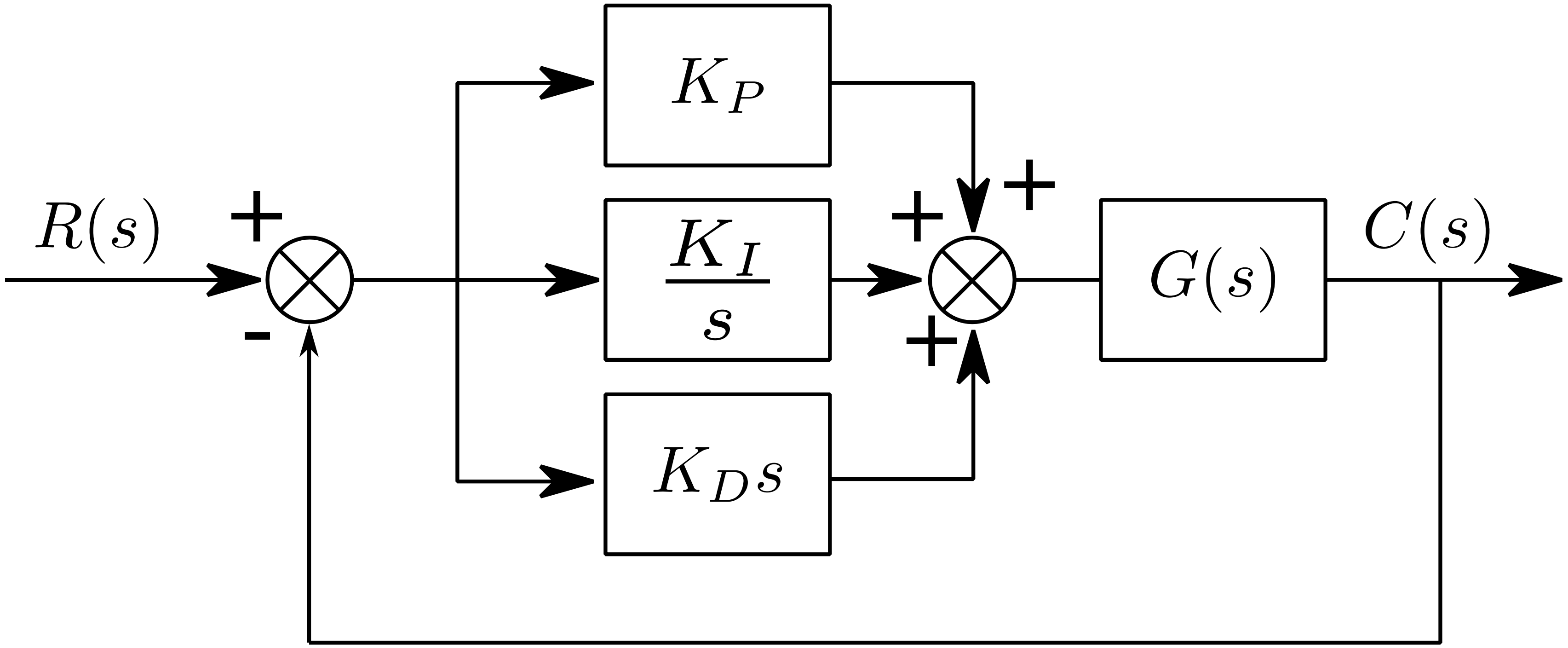
- Note that an equivalent closed-loop system may be represented in an open-loop form, using block-diagram reduction. It should be stated whether the system is open or closed-loop



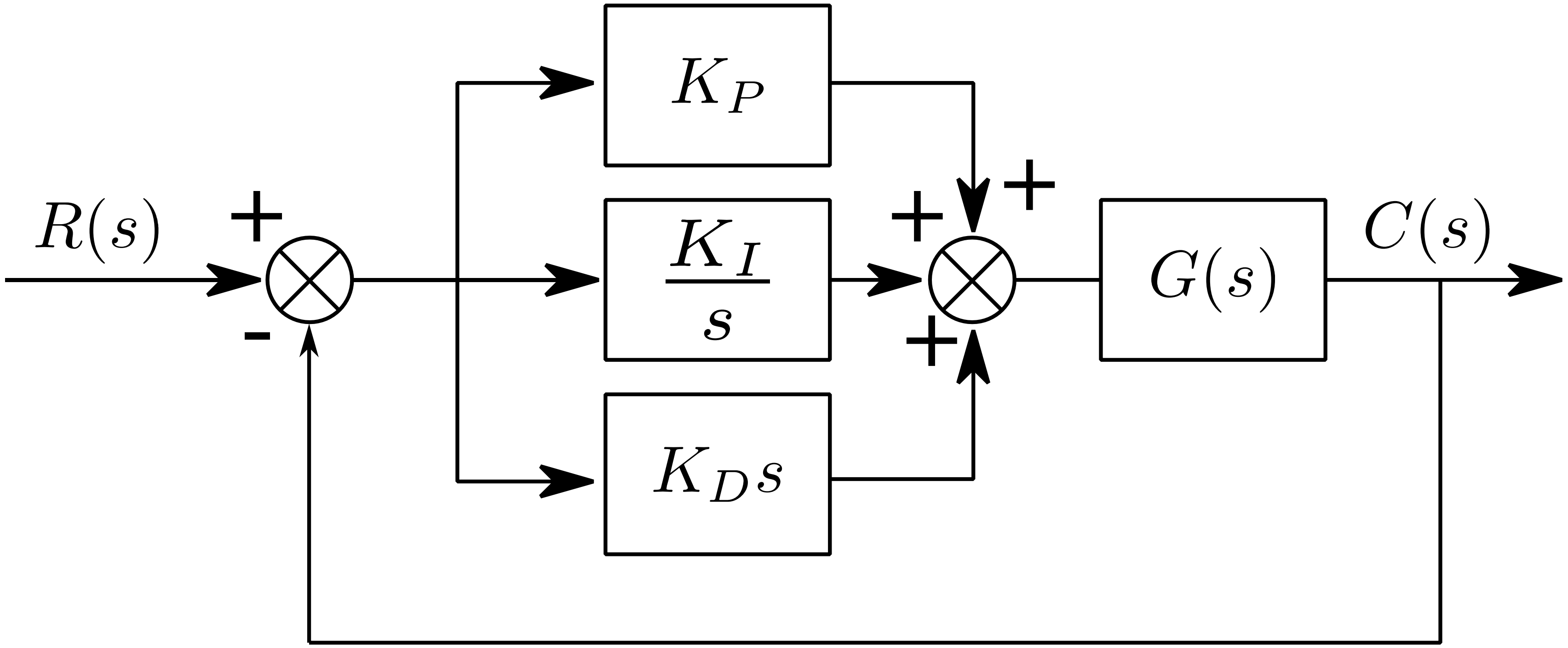
Reduce the block diagram shown to a single transfer function. Then find the equivalent transfer function if $G(s) = \frac{1}{(s+1)}$ and $K_P = 2, K_I = 2, K_D = 1$. What's the damping ratio of the step response of this system?



