CS 374 HW 2 Problem 3

Aldo Sanjoto, quddus2, Hieu Huynh

TOTAL POINTS

70 / 100

QUESTION 1

1 3A 10 / 10

✓ + **10 pts** Correct

+ **2.5 pts** IDK

+ **0 pts** wrong

+ **0 pts** did not state over a,b,c

+ **0 pts** ambiguous

QUESTION 2

2 3B 20 / 20

✓ + **20 pts** Correct

- **5 pts** Start state not identified

+ **5 pts** IDK

+ **0 pts** Incorrect NFA

+ **10 pts** Partially correct NFA

+ **10 pts** empty string not accepted

QUESTION 3

3 3C 30 / 30

+ **30 pts** Correct

+ **0 pts** Wrong

✓ + **15 pts** Correct contruction

✓ + **15 pts** Proof of correctness

+ **7.5 pts** IDK

- **5 pts** proof isnt rigorous

- **5 pts** proof only one side

+ **5 pts** Construction is not well defined

+ **7.5 pts** Prove f(L) is regular by regex

QUESTION 4

4 3D 10 / 40

+ **40 pts** Correct

+ **10 pts** construction not well defined

+ **20 pts** correct construction

+ **20 pts** proof of correctness

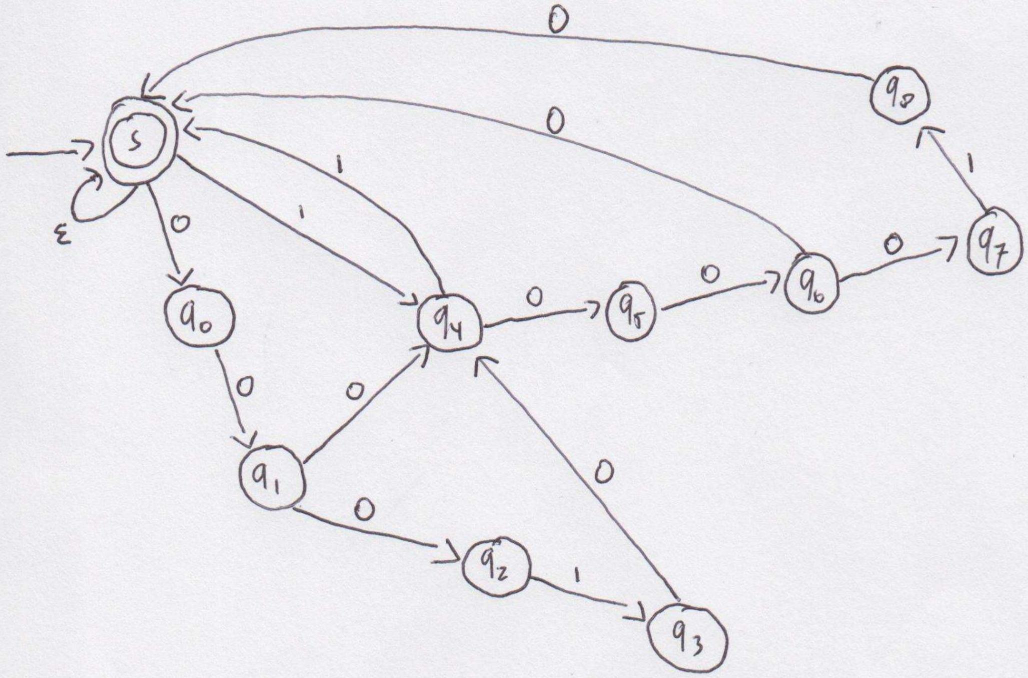
+ **10 pts** Proof only one side

+ **0 pts** wrong

✓ + **10 pts** IDK

3a) L: ∀string $w \in \{a, b, c\}^*$ with $|w|$ is even.

b)



Explanation:

The NFA above satisfies the regex of $((1+000+00010)(1+000+00010))^*$
An empty string is accepted thus our starting state is an accepting state.
For the first part of the expression: $(1+000+00010)$, the NFA provides a transition to q_4 if the input is 1. A transition from the S state to q_0 to q_1 to q_4 which satisfies the input 000. A transition from the s state to q_0 to q_1 to q_2 to q_3 to q_4 for the input 00010. Then, we still need to choose $(1+000+00010)$ to be accepted.
If the input is 1, it can use the transition in q_4 to the s state.
If the input is 000, it can use the path from q_4 to q_5 to q_6 to the S state.
If the input is 00010, it can use the path from q_4 to q_5 to q_6 to q_7 to q_8 to the s state. Our overall NFA thus accepts the expression.

