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PS 2 | **Stat231**

Contents

1 General Instructions

Answer the following:

2.14 (a) Let X be a continuous, nonnegative random variable [$f(x) = 0$ for $x < 0$]. Show that,

$$\mathrm{E}X = \int_0^\infty [1 - F_X(x)] dx$$

where $F_X(x)$ is the cdf of X.

(b) Let X be a discrete random variable whose range is the nonnegative integers. Show that

$$\mathrm{E}X = \sum_{k=0}^{\infty} (1 - F_X(x))$$

where $F_X(k) = P(X \leq k)$. Compare this with part (a).

2 Solutions

2.14

$$\begin{aligned} \mathrm{E}X &= \int_0^\infty [1 - F_X(x)]\, dx = \int_0^\infty P(X > x)\, dx \\ &= \int_0^\infty \int_x^\infty f_X(t)\, dt\, dx = \iint\limits_D f_X(t)\, \mathrm{d}t\, \mathrm{d}x \end{aligned}$$

3.11 (a) Given

$$P(X = x | N, M, K) = \frac{\binom{M}{x}\binom{N-M}{K-x}}{\binom{N}{K}},\ x = 0, 1, \cdots, K.$$

Note that, $\binom{N}{K} = \frac{N!}{k!(N-K)!}$, then

$$\lim_{M/N \to p, M \to \infty, N \to \infty} \frac{\binom{M}{x}\binom{N-M}{K-x}}{\binom{N}{K}}$$

$$\lim_{M/N \to p, M \to \infty, N \to \infty} \frac{\frac{M!}{x!(M-x)!} \cdot \frac{(N-M)!}{(K-x)!(N-M-K+x)!}}{\frac{N!}{K!(N-K)!}}$$