



One Loss for Quantization: Deep Hashing with Discrete Wasserstein Distributional Matching

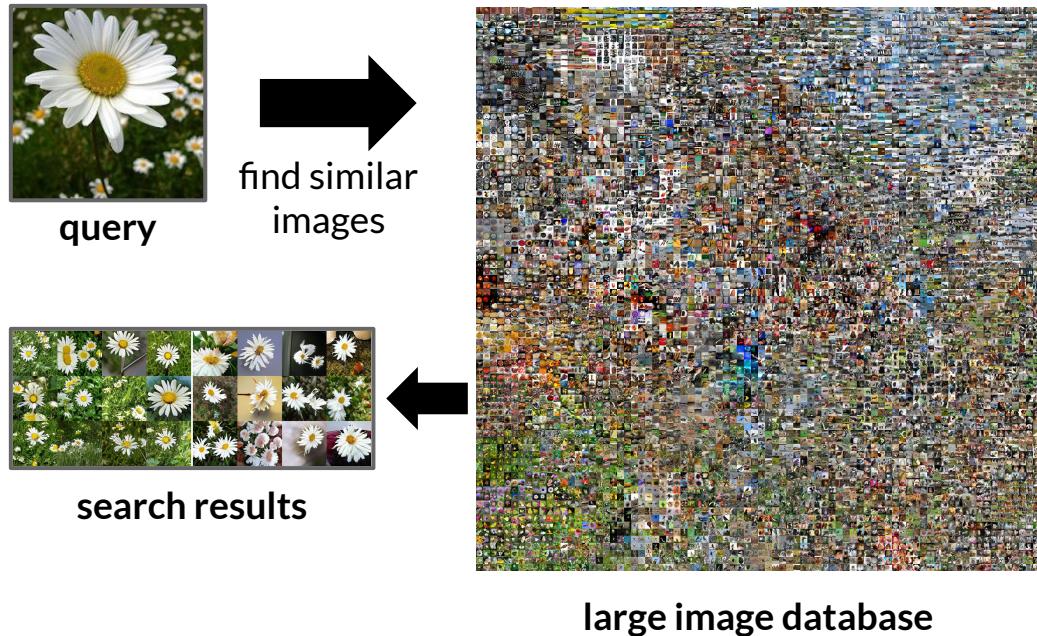
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Cognitive Computing Lab, **Baidu Research, USA**



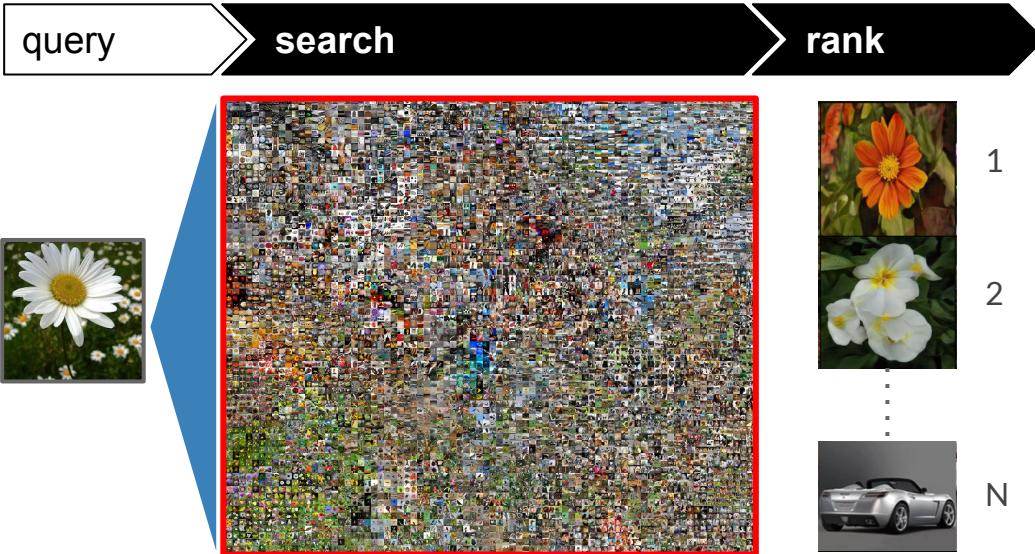
Similarity Search

Problem: Given a dataset of N items $X = \{x_1, x_2, \dots, x_N\}$ and a query q , we aim to find l items $R = \{x_1, x_2, \dots, x_l\}$ such that, for a similarity function **sim**, we have:

$$\begin{aligned} \text{sim}(q, x_i) &\geq \text{sim}(q, x_j) \\ \forall x_i \in R, \forall x_j \in X \setminus R \end{aligned}$$



