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REGRET THEORY: AN ALTERNATIVE THEORY OF RATIONAL CHOICE UNDER UNCERTAINTY*

Graham Loomes and Robert Sugden

The main body of current economic analysis of choice under uncertainty is built upon a small number of basic axioms, formulated in slightly different ways by von Neumann and Morgenstern (1947), Savage (1954) and others. These axioms are widely believed to represent the essence of rational behaviour under uncertainty. However, it is well known that many people behave in ways that systematically violate these axioms.[1]

We shall initially focus upon a paper by Kahneman and Tversky (1979) which presents extensive evidence of such behaviour. Kahneman and Tversky offer a theory, which they call 'prospect theory', to explain their observations. We shall offer an alternative theory which is much simpler than prospect theory and which, we believe, has greater appeal to intuition.

The following notation will be used throughout. The ith *prospect* is written as X_i. If it offers increments or decrements of wealth $x_1, \ldots, x_n$ with probabilities $p_1, \ldots, p_n$ (where $p_1 + \ldots + p_n = 1$) it may be denoted as $(x_1, p_1; \ldots; x_n, p_n)$. Null consequences are omitted so that the prospect $(x, p; 0, 1-p)$ is written simply as (x, p). Complex prospects, i.e. those which offer other prospects as consequences, may be denoted as $(X_i, p_i; \ldots; X_n, p_n)$. We shall use the conventional notation $\succ$, $\succcurlyeq$ and $\sim$ to represent the relations of strict preference, weak preference and indifference. We shall take it that for all prospects X_i and X_k, $X_i \succcurlyeq X_k$ or $X_i \preccurlyeq X_k$; but we shall not in general require that the relation $\succcurlyeq$ is transitive.

I. KAHNEMAN AND TVERSKY'S EVIDENCE

Kahneman and Tversky's experiments offered hypothetical choices between pairs of prospects to groups of university faculty and students. Table 1 lists a selection of their results, which reveal three main types of violation of conventional expected utility theory:

(*a*) The 'certainty effect' or 'common ratio effect', e.g. the conjunction of $X_5 \prec X_6$ and $X_9 \succ X_{10}$ and the conjunction $X_{13} \prec X_{14}$ and $X_{15} \succ X_{16}$. There is also a 'reverse common ratio effect', e.g. the conjunction of $X_7 \succ X_8$ and $X_{11} \prec X_{12}$.

(*b*) The original 'Allais Paradox' or 'common consequences effect', e.g. the conjunction of $X_1 \prec X_2$ and $X_3 \succ X_4$.

(*c*) The 'isolation effect' in two-stage gambles, e.g. the conjunction of $X_9 \succ X_{10}$ and $X_{17} \prec X_{18}$.

* We particularly wish to thank Michael Jones-Lee, Mark Machina and two anonymous referees for many helpful suggestions and criticisms.

[1] For a survey and discussion of much of the evidence, see Allais and Hagen (1979) and Schoemaker (1980, 1982).

Table 1 also reveals a 'reflection effect' where a change of sign on the consequences is associated with a reversal of the modal preference and the risk attitude that characterises it, e.g. $X_5 \prec X_6$ and $X_7 \succ X_8$. One instance of the reflection effect, revealed in Problems 14 and 14′, may be interpreted as an

Table 1

Kahneman and Tversky problem no.	Prospects offered†	Modal preference	Percentage of subjects with modal preference	Characterisation of modal preference
1	$X_1 = (2{,}500, 0{\cdot}33; 2{,}400, 0{\cdot}66)$ $X_2 = (2{,}400, 1{\cdot}00)$	$X_1 \prec X_2$	82*	Risk averse
2	$X_3 = (2{,}500, 0{\cdot}33)$ $X_4 = (2{,}400, 0{\cdot}34)$	$X_3 \succ X_4$	83*	Not clear
3	$X_5 = (4{,}000, 0{\cdot}80)$ $X_6 = (3{,}000, 1{\cdot}00)$	$X_5 \prec X_6$	80*	Risk averse
3′	$X_7 = (-4{,}000, 0{\cdot}80)$ $X_8 = (-3{,}000, 1{\cdot}00)$	$X_7 \succ X_8$	92*	Risk loving
4	$X_9 = (4{,}000, 0{\cdot}20)$ $X_{10} = (3{,}000, 0{\cdot}25)$	$X_9 \succ X_{10}$	65*	Not clear
4′	$X_{11} = (-4{,}000, 0{\cdot}20)$ $X_{12} = (-3{,}000, 0{\cdot}25)$	$X_{11} \prec X_{12}$	58	Not clear
7	$X_{13} = (6{,}000, 0{\cdot}45)$ $X_{14} = (3{,}000, 0{\cdot}90)$	$X_{13} \prec X_{14}$	86*	Risk averse
8	$X_{15} = (6{,}000, 0{\cdot}001)$ $X_{16} = (3{,}000, 0{\cdot}002)$	$X_{15} \succ X_{16}$	73*	Risk loving
10	$X_{17} = (X_5, 0{\cdot}25)$ $X_{18} = (X_6, 0{\cdot}25)$	$X_{17} \prec X_{18}$	78*	Risk averse
14	$X_{19} = (5{,}000, 0{\cdot}001)$ $X_{20} = (5, 1{\cdot}000)$	$X_{19} \succ X_{20}$	72*	Risk loving
14′	$X_{21} = (-5{,}000, 0{\cdot}001)$ $X_{22} = (-5, 1{\cdot}000)$	$X_{21} \prec X_{22}$	83*	Risk averse

* Statistically significant at the 0·01 level.
† Consequences are increments or decrements of wealth, measured in Israeli pounds.

example of simultaneous gambling and insurance, since $X_{19} \succ X_{20}$ indicates a willingness to enter an actuarially fair lottery offering a small probability of a large prize, while $X_{21} \prec X_{22}$ signifies a willingness to take out actuarially fair insurance against a small probability of a large loss. We also note an interesting mixture of risk attitudes. Sometimes risk aversion is associated with problems involving increments of wealth, e.g. $X_{13} \prec X_{14}$, and sometimes with problems involving decrements, e.g. $X_{21} \prec X_{22}$. Likewise, risk loving is sometimes associated with problems involving increments, e.g. $X_{15} \succ X_{16}$, and sometimes with problems involving decrements, e.g. $X_7 \succ X_8$.

Simultaneous gambling and insurance, the reflection effect, and the mixture of risk attitudes may all be accommodated by conventional expected utility theory, though only at the cost of certain fairly arbitrary assumptions and some rather unsatisfactory implications.[1] But no accommodation is possible for the effects listed in (*a*), (*b*) and (*c*) above – the observations here simply violate one or more of the conventional axioms.

[1] See Friedman and Savage (1948), Markowitz (1952) and Hirschleifer (1966).

However, in the next section we shall outline the framework of an alternative theory which not only explains the reflection effect and simultaneous gambling and insurance, but also predicts the behaviour described in (*a*), (*b*) and (*c*). We shall then argue that, besides being predictable, such behaviour can be defended as rational, and that our model therefore provides the basis for an alternative theory of rational choice under uncertainty.

