Statistical Inference Project1

ALT-Data

Question

1. In this project you will investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is 1/lambda and the standard deviation is also 1/lambda. Set lambda = 0.2 for all of the simulations. You will investigate the distribution of averages of 40 exponentials. Note that you will need to do a thousand simulations."   "Show the sample mean and compare it to the theoretical mean of the distribution.
2. Show how variable the sample is (via variance) and compare it to the theoretical variance of the distribution.
3. Show that the distribution is approximately normal.

Preparation/seed values

```
# set seed for reproducability
set.seed(9999901)


lambda <- 0.4


n <- 40


simulations <- 1000


simulated_exponentials <- replicate(simulations, rexp(n, lambda))

# calculate mean of exponentials
means_exponentials <- apply(simulated_exponentials, 2, mean)
```

Question 1

Show where the distribution is centered at and compare it to the theoretical center of the distribution.

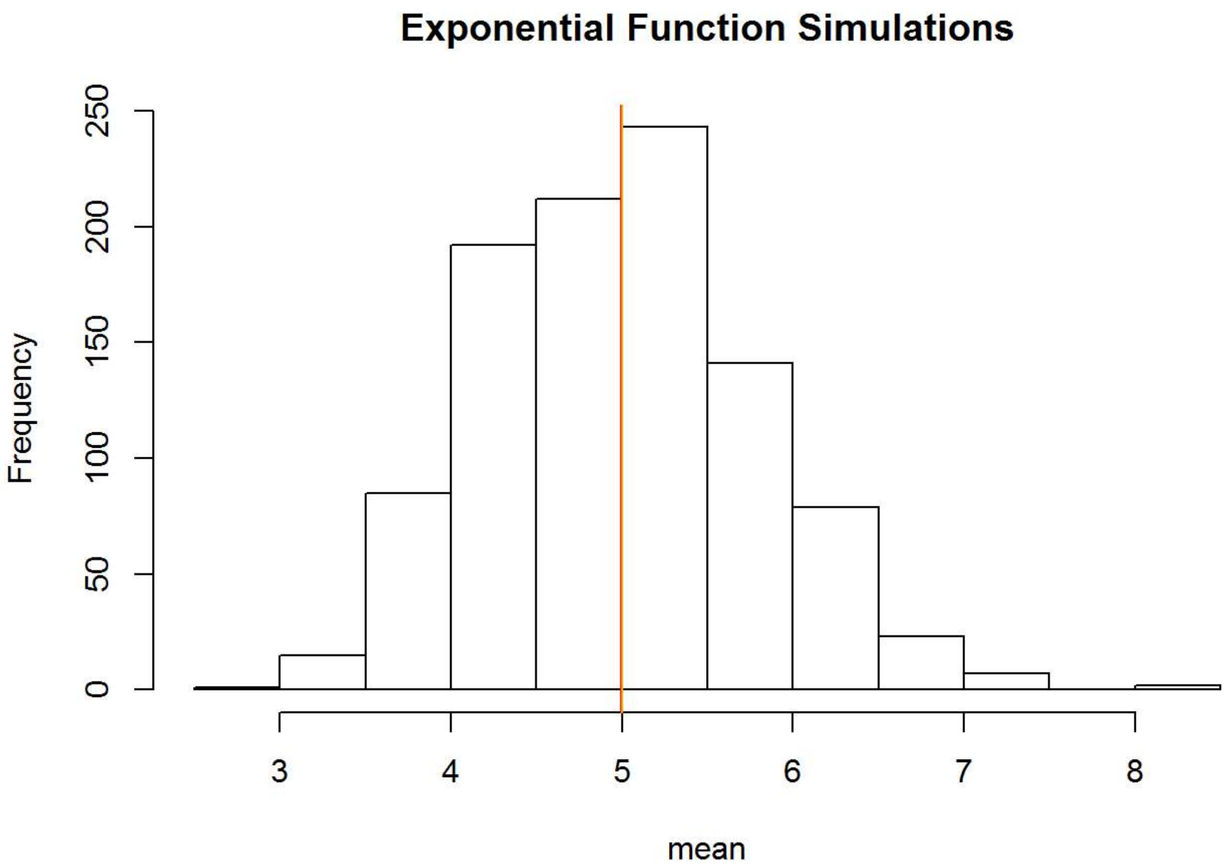
```
analytical_mean <- mean(means_exponentials)
analytical_mean
```

```
## [1] 4.993867
```

```
# analytical mean
theory_mean <- 1/lambda
theory_mean
```

```
## [1] 5
```

```
# visualization
hist(means_exponentials, xlab = "mean", main = "Exponential Function Simulations")
abline(v = analytical_mean, col = "red")
abline(v = theory_mean, col = "orange")
```



The analytics mean = 4.99 while theoretical mean = 5. The center of distribution of avg 40 exponentials is very close to the theoretical.

