Advertising Data Analysis Using Theory of Constraints

Environment Setup & Data Loading

In [45]:
```
import os
import warnings

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import statsmodels.api as sm

warnings.filterwarnings("ignore")
plt.style.use("default")
sns.set_context("notebook")
DATA_PATH = "advertising.csv"

if not os.path.exists(DATA_PATH):
    raise FileNotFoundError(f"Dataset not found at path: {DATA_PATH}")

df = pd.read_csv(DATA_PATH)
df.head()
```

Out[45]:

	Date	TV	Radio	Newspaper	Sales
0	Thursday, January 1, 2015	230.1	37.8	69.2	22.1
1	Thursday, January 8, 2015	44.5	39.3	45.1	10.4
2	Thursday, January 15, 2015	17.2	45.9	69.3	9.3
3	Thursday, January 22, 2015	151.5	41.3	58.5	18.5
4	Thursday, January 29, 2015	180.8	10.8	58.4	12.9

In [8]:
```
# Convert column index to list for readability
list(df.columns)
```

Out[8]: ['Date', 'TV', 'Radio', 'Newspaper', 'Sales']

In [9]:
```
# Overview of dataset structure and data types
df.info()

# Statistical summary of numerical features
df.describe()

# Check for missing or null values
df.isnull().sum()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 346 entries, 0 to 345
Data columns (total 5 columns):
 #   Column     Non-Null Count  Dtype  
---  ------     --------------  -----  
 0   Date       346 non-null    object 
 1   TV         346 non-null    float64
 2   Radio      346 non-null    float64
 3   Newspaper  346 non-null    float64
 4   Sales      346 non-null    float64
dtypes: float64(4), object(1)
memory usage: 13.6+ KB
```

Out[9]:
```
Date         0
TV           0
Radio        0
Newspaper    0
Sales        0
dtype: int64
```

Formatting Issue

> We can see that the date type is an object not an actual date format
>
> Lets correct this first

In [10]:
```python
# Convert it to a proper datetime format for time-based analysis

df['Date'] = pd.to_datetime(df['Date'])

# Verify the conversion
df.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 346 entries, 0 to 345
Data columns (total 5 columns):
 #   Column     Non-Null Count  Dtype         
---  ------     --------------  -----         
 0   Date       346 non-null    datetime64[ns]
 1   TV         346 non-null    float64       
 2   Radio      346 non-null    float64       
 3   Newspaper  346 non-null    float64       
 4   Sales      346 non-null    float64       
dtypes: datetime64[ns](1), float64(4)
memory usage: 13.6 KB
```

In [11]:
```python
# Preview the converted datetime values

df['Date'].head()
```

Out[11]:
```
0   2015-01-01
1   2015-01-08
2   2015-01-15
3   2015-01-22
4   2015-01-29
Name: Date, dtype: datetime64[ns]
```

In [12]:
```python
df['Date'].isnull().sum() # <---- Currently we are chcking the date format a
```

Out[12]:
```
0
```

Identify the Constraint (CORE TOC STEP)

In [13]:
```python
# Compute correlation of advertising channels with Sales
# This helps identify the directional strength of each channel's impact

sales_corr = (
    df[['TV', 'Radio', 'Newspaper', 'Sales']]
    .corr()['Sales']
    .sort_values(ascending=False)
    * 100
)

sales_corr
```

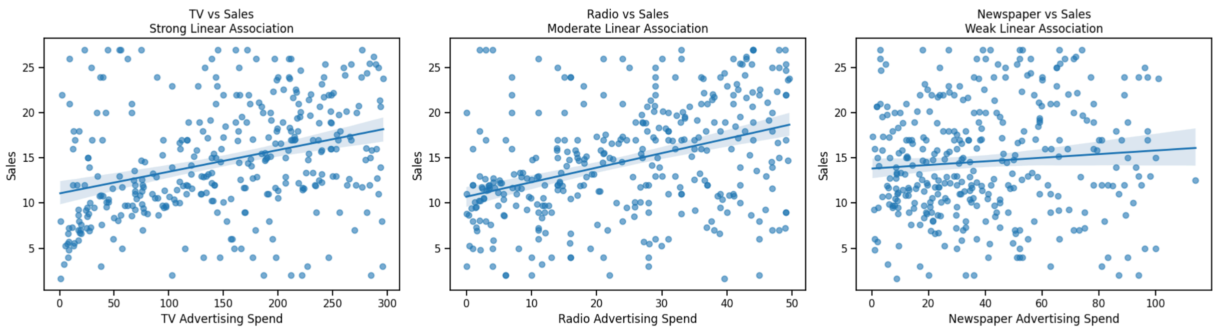
Out[13]:
```
Sales        100.000000
Radio         38.410415
TV            33.025625
Newspaper      8.529707
Name: Sales, dtype: float64
```