II. THE FRAMEWORK OF AN ALTERNATIVE THEORY

We consider an individual in a situation where there is a finite number, n, of alternative *states of the world*, any one of which might occur. Each state j has a probability p_j where $0 < p_j \leqslant 1$ and $p_1 + \ldots + p_n = 1$. These probabilities may be interpreted either as objective probabilities known to the individual or, in the absence of firm knowledge of this kind, as subjective probabilities which represent the individual's degree of belief or confidence in the occurrence of the corresponding states. The individual's problem is to choose between *actions*. Each action is an n-tuple of *consequences*, one consequence for each state of the world. We shall write the consequence of the ith action in the event that the jth state occurs as x_{ij}. Consequences need not take the form of changes in wealth, although in our applications of our theory, we shall interpret x_{ij} as an increment or decrement of wealth, measured relative to some arbitrary level (which need not be the individual's current wealth). Notice that actions, unlike prospects, associate consequences with particular states of the world. Thus a number of different actions might correspond with the same prospect. We shall recognise this difference by using the symbol A for actions, reserving X for prospects. Thus far, our theory has a close resemblance to Savage's, except in that we take probabilities as given, just as von Neumann and Morgenstern do.

A choice problem may involve any number of available actions, but we shall begin by analysing problems where there is only a pair of actions to choose between. All of Kahneman and Tversky's evidence concerns the behaviour of people choosing between pairs of prospects. Choices between three or more actions raise some additional issues, which we shall discuss in Section IV.

Our first assumption is that for any given individual there is a *choiceless utility function* $C(.)$, unique up to an increasing linear transformation, which assigns a real-valued utility index to every conceivable consequence. The significance of the word 'choiceless' is that $C(x)$ is the utility that the individual would derive from the consequence x if he experienced it *without having chosen it*. For example, he might have been compelled to have x by natural forces, or x might have been imposed on him by a dictatorial government. Thus – in contrast to the von Neumann–Morgenstern concept of utility – our concept of choiceless utility is defined independently of choice. Our approach is utilitarian in the classical sense. What we understand by 'choiceless utility' is essentially what Bernoulli and Marshall understood by 'utility' – the psychological experience of pleasure that is associated with the satisfaction of desire. We believe that it is possible to introspect about utility, so defined, and that it is therefore meaningful to talk about utility being experienced in choiceless situations.

Now suppose that an individual experiences a particular consequence as the result of an act of choice. Suppose that he has to choose between actions A_1 and A_2 in a situation of uncertainty. He chooses A_1 and then the jth state of the world occurs. He therefore experiences the consequence x_{1j}. He now knows that, had he chosen A_2 instead, he would be experiencing x_{2j}. Our introspection suggests to us that the psychological experience of pleasure associated with having the consequence x_{1j} in these circumstances will depend not only on the nature of x_{1j} but also on the nature of x_{2j}. If x_{2j} is a more desirable consequence than x_{1j}, the individual may experience *regret*: he may reflect on how much better his position would have been, had he chosen differently, and this reflection may reduce the pleasure that he derives from x_{1j}. Conversely, if x_{1j} is the more desirable consequence, he may experience what we shall call *rejoicing*. the extra pleasure associated with knowing that, as matters have turned out, he has taken the best decision.

We guess that many readers will recognise these experiences. For example, compare the sensation of losing £100 as the result of an increase in income tax rates, which you could have done nothing to prevent, with the sensation of losing £100 on a bet on a horse race. Our guess is that most people would find the latter experience more painful, because it would inspire regret. Conversely, compare the experience of gaining £100 from an income tax reduction with that of winning £100 on a bet. Now we should guess that most people would find the latter experience more pleasurable. This concept of regret resembles Savage's (1951) notion in some ways, but it will emerge that our theory is very different from his minimax regret criterion.

We shall incorporate the concepts of regret and rejoicing into our theory by means of a *modified utility function*. Suppose that an individual chooses action A_i in preference to action A_k, and that the jth state of the world occurs. The actual consequence is x_{ij} while, had he chosen differently, x_{kj} would have occurred. We shall write $C(x_{ij})$ as c_{ij} and we shall then say that the individual experiences the *modified utility* m^k_{ij} where:

$$m^k_{ij} = M(c_{ij}, c_{kj}). \qquad (1)$$

The function $M(.)$ assigns a real-valued index to every ordered pair of choiceless utility indices. The difference between m^k_{ij} and c_{ij} may be interpreted as an increment or decrement of utility corresponding with the sensations of rejoicing or regret. To formulate regret and rejoicing in this way is to assume that the degree to which a person experiences these sensations depends only on the choiceless utility associated with the two consequences in question – 'what is' and 'what might have been' – and is independent of any other characteristics of these consequences. Given this assumption, it is natural to assume in addition that if $c_{ij} = c_{kj}$ then $m^k_{ij} = c_{ij}$: if what occurs is exactly as pleasurable as what might have occurred, there is neither regret nor rejoicing. It is equally natural to assume that $\partial m^k_{ij}/\partial c_{kj} \leqslant 0$: the more pleasurable the consequence that might have been, the more regret – or less rejoicing – is experienced. (We include as a limiting case the possibility that a person might not experience regret or rejoicing at all.) We also make the uncontroversial assumption that $\partial m^k_{ij}/\partial c_{ij} > 0$: that, other things being equal, modified utility increases with choiceless utility.

Our theory is that the individual chooses between actions so as to maximise the mathematical expectation of modified utility. We may define the *expected modified utility* E_i^k of action A_i, evaluated with respect to action A_k, by:

$$E_i^k = \sum_{j=1}^{n} p_j m_{ij}^k. \tag{2}$$

Faced with a choice between A_i and A_k, the individual will prefer A_i, prefer A_k or be indifferent between them according to whether E_i^k is greater than, less than or equal to E_k^i.

Why, it may be asked, do we assume that people maximise the mathematical expectation of modified utility? Principally because this is a simple assumption which yields implications consistent with empirical evidence. We do not claim that maximising expected modified utility is the only objective that is consistent with a person being rational. However – and we shall say more about this in Section V – we believe that this is not *ir*rational, and that, given the utilitarian premises of our approach, there is at least a presumption that people who experience regret and rejoicing will seek to maximise expected modified utility. Notice that, in our theory, someone who does not feel regret or rejoicing at all will simply maximise expected choiceless utility. This special case of our theory corresponds with expected utility theory in its traditional or Bernoullian form, in which utility is interpreted as a psychological experience. To assume that people maximise expected modified utility is to generalise Bernoulli's theory in a very natural way, since the individual who *does* experience rejoicing and regret can be expected to try to anticipate those feelings and take them into account when making a decision under uncertainty.

We shall now show that all of the experimental evidence described in Section I is consistent with regret theory. We shall do this by taking a restricted form of our general theory and by showing that the experimental evidence is consistent with this restricted form.

