ME 417 - Homework #1

Control of Mechanical Systems - Summer 2020

Homework Due: Sun, 18 Oct 2020 23:59

Complete the following problems and submit a hard copy of your solutions. You are encouraged to work together to discuss the problems but submitted work **MUST** be your own. This is an **individually** submitted assignment.

Problem 1

System Modeling (25pts)

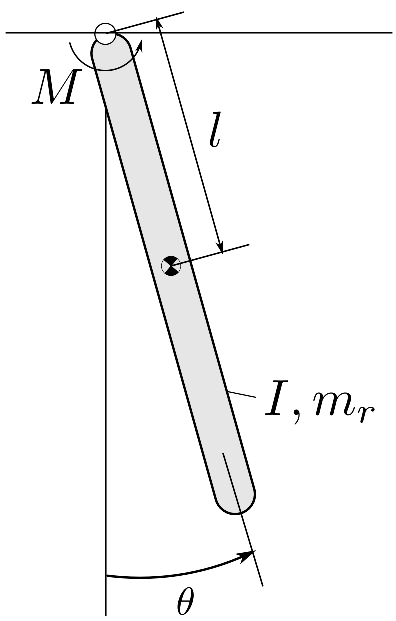
Given the following system

a. Derive the equations of motion for the system

b. Find the transfer function that relates M to $\dot{\theta}$

c. Find the pole locations of the transfer function derived in part (b)

Given: $l = 1m$, $m_r = 2.5kg$



Solution:

a.

By summing the moment about O, we can derive the equation of motion

$\sum M_o = M - m_r gsin\theta \cdot l = I_o\ddot{\theta}, I_o = \frac{4}{3}m_r l^2$

$\frac{4}{3}m_r l^2\ddot{\theta} + m_r gsin\theta \cdot l = M$, linearizing with small angle approximation assumption: $sin\theta \approx \theta$, we get

$0.208\ddot{\theta} + 24.5\theta = M$

b.

From the equations of motion we know that the transfer function

$\frac{\Theta(s)}{M(s)} = \frac{1}{1.33l^2 m_r s^2 + 9.81m_r} = \frac{1}{3.325s^2 + 24.525}$

To get the transfer function $\frac{\dot{\Theta}(s)}{M(s)}$ we differentiate the signal, by multiplying by s

$$G(s) = \frac{\dot{\Theta}(s)}{M(s)} = \frac{\Theta(s)}{M(s)}s = \frac{s}{3.325s^2 + 24.525}$$

c.

To find the poles of the transfer function, we find the roots of its denominator.

Poles are located at $\begin{bmatrix} 0.0 - 2.72i \\ \\ 0.0 + 2.72i \end{bmatrix}$

Problem 2

System Modeling (25pts)

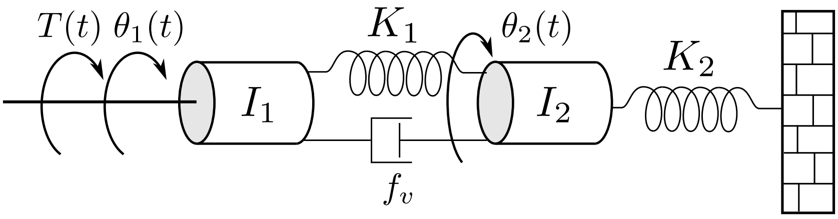
Given the following system

a. Derive the equations of motion for the system

b. Find the transfer function that relates T to θ_2

c. Find the steady state value of θ_2 given a step-input $T(t) = 2$

Given: $I_1 = 0.5kg \cdot m^2$, $I_2 = 0.25kg \cdot m^2$, $K_1 = 200N/m$, $K_2 = 300N/m$, $f_v = 50N \cdot s/m$



Solution:

a.

By using the impedance method

$[I_1s^2 + f_vs + K_1]\Theta_1(s) - [f_vs + K_1]\Theta_2(s) = T(s)$

$-[f_vs + K_1]\Theta_1(s) + [I_2s^2 + f_vs + K_1 + K_2]\Theta_2(s) = 0$

b.

Decoupling the EOM using Cramer's rule, we can find $G_2(s) = \dfrac{\theta_2(s)}{T(s)}$

$$G_2(s) = \frac{-a_2y_1}{\Delta} = \frac{-K_1 + f_vs}{I_1I_2s^4 + I_1K_1s^2 + I_1K_2s^2 + I_1f_vs^3 + I_2K_1s^2 + I_2f_vs^3 + K_1K_2 + 4K_1f_vs + K_2f_vs}$$

$$G_2(s) = \frac{-K_1 + f_vs}{I_1I_2s^4 + I_1K_1s^2 + I_1K_2s^2 + I_1f_vs^3 + I_2K_1s^2 + I_2f_vs^3 + K_1K_2 + 4K_1f_vs + K_2f_vs}$$

c.