Visual Validation of Directional Signal Using Regression Lines

- Now, lets check this out if we put some values/invest on any of this medium or channel
- which one is giving us most of the returns or generating the sales

In [14]:
```python
# Advertising Spend vs Sales
# Regression Visualization


fig, axes = plt.subplots(1, 3, figsize=(18, 5))

# TV vs Sales
sns.regplot(
    x='TV',
    y='Sales',
    data=df,
    ax=axes[0],
    scatter_kws={'alpha': 0.6},
    line_kws={'linewidth': 2}
)
axes[0].set_title('TV vs Sales\nStrong Linear Association')
axes[0].set_xlabel('TV Advertising Spend')
axes[0].set_ylabel('Sales')

# Radio vs Sales
sns.regplot(
    x='Radio',
```

```
    y='Sales',
    data=df,
    ax=axes[1],
    scatter_kws={'alpha': 0.6},
    line_kws={'linewidth': 2}
)
axes[1].set_title('Radio vs Sales\nModerate Linear Association')
axes[1].set_xlabel('Radio Advertising Spend')
axes[1].set_ylabel('Sales')

# Newspaper vs Sales
sns.regplot(
    x='Newspaper',
    y='Sales',
    data=df,
    ax=axes[2],
    scatter_kws={'alpha': 0.6},
    line_kws={'linewidth': 2}
)
axes[2].set_title('Newspaper vs Sales\nWeak Linear Association')
axes[2].set_xlabel('Newspaper Advertising Spend')
axes[2].set_ylabel('Sales')

plt.tight_layout()
plt.show()
```



- As we can see, for TV & Radio there is a upward line which means if we are spending money on both they are giving us incerase in sales compare to Newspaper

  > Takeaway

- While in the Newspaper the line is flat which means we are not able to generate much sales in the Newspaper.

- So our constraint is "Newspaper"

In [15]:
```
# Multivariate Regression Analysis

# While correlation analysis and scatter plots provide directional insight,
# they do not quantify marginal throughput or isolate the individual impact
# of each advertising channel.
#
# A multivariate OLS regression is therefore performed to estimate the
```

```python
# marginal contribution of each channel to Sales while controlling for
# the others.
#
# From a Theory of Constraints (TOC) perspective, this step is critical to
# distinguish true throughput drivers from channels that consume budget
# without meaningfully increasing output.

import statsmodels.api as sm

# Define predictors and target
X = df[['TV', 'Radio', 'Newspaper']]
X = sm.add_constant(X)  # Add intercept term
y = df['Sales']

# Fit OLS model
model = sm.OLS(y, X).fit()

# Model summary
model.summary()
```

Out[15]:

OLS Regression Results

Dep. Variable:	Sales	**R-squared:**	0.260
Model:	OLS	**Adj. R-squared:**	0.253
Method:	Least Squares	**F-statistic:**	39.97
Date:	Tue, 06 Jan 2026	**Prob (F-statistic):**	3.64e-22
Time:	01:41:02	**Log-Likelihood:**	-1072.2
No. Observations:	346	**AIC:**	2152.
Df Residuals:	342	**BIC:**	2168.
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	7.0526	0.823	8.566	0.000	5.433	8.672
TV	0.0244	0.003	7.194	0.000	0.018	0.031
Radio	0.1621	0.020	8.104	0.000	0.123	0.201
Newspaper	0.0009	0.011	0.081	0.935	-0.021	0.023

Omnibus:	9.896	**Durbin-Watson:**	2.118
Prob(Omnibus):	0.007	**Jarque-Bera (JB):**	15.851
Skew:	0.160	**Prob(JB):**	0.000361
Kurtosis:	3.998	**Cond. No.**	497.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

What can we understand from this?

> 1. this is basically a Ordinary Least Squares multiple linear regression
> 2. in Maths: Sales=β0+β1(TV)+β2(Radio)+β3(Newspaper)+ϵ

In [16]:

```
# Throughput Efficiency (Sales per Spend)

(df[['TV','Radio','Newspaper']]
 .div(df['Sales'], axis=0)
 .mean()
 .sort_values())
```

Out[16]:
```
Radio         1.989754
Newspaper     3.647249
TV           12.372442
dtype: float64
```

In [40]:
```
# We have found the Constraint
```

EXPLOIT THE CONSTRAINT

In [18]:
```
df['Newspaper_bin'] = pd.qcut(df['Newspaper'], q=4)
df.groupby('Newspaper_bin')['Sales'].mean()
```

