CS 374 HW 3 Problem 1

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TOTAL POINTS

95 / 100

QUESTION 1

1 1A **20 / 25**

- 0 pts Correct answer
- 10 pts Incorrect infinite fooling set (F)
- ✓ **- 5 pts Incorrect selection of arbitrary u,v F**
- 5 pts Incorrect distinguishing suffix x
- 5 pts Incorrect proof that exactly one of ux and vx belongs to language
- 5 pts For every minor error
- 25 pts We are unable to follow the logic of the answer, or the answer is just way too long. In the future, you might want to consider using "IDK"
- 25 pts The answer is unreadable
- 18.75 pts IDK

i,j should be >=1

QUESTION 2

2 1B **25 / 25**

- ✓ **- 0 pts Correct answer**
- 10 pts Incorrect infinite fooling set (F)
- 5 pts Incorrect selection of arbitrary u,v F
- 5 pts Incorrect distinguishing suffix x
- 5 pts Incorrect proof that exactly one of ux and vx belongs to language
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QUESTION 3

3 1C **25 / 25**

- ✓ **- 0 pts Correct answer**
- 18 pts Incorrect proof
- 7 pts Incorrect counter-example
- 5 pts For every minor error
- 25 pts We are unable to follow the logic of the answer, or the answer is just way too long. In the future, you might want to consider using "IDK"
- 25 pts The answer is unreadable
- 18.75 pts IDK

QUESTION 4

4 1D **25 / 25**

- ✓ **- 0 pts Correct answer**
- 10 pts Incorrect infinite fooling set (F)
- 5 pts Incorrect selection of arbitrary u,v F
- 5 pts Incorrect distinguishing suffix x
- 5 pts Incorrect proof that exactly one of ux and vx belongs to language
- 5 pts For every minor error
- 25 pts We are unable to follow the logic of the answer, or the answer is just way too long. In the future, you might want to consider using "IDK"
- 25 pts The answer is unreadable
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1a.)

$L = \{ 0^n w \bar{w} 1^n \mid 0 \le n \le 3,\ w \in \{0,1\}^+ \}$

let $F = \{ 0^n \mid n \ge 1 \}$

let x & $y \in F$
such that $x = 0^i$ where $i \neq j$
$y = 0^j$

then if $t = 1^i$

$$\begin{cases} 0^i \cdot t \in L \\ 0^j \cdot t \notin L \end{cases}$$

$\Downarrow$

$$\begin{cases} 0^i 1^i \in L \\ 0^j 1^i \notin L \text{ because } i \neq j \end{cases}$$

$\Rightarrow$ F is a fooling set for L & because F is infinite L cannot be regular

1b.) L is defined a string in $\{0,1\}^*$ such that ~~[illegible]~~ and two distinct runs of 0's are not of equal length

$F = \{ 0^i 1 \mid i \ge 1 \}$

x & $y \in$ ~~B~~F &
let $x = 0^i 1$ $t = 0^i$
$y =$ ~~[illegible]~~ $0^j 1$

F is a fooling set for L & because F is infinite L cannot be regular

then $x \cdot t \Rightarrow 0^i 1 0^i \notin L$ since $i = i$
$y \cdot t \Rightarrow 0^j 1 0^i \in L$ since $i \neq j$ which guarantees the run of 0's is not of the same length

1 1A 20 / 25

- 0 pts Correct answer
- 10 pts Incorrect infinite fooling set (F)

✓ - 5 pts **Incorrect selection of arbitrary u,v F**

- 5 pts Incorrect distinguishing suffix x
- 5 pts Incorrect proof that exactly one of ux and vx belongs to language
- 5 pts For every minor error
- 25 pts We are unable to follow the logic of the answer, or the answer is just way too long. In the future, you might want to consider using "IDK"
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1a.)

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1b.) L is defined a string in $\{0,1\}^*$ such that ~~[illegible]~~ and two distinct runs of 0's are not of equal length

$F = \{ 0^i 1 \mid i \geq 1 \}$

x & $y \in$ ~~B~~F &
let $x = 0^i 1$ $t = 0^i$
$y =$ ~~[illegible]~~ $0^j 1$

F is a fooling set for L & because F is infinite L cannot be regular

then $x \cdot t \Rightarrow 0^i 1 0^i \notin L$ since $i = i$
$y \cdot t \Rightarrow 0^j 1 0^i \in L$ since $i \neq j$ which guarantees the run of 0's is not of the same length

2 1B 25 / 25

✓ - **0 pts** Correct answer

- **10 pts** Incorrect infinite fooling set (F)
- **5 pts** Incorrect selection of arbitrary u,v F
- **5 pts** Incorrect distinguishing suffix x
- **5 pts** Incorrect proof that exactly one of ux and vx belongs to language
- **5 pts** For every minor error
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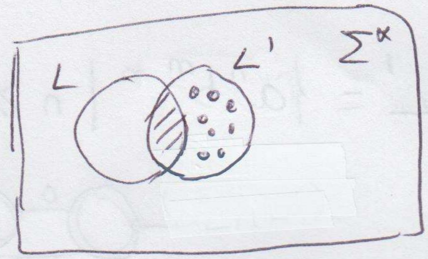
1)(C) Prove $(L \cup L')$ not regular