1 3A 10 / 10

✓ + **10 pts** Correct

+ **2.5 pts** IDK

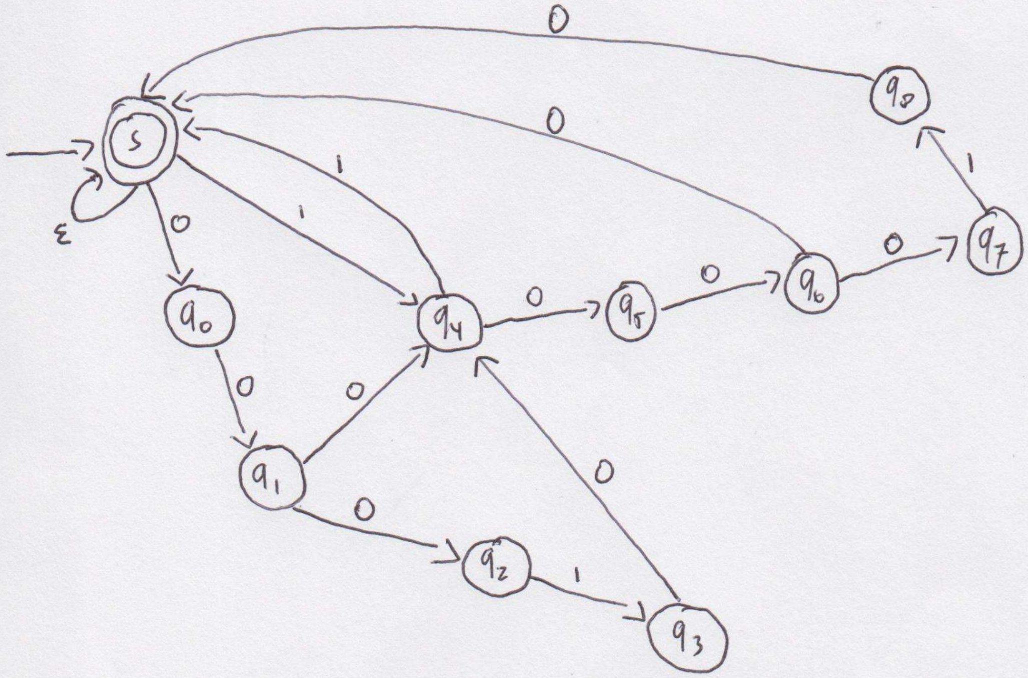
+ **0 pts** wrong

+ **0 pts** did not state over a,b,c

+ **0 pts** ambiguous

3a) L: ∀string $w \in \{a, b, c\}^*$ with $|w|$ is even.

b)



Explanation:

The NFA above satisfies the regex of $((1+000+00010)(1+000+00010))^*$
An empty string is accepted thus our starting state is an accepting state.
For the first part of the expression: $(1 + 000 + 00010)$, the NFA provides a transition to q_4 if the input is 1. A transition from the S state to q_0 to q_1 to q_4 which satisfies the input 000. A transition from the s state to q_0 to q_1 to q_2 to q_3 to q_4 for the input 0001 0. Then, we still need to choose $(1 + 000 + 00010)$ to be accepted.
If the input is 1, it can use the transition in q_4 to the s state.
If the input is 000, it can use the path from q_4 to q_5 to q_6 to the S state.
If the input is 00010, it can use the path from q_4 to q_5 to q_6 to q_7 to q_8 to the s state. Our overall NFA thus accepts the expression.

