Argument, $\arg(z) = \theta$

$$z = a+ib$$
$$= x+iy$$
$$|z| = \sqrt{x^2+y^2}$$

Addition: $(3+7i)+(8+11i)$

$$= 11+18i \text{ Ans.}$$

Substraction: $(a+bi)-(c+di)$

$$= a+bi-c-di$$
$$= (a-c)\ (b-d)i$$

Multiply: $(3-8i)(5+7i)$

$$= 15+21i-40i-56i^2$$
$$= 15-19i+56$$
$$= 71-19i \text{ Ans.}$$

Division: $\dfrac{3+4i}{6+7i} = \dfrac{3+4i}{6+7i} \times \dfrac{6-7i}{6-7i}$

$$= \frac{18-21i+24i-28i^2}{(6)^2-(7i)^2}$$

$$= \frac{18+3i+28}{36+49} = \frac{46+3i}{85}$$

$$= \frac{46}{85} + \frac{3i}{8} \text{ Ans.}$$

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2.

1. $\dfrac{-2}{1+i\sqrt{3}}$

$= \dfrac{-2(1-i\sqrt{3})}{(1+i\sqrt{3})(1-i\sqrt{3})}$

$= \dfrac{-2+i2\sqrt{3}}{(1)^2-(i\sqrt{3})^2}$

$= \dfrac{-2+i2\sqrt{3}}{1+1.3}$

$= \dfrac{-2+i2\sqrt{3}}{4}$

$= -\dfrac{2}{4}+\dfrac{2\sqrt{3}}{4}i$

$= -\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i$ Ans.

2. $|-4+2i| = \sqrt{(-4)^2+2^2}$ $\quad [|z| = \sqrt{x^2+y^2}]$

$= \sqrt{16+4}$

$= \sqrt{20}$

Ans: $\sqrt{20}$.

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3. $|z_1 z_2| = |z_1|\,|z_2|$ Prove this.

Soln:

Let, $z_1 = x_1 + iy_1$

$z_2 = x_2 + iy_2$

$$z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$$

$$= x_1x_2 + x_1y_2 i + x_2y_1 i + y_1y_2 i^2$$

$$= (x_1x_2 - y_1y_2) + (x_1y_2 + x_2y_1)i$$

$$L.H.S = \sqrt{(x_1x_2 - y_1y_2)^2 + (x_1y_2 + x_2y_1)^2}$$

$$= \sqrt{\{(x_1x_2)^2 - \cancel{2x_1x_2 \cdot y_1y_2} + (y_1y_2)^2\} + \{(x_1y_2)^2 + \cancel{2 \cdot x_1y_2 \cdot x_2y_1} + (x_2y_1)^2\}}$$

$$= \sqrt{(x_1x_2 + y_1y_2)^2 + (x_1y_2 + x_2y_1)^2}$$

$$= \sqrt{x_1^2x_2^2 + y_1^2y_2^2 + x_1^2y_2^2 + x_2^2y_1^2}$$

$$= \sqrt{x_1^2x_2^2 + x_1^2y_2^2 + y_1^2y_2^2 + x_2^2y_1^2}$$

$$= \sqrt{x_1^2(x_2^2 + y_2^2) + y_1^2(y_2^2 + x_2^2)}$$

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$$= \sqrt{(x_2^2+y_2^2)(x_1^2+y_1^2)}$$

$$= \sqrt{x_2^2+y_2^2} \cdot \sqrt{x_1^2+y_1^2}$$

$$= |z_1| \cdot |z_2|$$

4 $|z_1+z_2| \leq |z_1| + |z_2|$

Soln: Let,

$$z_1 = x_1 + iy_1$$

$$z_2 = x_2 + iy_2$$

Now,

$$z_1+z_2 = (x_1+iy_1) + (x_2+iy_2)$$

$$= x_1+iy_1+x_2+iy_2$$

$$= (x_1+x_2) + (y_1+y_2)i$$

$$|z_1+z_2| = \sqrt{(x_1+x_2)^2+(y_1+y_2)^2}$$

Now,

$$\sqrt{(x_1+x_2)^2+(y_1+y_2)^2} \leq \sqrt{x_1^2+y_1^2} + \sqrt{x_2^2+y_2^2}$$

$$\Rightarrow \left(\sqrt{(x_1+x_2)^2+(y_1+y_2)^2}\right)^2 \leq \left(\sqrt{x_1^2+y_1^2}\right)^2 + \left(\sqrt{x_2^2+y_2^2}\right)^2$$

$$\Rightarrow (x_1+x_2)^2+(y_1+y_2)^2 \leq x_1^2+y_1^2 + x_2^2+y_2^2 + 2\sqrt{x_1^2+y_1^2}\sqrt{x_2^2+y_2^2}$$