- A classic controller, and one heavily applied on real-world systems, is the proportional-integral-derivative controller. The feedback error signal is fed to up-to three components, for which there are three gains to tune.

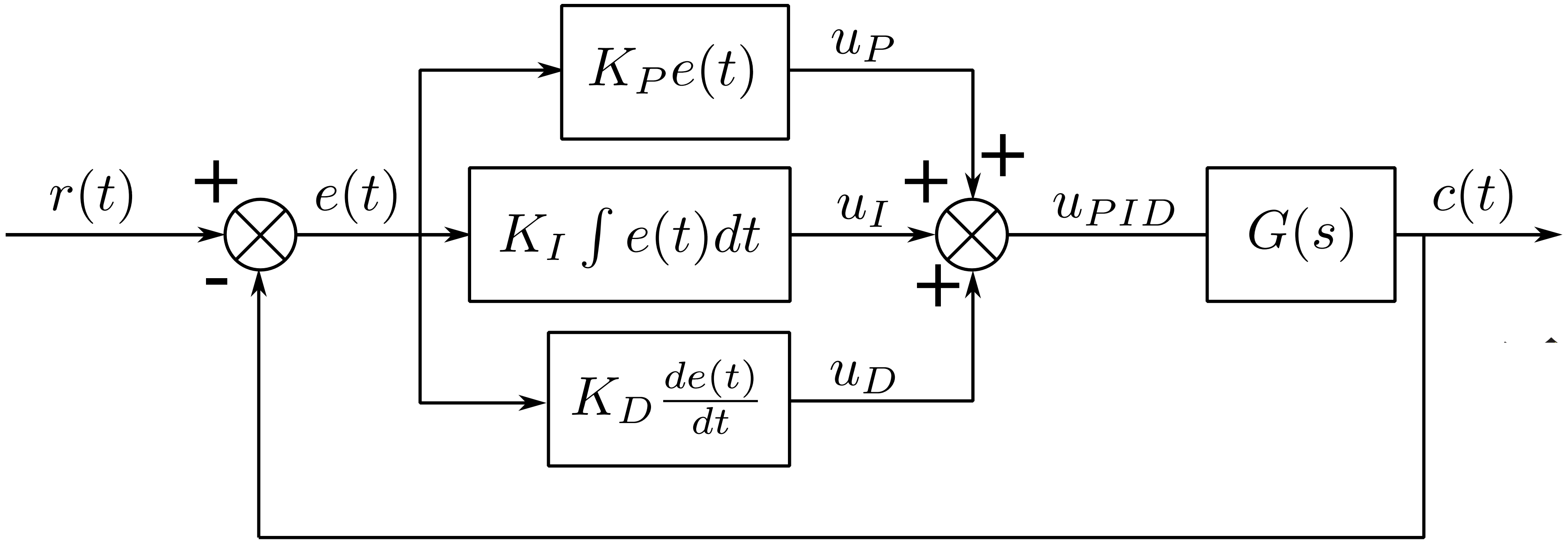


- The proportional gain K_P, which alone, is like a simple gain controller that scales the error signal $u_p(t) = K_p e(t)$.
- The integral gain K_I, which scales the integral of the error signal $u_I = K_I \int e(t)dt$
- The derivative gain K_D, which scales the derivative of the error signal $u_D = K_D \frac{de(t)}{dt}$



- If only the proportional gain K_p is used, then we call the controller a P controller.
  - This is the simple gain controller.
- If $K_P, K_I \neq 0, K_D = 0$, then a PI controller
- If $K_P, K_D \neq 0, K_I = 0$, then a PD controller
- If $K_P, K_I, K_D \neq 0$, then a PID controller
- We will treat the PID controller in detail in this course.