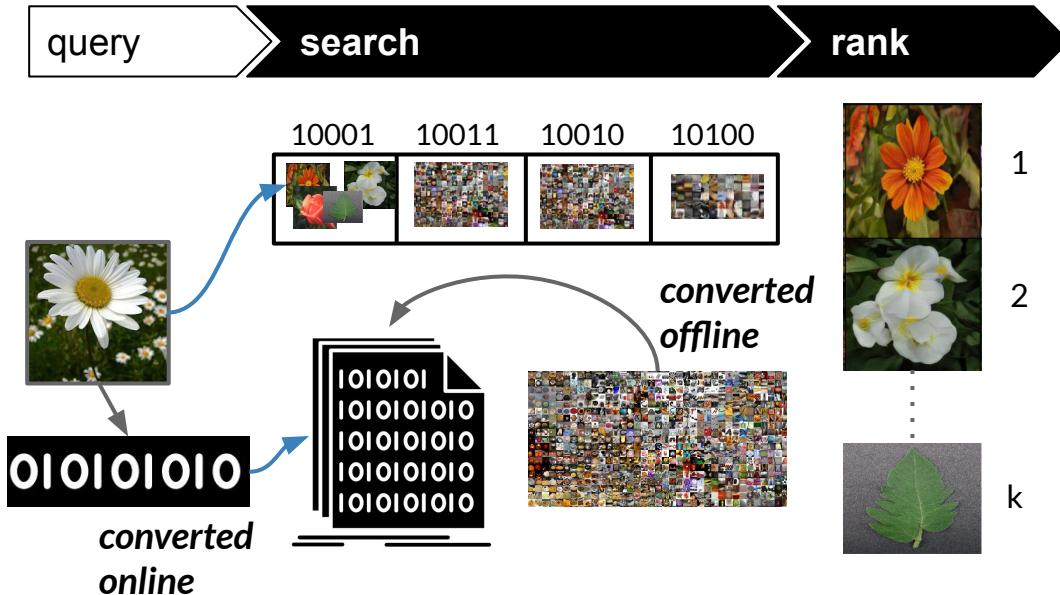
Linear Search



Exhaustive search

- ▷ Infeasible in large database of millions or billions of items.
- ▷ Wasteful of computation
 - only a small subset is relevant
 - real-time ranking is impossible

Approximate Nearest Neighbor (ANN)



Approximate Search (Hashing)

- ▷ Transforms images into binary vectors expressing their similarity.
- ▷ Search via table look-up
- ▷ Linear Search in Discrete space:
 - Memory efficient: 4MB for 1M items
 - Compute efficient: 2 instructions per distance computation

Hash-function Learning

- ▷ Learn a hash function

$$F : \mathcal{R}^n \rightarrow \{0, 1\}^m$$

discrete function



$$f : \mathcal{R}^n \rightarrow [0, 1]^m$$

continuous relaxation

$$F(x) = f(x) > 0.5$$

discretization

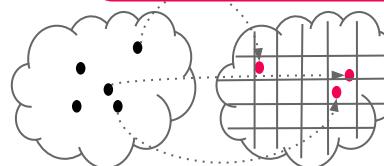
- ▷ Overall objective function of hashing methods

$$\arg \min_f E_{x \sim D_x} L(x, f(x))$$

locality-preserving loss

preserves the semantics
of **sim** in discrete space

$$+ E_{x \sim D_x} \sum_k \lambda_i \times H_k(f(x))$$



this work

hashing regularizer

minimizes gap between
continuous and discrete
optimizations.

