ME417 - Homework #3

Control of Mechanical Systems - Spring 2020

Homework Due: September 27th, 2020

Complete the following problems and submit a hard copy of your solutions. You are encouraged to work together to discuss the problems but submitted work **MUST** be your own. This is an **individually** submitted assignment.

Problem 1

(20 Pts)
For each of the following systems, derive an equivalent state-space representation. Then determine if the system is **controllable**.

1.1) $G(s) = \dfrac{1}{s^2 + 2s + 4}$

1.2) $G(s) = \dfrac{(s+5)}{s^2 + 2s + 4}$

1.3) $G(s) = \dfrac{(s^2 + 10s + 100)}{(s+2)(s^2 + 2s + 4)}$

Problem 2

(20 Pts)
For each of the following closed-loop systems, derive an equivalent transfer function representation. Then find the steady state error to a step input $r(t) = 12$

2.1) $\dot{x} = \begin{bmatrix} 0.0 & 1.0 \\ 1.0 & -2.0 \end{bmatrix} x + \begin{bmatrix} 0.0 \\ 1.0 \end{bmatrix} u$

$y = \begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix} x$

2.2) $\dot{x} = \begin{bmatrix} 0.0 & 1.0 & 1.0 \\ 0.0 & 1.0 & 0.0 \\ -1.0 & -2.0 & -3.0 \end{bmatrix} x + \begin{bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{bmatrix} u$

$y = \begin{bmatrix} 1.0 & 0.0 & 0.0 \end{bmatrix} x$

Problem 3

Given the following system, derive the closed-loop state space representation for each of the inputs given.

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 5 & -1 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

3.1) $u(t) = r - \mathbf{K}x$, $\mathbf{K} \in \mathbb{R}^{1\times 2}$

3.2) $u(t) = K_p e = K_p(r - y)$, $K_p \in \mathbb{R}^1$

Problem 4

(20 Pts)
A very simplified and linear model of an inverted pendulum on a cart is given.

$m_c = 5kg$, $m_r = 0.25kg$, $l = 30cm$

4.1) Determine the stability of the open-loop system

4.2) Show that the system is controllable

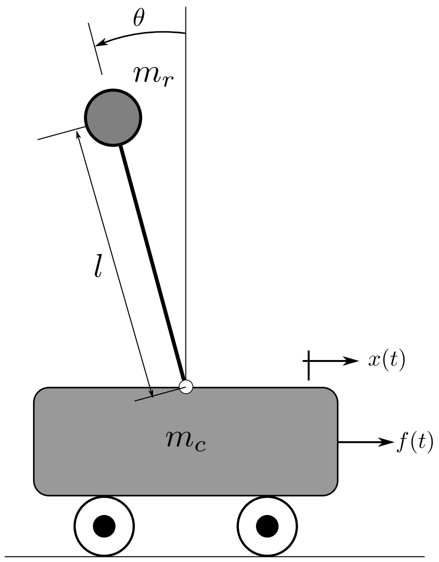
4.3) Design a full-state feedback controller to achieve a settling time of $T_s = 2s$ and $\zeta = 0.5$

4.4) Justify your choice for the additional poles' locations

You can use MATLAB Symbolic to aid in the calculation of the determinants and adjugate, but the rest of the solution should be carried by hand.

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{m_r \cdot g}{m_c} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{g}{l} & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{m_c} \\ 0 \\ \frac{-1}{l \cdot m_c} \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} x$$



Problem 5

(20 Pts)
Given the following open-loop system.

5.1) For what values of a is the system controllable?

5.2) Show that applying full-state feedback, for a step input of $r(t) = 5$, the system yields a finite steady-state error. Show the value of the steady-state error.

5.3) Show that adding an integral controller to the closed-loop system in state-space, results in the elimination of the steady-state error.

5.4) What is the order of the system with just a state feedback controller? What is the order of the system when you include the integrator?

$$\dot{x} = \begin{bmatrix} 0 & -5 \\ a & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$