ME 417 Control of Mechanical Systems

Spring 2020 Homework #2

Complete the following problems and submit a hard copy of your solutions. You are encouraged to work together to discuss the problems but submitted work **MUST** be your own. This is an **individually** submitted assignment.

Problem 1 (20pts)

For each of the following systems, sketch a **general shape** root-locus and includes, **as applicable**:

- Asymptotes intercept and angles
  a) $G(s) = \frac{(s+6)}{(s+1)(s^2+2s+4)}$
  b) $G(s) = \frac{(s-5)(s+20)}{s(s^2+4s+16)}$
  c) $G(s) = \frac{(s^2+2s+3)(s+20)}{s(s^2+2s+6)}$
  d) $G(s) = \frac{(s+3)(s-20)}{s^2(s^2+4s+100)}$

Problem 2 (20pts)

For each of the following systems, sketch a **refined** root-locus and includes, **as applicable**:

- Break-in and Break-away points
- jω crossing and the gain value at the crossing
- Angle of departures

Also, find the range of K for which the system is stable. SHOW YOUR WORK

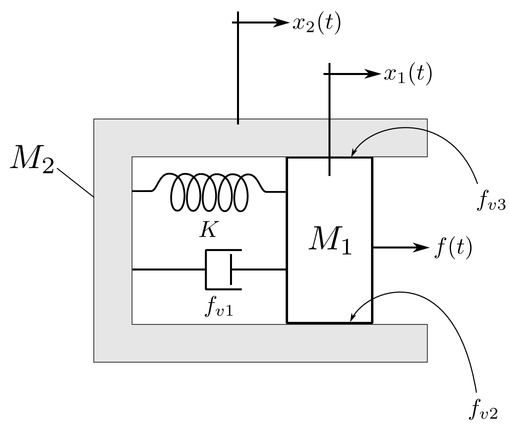
a) $G(s) = \frac{(s+6)}{(s+1)(s+2)(s^2+2s+4)}$
b) $G(s) = \frac{(s-5)(s+20)}{s^2(s^2+16)}$

Problem 3 (20 Pts)

Given the mechanical system shown on the figure. Use MATLAB to verify your work, but carryout the solutions and controller design by hand.

a) Derive the equations of motion for the system
b) Find the transfer function relating the input $f(t)$ to $x_1(t)$, $G_1(s) = \frac{x_1(s)}{f(s)}$
c) Analyze the stability of the system $G_1(s)$
d) Design a feedback controller, using the root-locus technique, around G_1 to achieve: Zero Steady-State Error, Settling time of $2s$, Peak Time of 2.5s, to a step input. Justify if the system can be approximated as second order.
e) Derive the transfer function relating the reference position to $x_1(t)$
f) Derive the transfer function relating the reference position to $x_2(t)$, with the feedback system derived above

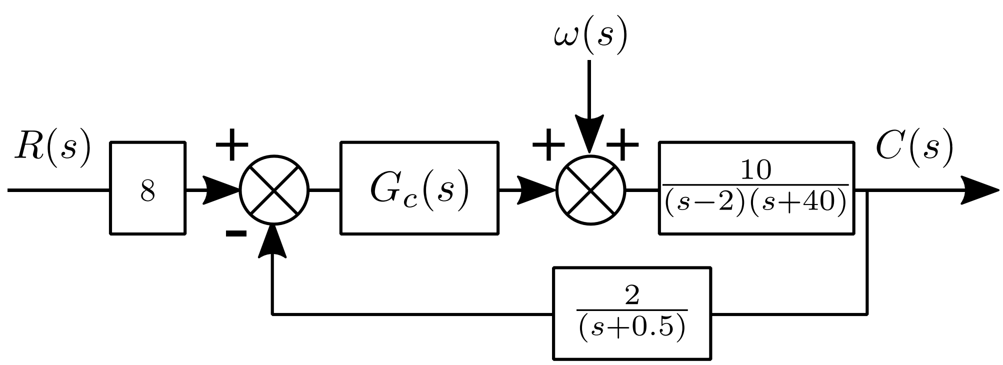
$$M_1 = 2kg, M_2 = 1kg, K = 10\ N/m\ , f_{v1} = 2N \cdot s/m, f_{v2} = f_{v3} = 2N \cdot s/m$$



Problem 4 (20 Pts)

A closed-loop system with input disturbance is shown.

a) Design a controller that results in stable response with a settling time of $T_s = 0.667s$. Is a second order approximation valid? Justify. Hint: Choose a convenient constraint for ζ or ω_d to simplify your calculations.
b) Show that the steady-state error of the closed-loop system, to an **impulse disturbance** input, is zero.
c) Given your designed controller, derive the transfer function relating the input $r(t)$ to the controller output $u(t)$, then find the time domain equation of $u(t)$ to an input $r(t) = 5$.



Problem 5 (20 Pts)

Given the following open-loop system

$$G_p = \frac{1}{(s+3)} + \frac{(s+5)}{(s-1)(s+2)}$$

a) Discuss the stability of the system
b) Design a controller that yields a stable and critically damped response with a gain less than 10
c) With the controller you designed, discuss whether the closed-loop system can be considered second-order.