Hash-function Learning

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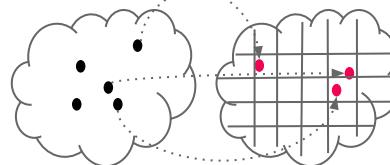
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$$F(x) = f(x) > 0.5$$

discretization

- ▷ Overall objective function of hashing methods

$$\arg \min_f E_{x \sim D_x} L(x, f(x)) + E_{x \sim D_x} \sum_k \lambda_i \times H_k(f(x))$$



Existing Objectives are Complex

\min_f [locality preserving loss]

$$+ |W^T W - I|_2 + \sum_{k=1}^m \bar{b}_k \log \bar{b}_k + (1 - \bar{b}_k) \log (1 - \bar{b}_k)$$

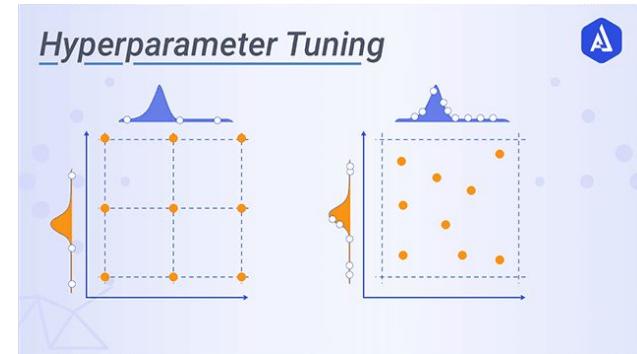
Bit Uncorrelation

$$+ \sum_x \sum_{k=1}^m -f(x) \log(f(x)) - (1 - f(x)) \log(1 - f(x))$$

Bit Balance

Low Quantization Error

Complex objective increases training complexity
(i.e., hyperparameter tuning)



[Source: [Online](#)]

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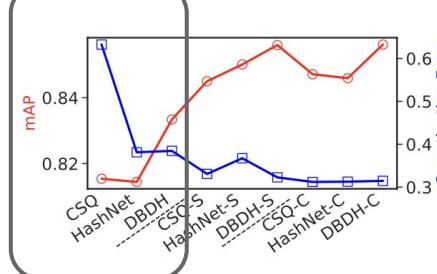
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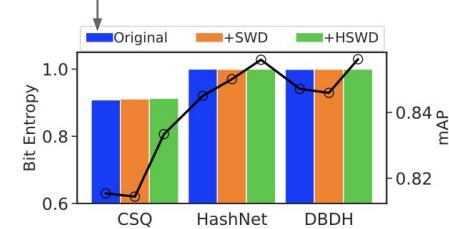
existing optimization

Complex objective increases training complexity
(i.e., hyperparameter tuning)

Complex objective results in sub-optimal quantization



(a) Quantization Error



(b) Bit Entropy

[Doan et al. 2022]

Existing Objectives are Complex

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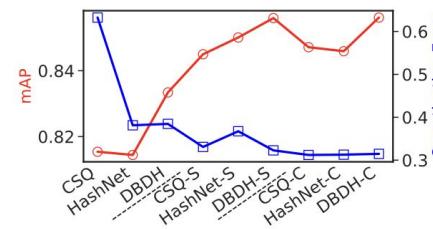
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Bit Balance

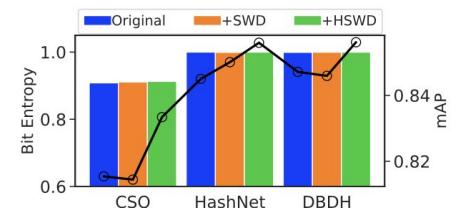
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Existing Objectives are Complex

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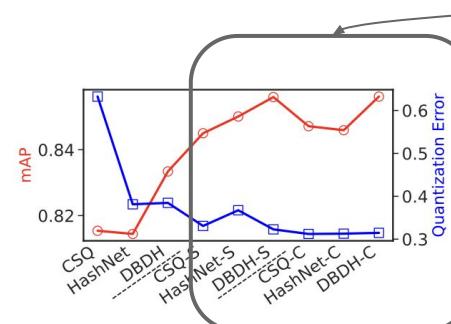
Bit Balance

Low Quantization Error

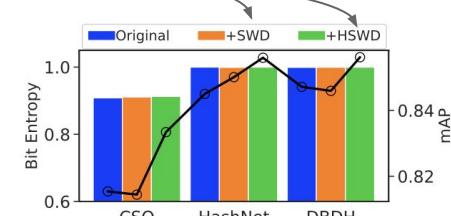
this work

Complex objective increases training complexity
(i.e., hyperparameter tuning)

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(a) Quantization Error



(b) Bit Entropy

[Doan et al. 2022]

Single-shot Quantization Loss

Our approach: single divergence loss

$$\arg \min_f d(q \parallel q^*) \quad f(x) \sim q \\ q^*: \text{fixed distribution}$$