The particular restriction involves a simplifying assumption about the function $M(.)$. We shall assume that the degree of regret or rejoicing that a person experiences depends only on the difference between the choiceless utility of 'what is' and the choiceless utility of 'what might have been'. This allows us to define a *regret–rejoice function* $R(.)$ which assigns a real-valued index to every possible increment or decrement of choiceless utility, and then to write:

$$m_{ij}^k = c_{ij} + R(c_{ij} - c_{kj}). \tag{3}$$

It follows from the assumptions we have made about $M(.)$ that $R(0) = 0$ and that $R(.)$ is non-decreasing. In the limiting case in which $R(\xi) = 0$ for all ξ, regret theory would yield exactly the same predictions as expected utility theory. Since we wish to emphasise the differences between the two theories we shall assume that $R(.)$ is strictly increasing and three times differentiable.

Now suppose, as before, that an individual has to choose between the actions A_i and A_k. The individual will have the weak preference $A_i \succcurlyeq A_k$ if and only if:

$$\sum_{j=1}^{n} p_j[c_{ij} - c_{kj} + R(c_{ij} - c_{kj}) - R(c_{kj} - c_{ij})] \geqslant 0. \tag{4}$$

It is convenient to define a function $Q(.)$ such that for all ξ,

$$Q(\xi) = \xi + R(\xi) - R(-\xi). \tag{5}$$

Thus $A_i \geqslant A_k$ if and only if:

$$\sum_{j=1}^{n} p_j[Q(c_{ij} - c_{kj})] \geqslant 0. \tag{6}$$

$Q(.)$ is an increasing function which has the following property of symmetry: for all ξ, $Q(\xi) = -Q(-\xi)$. Thus to know the value of $Q(\xi)$ for all $\xi \geqslant 0$ is to know the value of $Q(\xi)$ for all ξ.

Three alternative simplifying assumptions about $Q(.)$ can be distinguished:

Assumption 1. $Q(.)$ is linear or equivalently, for all ξ, $R''(\xi) = R''(-\xi)$. It follows immediately from (6) that in this case the individual will behave exactly as if he were maximising expected choiceless utility. Thus regret theory would yield the same predictions as expected utility theory and choiceless utility indices would be operationally indistinguishable from von Neumann–Morgenstern utility indices.

Assumption 2. $Q(.)$ is concave for all positive values of ξ or equivalently, for all $\xi > 0$, $R''(\xi) < R''(-\xi)$.

Assumption 3. $Q(.)$ is convex for all positive values of ξ or equivalently, for all $\xi > 0$, $R''(\xi) > R''(-\xi)$.

On the face of it, there seems to be no *a priori* reason for preferring any one of these assumptions to the others. They are simply alternative assumptions about human psychology and a choice between them should be made mainly on the basis of empirical evidence.[1] We shall therefore show that all the evidence listed in Table 1 is consistent with the restricted form of our theory under Assumption 3. In contrast, Assumption 1 would predict no violations of expected utility theory, while Assumption 2 would predict violations, but in the opposite direction to those generally observed.

III. SOME IMPLICATIONS OF REGRET THEORY

We shall now derive some implications of our theory concerning choices between pairs of *statistically independent* prospects. In our theory, a choice problem cannot be analysed unless a matrix of state-contingent consequences can be specified, and a given pair of prospects (i.e. probability distributions of consequences) may

[1] We say 'mainly' because there may be some theoretical reasons for expecting Assumption 3 to be true more often than either of the other two assumptions. Notice that it is a sufficient (but not a necessary) condition for Assumption 1 to hold that, for all ξ, $R'''(\xi) = 0$. Similarly it is sufficient for Assumption 2 to hold that, for all ξ, $R'''(\xi) < 0$; and it is sufficient for Assumption 3 to hold that, for all ξ, $R'''(\xi) > 0$. Consider the following three alternative cases: that $R(.)$ is linear, that it is everywhere convex, and that it is everywhere concave. Linearity entails that for all ξ, $R'''(\xi) = 0$ and so entails Assumption 1. Convexity entails that for all ξ, $R'(\xi) > 0$ and $R''(\xi) > 0$. Since, given these two conditions, $R'''(\xi) \leqslant 0$ cannot hold for all ξ, the simplest assumption to make about $R'''(.)$ is that for all ξ, $R'''(\xi) > 0$. This in turn entails Assumption 3. Concavity entails that for all ξ, $R'(\xi) > 0$ and $R''(\xi) < 0$. Since $R'''(\xi) \leqslant 0$ cannot hold for all ξ, the simplest assumption to make is again that for all ξ, $R'''(\xi) > 0$. So Assumption 3 fits with both convexity and concavity, while Assumption 1 is appropriate only for linearity – which is only one point on a continuous spectrum which ranges from extreme convexity to extreme concavity.

be capable of being represented by many different matrices. However, the assumption of statistical independence ensures that there is a unique matrix for each pair of prospects. In most of Kahneman and Tversky's experiments, subjects were simply asked to choose between pairs of prospects. In such cases, we suggest, the most natural assumption for subjects to make is that the prospects are independent. Given this assumption, we can show that the evidence of Table 1 is entirely consistent with regret theory. As before, we shall use x_1 and x_2 to represent consequences. We shall use c_1 and c_2 to represent the choiceless utility indices $C(x_1)$ and $C(x_2)$. For simplicity, we choose a transformation of $C(.)$ such that $C(0) = 0$; and we assume that $C(.)$ is an increasing function.

(a) The 'common ratio effect', and its reverse

Our theory yields the following prediction, which violates expected utility theory:

> Let $X_i = (x_1, \lambda p)$ and $X_k = (x_2, p)$ be independent prospects, where $1 \geqslant p > 0$ and $1 > \lambda > 0$. If there exists some probability $\bar{p}$ such that $X_i \sim X_k$ when $p = \bar{p}$, then (*i*) (*the common ratio effect*) if $x_1 > x_2 > 0$, then $p < \bar{p} \Rightarrow X_i \succ X_k$ and $p > \bar{p} \Rightarrow X_i \prec X_k$ and (*ii*) (*the reverse common ratio effect*) if $0 > x_2 > x_1$, then $p < \bar{p} \Rightarrow X_i \prec X_k$ and $p > \bar{p} \Rightarrow X_i \succ X_k$.

In proving this result, it is convenient to begin by stating a general property of our theory. Let $X' = (x_1, p_1)$ and $X'' = (x_2, p_2)$ be any two independent prospects. The choice between these prospects may be represented by the matrix given in

Table 2

Action corresponding with prospect	p_1p_2	$p_1(1-p_2)$	$(1-p_2)p_2$	$(1-p_1)(1-p_2)$
X'	x_1	x_1	0	0
X''	x_2	0	x_2	0

Table 2, where each column represents a different state of the world, and the probability that each state will occur is given at the top of its column. Applying Expression (6) to Table 2, we find: that

$$X' \underset{\prec}{\overset{\succ}{\sim}} X'' \text{ iff } p_1 Q(c_1) - p_2 Q(c_2) - p_1 p_2 [Q(c_1) - Q(c_1 - c_2) - Q(c_2)] \gtreqless 0. \quad (7)$$

Thus in the case where $X_i = (x_1, \lambda p)$ and $X_k = (x_2, p)$,

$$X_i \underset{\prec}{\overset{\succ}{\sim}} X_k \text{ iff } p\{\lambda Q(c_1) - Q(c_2) - \lambda p[Q(c_1) - Q(c_1 - c_2) - Q(c_2)]\} \gtreqless 0. \quad (8)$$

By assumption, $Q(c)$ is convex for all $c > 0$ so that when $c_1 > c_2 > 0$, $[Q(c_1) - Q(c_1 - c_2) - Q(c_2)] > 0$. Given this inequality, the common ratio effect follows straightforwardly from Expression (8). Conversely, when $0 > c_2 > c_1$, $[Q(c_1) - Q(c_1 - c_2) - Q(c_2)] < 0$; and this implies the reverse common ratio effect.