PID Feedback Control Block Diagram (figurative)



- When designing a feedback controller for a given system, the steps are:
  1. Select the form of the controller
     - We will explore the use of standard controllers (PI, Lag, PD, Lead, PID, Lag-Lead). Each one has its own pros/cons.
     - Each controller form produces a general set of behaviors on a system, an experienced control designer will know which to select for which application.
  2. Tune or compute the gains of the chosen controller
     - We will learn how to achieve this using alternate methods:
     1. A graphical technique of placing the closed-loop pool on the s-plane (Part II: Root-Locus)
     2. A graphical technique of compensating the system via the frequency response (Part III: Bode-Plots)
     3. A numerical technique of computing the gains using Linear Algebra (Part IV: State-Space)
  3. Test (simulate and/or experiment) and repeat until we are satisfied.

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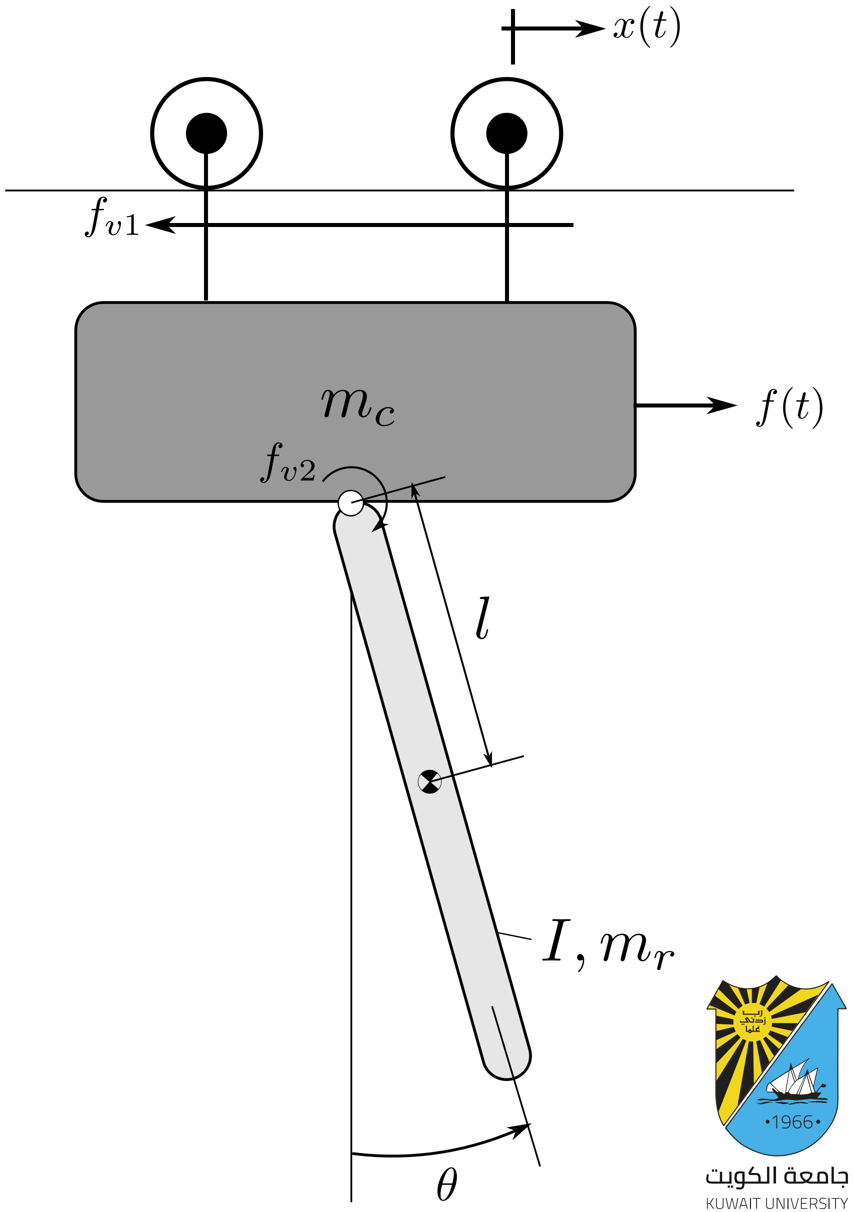
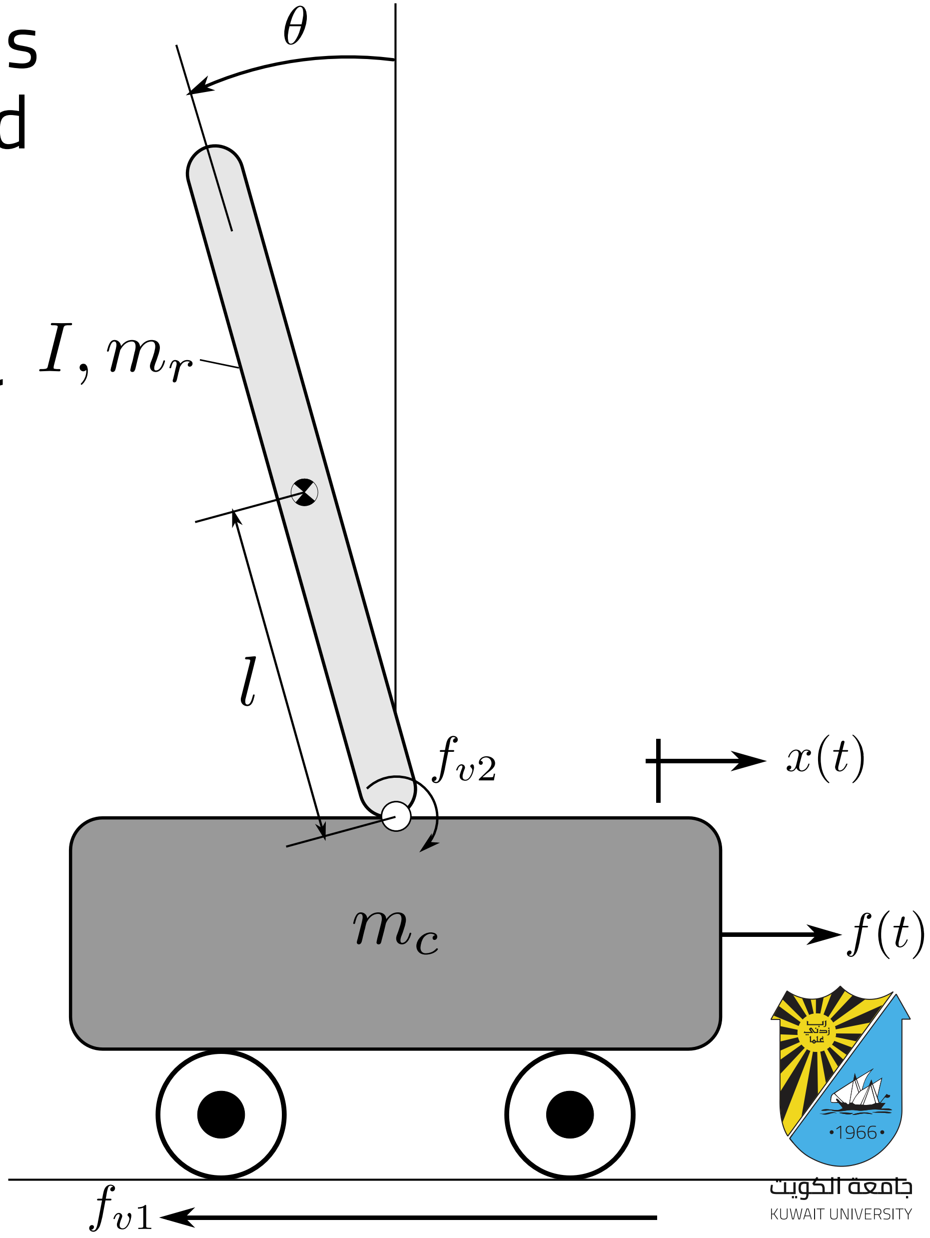
- Consider the simple crane system, what happens to the natural response, if the pendulum is released from $\theta \neq 0^o$?
- What happens to the response if $f_{v2} = 0$?
- Why design a controller for this system?
  - Increase speed of transient response
  - Minimize oscillation in transient response
  - Reject disturbances from the environment
  - Reduce steady-state errors
- Note that we would want to control $x(t)$ as well as $\theta(t)$



