ME 417 - Homework #2

Control of Mechanical Systems - Summer 2020

Homework Due: Sun, 25 Oct 2020 23:59

Complete the following problems and submit a hard copy of your solutions. You are encouraged to work together to discuss the problems but submitted work **MUST** be your own. This is an **individually** submitted assignment.

Problem 1

Stability Analysis (20pts)

For each of the following systems, find the poles of the system and determine the system's stability classification. Justify your answer.

a. $G(s) = \dfrac{s+19}{s^2+5s+9}$

b. $G(s) = \dfrac{s-5}{s^2+3s+100}$

c. $G(s) = \dfrac{s^2+20}{(s+3)\,(s^2+2s+20)}$

d. $G(s) = \dfrac{s\,(s-19)}{s^2-5s+20}$

Problem 2

Second Order Approximation (20pts)

For each of the following systems, determine if a 2nd-order approximation is valid. Justify your answer.

a. $G(s) = \dfrac{s+2}{(s+3)\,(s^2+2s+10)}$

b. $G(s) = \dfrac{s+1.99}{(s+1.9)\,(s^2+3s+10)}$

c. $G(s) = \dfrac{s+45}{(s+50)\,(s^2+5s+40)}$

d. $G(s) = \dfrac{s+1}{(s+3)\,(s^2+10s+200)}$

Hint: 5 times rule of thumb for higher order poles, or if zeros are present, compare the magnitude of the higher order term.

Problem 3

Stability and Feedback Form (20pts)

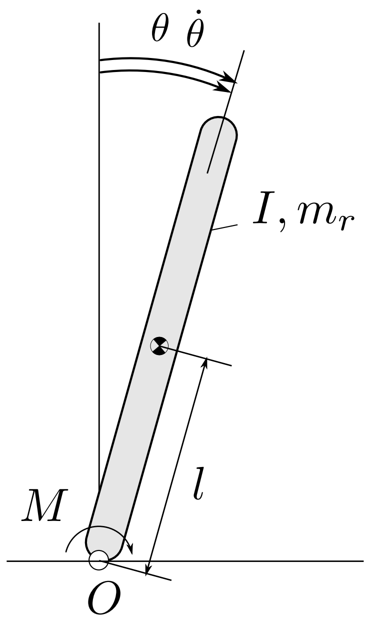
Given the following system representing a simple inverted pendulum with $m_r = 1.3kg, l = 0.3cm$. We wish to design a feedback controller to keep it balanced

a. Derive the equation of motion and the transfer function relating M to Θ

b. Analyze the stability of the plant

c. Draw a feedback diagram, highlighting the reference, error, plant input, plant output and the units of each signal. Include the sensor in the feedback block diagram. Assume the sensor is a first-order system with $\tau = 100\mu s$ with unity dc gain ($H(s \to 0) = 1$). What is the value of r, the reference?

d. If the controller chosen is $G_c = 30s + 60$, analyze the stability of the closed-loop system.



Problem 4

Block Diagram Manipulation (20pts)

Given the following block diagram, with

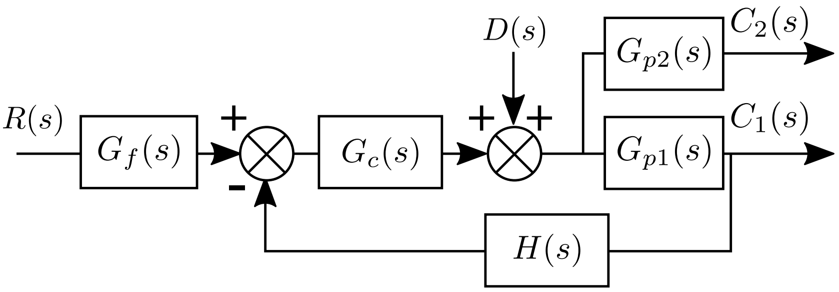
$G_f = 12,\ G_c = 3.0s + 16.5,\ G_{p1} = \dfrac{6.0}{1.0s^2 + 8.0s + 10.0},\ G_{p2} = \dfrac{1.0s + 1.0}{1.0s^2 + 8.0s + 20.0},\ H = \dfrac{1.0}{s}$

a. Derive the transfer function that relates the reference $R(s)$ to the output $C_1(s)$

b. Derive the transfer function that relates the reference $R(s)$ to the input to the plant $U(s)$

c. Derive the transfer function that relates the reference $R(s)$ to the output $C_2(s)$

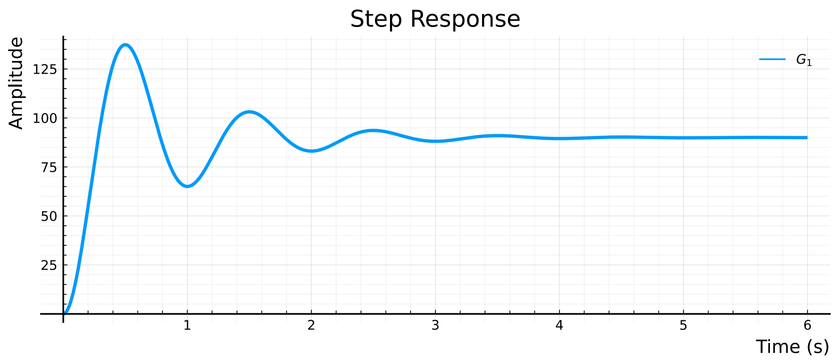
d. Derive the transfer function that relates the disturbance (noise) $D(s)$ to the output $C_1(s)$



Problem 5

Derive System from Response (20pts)

The following is the response of a second-order system to a step input $u = 9$



Derive, approximately, the transfer function of the system