ME 417 - Homework #2

Control of Mechanical Systems - Fall 2020

Homework Due: Thu, 07 Jan 2021 23:59

Complete the following problems and submit a hard copy of your solutions. You are encouraged to work together to discuss the problems but submitted work **MUST** be your own. This is an **individually** submitted assignment.

Problem 1

Stability Analysis (20pts)

For each of the following systems, find the poles of the system and determine the system's stability classification. Justify your answer.

a. $G(s) = \dfrac{s - 19}{s^2 + 2s + 6}$

b. $G(s) = \dfrac{s^2 - 25}{(s^2 + 2s + 20)\,(s^2 + 3s + 100)}$

c. $G(s) = \dfrac{s^2 + 20}{(s + 10)\,(s^2 + 100)}$

d. $G(s) = \dfrac{s\,(s - 19)}{s^2 - 5s + 20}$

Solution:

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a.

The poles of the system are $\begin{bmatrix} -1.0 - 2.24i \\ -1.0 + 2.24i \end{bmatrix}$

The system is stable.

There are no poles in the RHP plane nor on the imaginary axis

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b.

The poles of the system are $\begin{bmatrix} -1.5 - 9.89i \\ -1.5 + 9.89i \\ -1.0 - 4.36i \\ -1.0 + 4.36i \end{bmatrix}$

The system is stable.

There are no poles in the RHP plane nor on the imaginary axis

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c.

The poles of the system are $\begin{bmatrix} -10.0 \\ -10.0i \\ 10.0i \end{bmatrix}$

The system is marginally stable

There are poles in the imaginary axis with multiplicity 1 @ $\begin{bmatrix} -10.0i \\ 10.0i \end{bmatrix}$

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d.

The poles of the system are $\begin{bmatrix} 2.5 - 3.71i \\ 2.5 + 3.71i \end{bmatrix}$

The system is unstable.

There are poles in the RHP plane @ $\begin{bmatrix} 2.5 - 3.71i \\ 2.5 + 3.71i \end{bmatrix}$

Problem 2

Second Order Approximation (20pts)

For each of the following systems, determine if a 2nd-order approximation is valid. Justify your answer.

a. $G(s) = \dfrac{200s + 200}{(s+2)(s^2+2s+10)}$

b. $G(s) = \dfrac{20s + 220}{(s+10)(s^2+4)}$

c. $G(s) = \dfrac{45}{(s+11)(s^2+2s+40)}$

d. $G(s) = \dfrac{s + 10}{(s+1)(s^2+10s+200)}$

Hint: 5 times rule of thumb for higher order poles, or if zeros are present, compare the magnitude of the higher order term.

Solution:

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a.

First, finding the partial fraction expansion form. $G(s) = \dfrac{20.0\,(s+10.0)}{s^2+2.0s+10.0} - \dfrac{20.0}{s+2.0}$

The poles are close and the residues have similar magnitudes, 2nd order approximation can not me made.

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b.

First, finding the partial fraction expansion form. $G(s) = -\dfrac{0.192\,(s-114.0)}{s^2+4.0} + \dfrac{0.192}{s+10.0}$

The third pole is more than five times further to the left compared to the dominant poles. And the zero as well. A second-order approximation can be made.

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c.

First, finding the partial fraction expansion form. $G(s) = -\dfrac{0.324\,(s-9.0)}{s^2+2.0s+40.0} + \dfrac{0.324}{s+11.0}$

The third pole is more than five times further to the left compared to the dominant poles. Second-

Order approximation can be made

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d.

First, finding the partial fraction expansion form. $G(s) = -\frac{0.00524\,(9.0s - 110.0)}{s^2 + 10.0s + 200.0} + \frac{0.0471}{s + 1.0}$

The dominant pole is @ -1. The higher order poles and the zero are more than five times further to the left. A second order approximation can not be made.

