CS 374 HW 7 Problem 1

Aldo Sanjoto, quddus2, Hieu Huynh

TOTAL POINTS

77.5 / 100

QUESTION 1

1 1A 10 / 10

✓ - 0 pts Correct

- 4 pts Incorrect answer for the maximum value
- 3 pts Incorrect worst case
- 3 pts Incorrect running time of the algorithm
- 10 pts We are unable to follow the logic of the answer, or the answer is just way too long. In the future, you might want to consider using "IDK"
- 10 pts The answer is unreadable
- 7.5 pts IDK

QUESTION 2

2 1B 10 / 10

✓ - 0 pts Correct

- 2 pts Minor mistake in proof
- 5 pts Major mistake in proof, but the idea is correct overall
- 10 pts Incorrect idea and / or multiple major mistakes in proof
- 10 pts We are unable to follow the logic of the answer, or the answer is just way too long. In the future, you might want to consider using "IDK"
- 10 pts The answer is unreadable
- 7.5 pts IDK

QUESTION 3

3 1C 20 / 20

✓ - 0 pts Correct

- 4 pts Minor mistake in proof
- 10 pts Major mistake in proof, but the idea is correct overall
- 20 pts Incorrect idea and / or multiple major mistakes in proof
- 20 pts We are unable to follow the logic of the answer, or the answer is just way too long. In the future, you might want to consider using "IDK"
- 20 pts The answer is unreadable
- 15 pts IDK

QUESTION 4

4 1D 20 / 20

✓ - 0 pts Correct

- 4 pts Minor mistake in proof
- 10 pts Major mistake in proof, but the idea is correct overall
- 20 pts Incorrect idea and / or multiple major mistakes in proof
- 20 pts We are unable to follow the logic of the answer, or the answer is just way too long. In the future, you might want to consider using "IDK"
- 20 pts The answer is unreadable
- 15 pts IDK

QUESTION 5

5 1E 10 / 10

✓ - 0 pts Correct

- 2 pts Minor mistake in proof
- 5 pts Major mistake in proof, but the idea is correct overall
- 10 pts Incorrect idea and / or multiple major mistakes in proof
- 10 pts We are unable to follow the logic of the answer, or the answer is just way too long. In the future, you might want to consider using "IDK"
- 10 pts The answer is unreadable
- 7.5 pts IDK

QUESTION 6

6 1F 7.5 / 30

- 0 pts Correct

- **3 pts** [PROOF OF CORRECTNESS] - Minor mistake
- **8 pts** [PROOF OF CORRECTNESS] - Major mistake in proof, but the idea is correct overall
- **15 pts** [PROOF OF CORRECTNESS] - Incorrect idea and / or multiple major mistakes in proof
- **4 pts** [IMPLEMENTATION] - Inefficient data structures
- **4 pts** [IMPLEMENTATION] - Minor logical errors in implmentation
- **10 pts** [IMPLEMENTATION] - Incorrect implementation
- **2 pts** [RUNNING TIME] - Correct implementation, but slower running time than in the solutions
- **5 pts** Incorrect running time OR correct running time for incorrect implementation
- **30 pts** We are unable to follow the logic of the answer, or the answer is just way too long. In the future, you might want to consider using "IDK"
- **30 pts** The answer is unreadable

✓ **- 22.5 pts** **IDK**

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1 A)

Worst case: All cars choose the same 2 spots

⇒ For all i s.t $1 \le i \le n$:

$\langle i/s_i \rangle = \langle i/\delta \rangle$ and $\langle i/s_i' \rangle = \langle i/\delta \rangle$

- For each node $\langle i/s_i \rangle$, there are $(n-1)$ outgoing edges

⇒ For each pair $\langle i/s_i \rangle$ and $\langle i, s_i' \rangle$, there are $2(n-1)$ outgoing edges

⇒ For n pairs, there are $2n(n-1)$ edges

⇒ In worse case, there are $2n(n-1)$ edges

- To compute the graph:

For each vertex $\langle i, s_i \rangle$, need to check $2 \cdot (n-1)$ other vertices to find all outgoing edges of $\langle i, s_i \rangle$

There are $2 \cdot n$ vertices

⇒ Total running time: $O(n^2)$

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1b) Prove that if $c_i = s_i$ forces $c_j = s_j$ then $c_j = s_j'$ forces $c_i = s_i'$.

- Given $c_i = s_i$ forces $c_j = s_j$
- Prove by contradiction
- Assume if $c_j = s_j'$ and $c_i = s_i$
  - we have $c_i = s_i$ forces $c_j = s_j$
  - ⇒ This is contradiction because $c_j = s_j$ and $c_j = s_j'$

Therefore, $c_j = s_j'$ force $c_i = s_i'$

1C) - Given that $\langle i / s_i \rangle$ and $\langle i / s_i' \rangle$ are in the same SCC.