Question 2

Show how variable it is and compare it to the theoretical variance of the distribution..

```
# standard deviation of distribution
standard_deviation_dist <- sd(means_exponentials)
standard_deviation_dist
```

```
## [1] 0.7931608
```

```
# standard deviation from analytical expression
standard_deviation_theory <- (1/lambda)/sqrt(n)
standard_deviation_theory
```

```
## [1] 0.7905694
```

```
# variance of distribution
variance_dist <- standard_deviation_dist^2
variance_dist
```

```
## [1] 0.6291041
```

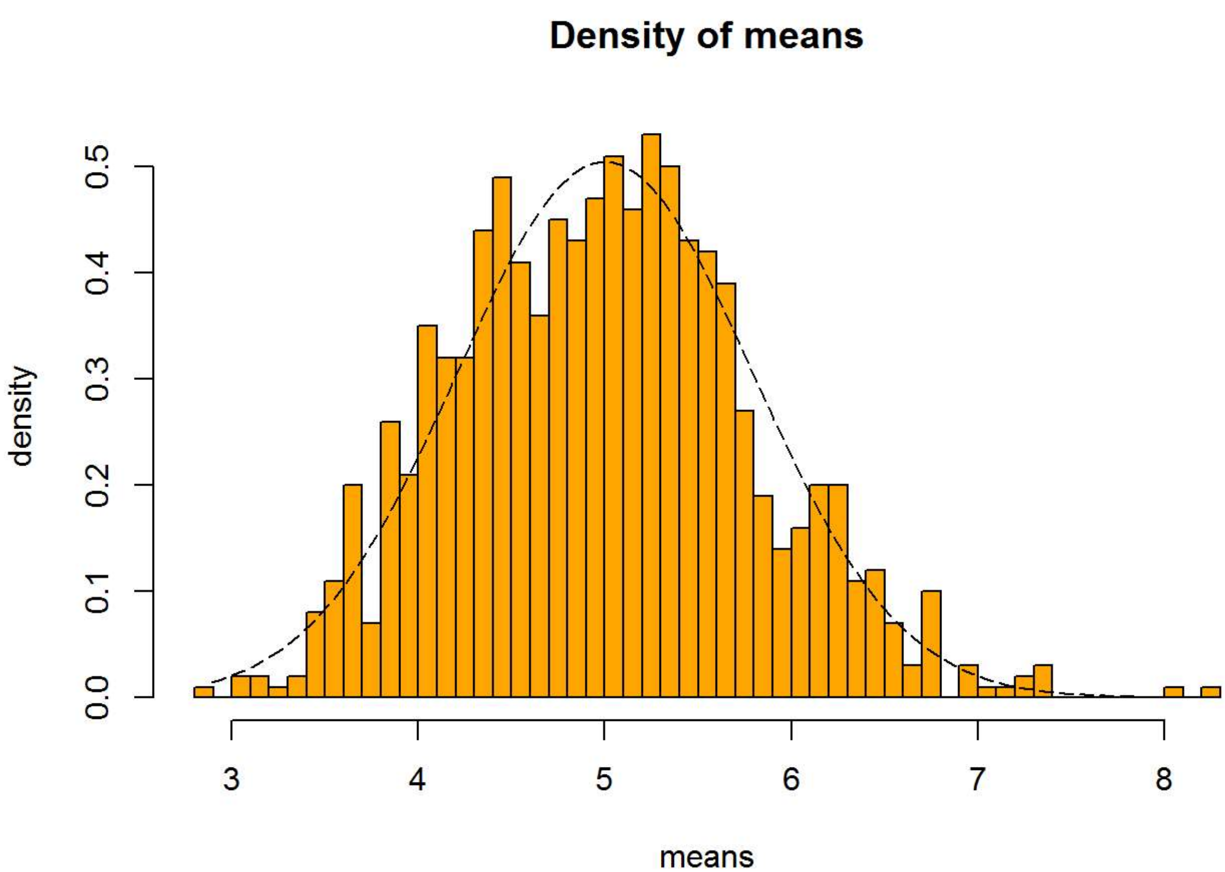
```
# variance from analytical expression
variance_theory <- ((1/lambda)*(1/sqrt(n)))^2
variance_theory
```

```
## [1] 0.625
```

