1 $x^4 + y^4 = u$ has lines of symmetry

B1 x-axis and y-axis

B1 $y = x$

B1 $y = -x$

$xy = v$ has lines of symmetry

B1 $y = \pm x$ but B0 if they include incorrect others also **4**

$A(\alpha, \beta) \Rightarrow$

B1 $B = (\beta, \alpha)$

B1 $C = (-\alpha, -\beta)$ Give both if C, D the wrong way round, but penalise

B1 $D = (-\beta, -\alpha)$ later gradients and/or distances incorrect as a result **3**

M1 Method for attempt at gradient of either/both CB, DA or BA, DC using α's and β's

A1 $= \dfrac{\alpha+\beta}{\alpha+\beta} = 1$ for CB, DA

A1 $= \dfrac{\beta-\alpha}{\alpha-\beta} = -1$ for BA, DC

E1 Adjacent sides perp^r. $\Rightarrow$ $ABCD$ a rectangle (noted or explained)

Give B1 for only proving //gm. using distances and/or equal vectors **4**

B1 Lengths CB, $DA = (\alpha + \beta)\sqrt{2}$ Can score these for //gm. bit

B1 Lengths BA, $DC = (\alpha - \beta)\sqrt{2}$

M1 Mult^g. these to get Area $= (\alpha + \beta)\sqrt{2} \times (\alpha - \beta)\sqrt{2}$

A1 $= 2(\alpha^2 - \beta^2)$ **4**

M1 A1 for $(\alpha^2 - \beta^2)^2 = \alpha^4 + \beta^4 - 2(\alpha^2 \beta^2) = u - 2v^2$

A1 so Area $ABCD = 2\sqrt{u - 2v^2}$ See Alt.1.1 **3**

M1 Subst^g. $u = 81$, $v = 4$ into their area expression

A1 Legitimately obtaining Area $= 2\sqrt{81 - 2\times16} = 14$ ANSWER GIVEN **2**

Alt. 1.1

M1 Eliminating (say) y from $x^4 + y^4 = u$, $xy = v$ to get $x^8 - ux^4 + v^4 = 0$

and using the quadratic formula to get expressions for x^4: $x^4 = \dfrac{u \pm \sqrt{u^2 - 4v^4}}{2}$

A1 Getting $\alpha = \sqrt[4]{\dfrac{u + \sqrt{u^2 - 4v^4}}{2}}$, $\beta = \sqrt[4]{\dfrac{u - \sqrt{u^2 - 4v^4}}{2}}$

Personally, I can't see them sorting this out any more simply … so A0 at the end. They can, however, proceed to subst. $u = 81$, $v = 4$ into their area expression for the final 2 marks.

2(i) **M1** Taking logs.: $\ln y = \sin(\pi e^x) . \ln a$

M1 Use of implicit diffn. Give M2 if done directly

A1 $\dfrac{1}{y}\dfrac{dy}{dx} = \pi e^x . \cos(\pi e^x) . \ln a$ **3**

M1 setting $\dfrac{dy}{dx} = 0$ and solving this eqn. $\cos(\pi e^x) = 0$ No need to note other bits $\neq 0$

i.e. $\pi e^x = (2n + 1)\tfrac{1}{2}\pi$ May be just $n = 0$, $n = 1$ to begin with

A1 $x = \ln\left(n + \tfrac{1}{2}\right)$

A1 $y = \begin{cases} a & n \text{ even} \quad \text{MAX's} \\ 1/a & n \text{ odd} \quad \text{MIN's} \end{cases}$ **3**

Alt.2.1

$y = a^{\sin(\pi . \exp x)}$

M1 Max's occur when $\sin(\pi e^x) = 1$ i.e. $\pi e^x = (2n + \tfrac{1}{2})\pi$

A1 for $x = \ln(2n + \tfrac{1}{2})$ $n = 0, 1, \ldots$ (be relaxed at which n's can be used) **3**

A1 for $y_{max} = a$

M1 Min's occur when $\sin(\pi e^x) = -1$ i.e. $\pi e^x = (2n - \tfrac{1}{2})\pi$

A1 for $x = \ln(2n - \tfrac{1}{2})$ $n = 1, 2, \ldots$ (be relaxed at which n's can be used)

A1 for $y_{min} = \dfrac{1}{a}$ **3**

(ii) **M1** for $\sin(\pi e^x) \approx \sin(\pi + \pi x)$ i.e. use of $e^x \approx 1 + x$ for small x

M1 $= -\sin(\pi x)$ [via $\sin(A + B)$ for instance]

$\approx -\pi x$ for small x, leading to

A1 $y \approx a^{-\pi x} = e^{-\pi x \ln a} \approx 1 - \pi x . \ln a$ legitimately obtained ANSWER GIVEN **3**

(iii) **B1** Asymptote $y = 1$ (as $x \to -\infty$, $y \to 1+$)

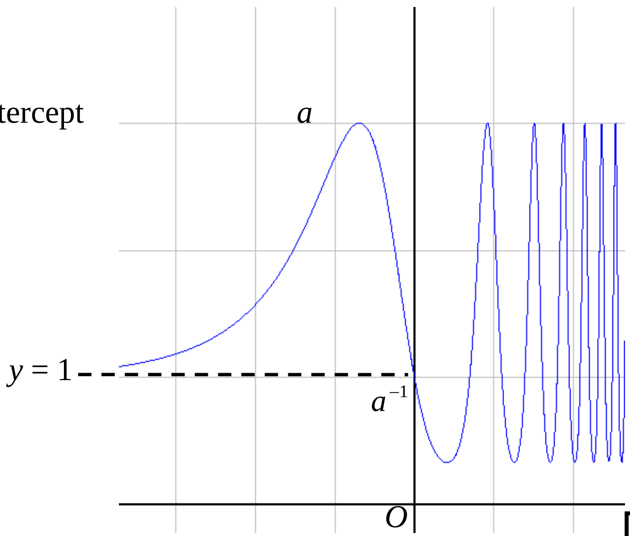
M1 For $x > 0$, curve oscillates between a and $\dfrac{1}{a}$ …

