1 $y = 1 - x + \tan x$

$\dfrac{dy}{dx} = -1 + \sec^2 x$ **M1** Differentiating **A1** $\dfrac{dy}{dx}$ ✓

$\dfrac{d^2y}{dx^2} = 2\sec^2 x \tan x$ **A1** $\dfrac{d^2y}{dx^2}$ ✓

When $x = \frac{1}{4}\pi$, $y = 2 - \frac{1}{4}\pi$ **B1** ***cao***

$\dfrac{dy}{dx} = 1$ **B1** ***ft***

$\dfrac{d^2y}{dx^2} = 4$ **B1** ***ft*** **Dealing with the curve** 6

Let circle have eqn. $(x-a)^2 + (y-b)^2 = r^2$ **M1** At any stage

Then $2(x-a) + 2(y-b)\dfrac{dy}{dx} = 0$ **M1 A1**

and $2 + 2(y-b)\dfrac{d^2y}{dx^2} + 2\left(\dfrac{dy}{dx}\right)^2 = 0$ **M1** (Product/Quotient Rule) **A1**

or $\dfrac{dy}{dx} = \dfrac{a-x}{y-b} \Rightarrow \dfrac{d^2y}{dx^2} = \dfrac{(y-b)(-1)-(a-x)\frac{dy}{dx}}{(y-b)^2}$ **Dealing with the circle** 5

When $x = \frac{1}{4}\pi$, $y = 2 - \frac{1}{4}\pi$, we have **Substitution**

$\left(\frac{1}{4}\pi - a\right)^2 + \left(2 - \frac{1}{4}\pi - b\right)^2 = r^2$ **M1 A1**

$\dfrac{dy}{dx} = -\dfrac{(x-a)}{(y-b)} = 1$ **M1**

$\Rightarrow -\frac{1}{4}\pi + a = 2 - \frac{1}{4}\pi - b$ or $a + b = 2$ **A1**

$2 + 2\left(2 - \frac{1}{4}\pi - b\right).4 + 2.(1)^2 = 0$ **M1 A1**

Matching the two up 6

$b = \frac{5}{2} - \frac{1}{4}\pi$ **A1 cao**

$a = \frac{1}{4}\pi - \frac{1}{2}$ **A1 cao**

$r^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \Rightarrow r = \frac{1}{\sqrt{2}}$ **A1 cso** **Answers** 3

2 $\cos 3x = \cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x$ **M1** $2x$ & x and sin or $\cos(A + B)$ used

$= (2c^2 - 1)c - 2sc.s$ **M1** Double-angles and $s^2 + c^2 = 1$

$= (2c^2 - 1)c - 2c(1 - c^2)$ used somewhere

$= 4c^3 - 3c$ **A1** (**ANSWER GIVEN**)

$\sin 3x = \sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x$

$= 2sc.c + (1 - 2s^2)s$

$= 2s(1 - s^2) + s(1 - 2s^2)$

$= 3s - 4s^3$ **A1**

ALT. $\cos 3x + \mathrm{i}\sin 3x = (c + \mathrm{i}\, s)^3$ **M1** *de Moivre* and equating Re. and Im. parts
M1 binomial expansion **A1 A1**

(If 2nd result just quoted, score M0 M0 A0 **A1**) ④

(i) $I(\alpha) = \int_0^\alpha \left(7\sin x - 8\sin^3 x\right) \mathrm{d}x$

↓ **M1** Use of above result to get rid of s^3

$= \int_0^\alpha \left(\sin x + 2\sin 3x\right) \mathrm{d}x$ **A1**

$= \left[-\cos x - \tfrac{2}{3}\cos 3x\right]_0^\alpha$ **A1** ***ft*** for both "$a \cos kx$" terms

$= -\cos\alpha - \tfrac{2}{3}(4\cos^3\alpha - 3\cos\alpha) + 1 + \tfrac{2}{3}$ **M1** Use of $\cos 3x$ to get expression in c

$= -\tfrac{8}{3}c^3 + c + \tfrac{5}{3}$ **A1** legitimately from correct unsimplified form (**ANSWER GIVEN**)