5

$$\Rightarrow 2(x_1x_2 + y_1y_2) \leq 2\left(\sqrt{x_1^2+y_1^2} \cdot \sqrt{x_2^2+y_2^2}\right)$$

$$\Rightarrow (x_1x_2 + y_1y_2)^2 \leq \left(\sqrt{x_1^2+y_1^2} \cdot \sqrt{x_2^2+y_2^2}\right)^2$$

$$\Rightarrow x_1^2x_2^2 + 2x_1x_2y_1y_2 + y_1^2y_2^2 \leq x_1^2x_2^2 + x_1^2y_2^2 + x_2^2y_1^2 + y_1^2y_2^2$$

$$\Rightarrow 2x_1x_2y_1y_2 \leq x_1^2y_2^2 + x_2^2y_1^2$$

$$\Rightarrow x_1^2y_2^2 - 2x_1x_2y_1y_2 + x_2^2y_1^2 \leq 0$$

$$\Rightarrow (x_1y_2 - x_2y_1)^2 \leq 0$$

There is true for all value of x & y.

So, Hence $|z_1 + z_2| \leq |z_1| + |z_2|$ (Proved)

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Again*

$|z_1+z_2| \leq |z_1|+|z_2|$

Soln Let,

$z_1 = x_1+iy_1$

$z_2 = x_2+iy_2$

and

$|z_1| = \sqrt{x_1^2+y_1^2}$

$|z_2| = \sqrt{x_2^2+y_2^2}$

Now,

$\cancel{|z_1+z_2|}\ z_1+z_2$

$= (x_1+iy_1)+(x_2+iy_2)$

$= (x_1+x_2)+(y_1+y_2)i$

$|z_1+z_2| = \sqrt{(x_1+x_2)^2+(y_1+y_2)^2}$

Then,

$|z_1+z_2| \leq |z_1|+|z_2|$

$\Rightarrow \sqrt{(x_1+x_2)^2+(y_1+y_2)^2} \leq \sqrt{x_1^2+y_1^2}+\sqrt{x_2^2+y_2^2}$

$\Rightarrow x_1^2+x_2^2+2x_1x_2+y_1^2+y_2^2+2y_1y_2 \leq x_1^2+y_1^2+x_2^2+y_2^2+2\sqrt{x_1^2+y_1^2}\sqrt{x_2^2+y_2^2}$

$\Rightarrow 2(x_1x_2+y_1y_2) \leq 2\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_2^2+y_2^2}$

$\Rightarrow (x_1x_2+y_1y_2)^2 \leq \left(\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_2^2+y_2^2}\right)^2$

$\Rightarrow x_1^2x_2^2+y_1^2y_2^2+2x_1x_2y_1y_2 \leq x_1^2x_2^2+x_1^2y_2^2+x_2^2y_1^2+y_1^2y_2^2$

$\Rightarrow 2x_1x_2y_1y_2 \leq x_1^2y_2^2+y_1^2y_2^2$

$\Rightarrow x_1^2y_2^2-2x_1x_2y_1y_2+y_1^2y_2^2 \leq 0$

$\Rightarrow (x_1x_2-y_1y_2)^2 \leq 0$

There is true for all value of x and y.
So Hence, $|z_1+z_2| \leq |z_1|+|z_2|$ (Proved).