A1 … getting ever closer together

B1 First max. for $x < 0$ at $y = a$

(since $n + \tfrac{1}{2} > 0$, least n is 0)

B1 Approx. negative linear through y-intercept

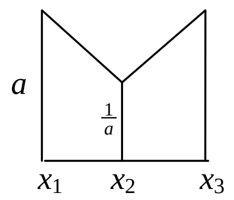


5

(iv) **B1** (1^{st} max at $n = 0$; 2^{nd} max at $n = 2$; …;) k^{th} max at $n = 2k - 2$; etc.

i.e. $x_1 = \ln\left(2k - \frac{3}{2}\right)$, $x_2 = \ln\left(2k - \frac{1}{2}\right)$, $x_3 = \ln\left(2k + \frac{1}{2}\right)$

M1 Area ≈ 2 trapezia = $\frac{1}{2}\left(a + \frac{1}{a}\right)(x_2 - x_1) + \frac{1}{2}\left(a + \frac{1}{a}\right)(x_3 - x_2)$



A1 $= \frac{1}{2}\left(a + \frac{1}{a}\right)(x_3 - x_1)$

M1 for $x_3 - x_1 = \ln\left(\dfrac{4k+1}{4k-3}\right)$ i.e. combining logs

M1 for $= \ln\left(\dfrac{4k-3+4}{4k-3}\right) = \ln\left(1 + \dfrac{1}{k - \frac{3}{4}}\right)$

A1 for $\left(\dfrac{a^2+1}{2a}\right)\ln\left(1 + (k - \frac{3}{4})^{-1}\right)$ legitimately ANSWER GIVEN **6**

Area may be found by rectangle – triangle, of course.

3 **B1** LHS $\equiv \tan\left(\frac{\pi}{4}-\frac{x}{2}\right) \equiv \dfrac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$

M1 $\equiv \dfrac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}}$

M1 $\equiv \dfrac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}} \times \dfrac{\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$

M1 $\equiv \dfrac{1-2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$ (since $c^2 + s^2 = 1$)

M1 $\equiv \dfrac{1-\sin x}{\cos x}$

A1 $\equiv \sec x - \tan x \equiv$ RHS **6**

Alt.3.1

B1 LHS $\equiv \dfrac{1-\tan\frac{x}{2}}{1+\tan\frac{x}{2}}$

M1 Using ½-angle formulae for cos in RHS $\equiv \dfrac{1-t^2}{1+t^2}$

M1 Using ½-angle formulae for tan in RHS $-\dfrac{2t}{1-t^2}$

M1 M1 Facts[g]. in N[r]. & D[r]. $\equiv \dfrac{(1-t)^2}{(1-t)(1+t)}$

A1 $\equiv \dfrac{1-t}{1+t}$ **6**

(i) **M1** Setting $x = \frac{\pi}{4}$ in (*) (**must** use (*)'s result)

A1 $\Rightarrow \tan\frac{\pi}{8} = \sqrt{2}-1$

M1 for use of $\tan(A + B)$ with $A = \frac{\pi}{3}$ and $B = \frac{\pi}{8}$; i.e. $\tan\frac{11\pi}{24} = \tan\left(\frac{\pi}{3}+\frac{\pi}{8}\right)$

M1 for use of $\tan\frac{\pi}{3} = \sqrt{3}$ and $\tan\frac{\pi}{8}$ = their above value

A1 $\tan\frac{11\pi}{24} = \dfrac{\sqrt{3}+\sqrt{2}-1}{1-\sqrt{3}\left(\sqrt{2}-1\right)} = \dfrac{\sqrt{3}+\sqrt{2}-1}{\sqrt{3}-\sqrt{6}+1}$ legitimately ANSWER GIVEN

Allow "or otherwise" approaches for the last 3 marks here **5**

(ii) **EITHER** **M1** Rationalising the denominator on RHS …

M1 … twice

e.g. $\dfrac{\sqrt{3}+\sqrt{2}-1}{1+\sqrt{3}-\sqrt{6}} = \dfrac{\sqrt{3}+\sqrt{2}-1}{1+\sqrt{3}-\sqrt{6}} \times \dfrac{1+\sqrt{3}+\sqrt{6}}{1+\sqrt{3}+\sqrt{6}} = \dfrac{1+2\sqrt{2}+\sqrt{3}}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1}$

A1 $= 2+\sqrt{2}+\sqrt{3}+\sqrt{6}$ legitimately ANSWER GIVEN

OR **M2 A1** Verification: $\left(\sqrt{3}-\sqrt{6}+1\right)\left(2+\sqrt{2}+\sqrt{3}+\sqrt{6}\right) = \sqrt{3}+\sqrt{2}-1$ **3**

(iii) **M1** Setting $x = \frac{11\pi}{24}$ in (*) $\Rightarrow$ $\tan\frac{\pi}{48} = \sec\frac{11\pi}{24} - \tan\frac{11\pi}{24} = \sqrt{1+t^2} - t$