$I(\alpha) = 0$ when $c = 1$ ($\alpha = 0$) **B1** ⑥

(ii) $J(\alpha) = \left[\tfrac{7}{2}\sin^2 x - \tfrac{8}{4}\sin^4 x\right]_0^\alpha$ **B1** both

$= \tfrac{7}{2}(1 - \cos^2\alpha) - 2(1 - \cos^2\alpha)^2$ **M1** Getting c's only

$= -2c^4 + \tfrac{1}{2}c^2 + \tfrac{3}{2}$ **A1** ✓ MUST be simplified (here or later)

M1 A1 for subst$^{\mathrm{g}}$. $c = -\tfrac{1}{6}$ into both sides: $\tfrac{245}{162}$ (N.B. may be done after following algebra)

M1 Equating two polynomials in c

$I(\alpha) = J(\alpha)$ when $0 = 2c^4 - \tfrac{8}{3}c^3 - \tfrac{1}{2}c^2 + c + \tfrac{1}{6}$ i.e. $0 = 12c^4 - 16c^3 - 3c^2 + 6c + 1$

M1 Full factorisation attempted: $0 = (c - 1)^2(2c + 1)(6c + 1)$ **A1**

$\cos\alpha = -\tfrac{1}{2}$ i.e. $\alpha = \tfrac{2}{3}\pi$ **A1**

$\cos\alpha = -\tfrac{1}{6}$ i.e. $\alpha = \pi - \cos^{-1}(\tfrac{1}{6})$ or $\cos^{-1}(-\tfrac{1}{6})$ and $\alpha = 0$ **A1** both ⑩

N.B. Unfortunately, the $\alpha \in (0, \pi)$ demand disappeared, so please ignore any work towards general solutions.

2 (ii) **Special Scheme for those who use** $\int \sin x \, dx = -\cos x$ rather than Eustace's $\frac{1}{2}\sin^2 x$

$J(\alpha) = \left[-7\cos x - 2\sin^4 x\right]_0^\alpha$ **B0**

$= 7 - 7\cos\alpha - 2(1 - \cos^2\alpha)^2$ **M1** Getting c's only

$= -2c^4 + 4c^2 - 7c + 5$ **A1 ft** MUST be simplified (here or later)

M1 A0 for subst^g. $c = -\frac{1}{6}$ into both sides: $\frac{245}{162} = \frac{4067}{648}$!

M1 Equating two polynomials in c

$I(\alpha) = J(\alpha)$ when $0 = 6c^4 - 8c^3 - 12c^2 + 24c - 10$

M1 Full factorisation attempted

A1 $0 = 2(c - 1)^3 (3c + 5)$

A0 A0 Answers Max ⑥

3 (i) **M1** Subst^{g}. n = 0, 1, (2), 3 into given formula

$F_0 = 0 \Rightarrow 0 = a + b$ or $b = -a$ **A1**

$F_1 = 1 \Rightarrow 1 = a(\lambda - \mu)$ **A1**

$[F_2 = 1 \Rightarrow 1 = a(\lambda^2 - \mu^2) \Rightarrow \lambda + \mu = 1]$

$F_3 = 2 \Rightarrow 2 = a(\lambda^3 - \mu^3) = a(\lambda - \mu)(\lambda^2 + \lambda\mu + \mu^2)$ **M1** Difference of 2 cubes

$= 1.(\lambda^2 + \lambda\mu + \mu^2) \Rightarrow \lambda^2 + \lambda\mu + \mu^2 = 2$ **A1** (**ANSWER GIVEN**) ⑤

$(\lambda + \mu)^2 - \lambda\mu = 1 - \lambda\mu \Rightarrow \lambda\mu = -1$

M1 Getting any two suitable eqns.; e.g. any two of $\lambda\mu = -1$, $\lambda - \mu = \dfrac{1}{a}$ and $\lambda + \mu = 1$

