CS 374 HW 3 Problem 3

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TOTAL POINTS

100 / 100

QUESTION 1

1 3A 40 / 40

✓ - **0 pts** Correct

- **- 30 pts** IDK
- **- 5 pts** empty string not accepted by grammar
- **- 10 pts** recursive rule for 0^i1^i(A) incorrect
- **- 10 pts** recursive rule for 1^i2^i(B) incorrect
- **- 5 pts** A: 0^i1^i does not terminate(epsilon rule not added)
- **- 5 pts** B: 1^i2^i does not terminate(epsilon rule not added)
- **- 10 pts** recursive rule to combine A and B wrong
- **- 40 pts** Ineligible: We cant read the answer/ follow the logic/ its way too long. You might want to consider the IDK option in future
- **- 40 pts** Late

QUESTION 2

2 3B 60 / 60

✓ - **0 pts** Correct

- **- 45 pts** IDK
- **- 20 pts** L in L(G) wrong
- **- 5 pts** induction variable to prove L(A) incorrect
- **- 5 pts** base case of L(A) induction incorrect
- **- 5 pts** hypothesis of L(A) induction incorrect
- **- 10 pts** inductive proof of L(A) incorrect
- **- 5 pts** any error in proof of L(B), or did not state L(B) belongs to 1^j2^j
- **- 10 pts** error in proving S->AB, or did not state and prove L(G) is in L, by combining L(A) and L(B)
- **- 60 pts** Unreadable/Illegible

(3)

(A) $S \rightarrow X \cdot Y$ // $\{0^i 1^j 2^k \mid j = i+k\}$

$X \rightarrow 0X1 \mid \varepsilon$ // $\{0^i 1^i \mid i \geq 0\}$

$Y \rightarrow 1Y2 \mid \varepsilon$ // $\{1^j 2^j \mid j \geq 0\}$

- NON-Terminals: S ; X ; Y
- Terminals: 0 ; 1 ; 2
- ~~WE~~ In Production rule S we move onto to production rule X and concatenate these results with the results of going into production rule Y
- In production rule ~~S~~ X we add a 1 for every 0 & we can ~~re~~ repeate productions rule X to repeate the number of 0's & 1's (ie 00...11) or we can be finished with this production by ~~going into ε~~ adding nothing
- In production rule Y we add a 2 for every 1 & we can repeate this production rule Y in order to repeate the number of 1's & 2's or we can be finished with the production by ~~go~~ adding nothing

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(Q3)

(A) ~~$L = \{0^i 1^j 2^k \mid j = i+k\}$~~

~~$S \to X \cdot Y$ // $\{0^i 1^j 2^k \mid j = i+k\}$~~

~~$X \to 0X1 \mid \varepsilon$ // $\{0^i 1^i \mid i \geq 0\}$~~

~~$Y \to 1Y2 \mid \varepsilon$ // $\{1^j 2^j \mid j \geq 0\}$.~~

(B) Prove that $w \in L$ iff $S \rightsquigarrow^* w$

($\Rightarrow$) Suppose $w \in L$, prove $S \rightsquigarrow^* w$:

- We have $w \in L \Rightarrow w = 0^i 1^j 2^k = 0^i 1^i 1^k 2^k$ (because $i+k=j$)
- We have $X \to 0X1 \mid \varepsilon$

$\Rightarrow X \rightsquigarrow^i 0^i X 1^i$ (Apply $X \to 0X1$ i times)

and $0^i X 1^i \to 0^i 1^i$ (Apply $X \to \varepsilon$)

$\Rightarrow X \rightsquigarrow^{i+1} 0^i 1^i$

- We have $Y \to 1Y2 \mid \varepsilon$

$\Rightarrow Y \rightsquigarrow^k 0^k Y 1^k$ (Apply $Y \to 0Y1$ k times)

and $0^k Y 1^k \to 0^k 1^k$ (Apply $Y \to \varepsilon$)

$\Rightarrow Y \rightsquigarrow^{k+1} 0^k 1^k$

$\Rightarrow S \to XY \rightsquigarrow^{i+1} 0^i 1^i Y \rightsquigarrow^{k+1} 0^i 1^i 1^k 2^k = w$

Therefore, $S \rightsquigarrow^* w$

($\Leftarrow$) Suppose $S \leadsto^* w$, prove $w \in L$:

- Prove that: if $X \leadsto^* x$, then $x \in \{0^i 1^i \mid i \geq 0\}$
  - Induction on number of steps in derivation of x.
  - Base case: Derivation with 1 step

    The only 1 step is $X \to \varepsilon \Rightarrow x = \varepsilon \Rightarrow x \in \{0^i 1^i \mid i \geq 0\}$

    $\Rightarrow$ Base case is true
  - IH: Assume if number of steps is smaller or equal n $(n \geq 1)$

    then $x \in \{0^i 1^i \mid i \geq 0\}$
  - Let number of steps be $(n+1)$, prove $X \leadsto^{n+1} x \in \{0^i 1^i \mid i \geq 0\}$

    number of steps $= (n+1) > 1$

    Because number of steps cannot be $X \to \varepsilon$

    $\Rightarrow$ First step cannot be $X \to \varepsilon$

    $\Rightarrow$ First step must be: $X \to 0X1$
  - Based on IH, we have:

    if $X \leadsto^n x_1$, then $x_1 \in \{0^i 1^i \mid i \geq 0\}$
  - Therefore:

    $$X \to 0X1 \leadsto^n 0x_1 1 = 0 0^i 1^i 1 = 0^{i+1} 1^{i+1}$$

    $$\Rightarrow X \leadsto^{n+1} x = 0^{i+1} 1^{i+1} \Rightarrow x \in \{0^i 1^i \mid i \geq 0\}$$
- Repeat the same argument, we have:

  if $Y \leadsto^* y$, then $y \in \{1^k 2^k \mid k \geq 0\}$
- We have: $S \to XY$ and $S \leadsto^* w$

  Therefore: $w = x \cdot y$ with $X \leadsto^* x$; $Y \leadsto^* y$

  And we have: $X \leadsto^* x \Rightarrow x \in \{0^i 1^i \mid i \geq 0\}$ (Proven above)

  $Y \leadsto^* y \Rightarrow y \in \{1^k 2^k \mid k \geq 0\}$

  $\Rightarrow w = 0^i 1^i 1^k 2^k$ with $i \geq 0$ and $k \geq 0$

  $\Rightarrow w = 0^i 1^{i+k} 2^k \in L$

  Therefore $w \in L$

Therefore, our grammar is correct!

2 3B 60 / 60

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