M1 $= \sqrt{1+t^2} - t$

M1 Good attempt at squaring : $\left(2+\sqrt{2}+\sqrt{3}+\sqrt{6}\right)^2$

$= 4+2+3+6+4\sqrt{2}+4\sqrt{3}+4\sqrt{6}+2\sqrt{6}+2\sqrt{12}+2\sqrt{18}$

A1 A1 $= 15+10\sqrt{2}+8\sqrt{3}+6\sqrt{6}$ (one each correct pair)

A1 Legitimately obtaining $\tan\frac{\pi}{48} = \sqrt{16+10\sqrt{2}+8\sqrt{3}+6\sqrt{6}} - \left(1+\sqrt{2}\right)\left(\sqrt{2}+\sqrt{3}\right)$
ANSWER GIVEN

6

4(i)	**M1**	Writing $p(x) - 1 \equiv q(x).(x-1)^5$, where $q(x)$ is a quartic polynomial	
	A1	for getting $p(1) = 1$	**2**
		Give B1 only if they get $p(1) = 1$ by having $q(x)$ constant, for instance	
(ii)	**M1**	Diff^g. using the product and chain rules	
	A1	$p'(x) \equiv q(x).5(x-1)^4 + q'(x).(x-1)^5$ correct unsimplified	
	A1	$\equiv (x-1)^4.\{5\,q(x) + (x-1)\,q'(x)\}$ so that $p'(x)$ is divisible by $(x-1)^4$	**3**
(iii)	**B1**	Similarly, we have that $p'(x)$ is divisible by $(x+1)^4$	**2**
	B1	$p(-1) = -1$	
	B1	Thus $p'(x)$ is divisible by $(x+1)^4.(x-1)^4 \equiv (x^2-1)^4$	
	M2	However, $p'(x)$ *is* a polynomial of degree 8	**5**
	A2	hence $p'(x) \equiv k(x^2-1)^4$ for some constant k Give A1 if $k = 1$ assumed	
	M1	for expansion of $(x^2-1)^4$	
	A1	$p'(x) \equiv k\left(x^8 - 4x^6 + 6x^4 - 4x^2 + 1\right)$	
	M1	for integrating term by term	
	A1	$p(x) \equiv k\left(\frac{1}{9}x^9 - \frac{4}{7}x^7 + \frac{6}{5}x^5 - \frac{4}{3}x^3 + x\right) + C$ ignore missing "+ C"here	
	M1	Use of $p(1) = 1$	
	M1	and $p(-1) = -1$ to find k and C	
	A1 A1	$k = \dfrac{315}{128}$, $C = 0$ **cao**	**8**

5 **B1** $\left(\sqrt{x-1}+1\right)^2 = x+2\sqrt{x-1}$ **1**

(i) **B1 B1** $\sqrt{x+2\sqrt{x-1}} = \sqrt{x-1}+1$ and $\sqrt{x-2\sqrt{x-1}} = \sqrt{x-1}-1$

M1 for integrating a constant: $I = \int_5^{10} 2\,\mathrm{d}x = \left[2x\right]_5^{10}$

A1 $= 10$ **4**

(ii) **B1** for noting (at any point) that the curve crosses the x-axis in (1¼, 10); at $x = 2$, in fact

or that $\sqrt{\left(\sqrt{x-1}-1\right)^2} = \left|\sqrt{x-1}-1\right|$

M1 Splitting area into two bits: Area = $\int_{1.25}^{2} \frac{1-\sqrt{x-1}}{\sqrt{x-1}}\,\mathrm{d}x + \int_{2}^{10} \frac{\sqrt{x-1}-1}{\sqrt{x-1}}\,\mathrm{d}x$

M1 M1 $= \int_{1.25}^{2}\left[(x-1)^{-\frac{1}{2}}-1\right]\mathrm{d}x + \int_{2}^{10}\left[1-(x-1)^{-\frac{1}{2}}\right]\mathrm{d}x$

A1 A1 ft correct integration $= \left[2\sqrt{x-1}-x\right]_{1.25}^{2} + \left[x-2\sqrt{x-1}\right]_{2}^{10}$ = ¼ + 4

A1 = 4¼

Note that integrating just one bit (usually from 1.25 to 10)
scores B0 M0 M0 M1 A1 A0 max. **7**

(iii) **B1** $\left(\sqrt{x+1}-1\right)^2 = x+2-2\sqrt{x+1}\ \ \forall\ x \geq 0$

M1 M1 Nr.; Facts[g]. Dr. $I = \int_{x=1.25}^{10} \frac{1+\sqrt{x-1}+\sqrt{x+1}-1}{\sqrt{x-1}\sqrt{x+1}}\,\mathrm{d}x$

A1 A1 $= \int_{x=1.25}^{10}\left((x+1)^{-\frac{1}{2}}+(x-1)^{-\frac{1}{2}}\right)\mathrm{d}x$

A1 A1 for correct integration $= \left[2\sqrt{x+1}+2\sqrt{x-1}\right]_{1.25}^{10}$

A1 $= 2\left(\sqrt{11}+1\right)$ **8**

<u>Alt.5.1</u>

M1 Use of substns. $u^2 = x-1$ and $v^2 = x+1$ (say)

M1 $(u+1)^2 = x-1+2\sqrt{x-1}+1 = x+2\sqrt{x-1}$

M1 and $(v-1)^2 = x+1-2\sqrt{x+1}+1 = x-2\sqrt{x+1}+2$

A1 A1 $I = \int_{x=1.25}^{10} \frac{(u+1)+(v-1)}{uv}\,\mathrm{d}x = \int\left(\frac{1}{u}+\frac{1}{v}\right)\mathrm{d}x$