M1 Solving simultaneously

A1 for $a = \dfrac{1}{\sqrt{5}}$, $b = -\dfrac{1}{\sqrt{5}}$ **A1** for $\lambda = \dfrac{1}{2}\left(1+\sqrt{5}\right)$, $\mu = \dfrac{1}{2}\left(1-\sqrt{5}\right)$ ④

(ii) **M1** Using the formula $F_n = a\,\lambda^n + b\,\mu^n = \dfrac{1}{2^n\sqrt{5}}\left\{(1+\sqrt{5})^n - (1-\sqrt{5})^n\right\}$ with $n = 6$

M1 Good attempt at a binomial expansion

$$F_6 = \frac{1}{2^6\sqrt{5}}\left\{\left(1+6\sqrt{5}+15.5+20.5\sqrt{5}+15.5^2+6.5^2\sqrt{5}+5^3\right.\right.$$ **A1** $576 + 256\sqrt{5}$

$$\left.-\left(1-6\sqrt{5}+15.5-20.5\sqrt{5}+15.5^2-6.5^2\sqrt{5}+5^3\right)\right\}$$ **M1** Conjugate of previous

$$= \frac{2}{2^6\sqrt{5}}\left(6\sqrt{5}+100\sqrt{5}+150\sqrt{5}\right) = \frac{2.2^8\sqrt{5}}{2^6\sqrt{5}} = 8$$ **A1** Legitimately shown ⑤

(iii) $\displaystyle\sum_{n=0}^{\infty}\frac{F_n}{2^{n+1}} = \frac{a}{2}\sum_{n=0}^{\infty}\left(\frac{\lambda}{2}\right)^n - \frac{a}{2}\sum_{n=0}^{\infty}\left(\frac{\mu}{2}\right)^n$ **M1** Use of formula

M1 Split into 2 series (& something useful done with them)

$$= \frac{1}{2\sqrt{5}}\left(\frac{1}{1-\frac{1}{4}\left(1+\sqrt{5}\right)}\right) - \frac{1}{2\sqrt{5}}\left(\frac{1}{1-\frac{1}{4}\left(1-\sqrt{5}\right)}\right)$$ **M1** $S\infty$ GP used (at least once)

$$= \frac{1}{2\sqrt{5}}\left(\frac{4}{3-\sqrt{5}}\right) - \frac{1}{2\sqrt{5}}\left(\frac{4}{3+\sqrt{5}}\right)$$ **M1** Simplifying

$$= \frac{2}{\sqrt{5}}\left(\frac{3+\sqrt{5}}{9-5}\right) - \frac{2}{\sqrt{5}}\left(\frac{3-\sqrt{5}}{9-5}\right)$$ **M1** Rationalising denominators (or equivalent)

$$= \frac{2}{\sqrt{5}}\left(\frac{2\sqrt{5}}{4}\right)$$

$= 1$ **A1** ***cao*** ⑥

Note: **(ii)** F_6 can be found by $a\,\lambda^6 + b\,\mu^6 = a\left(\lambda^6 - \mu^6\right) = a\left(\lambda^3 - \mu^3\right)\left(\lambda^3 + \mu^3\right) = F_3\left(\lambda^3 + \mu^3\right)$ etc.

4(i) **M1** Using the substn. $y = a - x$ **M1** Full substn. involving $\mathrm{d}y = -\,\mathrm{d}x$ and $(0, a) \to (a, 0)$

$$\int_0^a \frac{f(x)}{f(x) + f(a-x)}\,\mathrm{d}x = \int_a^0 \frac{f(a-y)}{f(a-y) + f(y)}\,.-\mathrm{d}y$$

$$= \int_0^a \frac{f(a-y)}{f(a-y) + f(y)}\,\mathrm{d}y = \int_0^a \frac{f(a-x)}{f(x) + f(a-x)}\,\mathrm{d}x$$ **A1** ③

Then $2\,I = \int_0^a \frac{f(x) + f(a-x)}{f(x) + f(a-x)}\,\mathrm{d}x = \int_0^a 1.\,\mathrm{d}x = [x]_0^a = a \Rightarrow I = \tfrac{1}{2}a$ **M1 A1** ②