Out[18]:
```
Newspaper_bin
(0.299, 18.3]      14.013636
(18.3, 34.45]      13.672941
(34.45, 57.45]     15.034884
(57.45, 114.0]     15.736782
Name: Sales, dtype: float64
```

In [19]:
```
df['Newspaper'].quantile([0.25, 0.50, 0.75])
```

Out[19]:
```
0.25    18.30
0.50    34.45
0.75    57.45
Name: Newspaper, dtype: float64
```

In [39]:
```
df_exploit = df[df['Newspaper'] <= 35]
```

In [21]:
```
df[['Sales']].mean(), df_exploit[['Sales']].mean()
```

Out[21]:
```
(Sales    14.617052
 dtype: float64,
 Sales    13.820904
 dtype: float64)
```

In [22]:
```
df['Newspaper'].mean() - df_exploit['Newspaper'].mean()
```

Out[22]:
```
21.315646125208186
```

In [23]:
```
df['Newspaper'].quantile([0.25, 0.50, 0.75])
```

Out[23]:
```
0.25    18.30
0.50    34.45
0.75    57.45
Name: Newspaper, dtype: float64
```

In [24]:
```
df_exploit = df[df['Newspaper'] <= 35]
```

In [25]:
```
df['Sales'].mean(), df_exploit['Sales'].mean()
```

Out[25]:
```
(14.617052023121387, 13.820903954802262)
```

In [26]:
```
df['Newspaper'].mean() - df_exploit['Newspaper'].mean()
```

Out[26]: 21.315646125208186

In [38]:
```
df['TP_ratio'] = df['Sales'] / df[['TV','Radio','Newspaper']].sum(axis=1)
df_exploit['TP_ratio'] = df_exploit['Sales'] / df_exploit[['TV','Radio','New
df['TP_ratio'].mean(), df_exploit['TP_ratio'].mean()
```

Out[38]: (0.08248236802063037, 0.09054665560726316)

In [28]:
```
df_toc = df[df['Newspaper'] <= 35]
```

In [29]:
```
df_toc[['TV','Radio','Sales']].corr()['Sales'].sort_values(ascending=False)
```

Out[29]:
```
Sales    1.000000
Radio    0.526301
TV       0.433305
Name: Sales, dtype: float64
```

In [30]:
```
X = df_toc[['TV','Radio']]
X = sm.add_constant(X)
y = df_toc['Sales']
sm.OLS(y, X).fit().summary()
```

Out[30]:

OLS Regression Results

Dep. Variable:	Sales	**R-squared:**	0.477
Model:	OLS	**Adj. R-squared:**	0.471
Method:	Least Squares	**F-statistic:**	79.41
Date:	Tue, 06 Jan 2026	**Prob (F-statistic):**	3.12e-25
Time:	01:41:02	**Log-Likelihood:**	-492.39
No. Observations:	177	**AIC:**	990.8
Df Residuals:	174	**BIC:**	1000.
Df Model:	2		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	5.5141	0.735	7.499	0.000	4.063	6.965
TV	0.0287	0.004	8.163	0.000	0.022	0.036
Radio	0.2053	0.021	9.815	0.000	0.164	0.247

Omnibus:	18.380	**Durbin-Watson:**	2.175
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	61.450
Skew:	0.230	**Prob(JB):**	4.53e-14
Kurtosis:	5.850	**Cond. No.**	425.

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

In [31]:

```
df_toc['TV_Radio'] = df_toc['TV'] * df_toc['Radio']
X = df_toc[['TV','Radio','TV_Radio']]
X = sm.add_constant(X)
sm.OLS(y, X).fit().summary()
```

Out[31]:

OLS Regression Results

Dep. Variable:	Sales	**R-squared:**	0.491
Model:	OLS	**Adj. R-squared:**	0.483
Method:	Least Squares	**F-statistic:**	55.73
Date:	Tue, 06 Jan 2026	**Prob (F-statistic):**	2.94e-25
Time:	01:41:02	**Log-Likelihood:**	-489.94
No. Observations:	177	**AIC:**	987.9
Df Residuals:	173	**BIC:**	1001.
Df Model:	3		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	7.0080	0.995	7.047	0.000	5.045	8.971
TV	0.0183	0.006	3.136	0.002	0.007	0.030
Radio	0.1318	0.039	3.356	0.001	0.054	0.209
TV_Radio	0.0005	0.000	2.202	0.029	5.35e-05	0.001

Omnibus:	28.856	**Durbin-Watson:**	2.210
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	90.929
Skew:	0.587	**Prob(JB):**	1.80e-20
Kurtosis:	6.310	**Cond. No.**	1.39e+04

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 1.39e+04. This might indicate that there are
strong multicollinearity or other numerical problems.