A1 A1 $= \int_{0.5}^{3}\left(\frac{1}{u}\right)2u\,\mathrm{d}u + \int_{1.5}^{\sqrt{11}}\left(\frac{1}{v}\right)2v\,\mathrm{d}v$

A1 $= 2\left(3-\tfrac{1}{2}\right)+2\left(\sqrt{11}-\tfrac{3}{2}\right) = 2+2\sqrt{11}$ **8**

6	**B1**	$(F_1 = 1, F_2 = 1), F_3 = 2, F_4 = 3, F_5 = 5, F_6 = 8,$	
	B1	$F_7 = 13, F_8 = 21, F_9 = 34, F_{10} = 55$	**2**

(i) **M1** For use of r.r. to get $\dfrac{1}{F_i} = \dfrac{1}{F_{i-1} + F_{i-2}} > \dfrac{1}{2F_{i-1}}$

E1 since $F_{i-2} < F_{i-1}$ for $i \ge 4$

M1 for splitting off 1st few terms:

$$S = \sum_{i=1}^{n} \frac{1}{F_i} > \frac{1}{F_1} + \frac{1}{F_2}\left(1 + \frac{1}{2} + \frac{1}{4} + \ldots\ldots\right) \text{ or } \frac{1}{F_1} + \frac{1}{F_2} + \frac{1}{F_3}\left(1 + \frac{1}{2} + \frac{1}{4} + \ldots\ldots\right)$$

M1 + next × S_∞(GP)

A1 $= 1 + 1 \times 2 = 3$ or $1 + 1 + \frac{1}{2} \times 2 = 3$

Condone non-"deduced" approaches which simply take 1st few terms to get a sum exceeding 3. **5**

M1 A1 Similarly, $\dfrac{1}{F_i} < \dfrac{1}{2}\left(\dfrac{1}{F_{i-2}}\right)$ for $i \ge 3$

M1 for splitting off 1st few terms

M1 then separating odds and evens (or equivalent)

M1 use of S_∞(GPs)

$$S = \sum_{i=1}^{n} \frac{1}{F_i} = \frac{1}{F_1} + \frac{1}{F_2} + \left(\frac{1}{F_3} + \frac{1}{F_5} + \ldots\ldots\right) + \left(\frac{1}{F_4} + \frac{1}{F_6} + \ldots\ldots\right)$$

$$< 1 + 1 + \frac{1}{2}\left(1 + \frac{1}{2} + \frac{1}{4} + \ldots\ldots\right) + \frac{1}{3}\left(1 + \frac{1}{2} + \frac{1}{4} + \ldots\ldots\right)$$

A1 $= 1 + 1 + \dfrac{1}{2} \times 2 + \dfrac{1}{3} \times 2 = 3\frac{2}{3}$ legitimately ANSWER GIVEN **6**

(ii) For showing $S > 3.2$. This can be done in an unsophisticated way by just taking the reciprocals of the F_i's (with or without helpful inequalities such as $\frac{1}{8} > \frac{1}{10}$) or by using the above method:

$$S > \frac{1}{F_1} + \frac{1}{F_2} + \frac{1}{F_3} + \frac{1}{F_4}\left(1 + \frac{1}{2} + \frac{1}{4} + \ldots\ldots\right) = 1 + 1 + \frac{1}{2} + \frac{1}{3} \times 2 = 3\tfrac{1}{6}$$

$$S > \frac{1}{F_1} + \frac{1}{F_2} + \frac{1}{F_3} + \frac{1}{F_4} + \frac{1}{F_5}\left(1 + \frac{1}{2} + \frac{1}{4} + \ldots\ldots\right) = 1 + 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{5} \times 2 = 3\tfrac{7}{30} > 3\tfrac{6}{30} = 3.2$$

Suggest
- **M1** for attempting to take more terms
- **M1** for sufficiently many
- **A1** for answer legitimately obtained

3

For showing $S < 3\frac{1}{2}$

Suggest **M1** for attempting to take more terms

M1 for sufficiently many

A2 for answer legitimately obtained

$$S < 1+1+\frac{1}{2}+\frac{1}{3}\left(1+\frac{1}{2}+\frac{1}{4}+.....\right)+\frac{1}{5}\left(1+\frac{1}{2}+\frac{1}{4}+.....\right) = 1+1+\frac{1}{2}+\frac{1}{3}\times 2+\frac{1}{5}\times 2 = 3\tfrac{17}{30}$$

$$S < 1+1+\frac{1}{2}+\frac{1}{3}+\frac{1}{5}\left(1+\frac{1}{2}+\frac{1}{4}+.....\right)+\frac{1}{8}\left(1+\frac{1}{2}+\frac{1}{4}+.....\right)$$

$$= 1+1+\frac{1}{2}+\frac{1}{3}+\frac{1}{5}\times 2+\frac{1}{8}\times 2 = 3\tfrac{29}{60} < 3\tfrac{1}{2}$$

See Alt.6.1

4

Alt.6.1

Note that, if done correctly at any stage, this gets 6 + 4 = 10 marks as it necessarily covers both RHS results.