One Single Quantization Loss

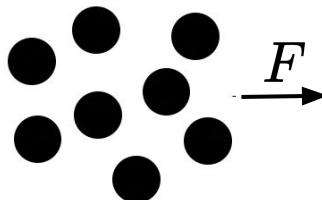
Disadvantages: challenging to optimize

\min_f [locality preserving loss]

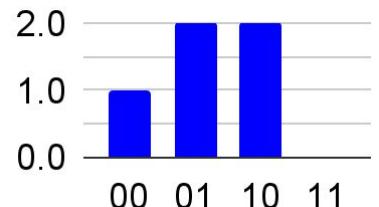
$$+ d(q \parallel q^*)$$

can be used to improve performance of
any existing Deep Supervised Hashing

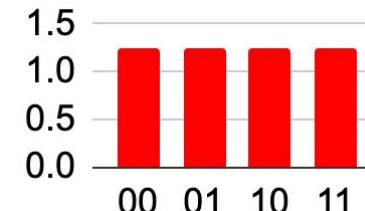
Task: learn 2-bit hash function



F



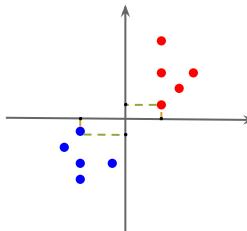
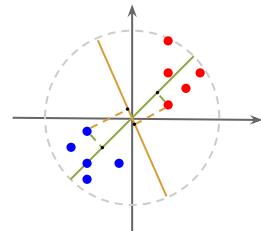
learned distribution q



optimal distribution q^*
(with maximum entropy)

$$q^* : b_i \sim \text{bernoulli}(0.5)$$

Choosing the “Right” Divergence $\mathcal{D}(q(b) \parallel q^*(z))$



Sliced Wasserstein Distance

- Lower sample complexity
- No minimax
- Several directions are discriminative

Hash-Sliced Wasserstein Distance

- Lower sample complexity
- No minimax
- Small number of discriminative projections

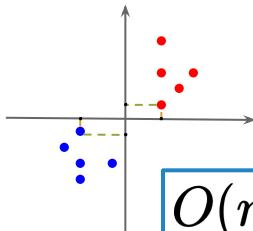
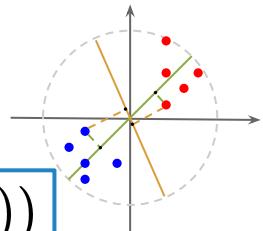
Wasserstein Distance

- Non-trivial to estimate
- High sample complexity
- Possibly minimax optimization (dual domain)

Other divergences (e.g. KL, JSD, etc...)

- Do not work with non-overlapping supports
- High sample complexity
- Minimax optimization

Choosing the “Right” Divergence $\mathcal{D}(q(b) \parallel q^*(z))$



$O(mN\log(Nd)), m \ll L$

Sliced Wasserstein Distance

- Lower sample complexity
- No minimax
- Several directions are discriminative

$$\mathcal{D}(h(X), B) \approx \left(\frac{1}{m} \sum_{l=1}^m [\mathcal{W}(h(X)_{l,:}, B_{l,:})]^2 \right)^{1/2}$$

no projection: averaging along
each hashing dimension

Proposed Hash-Sliced Wasserstein Distance

- Lower sample complexity
- No minimax
- Small number of discriminative projections

$$\mathcal{D}(h(X), B) \approx \left(\frac{1}{L} \sum_{l=1}^L \mathcal{W}(\omega_l^T h(X), \omega_l^T B) \right)^{1/2}$$