- If we look at an inverted pendulum, what happens to the natural response if the system is perturbed (non-zero i.c.)?
- Note that both the crane and inverted pendulum are structurally similar but the "Zero" position, or state, are defined differently.
  - $\Theta_{pendulum} = \theta_{crane} \pm \pi$
- Why design a controller for this system?
  - **Stabilize** the system
  - Once stable, achieve a desired transient and steady-state performance level.



Stability

- A **linear, time-invariant** (LTI) system can either be
  1. Stable (a.k.a Asymptotically Stable)
  2. Marginally Stable
  3. Unstable
- In other definitions, the classification "**stable** systems", include both marginally and asymptotically stable systems, in this course we will use Nise's classification above: *Stable*, *Marginally Stable* or *Unstable*.

- An **LTI** system is said to be **stable** if its response to an ***impulse*** (impulse-response) approaches zero as time approaches infinity.
- Note that the response is evaluated for an **IMPULSE** input. In other words, the stability measures the tendency of the natural response to decay.

- LTI Systems **Stability** conditions:
  - In the time domain $c(t) = c_{forced}(t) + c_{natural}(t)$:
    - $\lim_{t\to\infty} c_{natural}(t) = 0$
  - In the s-plane:
    - Its poles must lie in the left half of the s-plane.
  - In the transfer function:
    - The roots of its characteristic polynomial must all have negative real parts.

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- LTI Systems **Instability** conditions:
  - In the time domain $c(t) = c_{forced}(t) + c_{natural}(t)$:
    - $c_{natural}(t)$ grows as $t \to \infty$
  - In the s-plane:
    - It has a pole in the right half plane
    - It has a pole of multiplicity greater than 1 on the imaginary axis
  - In the transfer function:
    - At least one of the roots of its characteristic polynomial has a <u>positive real part</u>, or
    - There is an imaginary root with multiplicity greater than 1

- LTI Systems **Marginal Instability** conditions:
  - In the time domain $c(t) = c_{forced}(t) + c_{natural}(t)$:
    - $c_{natural}(t)$ neither grows nor decays as $t \to \infty$ (Pure Oscillation)
  - In the s-plane:
    - It has a pole/s on the imaginary axis with multiplicity no greater than 1
  - In the transfer function:
    - There is an imaginary root with multiplicity no greater than 1