M1 Since $F_{i-2} < F_{i-1}$ for $i \ge 4$,

M1 $F_i = F_{i-1} + F_{i-2} \Rightarrow F_i < 2F_{i-1}$

$\Rightarrow 3F_i < 2F_{i-1} + 2F_{i-2} = 2F_{i+1}$

A1 so that $F_{i+1} > \frac{3}{2}F_i$ and $\frac{1}{F_{i+1}} < \frac{2}{3}\left(\frac{1}{F_i}\right)$ for $i \ge 4$

M1 for splitting off 1st few terms: $S < 1+1+\frac{1}{2}\left(1+\frac{2}{3}+\left[\frac{2}{3}\right]^2+.....\right)$

M1 for use of S_∞(GPs) $= 2+\frac{1}{2}\left(\frac{1}{1-\frac{2}{3}}\right)$

A1 $= 3\frac{1}{2}$

6 + **4**

Other comparable approaches may also be valid for one or both part(s)

7 **M1** for use of product-within-a-product rule

$y = (x-a)^n e^{bx}\sqrt{1+x^2}$

A3 $\Rightarrow \dfrac{dy}{dx} = (x-a)^n e^{bx}\dfrac{x}{\sqrt{1+x^2}} + (x-a)^n b\, e^{bx}\sqrt{1+x^2} + n(x-a)^{n-1} e^{bx}\sqrt{1+x^2}$

(one each term correct, unsimplified)

M1 for factorising out the given terms: $\dfrac{(x-a)^{n-1}\,e^{bx}}{\sqrt{1+x^2}}\left\{x(x-a)+b(x-a)(1+x^2)+n(1+x^2)\right\}$

to get $q(x) = bx^3 + (n+1-ab)x^2 + (b-a)x + (n-ab)$ or noting its cubic-ity

Condone "slightly" incorrect cubics (coefficients)

5

(i) $\displaystyle\int \frac{(x-4)^{14}\,e^{4x}}{\sqrt{1+x^2}}\left(4x^3-1\right)dx$

M1 for noting/using $n = 15$, $a = b = 4$

A1 for $q(x) = 4x^3 - 1$ may be implicit

A1 for $I = (x-4)^{15}\,e^{4x}\sqrt{1+x^2}$ $(+\,C)$

3

(ii) $\displaystyle\int \frac{(x-1)^{21}\,e^{12x}}{\sqrt{1+x^2}}\left(12x^4-x^2-11\right)dx$

M1 A1 for $12x^4 - x^2 - 11 \equiv (x-1)(12x^3 + 12x^2 + 11x + 11)$

M1 for noting/using $n = 23$, $a = 1$, $b = 12$

A1 for $q(x) = 12x^3 + 12x^2 + 11x + 11$ may be implicit

A1 for $I = (x-1)^{23}\,e^{12x}\sqrt{1+x^2}$ $(+\,C)$

5

(iii) $\displaystyle\int \frac{(x-2)^{6}\,e^{4x}}{\sqrt{1+x^2}}\left(4x^4+x^3-2\right)dx$

M1 $n = 8$, $a = 2$, $b = 4$

A1 gives $\dfrac{dy_8}{dx} = \dfrac{(x-2)^7\,e^{4x}}{\sqrt{1+x^2}}\left\{4x^3+x^2+2x\right\}$

A1 $= \dfrac{(x-2)^6\,e^{4x}}{\sqrt{1+x^2}}\left\{4x^4-7x^3-4x\right\}$

M1 $n = 7$, $a = 2$, $b = 4$ Give both these M1s if they use $a = 2$, $b = 4$ *and* attempt to do something with both $n = 7, 8$

A1 gives $\dfrac{dy_7}{dx} = \dfrac{(x-2)^6\,e^{4x}}{\sqrt{1+x^2}}\left\{4x^3+2x-1\right\}$

M1 $I = \displaystyle\int\left(\frac{dy_8}{dx}+2\frac{dy_7}{dx}\right)dx = y_8 + 2\,y_7$

A1 $= x(x-2)^7\,e^{4x}\sqrt{1+x^2}$ $(+\,C)$

7

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8 Diagram

B1 for P on AB …

B1 … between A and B

B1 for Q on AC …

B1 … on other side of A to C **4**

B1 for $CQ = \mu AC$

B1 for $BP = \lambda AB$

M1 A1 Subst^g. into $CQ \times BP = AB \times AC \Rightarrow \mu AC \,.\, \lambda AB = AB \,.\, AC$

A1 $\Rightarrow \mu = \dfrac{1}{\lambda}$

Don't reward phoney vector work such as $\mu(\mathbf{a} - \mathbf{c}) \times \lambda(\mathbf{a} - \mathbf{b}) = (\mathbf{b} - \mathbf{a}) \times (\mathbf{c} - \mathbf{a})$ which then cancels to give the right result (taking vectors to be scalars), whether they treat the "×" as scalar multiplication, the scalar product or the vector product. However, if they have $|\,\mathbf{a} - \mathbf{c}\,|$, etc., then it is correct. **5**

M1 Attempt at eqn. of PQ, or equivalent, in the form

$\mathbf{r} = t\,\mathbf{p} + (1 - t)\,\mathbf{q}$ for some scalar parameter t

$\mathbf{r} = t\lambda\,\mathbf{a} + t(1 - \lambda)\mathbf{b} + (1 - t)\mu\,\mathbf{a} + (1 - t)(1 - \mu)\mathbf{c}$

M1 Subst^g. for μ in terms of λ **5**

A1 A1 A1 $= \left(t\lambda + \dfrac{1}{\lambda} - \dfrac{t}{\lambda}\right)\mathbf{a} + t(1-\lambda)\mathbf{b} + (1-t)\left(\dfrac{\lambda - 1}{\lambda}\right)\mathbf{c}$ one each component

M1 When $t = \dfrac{1}{1-\lambda}$ from the **b**-component, $1 - t = \dfrac{\lambda}{\lambda - 1}$ or equating to $-\mathbf{a} + \mathbf{b} + \mathbf{c}$

