CS 374 HW 1 Problem 3

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TOTAL POINTS

98 / 100

QUESTION 1

1 3A **28 / 30**

- **- 0 pts** Correct
- **- 30 pts** for incorrect description.
- **- 22.5 pts** for IDK
- **- 22.5 pts** if word prefix is not mentioned in the answer.
- **- 15 pts** for failing to provide correct proof.
- ✓ **- 2 pts minor error**
- **- 2 pts** minor error
- **- 30 pts** No submission
- **- 30 pts** IDK

Use prefix next time

QUESTION 2

2 3B **30 / 30**

- ✓ **- 0 pts Correct**
- **- 25 pts** if failing to follow the definition of L(M') (or an equivalent definition).
- **- 2 pts** for each minor error.
- **- 2 pts** minor error
- **- 15 pts** for other incorrect logic.
- **- 22.5 pts** idk
- **- 30 pts** No submission

QUESTION 3

3 3C **40 / 40**

- ✓ **- 0 pts Correct**
- **- 45 pts** for not using product construction or other correct construction strategies.
- **- 30 pts** for incorrect accept states construction.
- **- 15 pts** for each of other missing/incorrect dfa components.
- **- 2 pts** for each minor error.
- **- 2 pts** Minor error
- **- 40 pts** No submission

3A) $M' = (Q\ \Sigma\ \delta\ s\ B(M))$

$\Rightarrow L(M') = \{ w \mid$ there is no string x such that $w \cdot x \in L(M) \}$

To prove M' accepts $L(M')$, we need to prove that:

$\delta^*(s, w) \in B$ iff for all $x \in \Sigma^*$, $w \cdot x \notin L(M)$.

- Suppose $\delta^*(s,w) \in B$, prove for all $x \in \Sigma^*$, $w \cdot x \notin L(M)$:

  Let $q = \delta^*(s,w) \Rightarrow q \in B$

  Also, we have $\delta^*(s, w \cdot x) = \delta^*(\delta^*(s,w), x)$

  $= \delta^*(q, x)$

  We have $q \in B \Rightarrow \delta^*(q,x) \notin A$ for all $x \in \Sigma^*$ $\Rightarrow$ this is true because once we are in q we cannot leave q according to the definition of a bad state

  $\Rightarrow \delta^*(s, w \cdot x) \notin A$, $\forall x \in \Sigma^*$

  Therefore, $w \cdot x \notin L(M)$ for all $x \in \Sigma^*$

- Suppose for all $x \in \Sigma^*$, $w \cdot x \notin L(M)$, prove that $\delta^*(s,w) \in B$

  Let $\delta^*(s,w) = q$

  $\Rightarrow \delta^*(s, w \cdot x) = \delta^*(\delta^*(s,w), x)$ (can be proven by doing induction, and using the definition of transition function)

  $= \delta^*(q, x) \notin A$ for all $x \in \Sigma^*$ because $w \cdot x \notin L(M)$ for all x $\Rightarrow \delta^*(s, w \cdot x) \notin A$ for all x

  ~~We have $w \cdot x \notin L(M)$ for all~~

  Therefore, q is a bad state because of the definition of bad state

  $\Rightarrow \delta^*(s,w)$ is also a bad state because $\delta^*(s,w) = q$

  $\Rightarrow \delta^*(s,w) \in B$

Therefore, we can conclude that: M' accepts $L(M')$

1 3A 28 / 30

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- **- 22.5 pts** if word prefix is not mentioned in the answer.
- **- 15 pts** for failing to provide correct proof.
- ✓ **- 2 pts minor error**
- **- 2 pts** minor error
- **- 30 pts** No submission

Use prefix next time

3B) Prove if $x \in L(M')$, then $xy \in L(M')$ $\forall y \in \Sigma^*$

- We have: $x \in L(M')$; Let $q = \delta^*(s, x)$

  $\Rightarrow q \in B(M)$

- We have that: $\delta^*(s, xy) = \delta^*(\delta^*(s, x), y)$ (can be proven by using induction and the definition of transition function)

  $= \delta^*(q, y)$

- Now, we prove that if $q \in B(M)$, then $\delta^*(q, w) \in B(M)$ for all $w \in \Sigma^*$

  Prove by contradiction: Assume $q \in B(M)$ and $\delta^*(q, w) \notin B(M)$

  Let $q_1 = \delta^*(q, w)$ $\Rightarrow q \notin B(M)$

  Because q_1 is not a bad state, there exist string t such that: $\delta^*(q_1, t) \in A$.