The evidence of Problems 3 and 4 is consistent with the existence of the common ratio effect. Let $x_1 = 4{,}000$, $x_2 = 3{,}000$ and $\lambda = 0{\cdot}8$. Then if $p = 1{\cdot}0$, X_5

$= (x_1, \lambda p)$ and $X_6 = (x_2, p)$. If $p = 0{\cdot}25$, $X_9 = (x_1, \lambda p)$ and $X_{10} = (x_2, p)$. The conjunction of preferences $X_5 \prec X_6$ and $X_9 \succ X_{10}$ violates expected utility theory but is consistent with regret theory (corresponding with the case $1{\cdot}0 > \bar{p} > 0{\cdot}25$). Over half of Kahneman and Tversky's subjects had this conjunction of preferences. Further evidence of the common ratio effect is provided by Problems 7 and 8, while Problems 3′ and 4′ reveal the reverse common ratio effect.

(b) *The 'common consequences effect' or Allais paradox*

Our theory yields a further prediction, which also violates expected utility theory:

> Let $X_i = (x_1, p_1;\, x_2, \alpha)$ and $X_k = (x_2, p_2 + \alpha)$ be independent prospects where $1 \geqslant p_2 > p_1 > 0$ and $(1 - p_2) \geqslant \alpha \geqslant 0$. If there exists some probability $\bar{\alpha}$ such that $X_i \sim X_k$ when $\alpha = \bar{\alpha}$, then (*i*) (*the common consequences effect*) if $x_1 > x_2 > 0$, then $\alpha < \bar{\alpha} \Rightarrow X_i \succ X_k$ and $\alpha > \bar{\alpha} \Rightarrow X_i \prec X_k$ and (*ii*) (*the reverse common consequences effect*) if $0 > x_2 > x_1$, then $\alpha < \bar{\alpha} \Rightarrow X_i \prec X_k$ and $\alpha > \bar{\alpha} \Rightarrow X_i \succ X_k$.

According to regret theory,

$$X_i \gtrsim\!\!\!\!\!< X_k \text{ iff } p_1 Q(c_1) - p_2 Q(c_2) - p_1(p_2 + \alpha)\,[Q(c_1) - Q(c_1 - c_2) - Q(c_2)] \gtreqless 0. \quad (9)$$

Because $Q(c)$ is assumed to be convex for all $c > 0$, $[Q(c_1) - Q(c_1 - c_2) - Q(c_2)]$ is positive if $x_1 > x_2 > 0$ and negative if $0 > x_2 > x_1$. Given these two propositions, Expression (9) entails both the common consequences effect and the reverse common consequences effect.

The evidence of Problems 1 and 2 is consistent with the existence of the common consequences effect. Let $x_1 = 2{,}500$, $x_2 = 2{,}400$, $p_1 = 0{\cdot}33$ and $p_2 = 0{\cdot}34$. Then if $\alpha = (1 - p_2)$, $X_1 = (x_1, p_1;\, x_2, \alpha)$ and $X_2 = (x_2, p_2 + \alpha)$. If $\alpha = 0$, $X_3 = (x_1, p_1;\, x_2, \alpha)$ and $X_4 = (x_2, p_2 + \alpha)$. The conjunction of preferences $X_1 \prec X_2$ and $X_3 \succ X_4$ violates expected utility theory but is consistent with regret theory (corresponding with the case $0{\cdot}66 > \bar{\alpha} > 0$). At least 65% of Kahneman and Tversky's subjects had this conjunction of preferences. Kahneman and Tversky did not publish any results relevant to our prediction of a reverse common consequences effect.

(c) *The 'isolation effect' in the two-stage gambles*

In Kahneman and Tversky's Problem 10, their respondents were offered a two-stage gamble. In the first stage there was a 0·75 probability of the gamble ending with a null consequence and a 0·25 probability of going through to the second stage. Before embarking on the first stage, respondents were asked to choose which of X_5 or X_6 they would prefer if they got through to the second stage.

According to the compound probability axiom of expected utility theory, $X_{17} = (X_5, 0{\cdot}25)$ is equivalent to $(4{,}000, 0{\cdot}20)$ which is simply prospect X_9; and $X_{18} = (X_6, 0{\cdot}25)$ is equivalent to $(3{,}000, 0{\cdot}25)$ which is prospect X_{10}. Thus expected utility theory makes no distinction between Problem 10 and Problem 4.

However, regret theory does make a distinction. The simple prospects X_9 and X_{10} are regarded as statistically independent, and Problem 4 is therefore represented by the matrix of state-contingent consequences shown in Table 3*a*. By contrast, prospects X_{17} and X_{18} are not statistically independent: the first stage of the gamble is common to both, and if the state occurs under which the gamble comes to an end, the individual receives the same null consequence whichever prospect was chosen. Hence Problem 10 is represented by the matrix of state-contingent consequences shown in Table 3*b*. Since Tables 3*a* and 3*b* are

Table 3*a*

Action corresponding with prospect	0·60	0·20	0·15	0·05
X_9	0	0	4,000	4,000
X_{10}	0	3,000	0	3,000

Table 3*b*

Action corresponding with prospect	0·75	0·20	0·05
X_{17}	0	4,000	0
X_{18}	0	3,000	3,000

different, our theory provides no reason to suppose that an individual will have the same preferences between X_{17} and X_{18} as between X_9 and X_{10}.

Before analysing this example further, we present a result which holds for regret theory in its most general form, and which we shall call the *separability principle*.

> Let $S_1, \dots, S_n$ be mutually exclusive events (i.e. non-intersecting sets of states of the world) with the non-zero probabilities $p_1, \dots, p_n$ where $p_1 + \dots + p_n = 1$. Let $S'_1, \dots, S'_{n+1}$ be mutually exclusive events with the probabilities $\mu p_1, \dots, \mu p_n$, $1 - \mu$, where $0 < \mu < 1$. Let $A_i = (x_{11}, \dots, x_{1n})$ and $A_k = (x_{21}, \dots, x_{2n})$ be any two actions defined in relation to the events $S_1, \dots, S_n$. Let Let A_a and A_b be actions defined in relation to the events $S'_1, \dots, S'_{n+1}$, such that $A_a = (x_{11}, \dots, x_{1n}, y)$ and $A_b = (x_{21}, \dots, x_{2n}, y)$, y being any consequence common to both actions. Then $A_a \succcurlyeq A_b$ if and only if $A_i \succcurlyeq A_k$.