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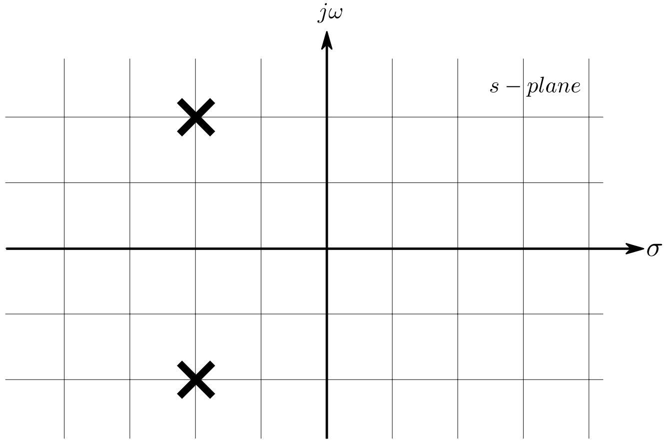
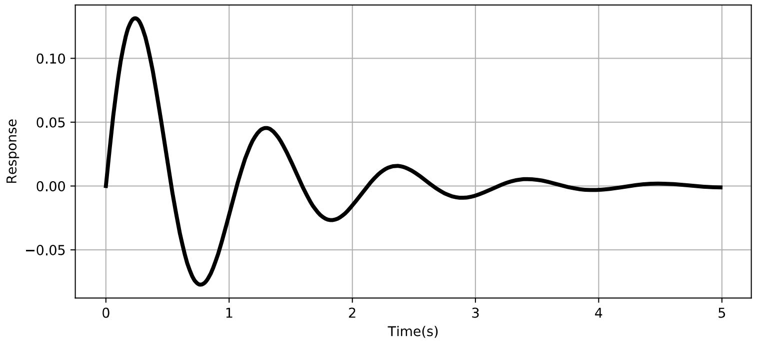
Stability

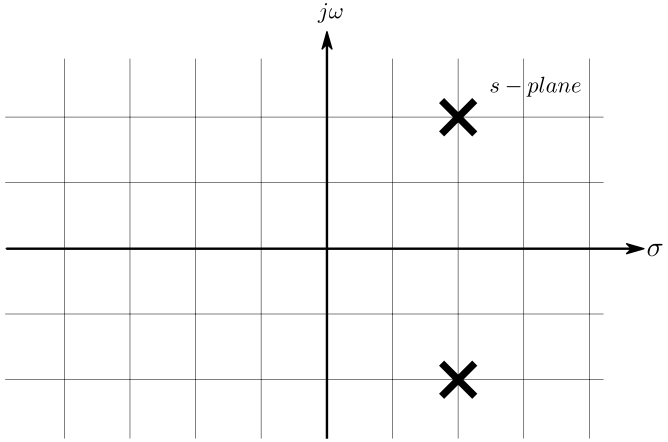
$$R(s) \rightarrow \boxed{G(s) = \frac{1}{s^2+2s+36}} \rightarrow C(s)$$

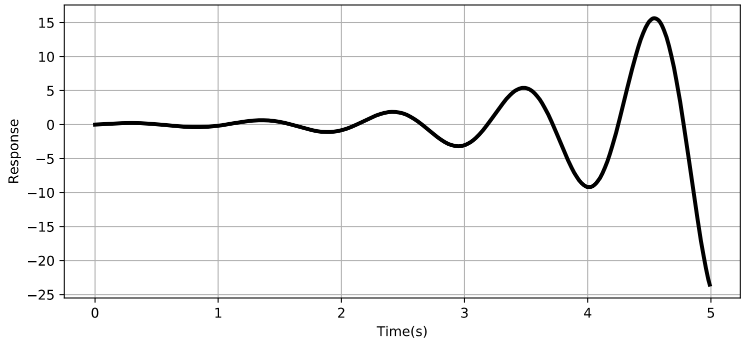




Stable

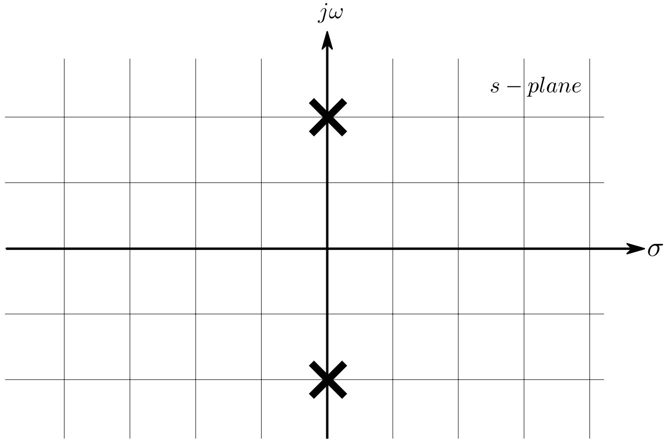
$$R(s) \rightarrow \boxed{G(s) = \frac{1}{s^2-2s+36}} \rightarrow C(s)$$

