CS 374 HW 8 Problem 1

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TOTAL POINTS

70 / 100

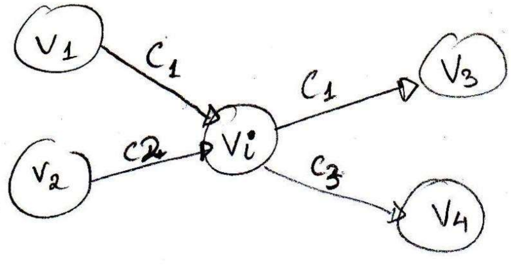
QUESTION 1

1 Flood it 70 / 100

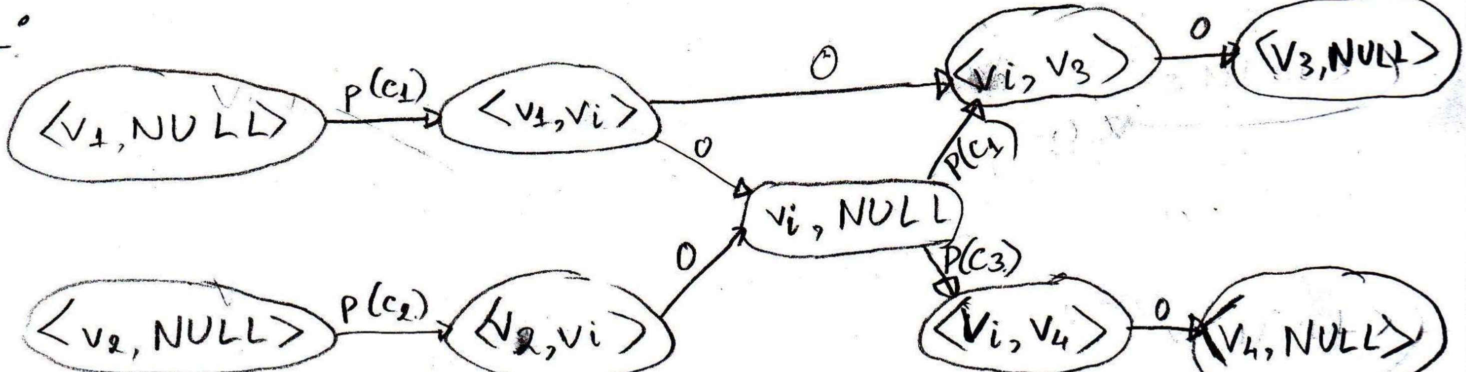
- **- 0 pts** Correct
- ✓ **- 30 pts** **Did not prove correctness of modified algorithm**
- **- 20 pts** Construction/Algorithm hard to follow but otherwise fine
- **- 20 pts** Minor flaws in Construction/Algorithm e.g. getting a factor Xi (#colors) in the edges.
- **- 30 pts** No runtime analysis, or simply stated runtime without explaining
- **- 20 pts** Described graph as pseudocode only...
- **- 75 pts** IDK
- **- 100 pts** wrong

Q1)

G :



G' :



Idea:

- We will create a new graph G' from the original graph
- Then we will use Dijktra algorithm to find the "shortest" path from <s, NULL> to <t, NULL> in G'
- The new graph G' will have: O(m) edges and O(m) vertices

Q1)

- Build new graph G' from original graph:
- G' has set of vertices: V'; and set of edge E'

Creating vertices:

For each v_i in $V(G)$ {
    For each $v_j \in In(v_i)$ { // $In(v_i)$ is the set of incoming edges of v_i in G
        Add vertex $\langle v_j, v_i \rangle$ to V'
    }
    Add vertex $\langle v_i, NULL \rangle$ to V'
}

Creating edges:

For each v_i in $V(G)$ {
    For $v_j \in In(v_i)$ {
        For $v_k \in Out(v_i)$ {
            if $(c(v_j, v_i) == c(v_i, v_k))$ {
                Add edge $(\langle v_j, v_i \rangle, \langle v_i, v_k \rangle)$ with weight $= 0$ to E'
            }
        }
        Add edge $(\langle v_j, v_i \rangle, \langle v_i, NULL \rangle)$ with weight $= 0$ to E'
    }
}

For each v_i in $V(G)$ {
    For $v_j \in Out(v_i)$ {
        Add edge $(\langle v_i, NULL \rangle, \langle v_i, v_j \rangle)$ with weight $=$ $p(c(v_i, v_j))$ to E'
    }
}

The graph G' has: $n + \sum_{i=1}^{n} deg(v_i) = (n+m)$ vertices

and #of edges $= \sum_{i=1}^{n} \left(|In(v_i)| + |Out(v_i)| + \text{Number of pair of incoming edge \& outgoing edge that has same color} \right)$

$\leq 3 \cdot m = O(m)$ edges

$\Rightarrow$ G' has $O(m)$ edges and $O(m)$ vertices

$\hookrightarrow$ since: $n < m \Rightarrow n+m \leq 2(m)$

- To create all vertices, it takes $O(n+m) = O(m)$
- To create all edges, it takes $O(n+m) = O(m)$
- $\Rightarrow$ To build new graph, it takes $O(n+m) = O(m)$

- After building the graph G', we use Dijktra algorithm to find "shortest" path from $\langle s, NULL \rangle$ to $\langle t, NULL \rangle$
- After finding the shortest path, we start at $\langle s, NULL \rangle$ and follow the path. If we traverse on an edge with weight > 0, Let say edge has weight $p(c_i)$, we will add color c_i to the sequence.
- Because graph G' has $O(m)$ edges and $O(m)$ vertices, the running time of Dijktra is $O(m + m\log m) = O(m \log m)$

Therefore, the total running time is $O(m\log m)$.

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