Let $\mathrm{f}(x) = \ln(1 + x)$ **M1**

Then $\ln(2 + x - x^2) = \ln[(1 + x)(2 - x)]$ **M1** Factorisation

$= \ln(1 + x) + \ln(2 - x)$ **M1** Log. work

and $\ln(2 - x) = \ln(1 + [1 - x]) = \mathrm{f}(a - x)$ with $a = 1$ **M1** Or shown via $x \to 1 - x$

so that $$\int_0^1 \frac{f(x)}{f(x) + f(1-x)}\,\mathrm{d}x = \tfrac{1}{2}$$ **A1** ⑤

$$\int_0^{\pi/2} \frac{\sin x}{\sin\left(x + \frac{1}{4}\pi\right)}\,\mathrm{d}x = \int_0^{\pi/2} \frac{\sin x}{\sin x.\frac{1}{\sqrt{2}} + \cos x.\frac{1}{\sqrt{2}}}\,\mathrm{d}x$$ **M1** $\sin(A + B)$ used; **A1** incl. the $\sqrt{2}$

$$= \sqrt{2}\int_0^{\pi/2} \frac{\sin x}{\sin x + \sin\left(\frac{1}{2}\pi - x\right)}\,\mathrm{d}x$$ **A1** **M1** $\cos = \sin(\tfrac{1}{2}\pi - \ )$

$$= \tfrac{1}{4}\pi\sqrt{2}$$ **A1** ④

(ii) **M1** for $u = \dfrac{1}{x}$ **M1** Full substn. involving $\mathrm{d}u = -\dfrac{1}{x^2}\,\mathrm{d}x$ and $\left(\tfrac{1}{2}, 2\right) \to \left(2, \tfrac{1}{2}\right)$

Then $$\int_{0.5}^2 \frac{1}{x}.\frac{\sin x}{\left(\sin x + \sin\left(\frac{1}{x}\right)\right)}\,\mathrm{d}x = \int_{0.5}^2 \frac{1}{x^2}.\frac{x\sin x}{\left(\sin x + \sin\left(\frac{1}{x}\right)\right)}\,\mathrm{d}x$$

$$= \int_2^{0.5} \frac{\frac{1}{u}.\sin\left(\frac{1}{u}\right)}{\left(\sin\left(\frac{1}{u}\right) + \sin u\right)}\,.-\mathrm{d}u$$ **M1**

$$= \int_{0.5}^2 \frac{1}{u}.\frac{\sin\left(\frac{1}{u}\right)}{\left(\sin u + \sin\left(\frac{1}{u}\right)\right)}\,\mathrm{d}u \quad \text{or} \quad \int_{0.5}^2 \frac{1}{x}.\frac{\sin\left(\frac{1}{x}\right)}{\left(\sin x + \sin\left(\frac{1}{x}\right)\right)}\,\mathrm{d}x$$ **A1**

Adding then gives $2\,I = \int_{0.5}^2 \frac{1}{x}\,\mathrm{d}x = [\ln x]_{0.5}^2 = 2\ln 2 \Rightarrow I = \ln 2$ **M1 A1** ⑥

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5 $\cos 2\alpha = \dfrac{(1,\ 1,\ 1)\bullet(5,\ -1,\ -1)}{\sqrt{3}.\sqrt{27}} = \dfrac{1}{3}$ **M1** Scalar product/product of moduli **A1** ②

(i) l_1 equally inclined to *OA* and *OB* iff

$$\frac{(m,\ n,\ p)\bullet(1,\ 1,\ 1)}{\sqrt{m^2+n^2+p^2}.\sqrt{3}} = \frac{(m,\ n,\ p)\bullet(5,\ -1,\ -1)}{\sqrt{m^2+n^2+p^2}.\sqrt{27}}$$ **M1** Two expressions of this form **A1 A1**

i.e. $3(m + n + p) = 5m - n - p$ or $m = 2(n + p)$ **M1** equated **A1** relationship ⑤

For l_1 the angle bisector, we also require $\dfrac{m+n+p}{\sqrt{m^2+n^2+p^2}.\sqrt{3}} = \cos\alpha$ **M1**