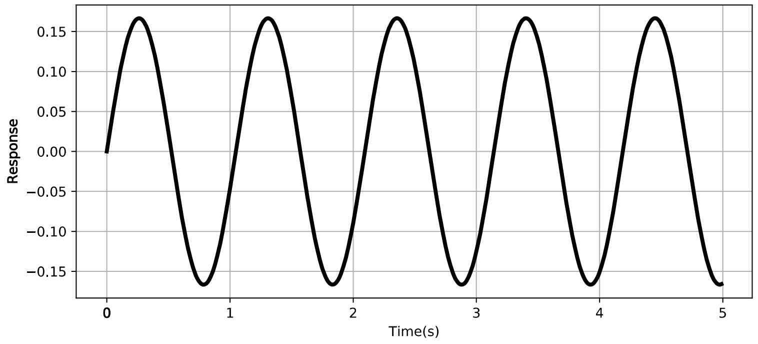




Unstable

$$R(s) \rightarrow \boxed{G(s) = \frac{1}{s^2+36}} \rightarrow C(s)$$





Marginally Stable

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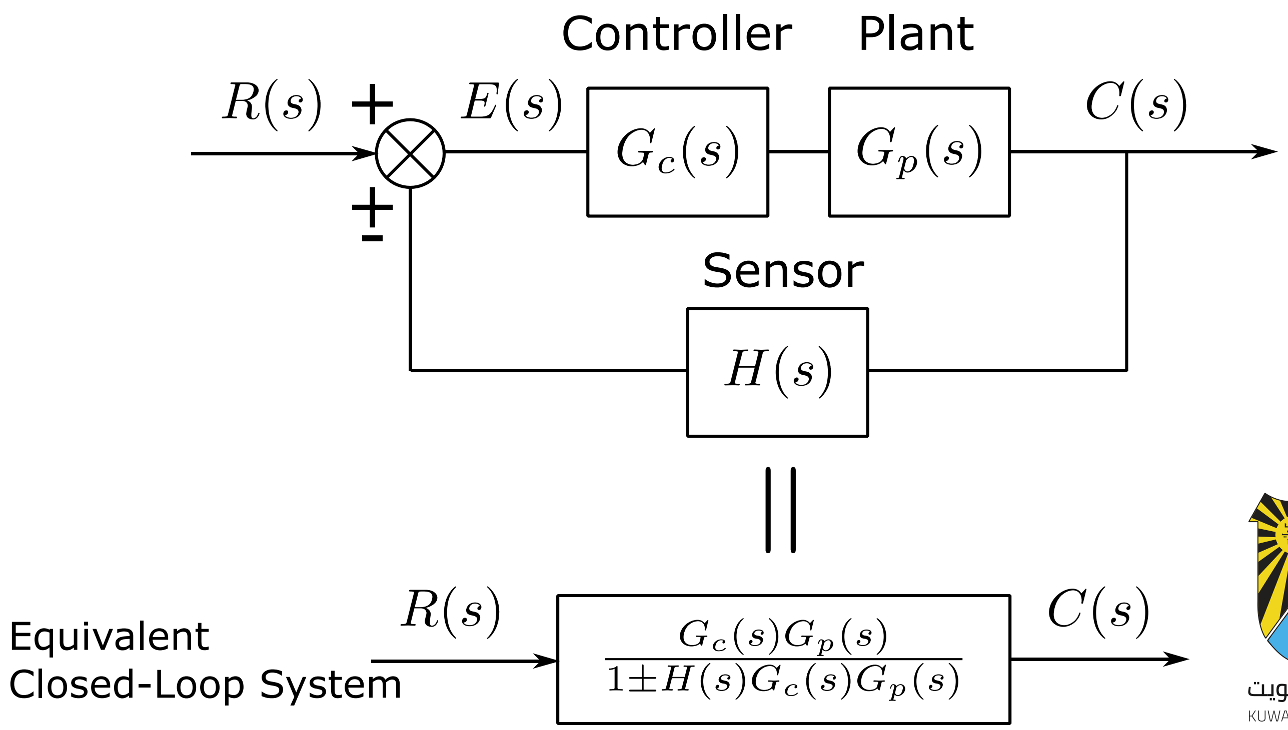
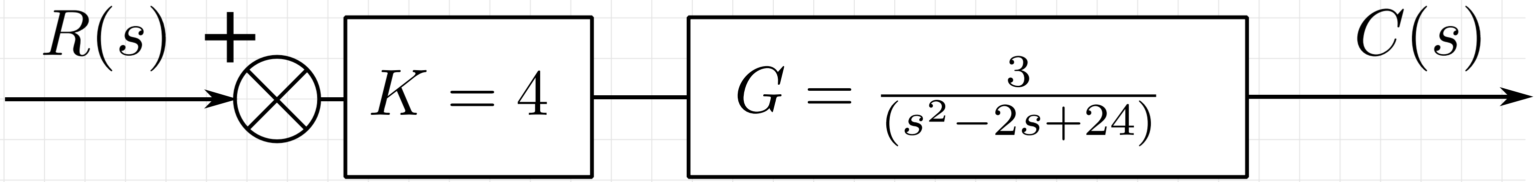
- The prior definitions were for LTI systems stability, but how do we apply them to feedback control systems?
- We look at the poles of the equivalent closed-loop system and apply the same conditions
- The characteristic polynomial of the closed-loop system:
  - $1 \pm H(s)G_c(s)G_p(s) = 0$

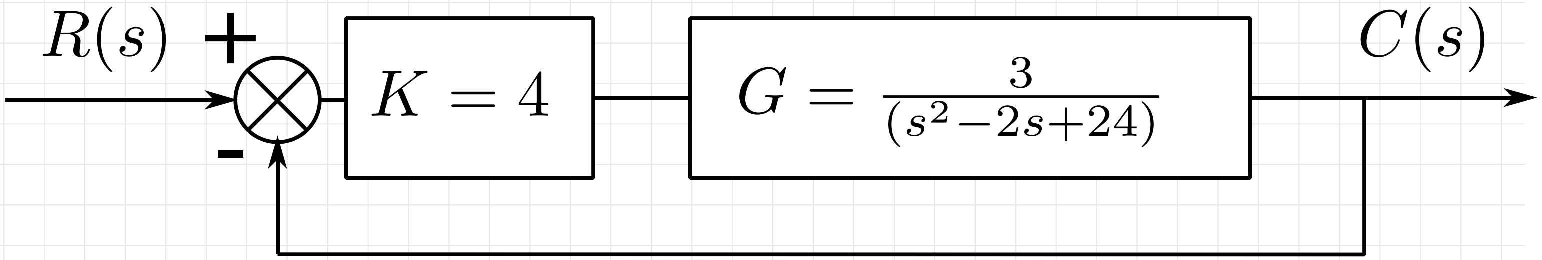


Determine the stability of the system shown.