The proof is straightforward. If E_i^k and E_k^i are the expected modified utilities of A_i and A_k, evaluated in relation to one another, then $E_a^b = \mu E_i^k + (1 - \mu) C(y)$ and $E_b^a = \mu E_k^i + (1 - \mu) C(y)$. Hence $E_i^k \geqslant E_k^i \Leftrightarrow E_a^b \geqslant E_b^a$, which entails $A_i \succcurlyeq A_k \Leftrightarrow A_a \succcurlyeq A_b$. The separability principle entails Savage's sure-thing principle as a special case. Let μ remain constant, and let us construct two new actions, A_c and A_d, which are the same as A_a and A_b except that the common consequence y is replaced by the common consequence z. It is clear that $A_i \succcurlyeq A_k \Leftrightarrow A_c \succcurlyeq A_d$, and hence it follows that $A_a \succcurlyeq A_b \Leftrightarrow A_c \succcurlyeq A_d$, which is Savage's sure-thing principle.

Returning to Kahneman and Tversky's evidence, let A_5 and A_6 be the actions corresponding to the independent prospects X_5 and X_6, and let A_{17} and A_{18} be the actions corresponding to X_{17} and X_{18} in Table 3 *b*. Since $E_{17}^{18} = \mu E_5^6 + (1-\mu)\,C(0)$ and $E_{18}^{17} = \mu E_6^5 + (1-\mu)\,C(0)$, it follows that $X_5 \prec X_6 \Leftrightarrow X_{17} \prec X_{18}$. We have already seen in (*a*) above that the conjunction $X_5 \prec X_6$ and $X_9 \succ X_{10}$ is consistent with our theory. Thus it follows that the conjunction $X_9 \succ X_{10}$ and $X_{17} \prec X_{18}$, which violates conventional expected utility, is also consistent with regret theory.

(*d*) *The 'reflection effect'*

The results in (*a*), (*b*) and (*c*) above were derived without making any assumption about $C(.)$ other than that it is monotonically increasing. We shall derive our results in (*d*) and (*e*) by making the additional assumption that $C(.)$ is linear; and, for convenience, we shall choose a transformation of that linear function such that for all x, $C(x) = x$.

Consider two independent prospects, $X_i = (x_1, p_1)$ and $X_k = (x_2, p_2)$. Their 'reflections' are denoted $X_i' = (-x_1, p_1)$ and $X_k' = (-x_2, p_2)$. From Expression (7) we know that $X_i \succcurlyeq X_k$ if and only if:

$$p_1 Q(x_1) - p_2 Q(x_2) - p_1 p_2 [Q(x_1) - Q(x_1 - x_2) - Q(x_2)] \geqslant 0. \tag{10}$$

Now exactly the same inequality is necessary and sufficient for $X_i' \preccurlyeq X_k'$. Hence $X_i \succcurlyeq X_k \Leftrightarrow X_i' \preccurlyeq X_k'$. Thus if $C(.)$ is linear, the reflection effect is always observed.

Our intuition is that $C(.)$ is not linear but concave. If this is correct, the reflection effect will not always be observed, and in particular, individuals will reject actuarially fair 50–50 gambles, rather than being indifferent towards them. This point is discussed further in Section V.

(*e*) *Mixed risk attitudes; simultaneous gambling and insurance*

Consider two independent prospects which offer an actuarially fair gamble: $X_i = (0, 1)$ and $X_k = (x, p;\ -px/(1-p),\ 1-p)$, where $0 < p < 1$ and $x > 0$. Maintaining our previous assumption about $C(.)$ we can apply Expression (7) and rearrange to give:

$$X_i \gtreqless X_k \text{ iff } Q\left(\frac{px}{1-p}\right) - \frac{p}{1-p}\,Q(x) \gtreqless 0. \tag{11}$$

From the assumption that $Q(x)$ is convex for all $x > 0$, it follows that $X_i \gtreqless X_k$ as $p \gtreqless 0{\cdot}5$. So the individual will accept small-stake large-prize fair gambles ($p < 0{\cdot}5$) but reject large-stake small-prize fair gambles ($p > 0{\cdot}5$). Insurance typically involves paying a small premium to avoid a small probability of a large loss; thus in terms of our theory – which does not use the concept of a 'reference point' – to buy actuarially fair insurance is to reject a large-stake small-prize fair gamble, and thus it is consistent with our theory that an individual may simultaneously insure and accept small-stake large-prize gambles. Moreover, we can construct both small-stake large-prize fair gambles, and large-stake small-prize fair gambles either with all consequences positive or with all consequences negative. Thus a mixture of risk attitudes in both the positive and the negative domain is also consistent with our theory.

These conclusions would require some modification if $C(.)$ were assumed to be concave rather than linear. In this case it can be shown that $X_i \succ X_k$ if $p \geqslant 0{\cdot}5$, but it is no longer possible to make a firm prediction when $p < 0{\cdot}5$. However, if an individual is more strongly influenced by the shape of $Q(.)$ than by the non-linearity of $C(.)$, simultaneous gambling and insurance is still consistent with our theory.

IV. TRANSITIVITY OF PREFERENCES AND MULTI-ACTION PROBLEMS

One controversial property of our theory is that $\succcurlyeq$, the relation of weak preference, is not necessarily transitive. Consider the three actions shown in Table 4 in relation to an individual for whom $C(.)$ is linear. Relative to A_1, A_2 is a large-stake small-prize fair gamble, so that the individual would have the preference $A_1 \succ A_2$ if he had to choose between these two actions. If, as our theory entails, the individual acts according to the separability principle outlined in Section III (c), state S_1 can be ignored in a comparison between A_2 and A_3. Thus, relative to A_2, A_3 is also a large-stake small-prize fair gamble, and so $A_2 \succ A_3$. However, relative to A_1, A_3 is a small-stake large-prize fair gamble, so that $A_3 \succ A_1$. This is not to say that our theory specifically predicts non-transitive pairwise choices (since the $C(.)$ function need not be linear); but such choices can be consistent with the theory.

Table 4

Action	S_1 0·4	S_2 0·2	S_3 0·4
A_1	6	6	6
A_2	0	10	10
A_3	0	0	15

The example shows that an individual will necessarily make non-transitive choices if (i) he acts according to the separability principle (or according to the sure-thing principle), (ii) he always accepts small-stake large-prize fair gambles and (iii) he always rejects large-stake small-prize fair gambles. In the light of the evidence that many people simultaneously gamble and insure one might well argue that a satisfactory theory of choice under uncertainty should encompass the case of the individual who acts according to (ii) and (iii). To say this is to say that either the sure-thing principle or the axiom of transitivity must be dropped. Our theory differs from many of its rivals by dropping transitivity rather than the sure-thing principle.

This raises two questions. One is whether a theory that allows non-transitive pairwise choices can be regarded as a theory of rational behaviour; this issue is discussed in Section V. The other question is how to extend our theory to deal with multi-action choice problems: since in our theory the relation $\succcurlyeq$ is not necessarily transitive, we cannot deal with choices from sets of three or more actions simply by invoking the idea of a preference ordering. We shall argue that the logic of regret and rejoicing points towards a different way of generalising a theory of pairwise choice.

