ME 417 - Num Assignment #4

Control of Mechanical Systems - Fall 2020

Num Assignment Due: Wed, 03 Mar 2021 23:59

Complete the following problems and submit your work as a working notebook and a saved pdf copy. *You can complete the numerical assignment using **Julia**, **Python** or **MATLAB**, and submit your work as a Jupyter Notebook (or MATLAB Livescript) + a pdf export*

Provide response plots as relevant, ensure that you label the figures, the axes, title plots and legends. Any controller design specifications given, should be met by observing the time response of the system. The Numerical Lessons provided will aid greatly in carrying out this assignment.

Collaboration is only allowed within the group members.

Problem 1

Controller Design via Pole Placement (20pts)

Given the state space model $\dot{\mathbf{x}} = \begin{bmatrix} 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \\ -23.0 & -5.0 & -1.0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0.0 \\ 1.0 \\ 0.0 \end{bmatrix} u$

$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{x}$

Design a controller via pole placement, to achieve the following performance specifications

- $T_s = 2s$

- $\zeta = 0.866$

- Zero steady-state error

Hint: Design the full-state feedback controller first for the transient response, then add the integral controller.

Problem 2

From Block diagram to State-Space (25pts)

Given the following block diagram, with

$G_f = 10$, $G_c = 20$, $G_{p1} = \dfrac{2.0}{1.0s^2 + 10.0s + 20.0}$, $G_{p2} = \dfrac{5.0}{1.0s^2 + 5.0s + 25.0}$, $H = 1.0s + 6.0$

a. (50%) Convert this system into state-space form, accounting for the multiple inputs and outputs.

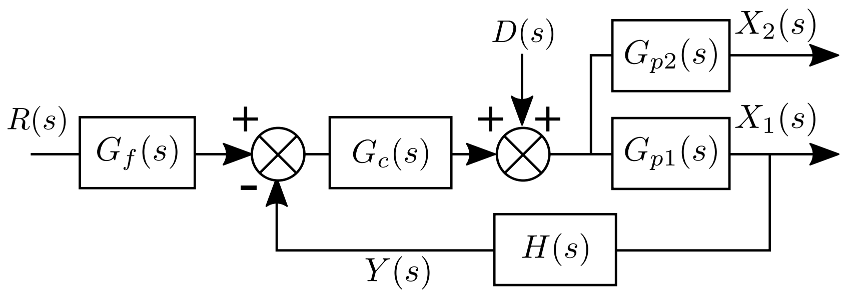
There are several ways to transfer the block diagram into state-space, but let's try to remodel the state-space from the beginning.

- The plant transfer functions give us the plant dynamics $\dot{x} = Ax + Bu$ and $\mathbf{u}$=[u d]^T"
- The sensor $H(s)$ relates the state to the output: $y = Cx$
- The feedback defines the control law: $U(s) = G_c(Gf * R(s) - Y(s))$, which can be substituted into the plant dynamics to find the closed-loop form.

c. (50%) Simulate the response of the system using basic numerical integration. To a

- Reference: r(t) = 5
- Random Noise, normally distributed, zero mean with $\sigma = 2$: d(t) = 2 * rand()

Note you already have the model of the system from the state-space, and this is a linear system. You can calculate the derivative and propagate the system directly.



Problem 3

Controlling a non-linear system using linear tools (20pts)

Given a 2-DOF system with the following nonlinear equations of motion

$$3x_1 - 2x_2 + 12\dot{x}_1\dot{x}_2 - 11\dot{x}_1 + 20\ddot{x}_1 = 5f_1 + 3f_2$$

$$2x_1 + 0.9x_2^2 + 12\dot{x}_2 + 11\ddot{x}_2 = f_1 + 5f_2$$

a. (25%) Put the nonlinear equations in a vector form $\dot{x} = f(x, t, u)$.

b. (25%) Simulate the natural response of the nonlinear system with the following initial conditions $x_0 = \begin{bmatrix} 0.0 & 0.05 & 0.05 & 0.0 \end{bmatrix}^T$ for $t = 0 : 15s$

c. (25%) Linearize the system (ignore the square and coupling terms) and put the system into the state space form $\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{B}u$

d. (25%) Design a full-state feedback controller using pole placement method, to stabilize the system. Test your controller on the linearized system first using lsim, then simulate the closed-loop system response using the linear and nonlinear model in a numerical integration setup. Assume you can control both inputs (What is the dimension of the gain matrix?)

Your controller is a regulator, so observe the closed-loop response with $r = 0$ and initial conditions $x_0 = \begin{bmatrix} 15.0 & 0.7 & 0.5 & 10.0 \end{bmatrix}^T$

Plot the response of all the 4 states, for both the linear and non-linear system in one subplot.

Can you control the non-linear (actual) system with the same controller? Explain why or why not.

Problem 4

Multi-Input Multi-Output Control (35pts)

On February 18th, 2021, NASA's Mars 2020 Preserverance Rover is planned for landing on Mars. A landing animation video can be seen here `http://bit.ly/Preserverance`

The Sky Crane, which is responsible for gracefuly landing the rover on the designated site, can be modeled as a rigid body with thruster forces being controlled by a gimbal to produce thrust at angles θ as shown.

a. (20%) Derive the equations of motion for the system (3 directions)

The thruster angles and thrust forces are all independent input variables now.

We wish to design a state-space controller to help the SkyCrane navigate to a desired position in space.

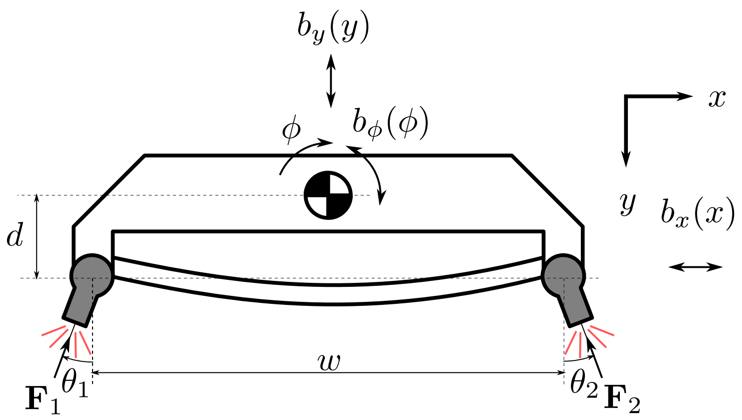
b. (30%) Linearize the system and put it in state-space form, then design a controller via pole placement to achieve the following transient response characteristics

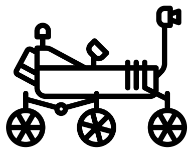
- $T_s = 2\,s$
- $\omega_d = 20\,rad/s$
- Zero steady-state error

c. (25%) Apply your controller on the nonlinear system (numerical integration)

d. (25%) In reality, there are limits to the inputs. The thrust can not be negative, and there is a minimum thrust once the engine is ignited. The thruster gimbal can only operate within a specific angular range. Repeat part c with the following saturation limits:

- Thrusts: $100N < F < 2000N$
- Gimbal Angle Range: $-45^o < \theta < 45^o$





Given $m = 500kg, I = 1125.0kg \cdot m^2, d = 1m, w = 3m, g = 3.711m/s^2, b_y = 12\dot{y}^2 + 4\dot{y}N \cdot s/m, b_x = 4\dot{x}^2 + 3\dot{x}N \cdot s/m, b_\phi = 12\dot{\phi}^2 + 4\dot{\phi}N \cdot s$