projection into 1-D space

Single-shot Quantization

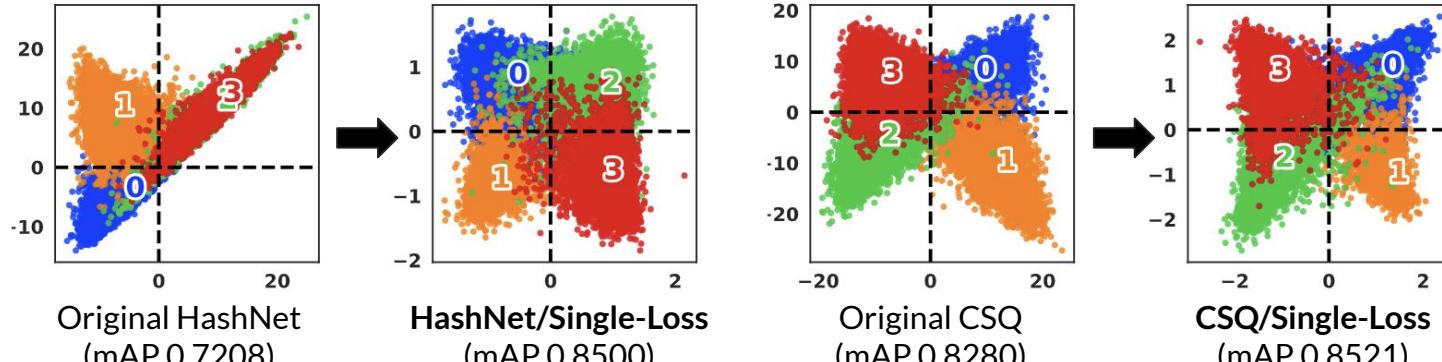


Figure. Learn 2-bit hash function on CIFAR10's data from 4 classes

Table. Averaged running time per epoch across different supervised hashing methods (in seconds).

Dataset	Original	SWD	HSDW
CIFAR-10	19.4	24.2	17.1/ 40%
NUS-WIDE	58.3	71.2	50.1/ 41%
COCO	55.6	68.1	49.5/ 37%

More computationally efficient even before intensive model selection

Performance Evaluation (Precision@1000)

Retrieve k items



Precision@k = number of / k

Blue: improvement over original methods

-S: Sliced Wasserstein Estimate | -C: Proposed Wasserstein Estimate

Method	CIFAR-10		NUS-WIDE	
	16 bits	32 bits	16 bits	32 bits
DSDH	0.8252	0.8406	0.8117	0.8294
DSDH-S	0.8526/ 3.3%	0.8543/ 1.6%	0.8162/ 0.6%	0.8312/ 0.2%
DSDH-C	0.8645/ 4.8%	0.8739/ 4.0%	0.8195/ 1.0%	0.8391/ 1.2%
HashNet	0.6193	0.8613	0.7581	0.8158
HashNet-S	0.8470/ 36.8%	0.8755/ 1.7%	0.7743/ 2.1%	0.8199/ 0.5%
HashNet-C	0.7698/ 24.3%	0.8715/ 1.2%	0.7456/-1.7%	0.8078/-1.0%
GreedyHash	0.8561	0.8616	0.7601	0.8009
GreedyHash-S	0.8583/ 0.3%	0.8656/ 0.5%	0.7657/ 0.7%	0.7973/-0.5%
GreedyHash-C	0.8517/-0.5%	0.8700/ 1.0%	0.7630/ 0.4%	0.7931/-1.0%
DCH	0.8621	0.8568	0.7843	0.7898
DCH-S	0.8622/0.0%	0.8761/ 2.3%	0.7846/0.0%	0.7923/ 0.3%
DCH-C	0.8654/ 0.4%	0.8635/ 0.8%	0.7893/ 0.6%	0.7914/ 0.2%
CSQ	0.8510	0.8571	0.7903	0.8285
CSQ-S	0.8661/ 1.8%	0.8732/ 1.9%	0.8034/ 1.7%	0.8318/ 0.4%
CSQ-C	0.8670/ 1.9%	0.8688/ 1.4%	0.8007/ 1.3%	0.8353/ 0.8%
DBDH	0.8440	0.8421	0.8122	0.8323
DBDH-S	0.8626/ 2.2%	0.8675/ 3.0%	0.8177/ 0.7%	0.8388/ 0.8%
DBDH-C	0.8658/ 2.6%	0.8731/ 3.7%	0.8135/ 0.1%	0.8380/ 0.7%

Single-Label Data

Multi-Label Data

Performance Evaluation (MAP)

Retrieve k items  MAP@k = Mean of Average Precisions from 1 to k (Area under PR Curve)