Now $\cos 2\alpha = 2\cos^2\alpha - 1 = \frac{1}{3} \Rightarrow \cos\alpha = \frac{\sqrt{2}}{\sqrt{3}}$ **M1 A1**

so $m + n + p = \sqrt{m^2+n^2+p^2}.\sqrt{2}$

Squaring both sides: $m^2 + n^2 + p^2 + 2mn + 2np + 2pm = 2(m^2 + n^2 + p^2)$ **M1**

$\Rightarrow 2mn + 2np + 2pm = m^2 + n^2 + p^2$ **A1**

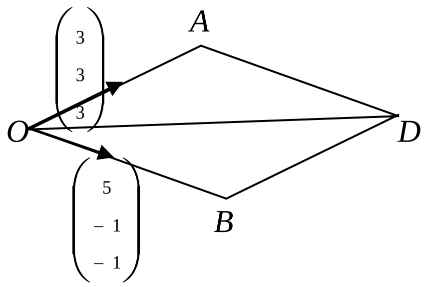
M1 Setting $m = 2n + 2p$ (or equivalent) then gives

$$2np + (2n + 2p)^2 = (2n + 2p)^2 + n^2 + p^2$$

which gives $(n - p)^2 = 0$ **M1** simplifying $\Rightarrow p = n,\ m = 4n$

and $\begin{pmatrix} m \\ n \\ p \end{pmatrix} = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$ (or any non-zero multiple) **A1** ⑧

ALT.



Form rhombus *OADB*.
Then angle bisector is in
the direction $\overrightarrow{OD} = 8\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ **M2 A1** this way

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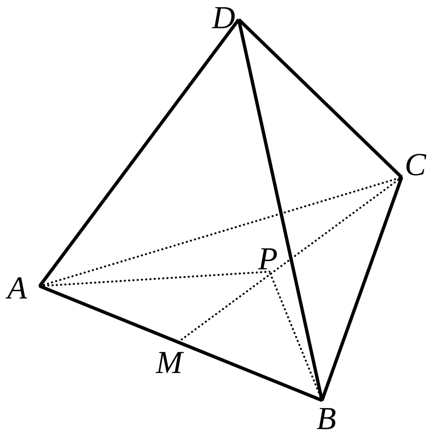
(ii) We already have this (if first method used above);
namely, $2uv + 2vw + 2wu = u^2 + v^2 + w^2$ **M1 A1**

In this case, $2xy + 2yz + 2zx = x^2 + y^2 + z^2$ gives
M1 all lines inclined at an angle $\cos^{-1}\frac{\sqrt{2}}{\sqrt{3}}$ to *OA*
describing the surface which is a (double-) cone **M1** Ignore lack of "double" here
vertex at *O*, having central axis *OA* **A1** Must say this & "double" ⑤

6(i)



Take M = midpt.AB = origin, **M1** set-up
the x-axis along AB and
the y-axis along MC.

Then $A = \left(-\tfrac{1}{2},\ 0,\ 0\right),\ B = \left(\tfrac{1}{2},\ 0,\ 0\right)$ (**A1**)

$C = \left(0,\ \tfrac{\sqrt{3}}{2},\ 0\right)$ by trig. or Pythagoras **M1 A1**

$P = \left(0,\ \tfrac{\sqrt{3}}{6},\ 0\right)$ **A1**

PA (or PB) = $\tfrac{\sqrt{3}}{3}$ by Pythagoras

and $PD = \tfrac{\sqrt{6}}{3}$ or $\sqrt{\tfrac{2}{3}}$ by Pythagoras **A1** ⑥

i.e. $D = \left(0,\ \tfrac{\sqrt{3}}{6},\ \tfrac{\sqrt{6}}{3}\right)$

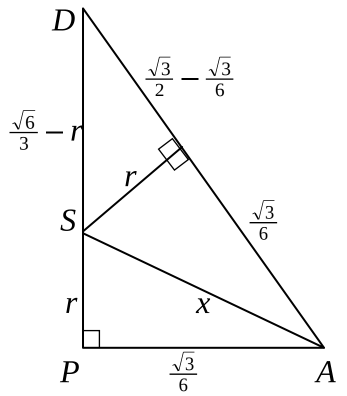
(ii) Angle betn. adjacent faces is $\angle DMP = \cos^{-1}\left(\dfrac{\frac{1}{6}\sqrt{3}}{\frac{1}{2}\sqrt{3}}\right)$ in Rt.∠d. ΔDMP