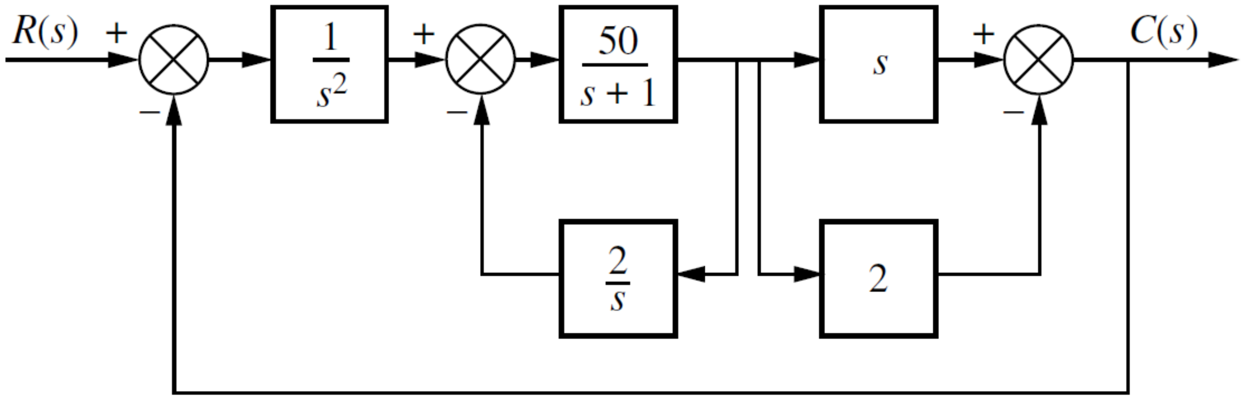
Example 7

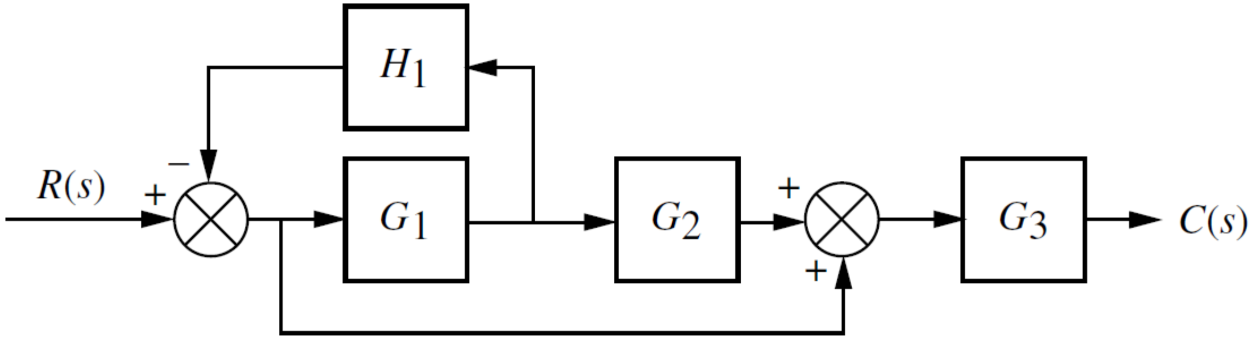
Determine the stability of the system shown.