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5 Given that,

$$|z_1+z_2| \geq |z_1| - |z_2|$$

Let,

$$z_1 = x_1 + iy_1$$

$$z_2 = x_2 + iy_2$$

$$|z_1| = \sqrt{x_1^2 + y_1^2}$$

$$|z_2| = \sqrt{x_2^2 + y_2^2}$$

and,

$$z_1 + z_2 = (x_1 + iy_1) + (x_2 + iy_2)$$

$$= (x_1 + x_2) + (y_1 + y_2)i$$

Then,

$$|z_1 + z_2| \geq |z_1| - |z_2|$$

$$\Rightarrow \sqrt{(x_1+x_2)^2 + (y_1+y_2)^2} \geq \left(\sqrt{x_1^2+y_1^2}\right) - \left(\sqrt{x_2^2+y_2^2}\right)$$

$$\Rightarrow x_1^2 + x_2^2 + 2x_1x_2 + y_1^2 + y_2^2 + 2y_1y_2 \geq x_1^2 + y_1^2 + x_2^2 + y_2^2 - 2\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_2^2+y_2^2}$$

$$\Rightarrow 2(x_1x_2 + y_1y_2) \geq -2\left(\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_2^2+y_2^2}\right)$$

$$\Rightarrow \cancel{x_1^2x_2^2} + \cancel{y_1^2y_2^2} + 2x_1x_2y_1y_2 \geq -\left(\cancel{x_1^2x_2^2} + x_1^2y_2^2 + x_2^2y_1^2 + \cancel{y_1^2y_2^2}\right)$$

$$\Rightarrow x_1^2y_2^2 + x_2^2y_2^2 + 2x_1x_2y_1y_2 \geq 0$$

$$\Rightarrow (x_1y_2 + x_2y_1)^2 \geq 0$$

This is for all values of x and y.

Hence $|z_1+z_2| \geq |z_1| - |z_2|$ (Proved)

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Problems of Absolute Value

Q $|z_1 z_2| = |z_1|\,|z_2|$

Soln: Let.

$$z_1 = x_1 + iy_1$$

$$z_2 = x_2 + iy_2$$

and $|z_1| = \sqrt{x_1^2 + y_1^2}$

$$|z_2| = \sqrt{x_2^2 + y_2^2}$$

Now,

$$z_1 \cdot z_2 = (x_1 + iy_1)(x_2 + iy_2)$$

$$= (x_1 x_2 + x_1 y_2 i) + (x_2 y_1 i + y_1 y_2)$$

$$= (x_1 x_2 + y_1 y_2) + (x_1 y_2 + x_2 y_1)\, i$$

Then,

$$L.H.S = |z_1 z_2|$$

$$= \sqrt{(x_1 x_2 - y_1 y_2)^2 + (x_1 y_2 + x_2 y_1)^2}$$

$$= \sqrt{x_1^2 x_2^2 + y_1^2 y_2^2 - 2x_1 x_2 y_1 y_2 + x_1^2 y_2^2 + x_2^2 y_1^2 + 2x_1 y_2 x_2 y_1}$$

$$= \sqrt{x_1^2(x_2^2 + y_2^2) + y_1^2(x_2^2 + y_2^2)}$$

$$= \sqrt{(x_1^2 + y_1^2) \cdot (x_2^2 + y_2^2)}$$

$$= \sqrt{x_1^2 + y_1^2} \cdot \sqrt{x_2^2 + y_2^2}$$

$$= |z_1| \cdot |z_2| \quad R.H.S \quad \text{(Showed)}$$

Again:

$$|z_1 z_2| = |z_1|\,|z_2|$$

Let, $z_1 = x_1 + iy_1$

$z_2 = x_2 + iy_2$

and $|z_1| = \sqrt{x_1^2 + y_1^2}$

$|z_2| = \sqrt{x_2^2 + y_2^2}$

Now,

$$z_1 \cdot z_2 = (x_1 + iy_1)(x_2 + iy_2)$$

$$= x_1 x_2 + x_1 i y_2 + x_2 i y_1 + y_1 y_2$$

$$= (x_1 x_2 - y_1 y_2) + (x_1 y_2 + x_2 y_1) i$$

Then,

$$L.H.S = |z_1 z_2|$$

$$= \sqrt{(x_1 x_2 - y_1 y_2)^2 + (x_1 y_2 + x_2 y_1)^2}$$

$$= \sqrt{x_1^2 x_2^2 + y_1^2 y_2^2 - 2x_1 x_2 y_1 y_2 + x_1^2 y_2^2 + x_2^2 y_1^2 + 2x_1 x_2 y_1 y_2}$$

$$= \sqrt{x_1^2 (x_2^2 + y_2^2) + y_1^2 (x_2^2 + y_2^2)}$$

$$= \sqrt{(x_1^2 + y_1^2) \cdot (x_2^2 + y_2^2)}$$

$$= \sqrt{x_1^2 + y_1^2} \cdot \sqrt{x_2^2 + y_2^2}$$

$$= |z_1|\,|z_2| \quad R.H.S \text{ (showed)}.$$

(b) Again.