To find the steady state value, we apply the final value theorem

$$c_{ss}(t) = \lim_{s\to 0} s\Theta_2(s) = \lim_{s\to 0} sG(s)R(s) = \lim_{s\to 0} \frac{2\,(50s - 200)}{0.125s^4 + 37.5s^3 + 300.0s^2 + 55000s + 60000} = -\frac{1}{150}$$

Problem 3

System Modeling (25pts)

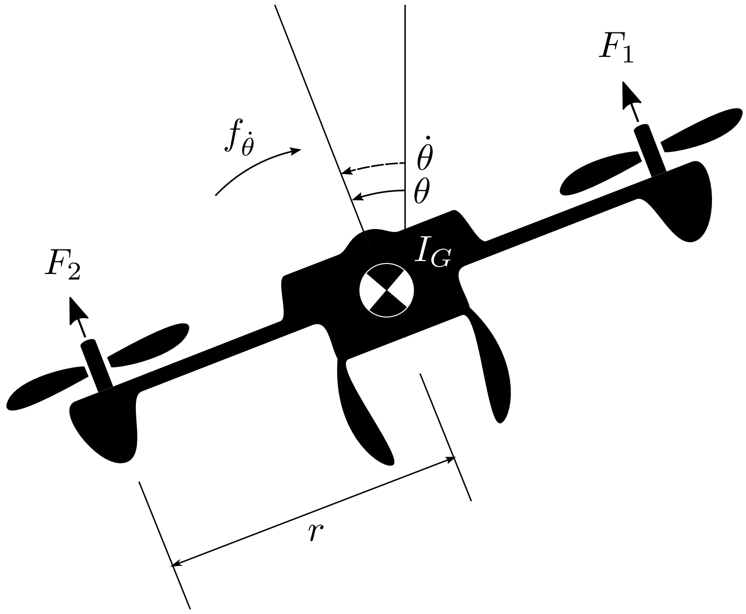
Consider a simplified quadrotor pitch control model. The Thrust F is a function of the rotor's speed $F = k_T\omega^2$, where k_T is the thrust constant

a. Derive the equation of motion for the system governing the pitch angle dynamics only

b. Find the transfer function that relates $\Delta F = F_1 - F_2$ to $\dot{\theta}$

c. Find the steady state value of $\dot{\theta}$ given a step-input $\omega_1 = 250rad/s, \omega_2 = 150rad/s$

Given: $I_G = 0.5kg \cdot m^2$, $K_T = 0.15kg \cdot m$, $r = 30.0cm$, $f_v = 0.25N \cdot s$



Solution:

a.

By summing the moment about G, we can derive the equation of motion

$\sum M_G = F_1 \cdot r - F_2 \cdot r - f_v\dot{\theta} = I_G\ddot{\theta}$

$I_G\ddot{\theta} + f_v\dot{\theta} = 0.5\ddot{\theta} + 0.25\dot{\theta} = (F_1 - F_2)r = \Delta F \cdot r = 0.3\Delta F$

b.

From the equations of motion we know that the transfer function

$$\frac{\Theta(s)}{\Delta F(s)} = \frac{r}{I_G s^2 + f_v s} = \frac{0.3}{0.5s^2 + 0.25s}$$

To get the transfer function $\frac{\dot{\Theta}(s)}{M(s)}$ we differentiate the signal, by multiplying by s

$$G(s) = \frac{\dot{\Theta}(s)}{M(s)} = \frac{\Theta(s)}{M(s)}s = \frac{0.3s}{0.5s^2 + 0.25s}$$

c.

To find the steady state value, we apply the final value theorem, with $R(s) = \frac{K_T(\omega_1^2 - \omega_2^2)}{s}$

$$c_{ss}(t) = \lim_{s\to 0} s\dot{\Theta}(s) = \lim_{s\to 0} sG(s)R(s) = \lim_{s\to 0} \frac{1800.0s}{0.5s^2 + 0.25s} = 7200.0rad/s$$

Problem 4

Transfer Function Components (25pts)

For each of the following 3rd order systems, perform a partial fraction expansion, then cancel the third pole term if it is real magnitude is five times or higher than the real magnitude of the other two poles

a. $G(s) = \dfrac{10}{(s+1)\,(s^2+2s+2)}$

b. $G(s) = \dfrac{23}{(s+2)\,(s+3)\,(s+20)}$

c. $G(s) = \dfrac{2}{(s+10)\,(s^2+6s+8)}$

d. $G(s) = \dfrac{1}{(s+40)\,(s^2+2s+100)}$

e. $G(s) = \dfrac{5}{(s+10)\,(s^2+8s+20)}$

Solution:

a.

Partial fraction expansion: $G(s) = -\dfrac{10\,(s+1)}{s^2+2s+2} + \dfrac{10}{s+1}$

The third pole @ -1 is not more than five times further away on the real-axis relative to the dominant poles @ -1, and the pole term is not cancelled.

b.

Partial fraction expansion: $G(s) = \dfrac{23}{306\,(s+20)} - \dfrac{23}{17\,(s+3)} + \dfrac{23}{18\,(s+2)}$

The third pole @ -20 is more than five times further away on the real-axis relative to the dominant poles @ -2 , and the pole term is cancelled.

c.

Partial fraction expansion: $G(s) = \dfrac{1}{24\,(s+10)} - \dfrac{1}{6\,(s+4)} + \dfrac{1}{8\,(s+2)}$

The third pole @ -10 is not more than five times further away on the real-axis relative to the dominant poles @ -2, and the pole term is not cancelled.

d.

Partial fraction expansion: $G(s) = -\dfrac{s-38}{1620\,(s^2+2s+100)} + \dfrac{1}{1620\,(s+40)}$

The third pole @ -40.0 is more than five times further away on the real-axis relative to the dominant poles @ -1.0 , and the pole term is cancelled.

e.

Partial fraction expansion: $G(s) = -\frac{s-2}{8\left(s^2+8s+20\right)} + \frac{1}{8\left(s+10\right)}$

The third pole @ -10 is not more than five times further away on the real-axis relative to the dominant poles @ -4, and the pole term is not cancelled.