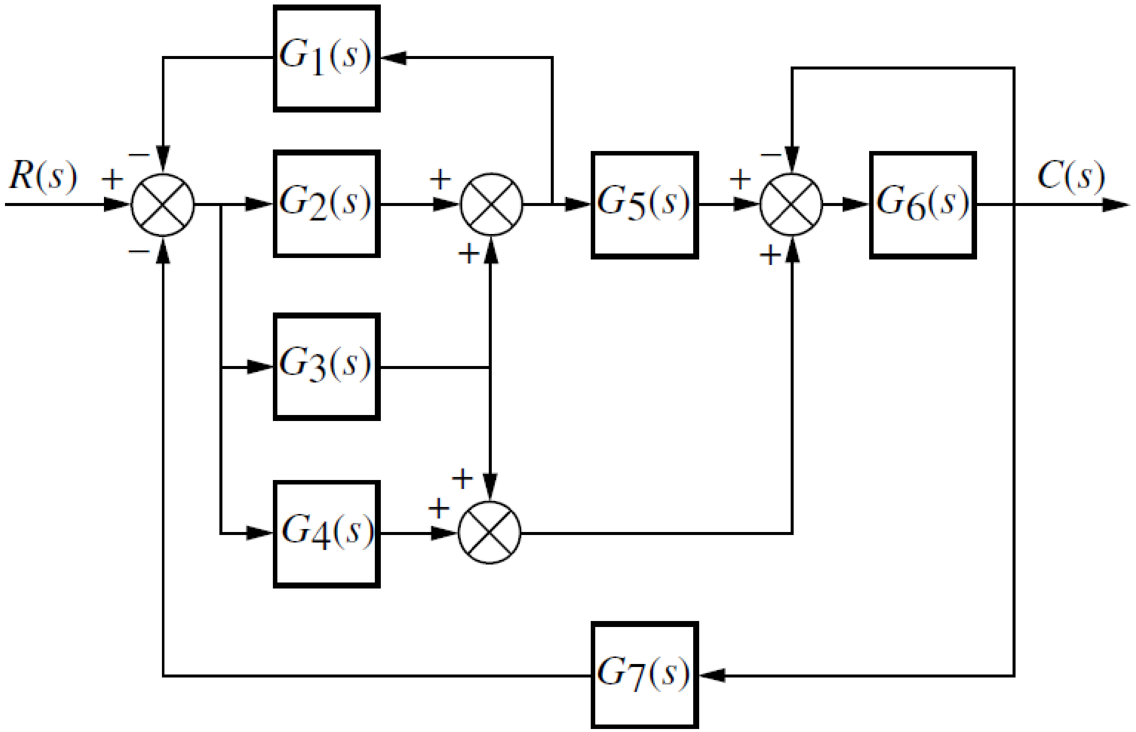


Find the equivalent transfer function for the following systems.

Nise: 5-1, 5-2, 5-3







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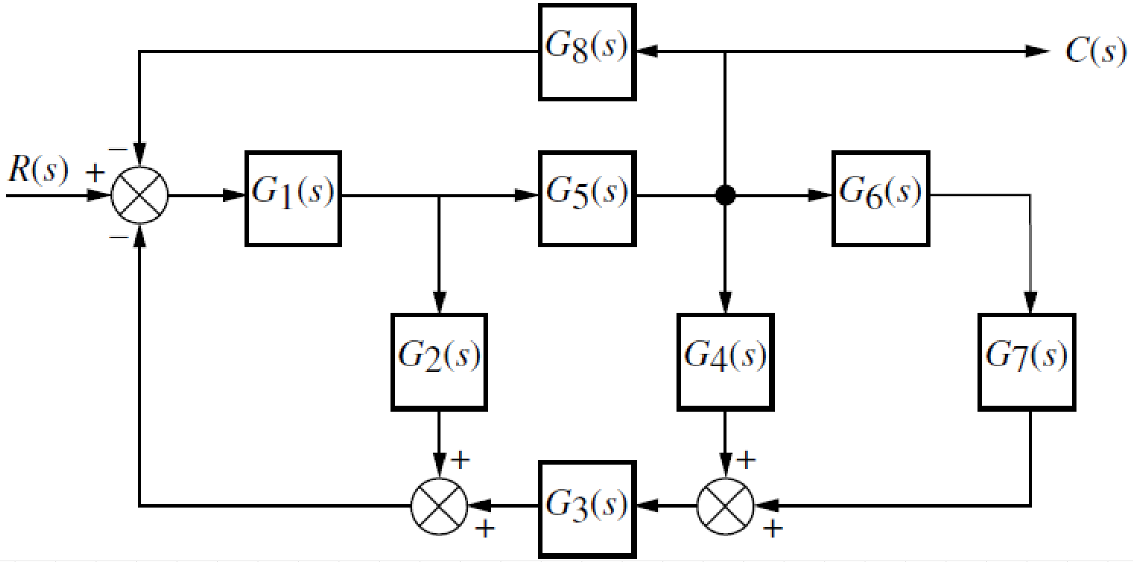
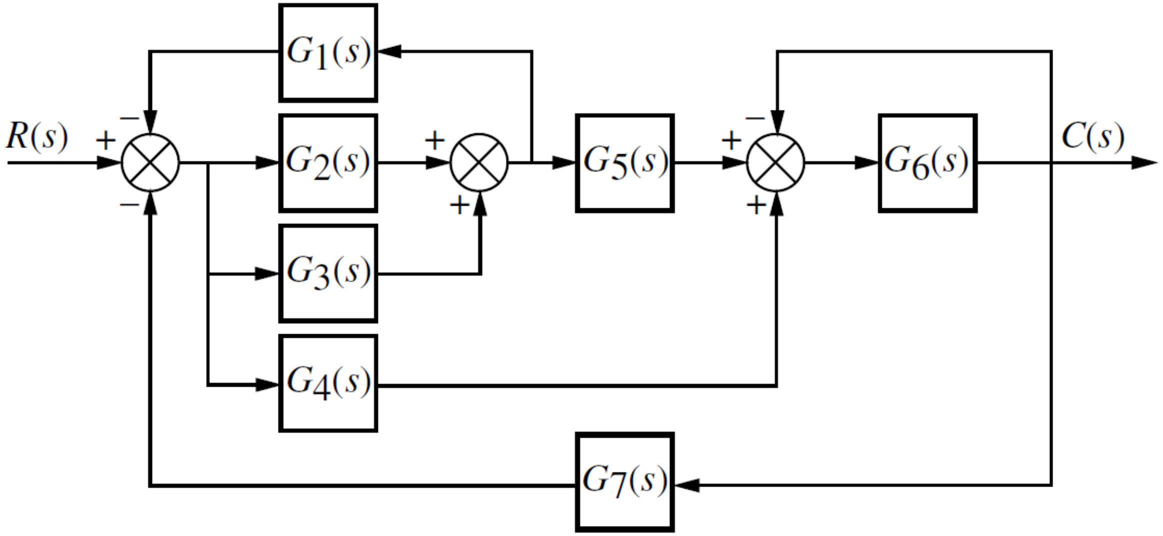
Practice Problem 2

Find the equivalent transfer function for the following systems.

Nise: 5-6, 5-10





Practice Problem 3

For each of the systems shown, determine the range of values (if they exist), for the proportional gain K that makes the system stable.