Given that,

$$|z_1+z_2| \le |z_1|+|z_2|$$

Let,

$$z_1 = x_1+iy_1$$

$$z_2 = x_2+iy_2$$

$$|z_1| = \sqrt{x_1^2+y_1^2}$$

$$|z_2| = \sqrt{x_2^2+y_2^2}$$

$$\therefore\ z_1+z_2 = (x_1+iy_1)+(x_2+iy_2)$$

$$= x_1+iy_1+x_2+iy_2$$

$$= (x_1+x_2)+(y_1+y_2)i$$

Then,

$$|z_1+z_2| \le |z_1|+|z_2|$$

$$\Rightarrow \sqrt{(x_1+x_2)^2+(y_1+y_2)^2} \le \sqrt{x_1^2+y_1^2}+\sqrt{x_2^2+y_2^2}$$

$$\Rightarrow x_1^2+x_2^2+2x_1x_2+y_1^2+y_2^2+2y_1y_2 \le x_1^2+y_1^2+x_2^2+y_2^2+2\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_2^2+y_2^2}$$

$$\Rightarrow 2(x_1x_2+y_1y_2) \le 2\left(\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_2^2+y_2^2}\right)$$

$$\Rightarrow x_1^2x_2^2+y_1^2y_2^2+2x_1x_2y_1y_2 \le x_1^2x_2^2+x_1^2y_2^2+x_2^2y_1^2+y_1^2y_2^2$$

$$\Rightarrow x_1^2y_2^2+y_1^2x_2^2-2x_1x_2y_1y_2 \le 0$$

$$\Rightarrow (x_1y_2-y_1x_2)^2 \le 0$$

This true for all value of x and y.

Hence $|z_1+z_2| \le |z_1|+|z_2|$

Rectangular to Polar

Write $2+i2\sqrt{3}$ in polar form.

Soln: Given that,

$$z = 2 + i2\sqrt{3} \quad \cdots (1)$$

Comparing equation (1) with $z = x+iy$ then,
We get,

$$x = 2$$
$$y = 2\sqrt{3}$$

We know,
Modulus of Absolute value.

$$r = \sqrt{x^2+y^2}$$
$$= \sqrt{2^2+(2\sqrt{3})^2}$$
$$= \sqrt{4+12}$$
$$= 4$$

Amplitude or Argument,

$$\theta = \tan^{-1}\frac{y}{x}$$
$$= \tan^{-1}\frac{2\sqrt{3}}{2}$$
$$= \tan^{-1}\sqrt{3}$$
$$= 60^\circ$$

then,

$$2+i2\sqrt{3} = r(\cos\theta + i\sin\theta)$$
$$= 4(\cos 60^\circ + i\sin 60^\circ)$$
$$= 4\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$$
$$= 4e^{i\frac{\pi}{3}}$$

Ans.

Again write $2+i2\sqrt{3}$ in Polar form.

Sol Given that.

$$z = 2+i2\sqrt{3} \quad \ldots (1)$$

Comparing with $z = x+iy$ then.

We get. $x = 2$

$y = 2\sqrt{3}$

We know,

$$r = \sqrt{x^2+y^2}$$
$$= \sqrt{2^2+(2\sqrt{3})^2}$$
$$= 4$$

Amplitud, $\theta = \tan^{-1}\frac{y}{x}$

$$= \tan^{-1}\frac{2\sqrt{3}}{2}$$
$$= \tan^{-1}\sqrt{3}$$
$$= 60^\circ$$

Then.

$$2+i2\sqrt{3} = r(\cos\theta + i\sin\theta)$$
$$= 4(\cos 60^\circ + i\sin 60^\circ)$$
$$= 4(\cos \pi/3 + i\sin \pi/3)$$
$$= 4e^{i\pi/3}$$

Ans.

Write $3e^{2i}$ in Rectangle form.

Given that,

$$3e^{2i}$$

We know,

$$re^{i\theta} = r(\cos\theta + i\sin\theta)$$

Therefore,

$$3e^{i2} = 3(\cos 2 + i\sin 2)$$
$$= 3(-0.41 + 0.09i)$$
$$= -1.23 + 2.72i$$

$$(x, y) = (-1.23 + 2.72i)$$ Ans.

