ME 417 - Homework #1

Control of Mechanical Systems - Fall 2020

Homework Due: Thu, 24 Dec 2020 23:59

Complete the following problems and submit a hard copy of your solutions. You are encouraged to work together to discuss the problems but submitted work **MUST** be your own. This is an **individually** submitted assignment.

Problem 1

System Modeling (25pts)

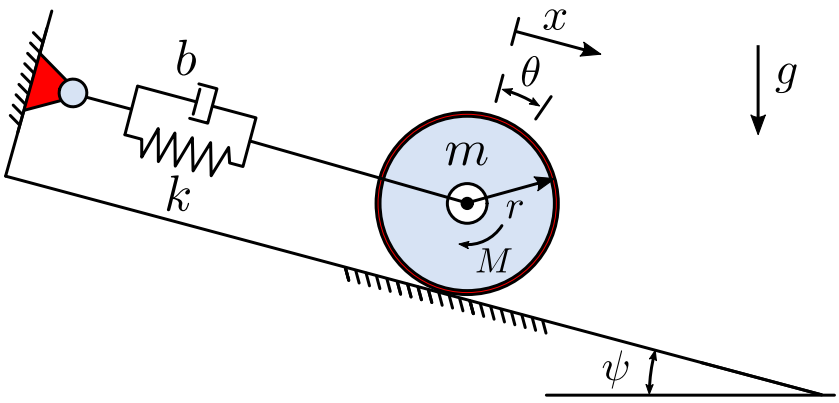
A disk of uniform mass rolls without slipping on an inclined surface as shown.

a. Derive the equations of motion for the system.

b. Find the transfer function that relates M to $\dot{\theta}$.

c. Find the pole locations of the transfer function derived in part (b)

Given: $r = 0.2m$, $m_r = 2.5kg$, $k = 150N/m$, $b = 60N \cdot s/m$, $\psi = 20^o$



Problem 2

System Modeling (25pts)

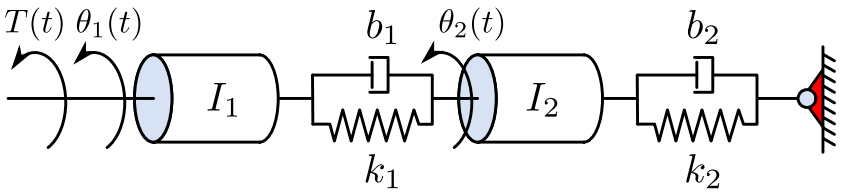
Given the following system

a. Derive the equations of motion for the system

b. Find the transfer function that relates T to θ_1

c. Find the steady state value of θ_1 given a step-input $T(t) = 10t$

Given: $I_1 = 0.2kg \cdot m^2$, $I_2 = 0.15kg \cdot m^2$, $k_1 = 280N/m$, $k_2 = 180N/m$, $b_1 = 35N \cdot s/m$, $b_2 = 25N \cdot s/m$



Problem 3

Time Response (25pts)

Given the following transfer function relating force to position

$$\frac{X}{F} = \frac{50.0}{s\,(s + 6.0)}$$

Derive the partial fraction expansion form for the output, sketch (by hand) the time response for position and velocity on the same figure, and find the steady-state output value for position for each of the following inputs.

a. $u_a(t) = 5$

b. $u_b(t) = 12.0t + 6.0$

c. $u_c(t) = 0.5e^{-4t}$

d. $u_d(t) = 3.0te^{-2t}$

Problem 4

Transfer Function Components (25pts)

For each of the following 3rd order systems, perform a partial fraction expansion, then cancel the third pole term if it is real magnitude is five times or higher than the real magnitude of the other two poles

a. $G(s) = \dfrac{20}{(s + 4)\,(s^2 + 3s + 20)}$

b. $G(s) = \dfrac{4}{(s + 1)\,(s + 2)\,(s + 20)^2}$

c. $G(s) = \dfrac{2}{(s+10)\,(s^2+2s+8)}$

d. $G(s) = \dfrac{1}{(s+10)\,(s^2+5s+100)}$

e. $G(s) = \dfrac{5}{(s+1)\,(s^2+3s+20)}$