Consider the problem of choosing one action from a set S. The logic of our approach requires that the individual should evaluate each action in turn by asking himself what sensations of regret or rejoicing he would experience in each state of the world, were he to choose that action. Since to choose one action is to reject all of the others, the individual could experience regret or rejoicing in contemplating any of the rejected actions. This idea might be formulated in the following way. As before, we use E_i^k to represent the expected modified utility of choosing action A_i in a situation where the only alternative is action A_k. Now let E_i^S represent the expected modified utility of choosing A_i from the set of actions S. It seems natural to make E_i^S a weighted average of the values of E_i^k for each of the actions A_k in S (other than A_i itself). One way of building this idea into our theory would be to assign *action weights* a_k^S to each action A_k in S, normalised so that these weights sum to unity. Then E_i^S could be defined as:

$$E_i^S = \sum_{k \in S} \frac{a_k^S}{1 - a_i^S} E_i^k \quad (k \neq i). \tag{12}$$

The individual's decision rule, as in the case of pairwise choice, would be to maximise expected modified utility. We hope in the future to formulate a theory of action weights, but in the example which follows we shall just make the simplest assumption – that each action has the same weight.

Table 5

Action	1/3	1/3	1/3
A_1	1	1	1
A_2	0	0	3
A_3	0	3	0

This illustrative example refers to the choice problem shown in Table 5. As before, we shall assume that $C(x) = x$, and we shall make a particular assumption about the regret–rejoice function, that over the relevant range, $R(\xi) = 1 - 0{\cdot}8^{\xi}$. In this case, and for these three actions, the relation $\succcurlyeq$ happens to be transitive; $A_2 \succ A_1$, $A_3 \succ A_1$, $A_2 \sim A_3$. It is tempting (but, we suggest, wrong) to conclude from this that A_1 will not be chosen from the set $\{A_1, A_2, A_3\}$. If the action weights are equal to one another then $E_1^S = 0{\cdot}946$, $E_2^S = 0{\cdot}899$ and $E_3^S = 0{\cdot}899$, so that, according to the decision rule, A_1 *will* be chosen. Whether or not such behaviour can be defended as rational will be discussed in Section V.

V. THE POSITIVE AND NORMATIVE STATUS OF REGRET THEORY

The experimental results published by Kahneman and Tversky, wide-ranging though they are, form only a small fraction of the evidence accumulated in the past 30 years to show consistent and repeated violations of certain axioms of expected utility theory. Regret theory is one of a number of alternative theories that have been proposed in the light of this evidence; other theories have been presented by, for example, Allais (1953), Kahneman and Tversky (1979), Fishburn (1981) and Machina (1982). We shall shortly compare our theory with these others, but first let us discuss a possible argument against regret theory.

It might be objected that regret theory is limited to cases where probabilities are known, and that it rests on assumptions about non-observable functions, whereas expected utility theory is built on clear behavioural axioms which make it possible, in principle, to construct a series of choice problems which will reveal the individual's von Neumann–Morgenstern utility function.

While we do not share the methodological position that the only satisfactory theories are those formulated entirely in terms of empirical propositions, we would point out that if an individual behaves according to our model, it is possible in principle to infer from observations of his choices: his subjective probabilities; his $C(.)$ function (unique up to a positive linear transformation); and his $Q(.)$ function (which, for any given transformation of $C(.)$, will be unique up to a positive linear transformation with a fixed point at the origin). Thus each of the assumptions about $C(.)$ and $Q(.)$ required to generate our predictions is in principle capable of empirical refutation. (For an outline of the procedures involved, see the Appendix.)

The other criteria that are commonly used to evaluate positive theories are predictive power, simplicity and generality. Regret theory yields a wide range of firm predictions that are supported by experimental evidence, and it does so on the basis of a remarkably simple structure. Only the two functions $C(.)$ and $Q(.)$ are required. As far as $C(.)$ is concerned, some of the most important predictions of our model – the common ratio effect, the common consequences effect, their reverses, and the isolation effect – require only that this function is monotonically increasing; the additional assumption of linearity yields clear predictions concerning the reflection effect and simultaneous gambling and insurance. In generating all these predictions, the other crucial assumption is simply that $Q(\xi)$ is convex for all $\xi > 0$.

Thus in comparison with Kahneman and Tversky's 'prospect theory' – which is also consistent with all the evidence in Table 1 – regret theory is very simple indeed. Kahneman and Tversky's theory superimposes on expected utility theory a theory of systematic violations. Among their many assumptions are: (*i*) the rounding of probabilities up or down, and the complete editing out of 'small' probabilities; (*ii*) a 'decision weight function' which overweights small probabilities, underweights large probabilities, involves 'subcertainty', 'subproportionality' and 'subadditivity', and which is discontinuous at both ends, thus implying certain 'quantal effects'; and (*iii*) a 'value function' (essentially a utility function) which *must* have at least one point of inflection (at the individual's 'reference point' – which may or may not move around) but which can, if required, have no less than five points of inflection. We believe that against the complex and somewhat ad hoc array of assumptions required by prospect theory the principle of Occam's Razor strongly favours the straightforwardness of regret theory.

Allais's and Machina's theories are considerably simpler than prospect theory, but they cannot explain all of the evidence in Table 1. Both of these theories assume that the individual has a preference ordering over prospects. Thus two of the fundamental principles of expected utility theory are retained: that pairwise choices are transitive and that courses of action associated with identical probability distributions of consequences are equivalent to one another. (We shall call this latter principle the *equivalence axiom.*) Allais and Machina break away from expected utility theory by dropping the independence axiom; given that the equivalence axiom is retained, this amounts to abandoning the sure-thing principle. Our strategy is radically different: we retain the sure-thing principle while jettisoning both the equivalence axiom and the transitivity axiom. As a result we are able to predict the isolation effect in two-stage gambles, a form of observed behaviour that contravenes the equivalence axiom and therefore cannot be explained by either Allais or Machina. We are also able to predict the systematic occurrence of the reflection effect. Although Allais's and Machina's theories are not contradicted by the reflection effect, they do not predict it.

Fishburn's model is more like regret theory (although he does not mention any notion of regret) in that he also drops the transitivity axiom. However, his model is presented in terms of prospects rather than actions, and therefore does not accommodate the isolation effect. On the other hand, if we restrict ourselves to statistically independent prospects (and Fishburn does so – see his p. 9), then our theory and his basic axioms are compatible, and provide an interesting example of how an axiomatic treatment and a more introspective psychologically-based approach may complement each other.[1]

However, having indicated that our theory provides certain predictions and explanations that the other theories mentioned do not, we should make it clear that we are not claiming that regret theory can explain *all* of the behavioural regularities revealed by experimental research into choice under uncertainty. So far we have focused on a number of patterns of behaviour observed by Kahneman and Tversky; but we have not dealt with every one of their observations, still less with the vast amount of evidence accumulated by other researchers.

Some of the experimental findings do not appear to be completely consistent. In relation to this paper, the most significant case concerns the reflection effect. Hershey and Schoemaker (1980*a*) and Payne *et al.* (1980) have published results that show this effect to be not nearly as strong or as general as Kahneman and Tversky's evidence suggests. However, this may not present any great difficulties for regret theory since, as we noted in Section III (*d*), the *general* prediction of the reflection effect requires $C(.)$ to be linear. Instances in which the reflection effect is weak or absent may well be explicable if $C(.)$ is assumed to be concave.