Again

Given that,

$$3e^{2i}$$

We know

$$re^{i\theta} = r(\cos\theta + i\sin\theta)$$
$$= 3(\cos 2 + i\sin 2)$$
$$= 3(-0.41 + 0.09i)$$
$$= -1.23 + 2.72i$$

$$(x, y) = (-1.23 + 2.72i)$$ Ans.

$4e^{i\pi/3}$

We know,

$$re^{i\theta} = r(\cos\theta + i\sin\theta)$$

$$= 4\left(\cos\pi/3 + i\sin\pi/3\right)$$

$$= 4(-0.95 - 1.21i)$$

$$= -3.81 - 1.21i$$

$\therefore (x, y) = (-3.81, -1.21i)$ Ans.

Again;

We know,

$$re^{i\theta} = r(\cos\theta + i\sin\theta)$$

$$= 4\left(\cos\pi/3 + i\sin\pi/3\right)$$

$$= 4(-0.95 - 1.21i)$$

$$= -3.81 - 1.21i$$

$\therefore (x, y) = (-3.81, -1.21i)$ Ans.

e^{-i}

We know

$$re^{i\theta} = r(\cos\theta + i\sin\theta)$$

$$= r\{\cos(-1) + i\sin(-1)\}$$

$$= 1.08 - 1.68i$$

$\therefore (x, y) = (1.08, -1.68i)$ Ans.

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State & Power DE MOVIER Theorem,

(a) Given that,

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

We know,

$$\cos n\theta + i\sin n\theta = (\cos\theta + i\sin\theta)^n$$

Here, $n = 5$

$$\cos 5\theta + i\sin 5\theta = (\cos\theta + i\sin\theta)^5$$

$$= \cos 5\theta + {}^5c_1\cos^4\theta(i\sin\theta) + {}^5c_2\cos^3\theta(i\sin\theta)^2 + {}^5c_3\cos^2\theta(i\sin\theta)^3 + {}^5c_4\cos\theta(i\sin\theta)^4 + (i\sin\theta)^5$$

$$= \cos^5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10i\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta$$

$$= (\cos 5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta) + (5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)i$$

Equation Real & Imaginary part.

$$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta \quad \text{---(i)}$$

$$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta \quad \text{---(ii)}$$

From equation (i)

$$\therefore \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$$

$$= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta(1 - 2\cos^2\theta + \cos^4\theta)$$

$$= \cos^5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta - 10\cos^3\theta + 5\cos^5\theta$$

$$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta \text{ (Proved)}$$

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Again:

(a) Given that,

$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$$

We know,

$$\cos n\theta + i\sin n\theta = (\cos\theta + i\sin\theta)^n$$

Here, $n = 5$

$$\cos 5\theta + i\sin 5\theta = (\cos\theta + i\sin\theta)^5$$

$$= \cos 5\theta + {}^5C_1\cos^4\theta\,(i\sin\theta) + {}^5C_2\cos^3\theta\,(i\sin\theta)^2 + {}^5C_3\cos^2\theta\,(i\sin\theta)^3 + {}^5C_4\cos\theta\,(i\sin\theta)^4 + i\sin^5\theta$$

$$= \cos 5\theta + 5i\cos^4\theta\sin\theta - 10\cos^3\theta\sin^2\theta - 10i\cos^2\theta\sin^3\theta + 5\cos\theta\sin^4\theta + i\sin^5\theta$$

$$= (\cos 5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta) + (5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$$

Equation Real & Imaginary number.

$$\cos 5\theta = \cos 5\theta - 10\cos^3\theta\sin\theta + 5\cos\theta\sin^4\theta \quad \text{....(i)}$$

$$\sin 5\theta = 5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta \quad \text{...(ii)}$$

From equation —— (i)

$$\therefore \cos 5\theta - 10\cos^3\theta\,\cancel{\sin\theta}\,(1-\cos^2\theta) + 5\cos\theta\,(1-\cos^2\theta)^2$$

$$= \cos 5\theta - 10\cos^3\theta + 10\cos^5\theta + 5\cos\theta - 10\cos^3\theta + 5\cos^5\theta$$

$$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta \quad \text{(Proved)}$$

(a) $z = -2+i$ (b) $z = 1-3i$ and show how the correspondence can be represented graphically