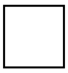
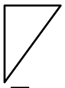
A1 A1 A1 $\mathbf{r} = -\mathbf{a} + \mathbf{b} + \mathbf{c}$ one for each component shown correct

N.B. If "t" is put the other way round in the line eqn., then $t = \dfrac{-\lambda}{1-\lambda}$ **4**

B1 $ABDC$ is a parallelogram

E1 Justified e.g. by observing that $\mathbf{d} - \mathbf{c} = \mathbf{b} - \mathbf{a} \Rightarrow$ one pair opp. sides equal and // **2**

9 (i)

Shape	Mass	Dist. c.o.m. from OZ	
	360ρ	$\frac{9}{2}$	**B1**
	180ρ	12	**B1**
Trapm.	540ρ	x	OR by subtraction

B1 for relative masses (2 **:** 1 **:** 3) N.B. ρ's immaterial throughout;

M1 for attempt at $\dfrac{\sum m_i x_i}{\sum m_i}$

A1 correct unsimplified: $x = \dfrac{360\rho \times \frac{9}{2} + 180\rho \times 12}{540\rho} = \dfrac{1620 + 2160}{540}$ or $\dfrac{3780}{540}$

A1 $= 7$ legitimately ANSWER GIVEN

6

(ii)

Shape	Mass	Dist. c.o.m. from OZ	
LH end	540ρ	7	
RH end	540ρ	7	
Front	$41d\rho$	$\frac{27}{2}$	**B1**
Back	$40d\rho$	0	This line may not appear explicitly
Base	$9d\rho$	$\frac{9}{2}$	

B1 all areas/masses correct

B1 all other distances correct

M1 for attempt at $\dfrac{\sum m_i x_i}{\sum m_i}$ with at least most of these

A1 correct unsimplified: $x_E = \dfrac{2 \times (540\rho) \times 7 + 41d\rho \times \frac{27}{2} + 0 + 9dp \times \frac{9}{2}}{1080\rho + 90d\rho}$

$= \dfrac{2 \times 9 \times 60 \times 7 + 41d \times \frac{27}{2} + 9d \times \frac{9}{2}}{90(12+d)} = \dfrac{2 \times 60 \times 7 + 41 \times \frac{3}{2}d + \frac{9}{2}d}{10(12+d)}$

M1 for decent factorising attempt $= \dfrac{2 \times 60 \times 7 + 66d}{10(12+d)}$

A1 $= \dfrac{3(140+11d)}{5(12+d)}$ legitimately ANSWER GIVEN

7

Object	Mass		Dist. c.o.m. from OZ	
Tank	2880ρ	**B1**	$\frac{27}{4}$	**B1**
Water	$10800k\rho$	**B1**	7	**B1**

M1 A1 $x_F = \dfrac{2880\rho \times \frac{27}{4} + 10800kp \times 7}{2880\rho + 10800k\rho}$ (A for✓ unsimplified)

$= \dfrac{72 \times 27 + 1080k \times 7}{288 + 1080k} = \dfrac{36(2 \times 27 + 30k \times 7)}{36(8+30k)}$

A1 $= \dfrac{27+105k}{4+15k}$

7

10 Collision $P_{1,2}$:

B1 for **<u>CLM</u>** $\rightarrow$ $m_1 u = m_1 v_1 + m_2 v_2$

B1 for **<u>NEL</u>** $eu = v_2 - v_1$

M1 for solving: $v_1 = \dfrac{(m_1 - em_2)}{m_1 + m_2}u$ **A1**; $v_2 = \dfrac{m_1(1+e)}{m_1 + m_2}u$ **A1** **ft** sign errors in NEL (e.g.) **5**

Collision $P_{4,3}$:

B1 for **<u>CLM</u>** $m_4 u = m_4 v_4 + m_3 v_3$

B1 for **<u>NEL</u>** $eu = v_3 - v_4$

M1 for solving: $v_3 = \dfrac{m_4(1+e)}{m_3 + m_4}u$ **A1**; $v_4 = \dfrac{(m_4 - em_3)}{m_3 + m_4}u$ **A1** **ft** sign errors in NEL (e.g.)

5

N.B. v_3, v_4 can be written straight down from the 1^{st} results (give **M3**, **A1**, **A1**)

Let $X = OP_2$ and $Y = OP_3$ initially.

M1 Calculating time to 1^{st} collision at O

A1 $$\frac{(m_1 + m_2)X}{m_1(1+e)u} = \frac{(m_3 + m_4)Y}{m_4(1+e)u}$$

M1 Calculating time to 2^{nd} collision at O

A1 $$\frac{(m_1 + m_2)X}{(m_1 - em_2)u} = \frac{(m_3 + m_4)Y}{(m_4 - em_3)u}$$

M1 Cancelling the u's and the $(1 + e)$'s

$$\Rightarrow \frac{(m_1 + m_2)X}{m_1} = \frac{(m_3 + m_4)Y}{m_4} \quad \text{and} \quad \frac{(m_1 + m_2)X}{(m_1 - em_2)} = \frac{(m_3 + m_4)Y}{(m_4 - em_3)} \qquad \textbf{(*)}$$

M1 Dividing these two (or equating for X / Y)

$$\Rightarrow \frac{m_1 - em_2}{m_1} = \frac{m_4 - em_3}{m_4}$$

M1 for simplifying: $1 - \dfrac{em_2}{m_1} = 1 - \dfrac{em_3}{m_4}$

A1 for $\dfrac{m_2}{m_1} = \dfrac{m_3}{m_4}$ or equivalent **cso**

M1 for subst^{g}. back into one eqn. in line **(*)**: $X\left(1 + \dfrac{m_2}{m_1}\right) = Y\left(1 + \dfrac{m_3}{m_4}\right)$