or $\angle DMC = \cos^{-1}\left(\dfrac{\frac{3}{4}+\frac{3}{4}-1}{2\times\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}}\right)$ by the Cosine Rule in ΔDMC

M1 Suitable Δ **M1** Appropriate method for chosen Δ **A1** correct unsimplified

$= \cos^{-1}\tfrac{1}{3}$ **A1** Legit. (**ANSWER GIVEN**) ④

(iii) Centre of sphere, S, is on PD **M1** equidistant from each vertex **M1**



M1 Valid Δ **A1 A1 A1** Correct relevant lengths

By Pythagoras, $x^2 = \tfrac{1}{12} + \left(\tfrac{6}{9} - 2\tfrac{\sqrt{6}}{3}x + x^2\right)$ **M1**

$\Rightarrow\ x = \tfrac{\sqrt{6}}{4}$ **A1**

Then $r = x\sin(90^\circ - \text{(ii)}) = \tfrac{1}{3}x = \tfrac{\sqrt{6}}{12}$ **M1 A1**

ALT.1: By similar Δs with same lengths.
ALT.2: By working with $\angle DAS = \angle PAS = \tfrac{1}{2}$ (answer to (ii)).
Then (e.g.) $\cos\theta = \tfrac{1}{3} \Rightarrow \tan\theta = 2\sqrt{2} \Rightarrow t = \tan\tfrac{1}{2}\theta$ g.b. $t^2\sqrt{2} + t - \sqrt{2} = 0$
and so $t = \tfrac{1}{\sqrt{2}}$ and $r = \tfrac{\sqrt{3}}{6}\tan\tfrac{1}{2}\theta = \tfrac{\sqrt{6}}{12}$
ALT.3: Of course, if they know that the sphere's centre is at the centre of mass of the tetrahedron $\left(\tfrac{1}{4}(\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d})\right)$ then the answer is just $\tfrac{1}{4}DP = \tfrac{\sqrt{6}}{12}$ ⑩

7(i) $y = x^3 - 3qx - q(1+q) \Rightarrow \frac{dy}{dx} = 3(x^2 - q) = 0$ **M1** Diff^g.

M1 setting $\frac{dy}{dx} = 0$ for TPs

M1 Subst^g. either/both x's back

When $x = +\sqrt{q}$, $y = -q\left(\sqrt{q}+1\right)^2$

< 0 since $q > 0$ **E1** Explained (or via all terms < 0)

When $x = -\sqrt{q}$, $y = -q\left(\sqrt{q}-1\right)^2$ **M1** Compl^g. the sq. attempted (or ≡)

< 0 since $q > 0$ **and** $q \neq 1$ **E1** Both needed

Since both TPs below x-axis,

the curve crosses the x-axis once only **E1** explained (possibly with sketch) ⑦

(ii) $x = u + \frac{q}{u} \Rightarrow x^3 = u^3 + 3uq + 3\frac{q^2}{u} + \frac{q^3}{u^3}$ **B1**

$0 = x^3 - 3qx - q(1+q) = u^3 + 3uq + 3\frac{q^2}{u} + \frac{q^3}{u^3} - 3qu - 3\frac{q^2}{u} - q - q^2$ **M1** substn.

