

laplace/body_ax - an excerpt from the book
C.Pozrikidis 'A Practical Guide to
Boundary-Element Methods'

with additional notes and commentaries

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Introduction

- This is a code that computes potential flow past an axisymmetric, compact (singly connected) or toroidal (doubly connected) body with arbitrary geometry, as illustrated in Figure
- original *Fortran77* code is here [CFDLAB code for laplace/body_ax](#)

Mathematical Formulation

The velocity u is decomposed into three parts as

$$u = u^\infty + v + u^D \quad (1)$$

where:

- u^∞ is the far-field component prevailing far from the body, expressing uniform (streaming) flow along the x axis of symmetry.
- v is the velocity due to a line vortex ring with specified strength situated in the interior of the body, generating circulation around the toroidal body. In the case of flow past a compact body, this component is inconsequential, and serves only to modify the disturbance velocity.
- u^D is a disturbance velocity expressed by the gradient of the single-valued harmonic potential Φ^D ,

$$u^D = \nabla \Phi^D \quad (2)$$

Requiring the no-penetration boundary condition $u \cdot n = 0$ around the contour of the body in a meridional plane of constant angle φ , where n is the unit vector normal to the body, we derive a boundary condition for the normal derivative of the disturbance potential,

$$n \cdot \nabla \phi^D \equiv \frac{\partial \phi^D}{\partial n} = -(u^\infty + v) \cdot n \quad (3)$$

Using the standard boundary-integral formulation, we find that the disturbance potential satisfies the integral equation of the second kind

$$\phi^D(x_0) = -2 \int_C G(x, x_0) [n(x) \cdot \nabla \phi^D(x)] dl(x) + 2 \int_C^{PV} \phi^D(x) [n(x) \cdot \nabla G(x, x_0)] dl(x) \quad (4)$$

where $G(x, x_0)$ is the free-space Green's function of Laplace's equation in an axisymmetric domain, and the point x_0 lies on the contour of the body C . Inserting the boundary condition (3) into (4) and rearranging, we obtain

$$\phi^D(x_0) - 2 \int_C^{PV} \phi^D(x) [n(x) \cdot \nabla G(x, x_0)] dl(x) = 2 \int_C (u^\infty + v) \cdot G(x, x_0) n(x) dl(x) \quad (5)$$

Numerical Method

The contour of the body in a meridional plane is discretized into a collection of