Kuwait University
College of Engineering and Petroleum

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ME417 CONTROL OF MECHANICAL SYSTEMS

PART II: CONTROLLER DESIGN VIA STATE-SPACE

LECTURE 5: STATE-SPACE CONTROLLER DESIGN PROBLEMS

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- Objectives:
    - Introduce the state-space integral controller
    - Work through additional controller design problems in state-space
- Reading:
    - *Nise: 12.8*

- To guarantee the elimination of steady-state error when applying a full state feedback controller, an integral controller can be added.
  - The state feedback controller can regulate the state output to zero (stabilize the system toward the equilibrium), but it cannot guarantee zero steady-state error to a non-zero reference input.
- Given an LTI open-loop system

$$\dot{\boldsymbol{x}} \in \mathcal{R}^{\boldsymbol{n}} = \mathbf{A}\boldsymbol{x} + \mathbf{B}u,\ u \in \mathcal{R}^1$$
$$\boldsymbol{y} \in \mathcal{R}^1 = \mathbf{C}\boldsymbol{x}$$

- Let us formulate the closed-loop state space form for a full state feedback with an integrator.

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- A state feedback controller with an integral can be expressed as

$$u = u_F + u_I = (r - \boldsymbol{Kx}) + K_I e_{int} = (r - \boldsymbol{Kx}) + K_I \int (r - y)dt$$

- Note that the integral term is not represented in the state vector, we can augment the state vector to include it.

$$\tilde{x} \in \mathcal{R}^{n+1} = \begin{bmatrix} \boldsymbol{x} \\ e_{int} \end{bmatrix}$$

- If we added a derivative controller, would we want to augment the state vector to include $\dot{e}$?
  - We can, but it would result in a redundant (non-minimal) state, for instance if:
    - $y = x_1$ and $x_2 = \dot{x}_1$ then $\dot{e} = \dot{r} - \dot{x}_1 = \dot{r} - x_2$

- Substituting u in the state-space equations, yields the closed-loop system:

$$\dot{\tilde{\boldsymbol{x}}} \in \mathcal{R}^{n+1} = \begin{bmatrix} \dot{\boldsymbol{x}} \\ e \end{bmatrix} = \begin{bmatrix} \mathbf{A}\boldsymbol{x} + \mathbf{B}(r - \boldsymbol{K}\boldsymbol{x} + K_I e_{int}) \\ r - \mathbf{C}\boldsymbol{x} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{B}\boldsymbol{K} & \mathbf{B}K_I \\ -\mathbf{C} & 0 \end{bmatrix} \tilde{\boldsymbol{x}} + \begin{bmatrix} \mathbf{B} \\ 1 \end{bmatrix} \mathrm{r}$$

- Note that we increased the number of states by 1, increasing the order of the system.
- This can also be seen in the transfer function form of the PI controller, $G_c = K_P \frac{(s+K_I/K_P)}{s}$, where the integrator increases the order of the closed-loop system.