10

A1 $\Rightarrow$ $X = Y$ legitimately ANSWER GIVEN

11 **M1 A1** for attempt at **<u>N2L</u>** with F_T (or P), R and a in $F_T - (n+1)R = (n+1)Ma$ correct

M1 $P = F_T \, . \, v$ used in their N2L statement

A1 $a = \dfrac{\dfrac{P}{v} - (n+1)R}{M(n+1)}$ or $\dfrac{P-(n+1)Rv}{M(n+1)v}$

B1 for noting that $a > 0 \Rightarrow P > (n+1)Rv$ (from correct working)

5

M1 for use of their $a = \dfrac{dv}{dt}$: $\dfrac{dv}{dt} = \dfrac{P-(n+1)Rv}{M(n+1)v}$

M1 for separating variables: $\dfrac{M(n+1)v}{P-(n+1)Rv}\,dv = dt$

M1 for suitable limits noted or used: $\displaystyle\int_0^V \frac{M(n+1)v}{P-(n+1)Rv}\,dv = \int_0^T 1.dt \quad (=T)$

M1 for method for sorting out LHS integral, e.g. by substn.

$s = P - (n+1)Rv \qquad ds = -R(n+1)\,dv$

M1 for completely eliminating the v's: $T = \dfrac{M}{R}\displaystyle\int \frac{P-s}{s} \times \frac{ds}{-R(n+1)}$

M1 for integrating to get a log. term and a linear one

$$\frac{-M}{(n+1)R^2}\int\left(\frac{P}{s}-1\right)ds = \frac{-M}{(n+1)R^2}\left[P\ln(s) - s\right]$$

M1 for correct use of limits and subst[g]. back to get T as a function of v

$$= \frac{-M}{(n+1)R^2}\left[P\ln(P-(n+1)Rv) - (P-(n+1)Rv)\right]_0^V$$

$$= \frac{-MP}{(n+1)R^2}\left\{\ln(P-(n+1)Rv) - P + (n+1)Rv - P\ln P + P - 0\right\}$$

A1 $$= \frac{-MP}{(n+1)R^2}\ln\left(\frac{P-(n+1)Rv}{P}\right) - \frac{MV}{R}$$

8

M1 for re-arranging into form $\ln(1-x)$: $T = \dfrac{-MP}{(n+1)R^2}\ln\left(1 - \dfrac{(n+1)Rv}{P}\right) - \dfrac{MV}{R}$

M1 for using given approxn.: $\approx \dfrac{-MP}{(n+1)R^2}\left(-\dfrac{(n+1)Rv}{P} - \dfrac{1}{2}\left(\dfrac{(n+1)Rv}{P}\right)^2 \ldots\ldots\right) - \dfrac{MV}{R}$

$= \dfrac{MV}{R} + \dfrac{(n+1)MV^2}{2P}\ldots\ldots - \dfrac{MV}{R}$

A1 $\Rightarrow PT \approx \tfrac{1}{2}(n+1)MV^2$ legitimately ANSWER GIVEN

3

E1 This is just "Work Done = Change in (K) Energy"

E1 in the case when $R = 0$

2

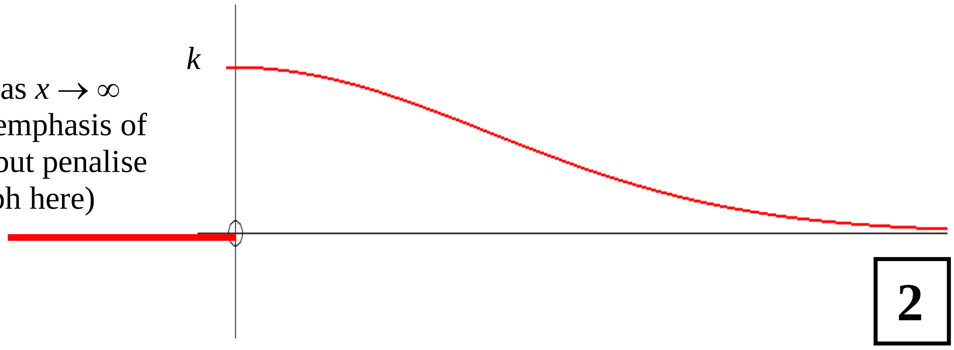
M1 When $R \neq 0$, WD against R = WD by engine – Gain in KE

A1 $\Rightarrow (n+1)RX = PT - \tfrac{1}{2}(n+1)MV^2$

2

12 (i) **B1** Shape at $(0, k)$

B1 Shape for $x > 0$ as $x \to \infty$
(ignore lack of emphasis of zero for $x < 0$, but penalise a non-zero graph here)



2

(ii) **M1** for attempted use of $\frac{1}{\sqrt{2\pi}}\int_0^\infty e^{-\frac{1}{2}t^2}\, dt$ from the standard Normal distribution

M1 Equating their integral to $\frac{1}{2}$

M1 Subst^g. $t = 2x$, $dt = 2\, dx$: $\frac{1}{\sqrt{2\pi}}\int_0^\infty 2e^{-2x^2}\, dx = \frac{1}{2} \Rightarrow \int_0^\infty e^{-2x^2} dx = \frac{\sqrt{2\pi}}{4}$