$\Rightarrow u^3 + \frac{q^3}{u^3} - q(1+q) = 0$ or $\left(u^3\right)^2 - q(1+q)\left(u^3\right) + q^3 = 0$ **M1** quadratic in u^3 **A1** ④

$u^3 = \frac{q(1+q) \pm \sqrt{q^2(1+q)^2 - 4q^3}}{2} = \frac{q}{2}\left\{1 + q \pm \sqrt{1 + 2q + q^2 - 4q}\right\}$ **M1** quadratic formula

$= \frac{q}{2}\left\{1 + q \pm \sqrt{(1-q)^2}\right\} = \frac{q}{2}\left\{1 + q \pm (1-q)\right\} = q$ or q^2 **M1** Compl^g. the sq.

giving $u = q^{\frac{1}{3}}$ or $q^{\frac{2}{3}}$ and $x = q^{\frac{1}{3}} + q^{\frac{2}{3}}$ **A1** ③

(iii) $\alpha + \beta = p$, $\alpha\beta = q \Rightarrow \alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$ **M1**

$= p^3 - 3qp$ **A1** legit. (**ANSWER GIVEN**)

ALT. $\alpha = \frac{1}{2}\left\{p + \sqrt{p^2 - 4q}\right\}$, $\beta = \frac{1}{2}\left\{p - \sqrt{p^2 - 4q}\right\}$

Then $\alpha^3 + \beta^3 = \frac{1}{8}\left\{p^3 + 3p^2\sqrt{p^2-4q} + 3p\left(p^2-4q\right) + \left(p^2-4q\right)\sqrt{p^2-4q}\right\}$

$+ \frac{1}{8}\left\{p^3 - 3p^2\sqrt{p^2-4q} + 3p\left(p^2-4q\right) - \left(p^2-4q\right)\sqrt{p^2-4q}\right\} = p^3 - 3qp$ ②

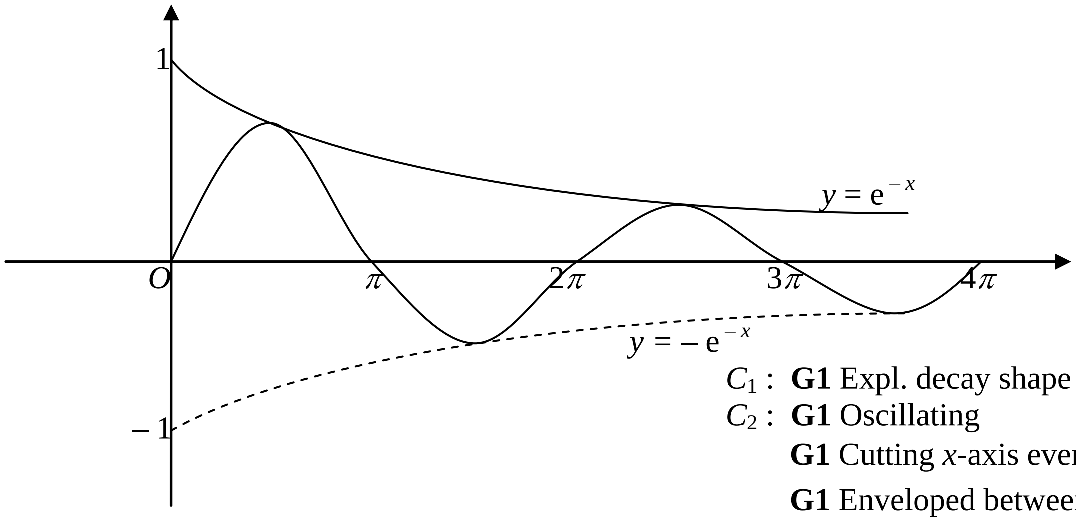
One root the square of the other $\Leftrightarrow \alpha = \beta^2$ or $\beta = \alpha^2 \Leftrightarrow 0 = \left(\alpha^2 - \beta\right)\left(\alpha - \beta^2\right)$ **E1**

$\left(\alpha^2 - \beta\right)\left(\alpha - \beta^2\right) = \alpha^3 + \beta^3 - \alpha\beta - (\alpha\beta)^2$ **M1** $= p^3 - 3qp - q(1+q)$ **A1**

$\Leftrightarrow p = q^{\frac{1}{3}} + q^{\frac{2}{3}}$ **A1** ***ft*** (ii)'s final answer only