There are nevertheless certain observations that simply cannot be accounted for by regret theory in the form presented here. One example is the 'framing' effect discussed by Tversky and Kahneman (1981) and the very similar 'context' effect observed by Hershey and Schoemaker (1980*b*). In these cases exactly the

[1] At a late stage, we have received a copy of a Working Paper by David E. Bell (1981) which is of great interest. Quite independently he has developed a model which also explicitly incorporates a notion of regret, using multi-attribute utility theory along the lines suggested by Keeney and Raiffa (1976). We note that when both models are applied to the same phenomena – the original Allais paradox, simultaneous insuring and gambling, and the reflection effect – the conclusions are strikingly similar.

same choice problem – that is, exactly the same when formulated in terms of a matrix of state-contingent consequences – receives markedly different responses, depending on the way the choice is presented. Another example is the 'translation' effect observed by Payne *et al.* (1980). This effect occurs when an individual prefers one prospect to another, but reverses his preference when the same sum of money is deducted from every consequence of both prospects. The observed pattern of reversal is not predicted by regret theory. Finally, systematic violations of the sure-thing principle have been observed (cf. Moskowitz (1974); Slovic and Tversky (1974)). And although there is some evidence that individuals violate the sure-thing principle much less often than they violate some other axioms (Tversky and Kahneman (1981, footnote 15)), as it stands our theory does not explain that behaviour.

On the other hand, there is some additional evidence that gives further support to regret theory. A particular instance is the form of 'preference reversal' observed by Lindman (1971) and Lichtenstein and Slovic (1971, 1973) and subsequently confirmed, after rigorous testing, by Grether and Plott (1979). This preference reversal occurs when an individual, faced with a pairwise choice between gambles A and B, chooses A; but when asked to consider the two gambles separately, places a higher certainty equivalent value on B. We have shown elsewhere (Loomes and Sugden (1982)) that the most commonly observed reversal pattern is predicted by regret theory even in its restricted form.

Of course, we acknowledge that there is no simple theory that gives a unified explanation of all the experimental evidence, and regret theory is no exception in this respect. But we have tried to construct a theory that explains as much of the evidence as possible on the basis of very few assumptions. We do not believe that choiceless utility and regret are the only factors that influence behaviour under uncertainty, but just that these two factors seem to be particularly significant. Indeed, we have become increasingly convinced by evidence of framing, context and translation effects that the notion of reference points deserves further consideration, although we have not tried to deal with that issue in this paper.

In constructing our theory we have avoided any assumptions of misperceptions or miscalculations by individuals. We do not doubt that in reality misperceptions and miscalculations occur, and sometimes in systematic rather than random ways. Nonetheless, our inclination as economists is to explain as much human behaviour as we can in terms of assumptions about rational and undeceived individuals. Thus we believe that regret theory does more than predict certain systematic violations of conventional expected utility theory: it indicates that such behaviour is not, in any meaningful sense of the word, irrational.

In claiming this we are breaking the terms of a truce that many theorists (with the notable exception of Allais) have tacitly accepted. Proponents of expected utility theory often concede that their theory has serious limitations as a predictive device but insist that its axioms have strong normative appeal as principles of rational choice. Thus Morgenstern (1979, p. 180) argues for expected utility theory on the grounds that 'if people deviate from the theory, an explanation of the theory and of their deviation will cause them to re-adjust their behaviour'. Similarly, Savage (1954, pp. 102–3) admits that when confronted with a pair of choice problems rather like Problems 1 and 2, he behaved in accordance with the common consequences effect and in violation of his own axioms. But, he says, he was able to convince himself that this behaviour was mistaken (though even after realising his 'mistake' he continued to feel an 'intuitive attraction' to that behaviour). At the other side of the truce, proponents of alternative theories have often been willing to accept these claims. Kahneman and Tversky (1979, p. 277) maintain that the departures from expected utility theory that prospect theory describes 'must lead to normatively unacceptable consequences' which a decision-maker would, if he realised the error of his ways, wish to correct. Similarly, Machina (1982, p. 277) notes the 'normative appeal' of the axioms of expected utility theory before going on to propose a positive theory that dispenses with one of these axioms.

However, we shall challenge the idea that the conventional axioms constitute the only acceptable basis for rational choice under uncertainty. We shall argue that it is no less rational to act in accordance with regret theory, and that conventional expected utility theory therefore represents an unnecessarily restrictive notion of rationality.

Regret theory rests on two fundamental assumptions: first, that many people experience the sensations we call regret and rejoicing; and second, that in making decisions under uncertainty, they try to anticipate and take account of those sensations.

In relation to the first assumption, it seems to us that psychological experiences of regret and rejoicing cannot properly be described in terms of the concept of rationality: a choice may be rational or irrational, but an experience is just an experience. As far as the second assumption is concerned, if an individual does experience such feelings, we cannot see how he can be deemed irrational for consistently taking those feelings into account.

We do not claim that acting according to our theory is the *only* rational way to behave. Nor do we suggest that all individuals who act according to our theory must violate the conventional axioms. Some individuals may experience no regret or rejoicing at all, while some others may have linear $Q(.)$ functions: in these special cases of our theory, we would predict that the individual's behaviour would conform with all the conventional axioms.

On the other hand, individuals with non-linear $Q(.)$ functions of the kind described in this paper may consistently and knowingly violate the axioms of transitivity and equivalence without ever accepting, even after the most careful reflection, that they have made a mistake. So these axioms do not necessarily have the self-evident or overwhelming normative appeal that many theorists suppose. We shall now try to show why we do not accept the idea that the transitivity and equivalence axioms are necessary conditions for rational choice under uncertainty.

Underlying those two axioms is a common idea: that the value placed on any action A_i depends only on the interaction between, on the one hand, the probability-weighted consequences offered by A_i and, on the other hand, the individual's pattern of tastes, including his attitude to risk.

That is what is symbolised when, for any individual, an expected utility number is assigned to an action, that expected utility number being quite independent of the range and nature of the available alternative actions. From this idea, that there is some value in 'having A_i' which is quite independent of the value of 'having A_k', and that if 'having A_i' gives more value than 'having A_k' then $A_i \succ A_k$, it follows that there must exist a complete and transitive preference ordering over all actions.

It also follows that the particular state pattern of consequences is of no special significance: if each action is evaluated independently, it does not matter how the consequence of that action under any state of the world compares with the consequence(s) of any other action(s) under the same state. Thus only the probability distribution of consequences matters, and all actions, simple or complex, which share the same probability distribution will be assigned the same expected utility number and must be regarded as equivalent for the purposes of choice decisions.

But if people experience regret and rejoicing, these arguments are illegitimate. In regret theory the proposition $A_i \succcurlyeq A_k$ cannot be read as 'having A_i is at least as preferred as having A_k'; it should rather be read as 'choosing A_i and simultaneously rejecting A_k is at least as preferred as choosing A_k and simultaneously rejecting A_i'. Thus the transitivity of the relation 'is at least as preferred as' (which we do not dispute) does not entail the transitivity of our relation $\succcurlyeq$; and so non-transitive choices do not indicate any logical inconsistency on the part of the decision-maker.