⇒ The size of that SCC must be greater than 2 because there cannot be a direct edge from $\langle i/s_i \rangle$ to $\langle i/s_i' \rangle$

Let $\langle j/s_j \rangle$ be another vertex in the same SCC

⇒ There is a path from $\langle i/s_i \rangle$ to $\langle j/s_j \rangle$ and there is a path from $\langle j/s_j \rangle$ to $\langle i/s_i' \rangle$

⇒ If $c_i = s_i$ forces $c_j = s_j$ and $c_j = s_j$ forces $c_i = s_i'$

⇒ There is a contradiction because: $c_i = s_i$ forces $c_i = s_i'$

- There is a path from $\langle i/s_i' \rangle$ to $\langle j/s_j \rangle$ and there is a path from $\langle j/s_j \rangle$ to $\langle i/s_i \rangle$

⇒ If $c_i = s_i'$ forces $c_j = s_j$ and $c_j = s_j$ forces $c_i = s_i$

⇒ There is a contradiction because $c_i = s_i'$ forces $c_i = s_i'$

Therefore, there is no legal parking way if $\langle i/s_i \rangle$ and $\langle i/s_i' \rangle$ in same SCC.

2 1B 10 / 10

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⇒ The size of that SCC must be greater than 2 because there cannot be a direct edge from $\langle i/s_i \rangle$ to $\langle i/s_i' \rangle$

Let $\langle j/s_j \rangle$ be another vertex in the same SCC

⇒ There is a path from $\langle i/s_i \rangle$ to $\langle j/s_j \rangle$ and there is a path from $\langle j/s_j \rangle$ to $\langle i/s_i' \rangle$

⇒ If $c_i = s_i$ forces $c_j = s_j$ and $c_j = s_j$ forces $c_i = s_i'$

⇒ There is a contradiction because: $c_i = s_i$ forces $c_i = s_i'$

- There is a path from $\langle i/s_i' \rangle$ to $\langle j/s_j \rangle$ and there is a path from $\langle j/s_j \rangle$ to $\langle i/s_i \rangle$

⇒ If $c_i = s_i'$ forces $c_j = s_j$ and $c_j = s_j$ forces $c_i = s_i$

⇒ There is a contradiction because $c_i = s_i'$ forces $c_i = s_i'$

Therefore, there is no legal parking way if $\langle i/s_i \rangle$ and $\langle i/s_i' \rangle$ in same SCC.

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- 15 pts IDK

1D) For any i, j such that $1 \le i, j \le t$ and $i \neq j$

$\langle i/x_i \rangle, \langle j/x_j \rangle \in Y \Rightarrow$ there is a path from $\langle i/x_i \rangle$ to $\langle j/x_j \rangle$

$\Rightarrow$ There is a path from $\langle j/x_j' \rangle$ to $\langle i/x_i' \rangle$ (because of part (B))

- We also know that there is a path from $\langle j/x_j \rangle$ to $\langle i/x_i \rangle$

$\Rightarrow$ There's a path from $\langle i/x_i' \rangle$ to $\langle j/x_j' \rangle$ (because of part (B))

Therefore, there is a path from $\langle j/x_j' \rangle$ to $\langle i/x_i' \rangle$

and there is a path from $\langle i/x_i' \rangle$ to $\langle j/x_j' \rangle$

$\Rightarrow \langle i/x_i' \rangle$ and $\langle j/x_j' \rangle$ are in a SCC

$\Rightarrow \langle 1/x_1' \rangle, \langle 2/x_2' \rangle, \dots, \langle t/x_t' \rangle$ form SCC $\overline{Y}$ in G

1E) Prove if X is a sink, then $\overline{X}$ is a source

- Prove by contradiction: Assume $\overline{X}$ is not a source

$\Rightarrow$ There is a vertex $\langle c_k, x_k \rangle \notin \overline{X}$ s.t there is an edge from $\langle c_k, x_k \rangle$ to a vertex in $\overline{X}$

WLOG, $(\langle c_k, x_k \rangle, \langle c_i, x_i \rangle) \in E(G)$ && $\langle c_i, x_i \rangle \in \overline{X}$

- By (1B), $\langle c_k, x_k \rangle$ forces $\langle c_i, x_i \rangle$, then $\langle c_i, x_i' \rangle$ forces $\langle c_k, x_k' \rangle$
- By (1D), $\langle c_i, x_i \rangle \in \overline{X} \Rightarrow \langle c_i, x_i' \rangle \in X$

And we have $\langle c_k, x_k' \rangle \notin X$ because $\langle c_k, x_k \rangle \notin \overline{X}$

Therefore, we have: that X has an outgoing edge

$\Rightarrow X$ is not the sink $\Rightarrow$ Contradiction

Therefore if X is a sink $\Rightarrow$ $\overline{X}$ must be a source.

4 1D **20 / 20**

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- By (1D), $\langle c_i, x_i \rangle \in \overline{X} \Rightarrow \langle c_i, x_i' \rangle \in X$

And we have $\langle c_k, x_k' \rangle \notin X$ because $\langle c_k, x_k \rangle \notin \overline{X}$

Therefore, we have: that X has an outgoing edge

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Therefore if X is a sink $\Rightarrow$ $\overline{X}$ must be a source.

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- 7.5 pts IDK

F) IDK

6 1F **7.5 / 30**

- **0 pts** Correct
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