- Let A and B be any set, we have that

$$A \setminus B = A \cap \overline{B}$$



- We have:

$$L = (L \cup L') \setminus (L' \setminus (L \cap L'))$$

$\boxed{/}$: $L \cap L'$

$\boxed{\because}$ $L' \setminus (L \cap L')$

$$= (L \cup L') \setminus (L' \cap \overline{(L \cap L')})$$

(Because $A \setminus B = A \cap \overline{B}$)

$$= (L \cup L') \cap \overline{(L' \cap \overline{(L \cap L')})} \text{ (Because } A \setminus B = A \cap \overline{B} \text{)}$$

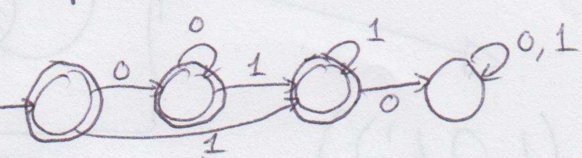
- Assume that $(L \cup L')$ is regular.
  - We have $(L \cap L')$ is regular
    - $\Rightarrow \overline{(L \cap L')}$ is regular (closure under complement)
    - $\Rightarrow (L' \cap \overline{(L \cap L')})$ is regular (closure under intersection)
    - $\Rightarrow \overline{(L' \cap \overline{(L \cap L')})}$ is regular (closure under complement)
    - $\Rightarrow (L \cup L') \cap \overline{(L' \cap \overline{(L \cap L')})}$ is regular (closure under complement
    - $\Rightarrow L$ is regular. This is contradiction!

Therefore, $(L \cup L')$ is not regular

We have:

- $L = \{0^{2n}1^n \mid n \geq 0\}$ is not regular (question 2 Discussion 9/14)

- $L' = \{0^n 1^m \mid n \geq 0, m \geq 0\}$ is regular



DFA for L'

Because $L \subseteq L'$

$\Rightarrow$ • $L \cup L' = L'$: regular

• $L \cap L' = L$: Not regular.

3 1C 25 / 25

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Q1)

(D) $F = \{ 0^{n\lg n - \lg n} \mid n = 2^m \text{ and } m \geq 3 \}$

- Let $x, y \in F$

$\Rightarrow x = 0^{i\lg i - \lg i}$ with i, k are power of 2 and $i, k \geq 2^3$

$y = 0^{k\lg k - \lg k}$

- WLOG, assume $i < k$
- Let $z = 0^{\lg i}$

$\Rightarrow x \cdot z = 0^{i\lg i - \lg i} 0^{\lg i} = 0^{i\lg i - \lg i + \lg i} = 0^{i\lg i}$

We have i be power of 2

$\Rightarrow \lg(i)$ is an integer

$\Rightarrow i\lg(i)$ is an integer $\Rightarrow i\lg(i) = \lceil i\lg(i) \rceil$

$\Rightarrow x \cdot z = 0^{\lceil i\lg(i) \rceil} \in L$

- $y \cdot z = 0^{k\lg k - \lg k} 0^{\lg(i)} = 0^{k\lg k - \lg k + \lg i}$
- Prove that $y \cdot z \notin L$ means prove $(k\lg k - \lg k + \lg i)$ not have the form $\lceil n\lg n \rceil$.

$\lceil (k-1)\cdot\lg(k-1) \rceil < k\lg k - \lg k + \lg(i) < \lceil k \cdot \lg(k) \rceil$

- Prove that: $\lceil (k-1)\cdot\lg(k-1) \rceil < k\lg k - \lg k + \lg(i)$
  - By ceiling definition, for any j: $j + 1 \geq \lceil j \rceil$

$\Rightarrow (k-1)\lg(k-1) + 1 \geq \lceil (k-1)\lg(k-1) \rceil$

$\Rightarrow (k-1)\lg(k-1) + \lg(i) > \lceil (k-1)\lg(k-1) \rceil$

(Because $i \geq 2^3 \Rightarrow \lg(i) \geq 3 > 1$

$\Rightarrow (k-1)\lg(k) + \lg(i) > \lceil (k-1)\lg(k-1) \rceil$

(Because $k > k-1 \Rightarrow \lg(k) > \lg(k-1)$)

$\Rightarrow k \cdot \lg(k) - \lg(k) + \lg(i) > \lceil (k-1)\lg(k-1) \rceil$

- We also have: $i < k \Rightarrow \lg(i) - \lg(k) < 0$

$\Longrightarrow k\lg(k) - \lg(k) + \lg(i) < k\lg(k) = \lceil k\lg(k) \rceil$

(Because k is power of 2 $\Rightarrow k\lg(k)$ is integer)

· Therefore, we have:

$\lceil (k-1)\lg(k-1) \rceil < k\lg k - \lg k + \lg(i) < \lceil k \lg k \rceil$

And we have k and $(k-1)$ are 2 consecutive integer

$\Rightarrow (k\lg k - \lg k + \lg(i))$ cannot have form $\lceil n\lg(n) \rceil$

$\Rightarrow y \circ z = 0^{k\lg k - \lg k + \lg(i)} \notin L$

Therefore ~~weh~~ we have:
$x \circ z \in L$ and $y \circ z \notin L$

Thus, F is a fooling set of L

· Because F is infinite $\Rightarrow L$ cannot be regular.

4 1D 25 / 25

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