2 3B 20 / 20

✓ + **20 pts** Correct

- **5 pts** Start state not identified

+ **5 pts** IDK

+ **0 pts** Incorrect NFA

+ **10 pts** Partially correct NFA

+ **10 pts** empty string not accepted

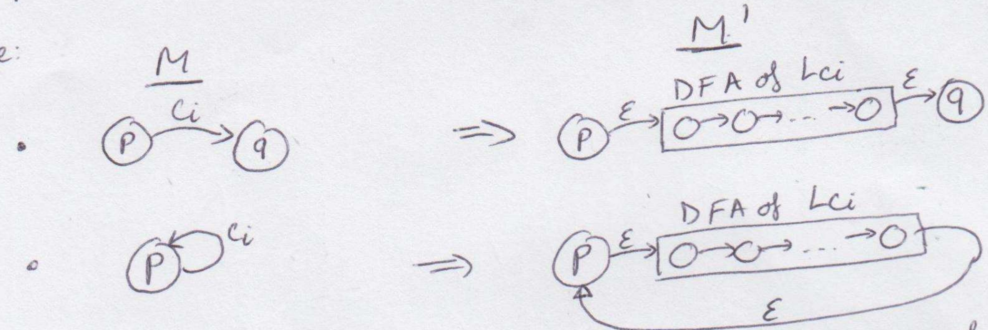
Q 3C) Given $M = (Q_M, \Sigma_M, \delta_M, S_M, A_M)$

How to construct $M' = (Q_{M'}, \Sigma_{M'}, \delta_{M'}, S_{M'}, A_{M'})$?

Step1: For each character c_i in Σ_M, construct DFA D_i of language L_{c_i} that accepts only string $f(c_i)$: $L_{c_i} = \{f(c_i)\}$

Step2: For any state $p, q \in Q_M$, whenever there is a transition from state p to q with input being character c_i, insert in between 2 states p and q the DFA D_i of Language L_{c_i} (described above)

Example:



- Step3: The accepting states of M' is the accepting states of M $(A_{M'} = A_M)$
- Step4: The starting state of M' is the starting state of M $(S_{M'} = S_M)$
- Step5: $Q_{M'}$ includes Q_M and all other states that are needed to construct all DFA of all languages L_{c_i} corresponding to character c_i in Σ_M

∘ Upper bound of # state of M'

- Let n be the number of state in M $(n = |Q|)$
- For each state in M, the maximum number of transition starting from that state is $|\Sigma_M|$, with $|\Sigma_M|$ be the number of character in alphabet Σ_M
- Each transition in DFA M is replace by a DFA of L_{ci} with c_i be the input causing the transition. Let x_i be the number of state in DFA of L_{ci}

⇒ Maximum number of states needed for all transitions ~~is~~ starting from one state is: $\sum_{i=1}^{|\Sigma_M|} x_i$ (sum of all States needed to build all DFA ~~for~~ L_{ci} for all character c_i in Σ)

- And we have M' also includes all states of M

$|\Sigma_M|$ → # of element in alphabet Σ_M.

$$\Rightarrow \text{\# of states of } M' \leq n + n \cdot \sum_{i=1}^{|\Sigma_M|} x_i$$

$$\Rightarrow \text{Upperbound} = \boxed{n + n \cdot \sum_{i=1}^{|\Sigma_M|} x_i}$$

(TO)

- Prove NFA M' accepts language $f(L)$

- Before proving, from the way we construct M', we have the following properties:

With the assumption that: $f(a) \neq f(b)$ if $a \neq b$, we have:

1) $\delta^*_{M'}(p, f(a)) \cap Q_M = q$ iff $\delta_M(p,a) = q$ for $q, p \in Q_M$, with $a \in \Sigma_M$. This is true because of the deterministic of DFA M.

2) If $\delta_M(p,a) = q$, then $q \in \delta^*_{M'}(p, f(a))$

Prove that $w \in L$ iff. $\delta^*_{M'}(s, f(w)) \cap A \neq \emptyset$

($\Rightarrow$): Suppose $w \in L$, prove $\delta^*_{M'}(s, f(w)) \cap A \neq \emptyset$

- Assumption: $f(\varepsilon) = \varepsilon$.
- Prove that if $\delta^*_M(p, w) = q$, then $q \in \delta^*_{M'}(p, f(w))$ for $q, p \in Q_M$
  - Induction on $|w|$.
  - Base case: $|w| = 0 \Rightarrow w = \varepsilon$
    $\Rightarrow \delta^*_M(p, \varepsilon) = p$ and $\delta^*_{M'}(p, \varepsilon) = \varepsilon reach(p) \ni p$
    $\Rightarrow p \in \delta^*_{M'}(p, \varepsilon) \Rightarrow$ Base case is true
  - IH: Assume for w with $|w| \leq n$, we have that: if $\delta^*_M(p, w) = q$, then $q \in \delta^*_{M'}(p, f(w))$
  - Let $|w| = n+1 \Rightarrow w = a \cdot x$, and $\delta^*_M(p, w) = q$
    Let $\delta_M(p, a) = p1$.
    - We have $p1 \in \delta^*_{M'}(p, f(a))$ from property (2).
      $\Rightarrow \delta^*_{M'}(p1, f(x)) \in \delta^*_{M'}(p, f(a) \cdot f(x)) = \delta^*_{M'}(p, f(w))$
    - Also, we have that: $\delta^*_M(p, w) = \delta^*_M(\delta(p,a), x) = \delta^*_M(p1, x)$
      and $\delta^*_M(p, w) = q \Rightarrow$ ~~and~~ $\delta^*_M(p1, x) = q$