  And we have that:

  $\delta^*(q, w \cdot t) = \delta^*(\delta^*(q, w), t)$ (can be proven by using induction and the definition of transition function

  $= \delta^*(q_1, t)$

  $\Rightarrow \delta^*(q, w \cdot t) \in A$ because $\delta^*(q_1, t) \in A$

  $\Rightarrow q$ is not a bad state

  This contradict with the assumption that $q \in B(M)$

  Therefore, we have if $q \in B(M)$, then $\delta^*(q, w) \in B(M)$ for all w

- We have $\delta^*(s, xy) = \delta^*(q, y)$ (proven above)

  And q is a bad state $\Rightarrow \delta^*(q, y) \in B(M) \Rightarrow \delta^*(s, xy) \in B(M)$

Therefore, $xy \in L(M')$ because $\delta^*(s, xy) \in B(M)$

2 3B 30 / 30

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3C)

Let define DFA $M_3 = \{Q_3, \Sigma, \delta_3, s_3, A_3\}$ accepts language L_3 such that

$L_3 = \{w \mid \text{no prefix of } w \text{ is in } L_1\}$

$\Rightarrow$ • $s_3 = s_1$

• $\delta_3(q,a) = \begin{cases} \delta_1(q,a) & \text{if } q \notin A_1 \\ q, & \text{if } q \in A_1 \end{cases}$

• $Q_3 = \{q \in Q_1 \mid \text{there exist } w \in \Sigma^* \text{ such that } \delta_3^*(s_3, w) = q\}$

• $A_3 = Q_3 \setminus A_1$

• We have : $L = \{w \mid w \in L_2 \text{ and no prefix of } w \text{ is in } L_1\}$

$\Rightarrow L = \{w \mid w \in L_2 \text{ and } w \in L_3\}$

$\Rightarrow L(M) = L(M_2) \cap L(M_3)$

$\Rightarrow$ With $M_2 = (Q_2, \Sigma, \delta_2, s_2, A_2)$ and $M_3 = (Q_3, \Sigma, \delta_3, s_3, A_3)$

$\Rightarrow M = (Q, \Sigma, \delta, s, A)$, where:

• $s = (s_2, s_3) = (s_2, s_1)$

• $Q = Q_2 \times Q_3 = \{(q_2, q_1) \mid q_2 \in Q_2 \text{ and } q_1 \in Q_1 \text{ such that exist } w \in \Sigma^* \text{ such that } \delta_3^*(s_1, w) = q\}$

• $\delta = Q \times \Sigma \rightarrow Q$ where:

$\delta((q_2, q_1), a) = (\delta_2(q_2, a), \delta_3(q_1, a))$

$= \begin{cases} (\delta_2(q_2, a), \delta_1(q_1, a)) & \text{if } q_1 \notin A_1 \\ (\delta_2(q_2, a), q_1) & \text{if } q_1 \in A_1 \end{cases}$

• $A = A_2 \times A_3 = \{(q_2, q_1) \mid q_2 \in A_2 \text{ and } q_1 \in A_3\}$

$= \{(q_2, q_1) \mid q_2 \in A_2 \text{ and } q_1 \in Q_3 \setminus A_1\}$

with Q_3 is described above

~~$(Q_3 = \{q$~~

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- **- 2 pts** for each minor error.
- **- 2 pts** Minor error
- **- 40 pts** No submission
- **- 30 pts** IDK