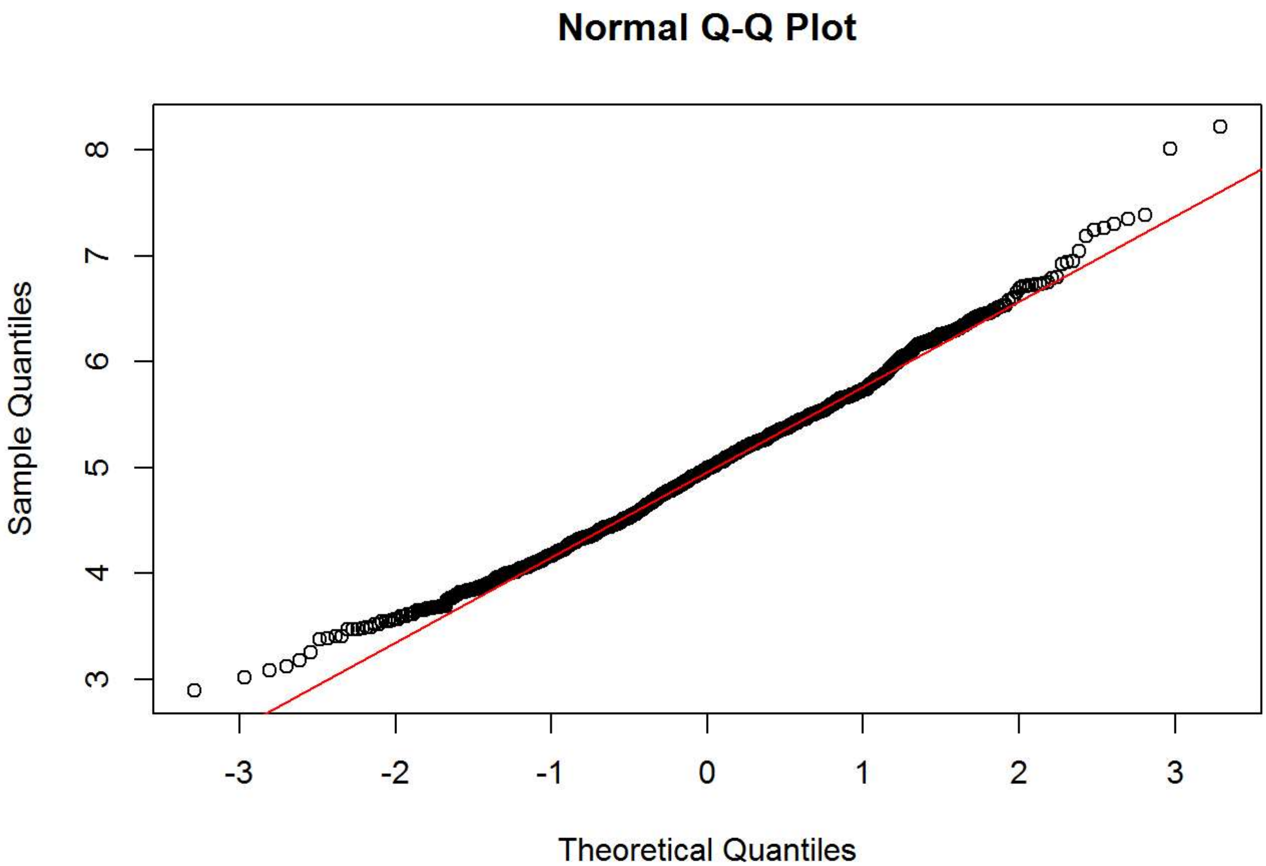
Question 3

Show that the distribution is approximately normal.

```
xfit <- seq(min(means_exponentials), max(means_exponentials), length=100)
yfit <- dnorm(xfit, mean=1/lambda, sd=(1/lambda/sqrt(n)))
hist(means_exponentials,breaks=n,prob=T,col="orange",xlab = "means",main="Density of means",ylab="density")
lines(xfit, yfit, pch=22, col="black", lty=5)
```




```
# compare the distribution of averages of 40 exponentials to a normal distribution
qqnorm(means_exponentials)
qqline(means_exponentials, col = 2)
```


Normal Q-Q Plot
Sample Quantiles
3
4
5
6
7
8
-3
-2
-1
0
1
2
3
Theoretical Quantiles