The idea that non-transitive choices are irrational is sometimes argued as follows. Suppose (as in the example discussed in connection with Table 4 in Section IV) that there are three actions A_1, A_2, A_3, such that $A_1 \succ A_2$, $A_2 \succ A_3$, and $A_3 \succ A_1$. Then, it is said, no choice can be made from the set $\{A_1, A_2, A_3\}$ without there being an inconsistency with one of the original preference statements: whichever action is chosen, another is preferred to it (cf. MacKay (1980, p. 90)). The principle that is being invoked here is Chernoff's axiom: if A_i is chosen from some set S, and if S' is a subset of S that contains A_i, then A_i must be chosen from S'. But we suggest the appeal of this axiom derives from the supposition that the value of choosing an action is independent of the nature and combination of the actions simultaneously rejected; and regret theory does not accept this supposition. Since $A_1 \succ A_2$ means only that choosing A_1 from the set $\{A_1, A_2\}$ is preferred to choosing A_2 from the set $\{A_1, A_2\}$ there is no implication that choosing A_1 from the set $\{A_1, A_2, A_3\}$ is preferred to choosing A_2 from the set $\{A_1, A_2, A_3\}$. A similar argument applies to the example discussed in connection with Table 5 in Section IV, where (despite the fact that the relation $\succcurlyeq$ happens to be transitive) there is another violation of Chernoff's axiom.

A second common objection to non-transitivity runs like this. If someone prefers A_1 to A_2, A_2 to A_3, and A_3 to A_1, every one of the actions is less preferred than another; so might he not get locked into an endless chain of choice in which he can never settle on any one action? Worse, might not a skilful bookmaker capture all his wealth by confronting him with a suitably constructed sequence of pairwise choices? But these objections rest on a fallacy. To suppose that the individual can get locked into a cycle of choices, it is necessary to suppose that all three actions are feasible. But if this is indeed the case, then propositions about pairwise choices – about how choices are made when there are only two feasible actions – are not relevant. The bookmaker can bankrupt his client only if he can successively persuade him to believe in each of a long chain of mutually inconsistent propositions about the feasible set.

Finally, there is no reason why the equivalence axiom should be regarded as a necessary condition for rational choice, even when the choice is between two simple actions with identical probability distributions of consequences. Consider

Table 6

Action	0·25	0·25	0·25	0·25
A_i	3	2	1	0
A_k	0	3	2	1

A_i and A_k in Table 6. If each action were evaluated independently, there would be no grounds for preferring 'having A_i' to 'having A_k', or vice versa. But in our model the decision is between 'choosing A_i and simultaneously rejecting A_k' and 'choosing A_k and simultaneously rejecting A_i'. These two alternatives are associated with different probability mixes of regret and rejoicing. (In terms of our theory, to choose A_i and reject A_k is to incur a 0·25 probability of $R(+3)$ and a 0·75 probability of $R(-1)$, while to choose A_k and reject A_i is to incur a 0·25 probability of $R(-3)$ and a 0·75 probability of $R(+1)$.) So for an individual who experiences regret and rejoicing, the two courses of action cannot be regarded as identical. It would therefore not be unreasonable for such an individual to prefer one to the other.

VI. CONCLUSION

The evidence presented by Kahneman and Tversky and many others points to a number of cases where commonly observed patterns of choice violate conventional expected utility axioms. The fact that these violations are neither small-scale nor randomly distributed may indicate that there are some important factors affecting many people's choices which have been overlooked or misspecified by conventional theory.

We suggest that one significant factor is an individual's capacity to anticipate feelings of regret and rejoicing. We therefore offer an alternative model which takes those feelings into consideration. This model yields a range of predictions consistent with the behaviour listed in Table 1 and provides an account of these and other choice phenomena which conventional theory has so far failed to explain.

That is the positive side of regret theory. But we believe that our approach also has strong normative implications. We have argued that our theory describes a form of behaviour which, although contravening the axioms of expected utility theory, is rational. Thus, while we do not suggest that behaving according to those conventional axioms is irrational, we *do* suggest that those axioms constitute an excessively restrictive definition of rational behaviour.

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Appendix: Inferring subjective probabilities and $C(.)$ and $Q(.)$ functions from choices

The following procedure will reveal, for any individual, which of two events has the higher subjective probability. Let S_1 and S_2 be any two non-intersecting and non-empty events (i.e. sets of states of the world). Let S_3 be the event that comprises all those states of the world not in S_1 or S_2. Let x, y, z be any three consequences such that the person in question prefers x to y (under certainty). Consider the two actions $A_i = (x, y, z)$ and $A_k = (y, x, z)$, which are defined in relation to the events S_1, S_2, S_3. It then follows from the separability principle (see Section III) that A_i is preferred to, indifferent to, or less preferred than A_k as the subjective probability of S_1 is greater than, equal to, or less than that of S_2. This procedure is broadly similar to the one proposed by Savage (1954) for inferring subjective probabilities for individuals who behave according to his postulates.

The restricted form of our theory (see Section III) uses two functions for the analysis of modified utility: $C(.)$ and $Q(.)$. $C(.)$ can be identified, up to a positive linear transformation, by confronting the individual with choices involving 50–50 gambles. Consider any two prospects of the form $X_i = (x_1, 1)$, $X_k = (x_2, 0{\cdot}5; x_3, 0{\cdot}5)$ where $x_3 > x_1 > x_2$, so that the corresponding choiceless utility indices are $c_3 > c_1 > c_2$. Then:

$$X_i \begin{smallmatrix}\prec\\\sim\\\succ\end{smallmatrix} X_k \text{ iff } 0{\cdot}5\,Q(c_1 - c_2) - 0{\cdot}5\,Q(c_3 - c_1) \lesseqqgtr 0.$$

But since $Q(.)$ is increasing, it follows that:

$$X_i \begin{smallmatrix}\prec\\\sim\\\succ\end{smallmatrix} X_k \text{ iff } 0{\cdot}5\,(c_1 - c_2) - 0{\cdot}5\,(c_3 - c_1) \lesseqqgtr 0.$$

Thus in this case, the individual chooses *as though* maximising expected *choiceless* utility. So $C(.)$ can be identified from experiments in much the same way as von Neumann–Morgenstern utility functions are identified.

If $C(.)$ is known, and if a particular transformation has been chosen, it is possible to define consequences in terms of their choiceless utilities. Let x_1 and x_2 be consequences such that $c_1 = 0$ and $c_2 = -1$. Let x_3 be any consequence such that $c_3 = \xi$ where $\xi > 0$ and $\xi \neq 1$. Consider the two prospects $X_i = (x_1, 1)$ and $X_h = (x_2, p; x_3, 1 - p)$. Then:

$$X_i \begin{smallmatrix}\succ\\\sim\\\prec\end{smallmatrix} X_h \text{ iff } \frac{Q(\xi)}{Q(1)} \lesseqqgtr \frac{p}{1-p}.$$

Thus if one can find a value of p such that the individual is indifferent between X_i and X_h it is possible to infer the value of $Q(\xi)/Q(1)$. So if $Q(1)$ is set equal to any arbitrary positive value, the value of $Q(\xi)$ can then be determined by experiment for all $\xi > 0$; hence the concavity, convexity or linearity of $Q(.)$ over any interval can be established.

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