      By IH, with $|x| = n$ and $\delta^*_M(p1, x) = q$
      $\Rightarrow q \in \delta^*_{M'}(p1, f(x))$

      And we have $\delta^*_{M'}(p1, f(x)) \in \delta^*_{M'}(p, f(w))$ (proven above)

Therefore, $q \in \delta^*_{M'}(p, f(w)) \Rightarrow$ Claim is true for all w

- We have $w \in L \Rightarrow \delta_M^*(s_M, w) \in A$

Let $q = \delta_M^*(s_M, w) \Rightarrow q \in \delta_{M'}^*(s_M, f(w))$

$\Rightarrow \delta_{M'}(s_M, f(w)) \cap A = q$ because $q \in A$

Therefore: $\delta_{M'}(s_M, f(w)) \cap A \neq \emptyset$

($\Leftarrow$) ~~Prove~~ Suppose $\delta^*(q, f(w)) \cap A \neq \emptyset$, prove $w \in L$.

Prove: with $\underline{p \,\&\, q \in Q_M}$, if $q \in \delta_N^*(p, f(w))$, then $\delta_M^*(p, w)\} = q$

- Induction on $|w|$ $\quad \delta_M^*(p, \varepsilon) = p$ with all state $p \in Q_M$
  - $|w| = 0 \Rightarrow w = \varepsilon \Rightarrow$
  - $\Rightarrow$ Base case is true
  - IH: Assume for $|w| \leq n$, if $q \in \delta_N^*(p, f(w))$, then $\delta_M^*(p, w) = q$
  - For $|w| = n+1 \Rightarrow w = a \cdot x$; and $q \in \{\delta_N^*(p, f(w)) \cap Q_M\}$

Let $\delta_M(p, a) = p_1 \Rightarrow \delta_M^*(p, w) = \delta^*(p_1, x)$ (because $p_1 = \delta(p, a)$)

Based on the property (1), we have

$\delta_{M'}^*(p, f(a)) \cap Q_M = p_1$

$\Rightarrow \delta_{M'}^*(p, f(a) \cdot f(x)) \cap Q_M = \delta_{M'}^*(p_1, f(x)) \cap Q$

$\Rightarrow$ From IH, and $|x| = n$, and $q \in \delta_{M'}^*(p_1, f(x))$

$\Rightarrow \delta_M^*(p_1, x) = q$

We have: $\delta_{M'}^*(p, f(w)) \cap Q_M = \delta_{M'}^*(p_1, f(x)) \cap Q$

$\Rightarrow q \in \delta_{M'}^*(p, f(w)) \cap Q_M$

$\Rightarrow \delta_M^*(p, w) = q$

Therefore the claim is true for all w

From the proof above, we have:

$$\delta^*_{M'}(s, f(w)) \cap A \neq \emptyset$$

Let $q \in \{\delta^*_{M'}(s, f(w)) \cap A\}$

$\Rightarrow \delta^*_{M}(s, w) = q \Rightarrow \delta^*_{M}(s, w) \in A$

(proven above)

$\Rightarrow w \in L$

Therefore NFA M' accept Language $f(L)$

3 3C 30 / 30

- + **30 pts** Correct
- + **0 pts** Wrong
- ✓ + **15 pts** Correct contruction
- ✓ + **15 pts** Proof of correctness
- + **7.5 pts** IDK
- - **5 pts** proof isnt rigorous
- - **5 pts** proof only one side
- + **5 pts** Construction is not well defined
- + **7.5 pts** Prove f(L) is regular by regex

Q.3D

I DON’T KNOW!!!!!

4 3D 10 / 40

+ **40 pts** Correct
+ **10 pts** construction not well defined
+ **20 pts** correct construction
+ **20 pts** proof of correctness
+ **10 pts** Proof only one side
+ **0 pts** wrong

✓ + **10 pts** IDK