Lessons in Maths Olympiads

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1 Introduction

This document aims to summarise all of the lessons that I've learnt and the mistakes I've made during my preparation for the 2025 IMO.

2 Lessons

2.1 Hands Dirty

This is perhaps the most important lesson that any aspiring Maths Olympians are taught as they are starting out, so it's only fitting that it is placed as the first lesson here too.

2.2 Wishful Thinking

"Ohhhh... wouldn't it be SOOOOOO nice if this was true?"

Often times it is beneficial to employ this technique to look "into the future", to try and find conjectures that we wish to proof that would further our investigation.

This comes in many forms: Hoping that four points are cyclic, this sequence is bounded, the function is injective ...

Let's look at an example.

Example 2.1 – 2021 ISL N1 Find all positive integers $n \geq 1$ such that there exists a pair (a, b) of positive integers, such that $a^2 + b + 3$ is not divisible by the cube of any prime, and

$$n = \frac{ab + 3b + 8}{a^2 + b + 3}.$$

My first thought was to write $a^2+b+3 \mid ab+3b+8$, and start cancelling terms. This was certainly the right idea, however without the following motivation, I would be running into deadends left right and center not knowing what I should cancel.

"Huh.. the degree of b on both parts of the fraction is 1, so wouldn't it be nice if I had a fraction in just a?"

With this motivation we do the following:

$$
\begin{aligned}
&a^2+b+3 \mid b(a+3)+8 \\
&a^2+b+3 \mid b(a+3)+8-(a+3)(a^2+b+3) \\
&a^2+b+3 \mid b(a+3)+8-a^2(a+3)-b(a+3)-3(a+3) \\
&a^2+b+3 \mid -(a+1)^3
\end{aligned}
$$

Which is actually really nice, as we know a^2+b+3 is cube free. So hence we have $a^2+b+3 \mid (a+1)^2$. But $a^2+2b+5 > 2a$ for any positive integers a, which gives $2(a^2+b+3) > (a+1)^2$. therefore we must have $(a+1)^2 = a^2+b+3 \iff a = 2b-2$. We can now sub back into the original equation to get $n = 2$ is the only solution, and we're done.

My thoughts: I would like to think that after we realise the nice cancellation of b, the remaining steps of the problem are quite routine. This ties nicely to Section 3.2.

2.3 Visualisation

This will come pretty naturally in most combinatorics questions, however we are not restricted to just that. As a learn-by-picture guy myself I often times find turning things such as sequences into something that I can see in my head helps me solve problems much faster.

Example 2.2 – ISL 2014 A1 Let $a_0 < a_1 < a_2 \ldots$ be an infinite sequence of positive integers. Prove that there exists a unique integer $n \geq 1$ such that

$$a_n < \frac{a_0+a_1+a_2+\cdots+a_n}{n} \leq a_{n+1}.$$

Proposed by Gerhard Wöginger, Austria.

3 Reflections

Unfortunately this is another lesson that I learnt way too late. After finishing a problem, the reflections on my motivation, ideas, and key steps are almost as important if not more important than learning the solution to a problem itself.

3.1 Think about your motivations

3.2 Notice the Key Step