Problem 3

Stability and Feedback Form (25pts)

On February 18th, 2021, NASA's Mars 2020 Preserverance Rover is planned for landing on Mars. A landing animation video can be seen here `http://bit.ly/Preserverance`

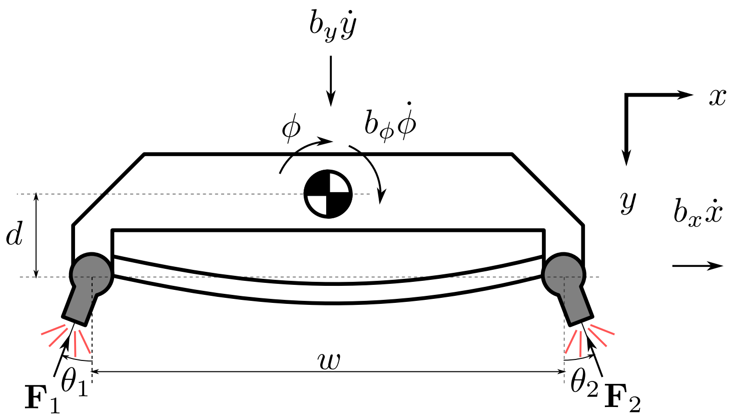
The Sky Crane, which is responsible for gracefuly landing the rover on the designated site, can be modeled as a rigid body with thruster forces being controlled by a gimbal to produce thrust at angles θ as shown.

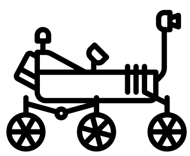
a. Derive the equations of motion for the system (3 directions)

b. List the transfers functions (just the input output signal expression) required to express all the dynamics of the system.

c. With the thrust values are constant with $F_1 - F_2 = \Delta F > 0$, and the thrust angles are equal but varying $\theta_1 = \theta_2 = \theta$. Derive the transfer function relating θ to $\dot{\phi}$, the angular velocity of the skycrane.

d. Assess the stability of this system $\frac{\dot{\Phi}}{\Theta}$





Given $m = 500kg, I = 1125.0kg \cdot m^2, d = 1m, w = 3m, g = 3.711m/s^2, b_y = 25N \cdot s/m, b_x = 10N \cdot s/m, b_\phi = 12N \cdot s$

Solution:

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a.

$\rightarrow + \sum F_x = F_1 sin\theta_1 - F_2 sin\theta_2 - b_x\dot{x} = m\ddot{x}$

$\downarrow + \sum F_y = -F_1 cos\theta_1 - F_2 cos\theta_2 + mg_M - b\dot{y} = m\ddot{y}$

$cw + \sum M_G = -F_1 sin\theta_1 \cdot d + F_2 sin\theta_2 \cdot d + F_1 cos\theta_1 \cdot \frac{w}{2} - F_2 cos\theta_2 \cdot \frac{w}{2} - b_\phi\dot{\phi} = I_G\ddot{\phi}$

b. We have thrust F_1, F_2 as well as thrust angles θ_1, θ_2 as inputs. As outputs we have the three directions x, y, ϕ, each of the input/outputs can be represented by a transfer function. With some assumptions(such as $\Delta F = F_1 - F_2$) the relationships can be further reduced.

c.

With the assuming that $\theta_1 = \theta_2 = \theta = \theta$, from the EOM we get, after linearizing and ignoring initial conditions:

$[\text{U+21BB}] + \sum M_G = \Delta F\theta \cdot d - b_\phi\dot{\phi} = I_G\dot{\phi}$

$$G(s) = \frac{d\Delta F}{I_G s + b_\phi} = \frac{\Delta F}{1125.0s + 12}$$

d. The poles of the system are $\left[-0.0107\right]$

The system is stable.