ALT. Let roots be α and α^2. Then $p = \alpha + \alpha^2$ and $q = \alpha^3$; i.e. $p = q^{\frac{1}{3}} + q^{\frac{2}{3}}$ ④

8



C_1 : **G1** Expl. decay shape

C_2 : **G1** Oscillating

G1 Cutting x-axis every π

G1 Enveloped between C_1 and $-C_1$ ④

The curves meet each time $\sin x = 1$ **M1** when $x = 2n\pi + \frac{\pi}{2}$ ($n = 0, 1, 2, \ldots$) **M1**

(These two M's might be implicit) ③$\Rightarrow$ $x_n = \dfrac{(4n-3)\pi}{2}$, $x_{n+1} = \dfrac{(4n+1)\pi}{2}$ **A1 A1 Limits** ④

$\int \left(\mathrm{e}^{-x} \sin x\right) \mathrm{d}x$ **M1** attempted by parts

$$= -\mathrm{e}^{-x}.\cos x - \int \left(\mathrm{e}^{-x}.\cos x\right) \mathrm{d}x \text{ or } -\mathrm{e}^{-x}.\sin x - \int \left(\mathrm{e}^{-x}.\sin x\right) \mathrm{d}x \quad \textbf{A1}$$

$$= -\mathrm{e}^{-x}.\cos x - \left\{\mathrm{e}^{-x}.\sin x + \int \left(\mathrm{e}^{-x}.\sin x\right) \mathrm{d}x\right\} \quad \textbf{M1 } 2^{\text{nd}} \text{ round of parts}$$

$\Rightarrow I = -\mathrm{e}^{-x}(\cos x + \sin x) - I$ **M1** by "looping"

$= -\frac{1}{2}\mathrm{e}^{-x}(\cos x + \sin x)$ **A1** **Anywhere it appears** ⑤

$A_n = \displaystyle\int_{x_n}^{x_{n+1}} \left(\mathrm{e}^{-x} - \mathrm{e}^{-x} \sin x\right) \mathrm{d}x$ **M1** (ignore limits for now)

$A_n = \left[-\mathrm{e}^{-x} + \frac{1}{2}\mathrm{e}^{-x}\left(\cos x + \sin x\right)\right]_{x_n}^{x_{n+1}}$ or $\left[\frac{1}{2}\mathrm{e}^{-x}\left(\cos x + \sin x - 2\right)\right]_{x_n}^{x_{n+1}}$ **M1** use of insert working

$= \frac{1}{2}\mathrm{e}^{-\frac{1}{2}\pi(4n+1)}\left(0+1-2\right) - \frac{1}{2}\mathrm{e}^{-\frac{1}{2}\pi(4n-3)}\left(0+1-2\right)$ **M1** use of limits

$= \frac{1}{2}\mathrm{e}^{-\frac{1}{2}\pi(4n+1)}\left(-1+\mathrm{e}^{2\pi}\right)$ **A1 (ANSWER GIVEN)** ④

Note that $A_1 = \frac{1}{2}\mathrm{e}^{-\frac{5}{2}\pi}\left(\mathrm{e}^{2\pi} - 1\right)$ and $A_{n+1} = \mathrm{e}^{-2\pi} A_n$ **M1**

so that $\displaystyle\sum_{n=1}^{\infty} A_n = A_1\left\{1 + \left(\mathrm{e}^{-2\pi}\right) + \left(\mathrm{e}^{-2\pi}\right)^2 + \ldots\right\}$

$$= \frac{1}{2}\mathrm{e}^{-\frac{5}{2}\pi}\left(\mathrm{e}^{2\pi} - 1\right) \times \frac{1}{1 - \mathrm{e}^{-2\pi}} = \frac{1}{2}\mathrm{e}^{-\frac{5}{2}\pi}\left(\mathrm{e}^{2\pi} - 1\right) \times \frac{\mathrm{e}^{2\pi}}{\mathrm{e}^{2\pi} - 1} \quad \textbf{M1 } S_\infty \text{ GP used}$$

$= \frac{1}{2}\mathrm{e}^{-\frac{1}{2}\pi}$ **A1** ③