In [32]:

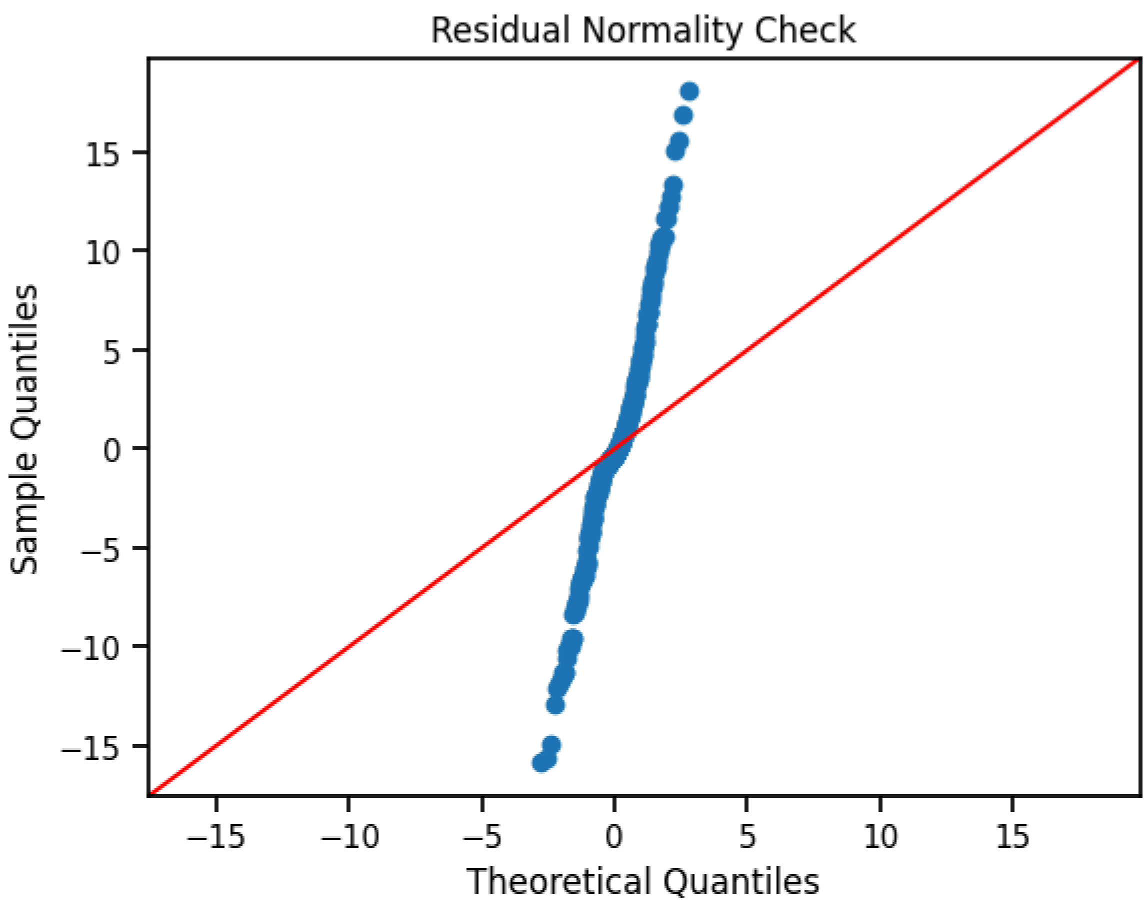
```
df_sim = df_exploit.copy()
df_sim['TV'] += 0.7 * 21.3
df_sim['Radio'] += 0.3 * 21.3

df_sim['Sales_pred'] = (
    7.008 +
    0.0183 * df_sim['TV'] +
    0.1318 * df_sim['Radio'] +
    0.0005 * df_sim['TV'] * df_sim['Radio']
)
df_sim['Sales_pred'].mean()
```

Out[32]: 15.552941232485875

In [33]:
```
model.conf_int().loc['Newspaper']
```

Out[33]:
```
0   -0.021150
1    0.022974
Name: Newspaper, dtype: float64
```

In [34]:
```
sm.qqplot(model.resid, line='45')
plt.title("Residual Normality Check")
plt.show()
```



In [41]:
```
(df_sim['Sales_pred'].mean() - df_exploit['Sales'].mean()) / 21.3
```

Out[41]: 0.08131630411660155

CONCLUSION

> The analysis identifies Newspaper advertising as a low-impact constraint, while TV and Radio act as true sales drivers. Reallocating budget from Newspaper to TV and Radio improves sales throughput without increasing total spend, supporting data-driven decision-making using the Theory of Constraints.