Exercise 2.1

Let

$$x_1(n) = e^{j\omega_0 n} \tag{1}$$

$$x_2(n) = \cos(\omega_0 n) \tag{2}$$

for $n = 0, 1, \ldots, N-1$ with $N = 100$ and $\omega_0 = 0.1\pi$.

(a) Using a computer, compute the K-point amplitude spectra of $x_1(n)$ and $x_2(n)$. Experiment with the size of K. Start with a value of $K = N$ and increase it. What changes when you change K? Why?

Solution: A MATLAB-implementation can be found in `ex1a.m`. Here are some observations. If we set $K = N$, we only see one non-zero value for $x_1(n)$ and two for $x_2(n)$. When we increase K, however, we see a lot of non-zero values. These appear since the DTFT of the windowed signal is given by

$$X_1(\omega) = W(\omega - \omega_0) = \begin{cases} N & \omega = \omega_0 \\ \frac{\sin((\omega-\omega_0)N/2)}{\sin((\omega-\omega_0)/2)} e^{-j(\omega-\omega_0)\frac{N-1}{2}} & \text{otherwise} \end{cases} \tag{3}$$

The DFT is a sampled version of the DTFT where we sample the frequency uniformly on a K-point grid as

$$\omega_k = 2\pi(k-1)/K \; . \tag{4}$$

If $K = N$ and $N\omega_0/(2\pi)$ is an integer, then

$$\sin((\omega_k - \omega)N/2) = 0 \qquad \forall k = \{0, 1, \ldots, N-1\} \tag{5}$$

so that

$$X_1(\omega_k) = W(\omega_k - \omega_0) = \begin{cases} N & \omega = \omega_0 \\ 0 & \text{otherwise} \end{cases} \; . \tag{6}$$

However, when $K > N$ and/or $N\omega_0/(2\pi)$ is not an integer, we see the effects of the rectangular window.

We now consider the trumpet signal `trumpet.wav` which you can find on Moodle.

(b) Using a computer, compute the K-point amplitude spectrum of the trumpet signal. What can you say about the trumpet signal? Which model is appropriate for describing such a trumpet signal?

Solution: A MATLAB-implementation can be found in `ex1b.m`. Clearly, the spectrum reveals that the trumpet signal only consists of a few sinusoidal components. A sum of sinusoids would therefore be a good model for the trumpet signal.

Exercise 2.2

Linear convolution between two sequences $x(n)$ and $h(n)$ can be performed in both the time- and the frequency-domain. Consider the two sequences

$$x(n) = \cos(\omega_0 n) \tag{7}$$

$$h(n) = \begin{cases} a^n & n \geq 0 \\ 0 & n < 0 \end{cases} \tag{8}$$

for $n = 0, 1, \ldots, N-1$ where $N = 10$, $\omega_0 = 1.1$, and $a = -0.9$.

(a) Using a computer, compute

$$y(n) = (h * x)(n) \tag{9}$$

in the time-domain by summation.

Solution: A MATLAB-implementation can be found in `ex2.m`.

(b) Same as in question 1, but now do the convolution in the frequency domain via two FFTs and one iFFT.

Solution: A MATLAB-implementation can be found in `ex2.m`.

(c) Same as in question 1, but now form the convolution matrix $\boldsymbol{H}$ and compute the convolution via

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} \ . \tag{10}$$

Solution: A MATLAB-implementation can be found in `ex2.m`.

(d) Same as in question 1, but now form the DFT matrix and implement the convolution in the frequency domain via matrix-vector algebra.

Solution: A MATLAB-implementation can be found in `ex2.m`.