

## EE2211 Tutorial 6

(Ridge Regression in Dual Form)

### Question 1:

Derive the solution for linear ridge regression in dual form (see Lecture 6 notes page 16).

(Polynomial Regression, 1D data)

### Question 2:

Given the following data pairs for training

$$\begin{aligned}\{x = -10\} &\rightarrow \{y = 5\} \\ \{x = -8\} &\rightarrow \{y = 5\} \\ \{x = -3\} &\rightarrow \{y = 4\} \\ \{x = -1\} &\rightarrow \{y = 3\} \\ \{x = 2\} &\rightarrow \{y = 2\} \\ \{x = 8\} &\rightarrow \{y = 2\}\end{aligned}$$

- (a) Perform a 3<sup>rd</sup>-order polynomial regression and sketch the result of line fitting.
- (b) Given a test point  $\{x = 9\}$  predict  $y$  using the polynomial model.
- (c) Compare this prediction with that of a linear regression.

(Polynomial Regression, 3D data, Python)

### Question 3:

- (a) Write down the expression for a 3<sup>rd</sup> order polynomial model having a 3-dimensional input.
- (b) Write down the  $\mathbf{P}$  matrix for this polynomial given  $\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ .
- (c) Given  $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ , can a unique solution be obtained in dual form? If so, proceed to solve it.
- (d) Given  $\mathbf{y} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ , can the primal ridge regression be applied to obtain a unique solution? If so, proceed to solve it.
- (e)

(Binary Classification, Python)

### Question 4:

Given the training data:

$$\begin{aligned}\{x = -1\} &\rightarrow \{y = \text{class1}\} \\ \{x = 0\} &\rightarrow \{y = \text{class1}\} \\ \{x = 0.5\} &\rightarrow \{y = \text{class2}\} \\ \{x = 0.3\} &\rightarrow \{y = \text{class1}\} \\ \{x = 0.8\} &\rightarrow \{y = \text{class2}\}\end{aligned}$$

Predict the class label for  $\{x = -0.1\}$  and  $\{x = 0.4\}$  using linear regression with signum discrimination.

(Multi-Category Classification, Python)

**Question 5:**

Given the training data:

$$\begin{aligned}\{x = -1\} &\rightarrow \{y = \text{class1}\} \\ \{x = 0\} &\rightarrow \{y = \text{class1}\} \\ \{x = 0.5\} &\rightarrow \{y = \text{class2}\} \\ \{x = 0.3\} &\rightarrow \{y = \text{class3}\} \\ \{x = 0.8\} &\rightarrow \{y = \text{class2}\}\end{aligned}$$

- (a) Predict the class label for  $\{x = -0.1\}$  and  $\{x = 0.4\}$  based on linear regression towards a one-hot encoded target.
- (b) Predict the class label for  $\{x = -0.1\}$  and  $\{x = 0.4\}$  using a polynomial model of 5<sup>th</sup> order and a one-hot encoded target.

(Multi-Category Classification, Python)

**Question 6 (continued from Q3 of Tutorial 2):**

Get the data set “`from sklearn.datasets import load_iris`”. Use Python to perform the following tasks.

- (a) Split the database into two sets: 74% of samples for training, and 26% of samples for testing. Hint: you might want to utilize `from sklearn.model_selection import train_test_split` for the splitting.
- (b) Construct the target output using one-hot encoding.
- (c) Perform a linear regression for classification (without inclusion of ridge, utilizing one-hot encoding for the learning target) and compute the number of test samples that are classified correctly.
- (d) Using the same training and test sets as in above, perform a 2<sup>nd</sup> order polynomial regression for classification (again, without inclusion of ridge, utilizing one-hot encoding for the learning target) and compute the number of test samples that are classified correctly. Hint: you might want to use `from sklearn.preprocessing import PolynomialFeatures` for generation of the polynomial matrix.

**Question 7**

MCQ: there could be more than one answer. Given three samples of two-dimensional data points  $\mathbf{X} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 3 & 3 \end{bmatrix}$  with

corresponding target vector  $\mathbf{y} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ . Suppose you want to use a full third-order polynomial model to fit these data.

Which of the following is/are true?

- a) The polynomials model has 10 parameters to learn
- b) The polynomial learning system is an under-determined one
- c) The learning of the polynomial model has infinite number of solutions
- d) The input matrix  $\mathbf{X}$  has linearly dependent samples
- e) None of the above

### Question 8

MCQ: there could be more than one answer. Which of the following is/are true?

- a) The polynomial model can be used to solve problems with nonlinear decision boundary.
- b) The ridge regression cannot be applied to multi-target regression.
- c) The solution for learning feature  $\mathbf{X}$  with target  $\mathbf{y}$  based on linear ridge regression can be written as  $\hat{\mathbf{w}} = (\mathbf{X}^T\mathbf{X} + \lambda\mathbf{I})^{-1}\mathbf{X}^T\mathbf{y}$  for  $\lambda > 0$ . As  $\lambda$  increases,  $\hat{\mathbf{w}}^T\hat{\mathbf{w}}$  decreases.
- d) If there are four data samples with two input features each, the full second-order polynomial model is an over-determined system.