Solution:

(a) $w = f(-2+i)$

$= (-2+i)^2$

$= 4-4i-1$

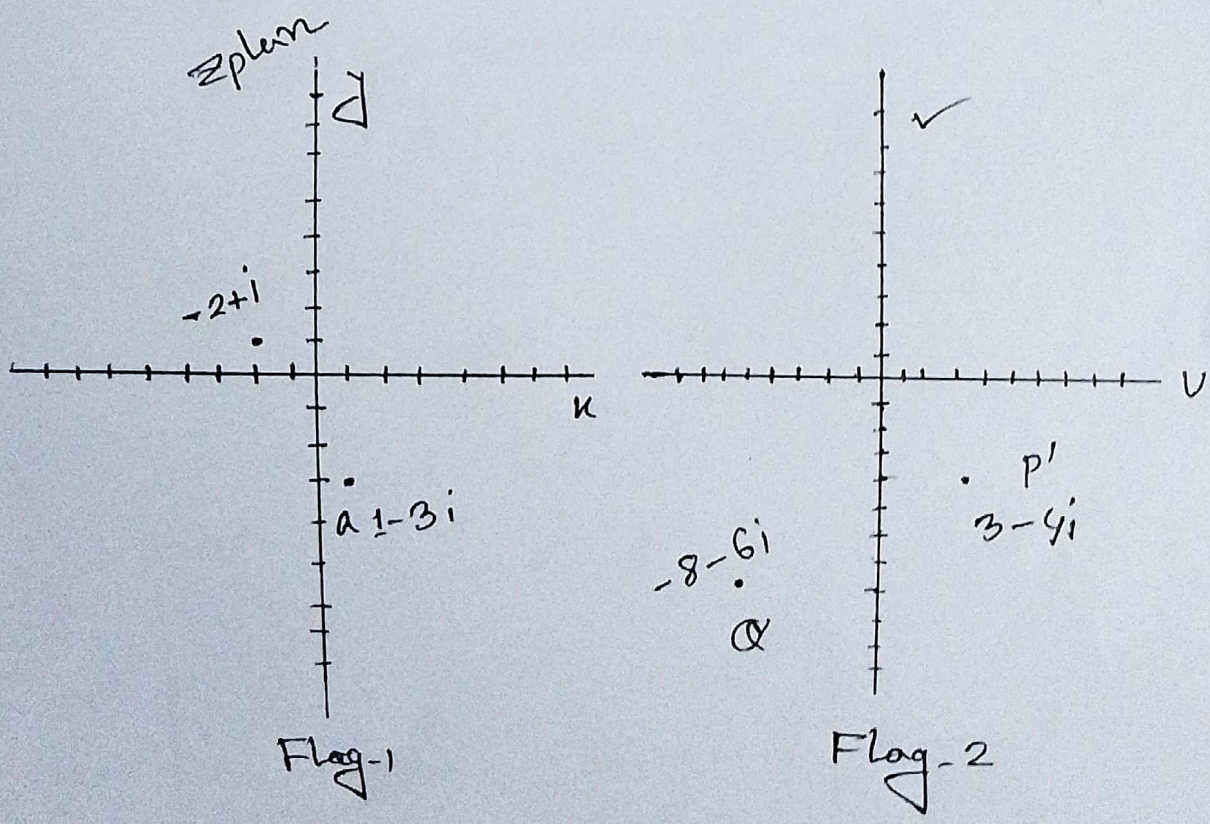
$= 3-4i$

(b) $w = f(1-3i)$

$= (1-3i)^2$

$= 1-6i-9$

$= -8-6i$



Flag-1 Flag-2

The point $z = -2+i$ represented by pointed P in the z plan of Flag-1 has the image point $w = 3-4i$ [point Q. of fig-1]. Is mapped into $w = -8-6i$ [point Q' of fig-2]. To each point in the z plane. there corresponds one and only point in the w plan. So that w is a single-valued function of z.