-S: Sliced Wasserstein Estimate | -C: Proposed Wasserstein Estimate

Method	CIFAR-10			NUS-WIDE			COCO		
	16 bits	32 bits	64 bits	16 bits	32 bits	64 bits	16 bits	32 bits	64 bits
DSDH [40]	0.7909	0.8072	0.8278	0.8270	0.8455	0.8640	0.7331	0.7853	0.8074
DSDH-S	0.8187/ 3.5%	0.8439/ 4.6%	0.8517/ 2.9%	0.8282/ 0.1%	0.8461/ 0.1%	0.8712/ 0.8%	0.7330/ 0.0%	0.8030/ 2.3%	0.8404/ 4.1%
DSDH-C	0.8531/ 7.9%	0.8620/ 6.8%	0.8658/ 4.6%	0.8433/ 2.0%	0.8631/ 2.1%	0.8749/ 1.3%	0.7424/ 1.3%	0.8032/ 2.3%	0.8408/ 4.1%
HashNet [6]	0.6922	0.8311	0.8566	0.7728	0.8336	0.8654	0.6899	0.7666	0.8098
HashNet-S	0.8131/ 17%	0.8573/ 3.2%	0.8749/ 2.1%	0.8062/ 4.3%	0.8438/ 1.2%	0.8713/ 0.7%	0.7215/ 4.6%	0.7764/ 1.3%	0.8189/ 1.1%
HashNet-C	0.7939/ 14%	0.8467/ 1.9%	0.8691/ 1.5%	0.8002/ 3.5%	0.8437/ 1.2%	0.8791/ 1.6%	0.7202/ 4.4%	0.7789/ 1.6%	0.8202/ 1.3%
GreedyHash [50]	0.8223	0.8474	0.8646	0.7802	0.8081	0.8328	0.6533	0.7219	0.7561
GreedyHash-S	0.8280/ 0.7%	0.8497/ 0.3%	0.8653/ 0.1%	0.7815/ 0.1%	0.8083/ 0.0%	0.8390/ 0.7%	0.6668/ 2.1%	0.7291/ 1.0%	0.7618/ 0.8%
GreedyHash-C	0.8375/ 1.9%	0.8536/ 0.7%	0.8722/ 0.9%	0.7890/ 1.1%	0.8179/ 1.2%	0.8477/ 1.8%	0.6637/ 1.6%	0.7299/ 1.1%	0.7712/ 2.0%
DCH [5]	0.8302	0.8432	0.8558	0.8015	0.8061	0.8040	0.7578	0.7792	0.7723
DCH-S	0.8372/ 0.8%	0.8515/ 1.0%	0.8602/ 0.5%	0.8058/ 0.5%	0.8079/ 0.2%	0.8067/ 0.3%	0.7657/ 1.1%	0.7831/ 0.5%	0.7803/ 1.0%
DCH-C	0.8446/ 1.7%	0.8596/ 1.9%	0.8711/ 1.8%	0.8159/ 1.8%	0.8145/ 1.0%	0.8155/ 1.4%	0.7702/ 1.6%	0.7892/ 1.3%	0.7807/ 1.1%
CSQ [58]	0.8069	0.8291	0.8366	0.7992	0.8384	0.8596	0.6783	0.7550	0.8146
CSQ-S	0.8401/ 4.1%	0.8555/ 3.2%	0.8554/ 2.3%	0.8044/ 0.7%	0.8495/ 1.3%	0.8626/ 0.4%	0.7036/ 3.7%	0.7765/ 2.8%	0.8234/ 1.0%
CSQ-C	0.8457/ 4.8%	0.8558/ 3.2%	0.8652/ 3.4%	0.8054/ 0.8%	0.8511/ 1.5%	0.8701/ 1.2%	0.6989/ 3.0%	0.7752/ 2.7%	0.8255/ 1.3%
DBDH [60]	0.7660	0.8223	0.8492	0.8305	0.8552	0.8666	0.7202	0.7826	0.8042
DBDH-S	0.8458/ 10%	0.8587/ 4.4%	0.8603/ 1.3%	0.8387/ 1.0%	0.8577/ 0.3%	0.8680/ 1.8%	0.7461/ 2.2%	0.7996/ 3.7%	0.8336/ 4.3%
DBDH-C	0.8466/ 10%	0.8593/ 4.5%	0.8668/ 2.1%	0.8395/ 1.1%	0.8633/ 0.9%	0.8760/ 1.1%	0.7389/ 2.6%	0.7889/ 0.8%	0.8308/ 3.9%

Single-Label Data

Multi-Label Data

Summary

- ▷ Show that **better quantization** results in better retrieval.
- ▷ Learn better quantization with a **single loss**.
- ▷ Propose an **efficient divergence estimate** for single-loss.

Our approach can be used with any existing Deep Supervised Hashing techniques to learn better-quantized hash functions!

Thank You!

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Code: https://github.com/khoadoan106/single_loss_quantization