M1 for use of total prob. = 1: $\frac{1}{k} = \frac{\sqrt{2\pi}}{4}$

A1 $k = \frac{4}{\sqrt{2\pi}}$

5

(iii) **M1** $E(X) = k\int_0^\infty xe^{-2x^2} dx$

A1 $= k\left[-\frac{1}{4}e^{-2x^2}\right]_0^\infty$

A1 **ft** $= \frac{1}{4}k = \frac{1}{\sqrt{2\pi}}$

3

M1 $E(X^2) = k\int_0^\infty x \times x\, e^{-2x^2} dx$

M1 for use of parts (or equivalent): $= k\left\{\left[-\frac{1}{4}xe^{-2x^2}\right]_0^\infty + \int_0^\infty \frac{1}{4}e^{-2x^2} dx\right\}$

A1 $= k\left\{0 + \frac{1}{4} \times \frac{\sqrt{2\pi}}{4}\right\} = \frac{1}{4}$ **ft** k (should still be ¼)

M1 for use of $\mathrm{Var}(X) = E(X^2) - E^2(X)$

A1 $\mathrm{Var}(X) = \frac{1}{4} - \frac{1}{2\pi}$ or $\frac{\pi - 2}{4\pi}$ **cao**

5

(iv) **M1** for $\frac{1}{2} = \frac{4}{\sqrt{2\pi}}\int_0^m e^{-2x^2}\,dx$

M1 for transforming back (or ≡) $= \frac{2}{\sqrt{2\pi}}\int_0^m 2e^{-2x^2}\,dx = 2\times\frac{1}{\sqrt{2\pi}}\int_0^{2m} e^{-\frac{1}{2}t^2}\,dt$

M1 correct use of Std. Nml. Distn.: $\frac{1}{2} = 2\{\Phi(2m)-\tfrac{1}{2}\}$ or $\Phi(\tfrac{1}{2}m)=\frac{3}{4}$

M1 Use of $Z(0, 1)$ tables: $2m = 0.6745$ **(**0.675-ish)

A1 m = 0.337 or 0.338

$\boxed{5}$

13	**M1**	For A: p(launch fails) = p(>1 fail)	
	M1	$= 1 - p_0 - p_1$	
	A1	$= 1 - q^4 - 4q^3p$	
	M1	for E(repair) = Σx p(x)	
	M1	for use of above result	
		$= 0.q^4 + K.4q^3p + 4K(1 - q^4 - 4q^3p)$	
		$= 4K\left[q^3p + (1-q)(1+q+q^2+q^3) - 4q^3p\right]$	
	A1	$= 4Kp\left[1+q+q^2-2q^3\right]$ legitimately ANSWER GIVEN See Alt.13.1	**6**

Alt.13.1

M1	for E(repair) = Σx p(x)	
	$= 0.q^4 + K.4q^3p + 4K(6p^2q^2 + 4p^3q + p^4)$	
M2 A1	for these terms	
M1	for facts[g]. and using $p = 1 - q$	
A1	$= 4Kp\left[1+q+q^2-2q^3\right]$ legitimately ANSWER GIVEN	**6**

M1	For B: p(launch fails) = p(>2 fail)	
M1	$= 1 - p_0 - p_1 - p_2$	
A1	$= 1 - q^6 - 6q^5p - 15q^4p^2$	
M1	for E(repair) = Σx p(x)	
M1	for use of above result	
	$= 0.q^6 + K.6q^5p + 2K.15q^4p^2 + 6K(1 - q^6 - 6q^5p - 15q^4p^2)$	
M1	Decent attempt to find a factor	
	$= 6K\left[q^5p + 5q^4p^2 + (1-q)(1+q+q^2+q^3+q^4+q^5) - 6q^5p - 15q^4p^2\right]$	
M1	Extracting the p and obtaining remaining in terms of q only	
	$= 6Kp\left[q^5 + 5q^4(1-q) + 1 + q + q^2 + q^3 + q^4 + q^5 - 6q^5 - 15q^4(1-q)\right]$	
A1	$= 6Kp\left[1+q+q^2+q^3-9q^4+6q^5\right]$ See Alt.13.2	**8**

Alt.13.2

M1	for E(repair) = Σx p(x)	
	$= 0.q^6 + K.6q^5p + 2K.15q^4p^2 + 6K(20q^3p^3 + 15q^2p^4 + 6qp^5 + p^6)$	
M2 A1	for these terms	
M1	Use of $p = 1 - q$ throughout	
	$= 6Kp\left[q^5 + 5q^4(1-q) + 20q^3(1-2q+q^2) + 15q^2(1-3q+3q^2-q^3) + \ldots\right.$	
M1	for the extra terms $\left.\ldots + 6q(1-4q+6q^2-4q^3+q^4) + (1-5q+10q^2-10q^3+5q^4-q^5)\right]$	
M1	Good attempt at collecting up terms	
A1	$= 6Kp\left[1+q+q^2+q^3-9q^4+6q^5\right]$	**8**

M1	Setting Rep(A) = $\frac{2}{3}$ Rep(B) $\Rightarrow$ $12Kp\left[1+q+q^2-2q^3\right] = 2Kp\left[1+q+q^2+q^3-9q^4+6q^5\right]$	
M1	$p = 0$ case noted or explained	
M1	Factrs[g]. rest: $0 = 3q^3(1 - 3q + 2q^2)$	
M1	Factrs[g]. quadratic bit:	
A1	All correct: $0 = 3q^3(1 - q)(1 - 2q)$	
A1	getting $p = 1, 0, \frac{1}{2}$ (Allow those who ditch p = 0, 1)	**6**