Again:

(a) $= w = f(-2+i)$

$= (-2+i)^2$

$= 4-4i-1$

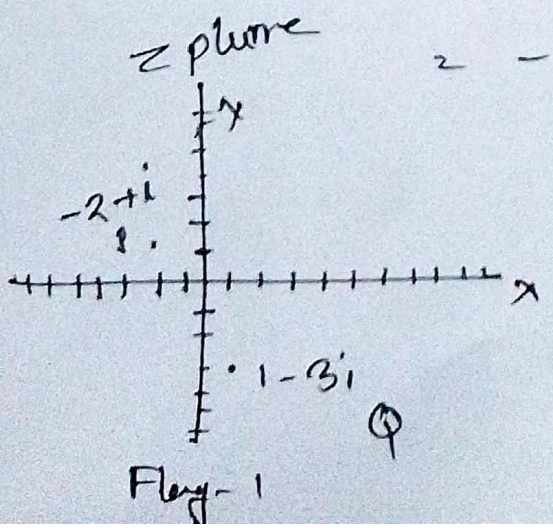
$= 3-4i$

(b) $w = f(1-3i)$

$= (1-3i)^2$

$= 1-6i-9$

$= -8-6i$



Flag-1

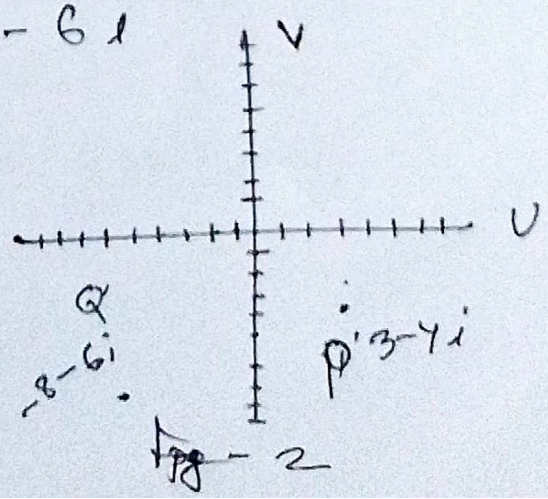


Fig-2

If $f(z) = z^2$ proved that $\lim_{z \to z_0} f(z) = z_0^2$

Soln: We must show that given any $\varepsilon > 0$ we can find δ. such that $|z^2 - z_0^2| < \varepsilon$ whenever

$0 < |z - z_0| < \delta$

If $\delta \leq 1$ then $0 < |z - z_0| < \delta$ implies that

$$|z^2 - z_0^2| = |z - z_0|\,|z + z_0|$$

$$< \delta |z - z_0 + 2z_0| \quad [\text{since } 0 < |z - z_0| < \delta]$$

$$< \delta \{ |z - z_0| + |2z_0| \}$$

$$< \delta (1 + |2z_0|)$$

Take δ as 1 or $\varepsilon / (1 + 2|z_0|)$ which ever is smaller, Then, we have $|z^2 - z_0^2| < \varepsilon$ whenever $|z - z_0| < \delta$ and the required result is proved.

Again:

We must show that given any $\varepsilon > 0$ We can find δ such that $|z^2 - z_0^2| < \varepsilon$ whenever $0 < |z - z_0| < \delta$ if $\delta \leq 1$ then $0 < |z - z_0| < \delta$ implies that,

$|z^2 - z_0^2| = |z - z_0|\,|z + z_0|$

$< \delta\,|z - z_0 + 2z_0|$ [since $0 < |z - z_0| < \delta$

$< \delta\{|z - z_0| + |2z_0|\}$

$< \delta\,(1 + |2z_0|)$

Take δ as 1 or $\varepsilon/(1+2|z_0|)$ whichever is smaller. Then we have $|z^2 - z_0^2| < \varepsilon$ when $|z - z_0| < \delta$ and the result is proved.

If $\lim_{z \to z_0} f(z)$ exists, prove that it must be unique

<u>Solution</u> We must show $\lim f(z) = l_1$ and $\lim_{z \to z_0} f(z) = l_2$ then $l_1 = l_2$

By hypotesis given only $\varepsilon > 0$ we find $\delta > 0$ such that. $|f(z) - l_1| < \varepsilon/2$ when $0 < |z - z_0| < \delta$

$|f(z) - l_2| < \varepsilon/2$ when $0 < |z - z_0| < \delta$

Then,

$$|l_1 - l_2| = |l_1 - f(z) + f(z) - l_2|$$

$$\leq |l_1 - f(z)| + |f(z) - l_2|$$

$$\leq \frac{\varepsilon}{2} + \frac{\varepsilon}{2}$$

$\therefore |l_1 - l_2| < \varepsilon$ (Proved).