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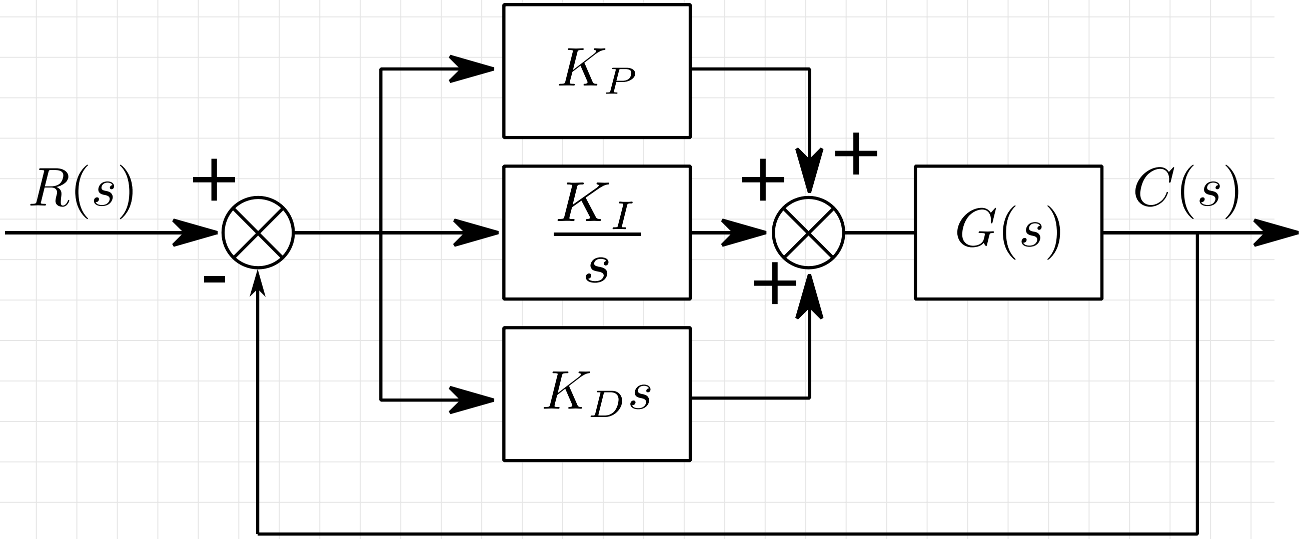
Find an equivalent state-space form for the closed-loop system shown.

Where $G(s) = \frac{16}{s^2}$



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A closed-loop system with a proportional feedback controller is shown in a state-space representation.

a. Design the proportional controller gain to achieve a damping ratio of $\zeta = 0.25$.

b. Compute the steady state error, with your choice of controller for a step input of $r(t) = 10$

c. Find the range of K_p values for which the system is stable

d. How different is the proportional controller than a full state feedback controller?

$$u = K_p(r - \boldsymbol{C}\boldsymbol{x})$$

$$\dot{\boldsymbol{x}} = (\boldsymbol{A} - \mathbf{B}K_p\mathbf{C})\boldsymbol{x} + \mathbf{B}K_p r = \begin{bmatrix} 0 & 1 \\ -2K_p & -3 \end{bmatrix}\boldsymbol{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} K_p r$$

$$\boldsymbol{y} \in \mathcal{R}^1 = \mathbf{C}\boldsymbol{x} = \begin{bmatrix} 1 & 0 \end{bmatrix}\boldsymbol{x}$$

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Example 2 Continue

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For the mechanical system shown.

Example 3

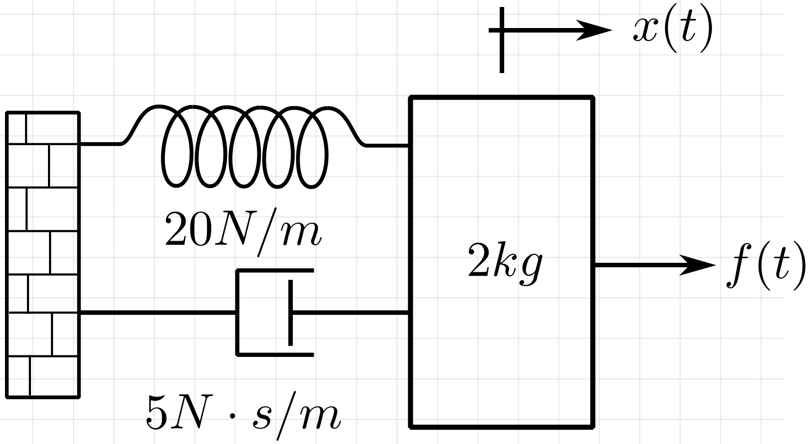
a) Derive the state space model of the system, assume the position x is the measured output.

b) Show that the system is controllable.

c) Design a full state feedback controller to achieve a critically damped response at $T_s = 0.1s$

d) Add an integral controller to eliminate steady-state error
   - How would you choose the value of K_I?

e) Find the closed-loop transfer function of the system.



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