There are no poles in the RHP plane nor on the imaginary axis

Problem 4

Block Diagram Manipulation (15pts)

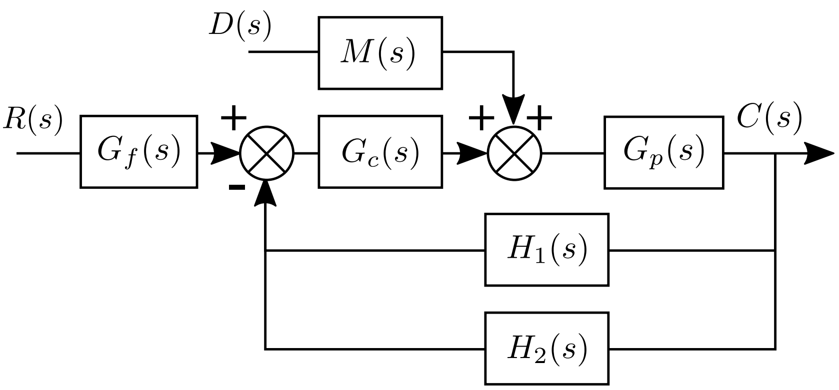
Given the following block diagram, with

$G_f = 12,\ G_c = 0.5s + 5.0,\ G_p = \frac{6.0}{1.0s^2 + 10.0},\ H_1 = \frac{1.0}{s},\ H_2 = 5,\ M = 5.0s$

a. Derive the transfer function that relates the reference $R(s)$ to the output $C(s)$

b. Derive the transfer function that relates the reference $R(s)$ to the input to the plant $U(s)$

c. Derive the transfer function that relates the disturbance (noise) $D(s)$ to the output $C(s)$



Solution:

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a.

This is the closed loop transfer function of the system

$$G_{cl} = \frac{C}{R} = G_f \frac{G_c G_p}{1 + G_c G_p (H_1 + H_2)} = \frac{36.0s^2 + 360.0s}{1.0s^3 + 15.0s^2 + 163.0s + 30.0}$$

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b.

With $G_{cl} = \frac{C}{R}$, we observe that $\frac{U}{R} = G_{cl}\frac{1}{G_p} = G_f \frac{G_c}{1 + G_c G_p (H_1 + H_2)} = \frac{6.0s^4 + 60.0s^3 + 60.0s^2 + 600.0s}{1.0s^3 + 15.0s^2 + 163.0s + 30.0}$

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c.

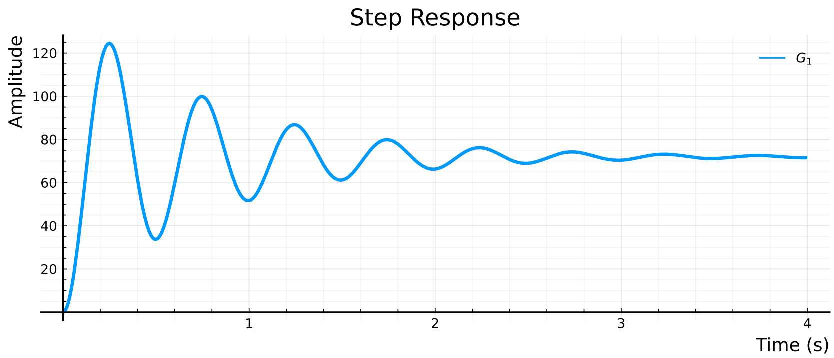
The disturbance signal forward path is only through MG_p. The feedback path is the same as before

$$\frac{C}{D} = M \frac{G_p}{1 + G_c G_p (H_1 + H_2)} = \frac{30.0s^2}{1.0s^3 + 15.0s^2 + 163.0s + 30.0}$$

Problem 5

Derive System from Response (20pts)

The following is the response of a second-order system to a step input $u = 18$



Derive, approximately, the transfer function of the system

Solution:

Peak time can be approximated from the response and this gives ω_d. $T_p = \frac{\pi}{\omega_d} = 0.249 \longrightarrow \omega_d = 12.6$

The percent overshoot can give us the damping coefficient ζ.

$\%OS = 0.729 \longrightarrow \zeta = \frac{-ln(\%OS)}{\sqrt{\pi^2 + ln(\%OS)^2}} = 0.1$

We know from F.V.T that $c(\infty) = |u|K\omega_n^2/\omega_n^2$, where K is the system gain s.t. $G(s) = K\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$K = 4$

And so the transfer function of the system is $G(s) = 4\frac{12.7^2}{s^2 + 2 \cdot 0.1 \cdot 12.7s + 12.7^2} = \frac{645.0}{1.0s^2 + 2.54s + 161.0}$