UFMG

Magic Squares

João Antonio Oliveira Pedrosa
31/10/2019

To construct the square, three different algorithms are used, depending on the input value:

1. $2n + 1 \mid n \in \mathbb{Z}$.
2. $4n + 2 \mid n \in \mathbb{Z}$.
3. $4n \mid n \in \mathbb{Z}$.

The algorithms works as follows:

1. **The Siamese Method:**

➔ Let the square be initialized with the number 1 on the middle row and in the rightiest column.

➔ Place the successor of the current number in the column to the right and in the above row. (If the current column is already as far right as possible, the right column becomes the leftiest column of the matrix. If the current row is already as high as possible, the line above becomes the lowest line of the matrix.)

➔ If the desired box is already occupied, place the successor in the box to the left of the current number.

➔ Repeat the process until every number is in it's place.

2. **Simple pair squares:**

➔ Let the square be divided into four quadrants. (Image 1).

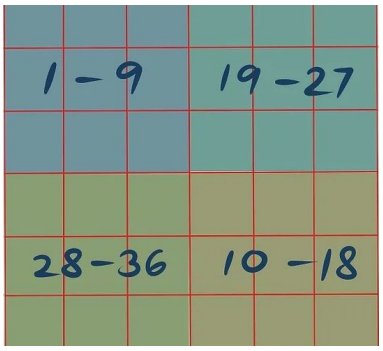

Image 1 - Square Quadrants.

➔ The algorithm will do the Siamese method for each of these squares. The first contains the numbers from 1 to $\frac{n}{4}$, the second contains from $\frac{n}{2}$ to $\frac{3n}{4}$, the third contains from $\frac{3n}{4}$ to n, and the fourth contains from $\frac{n}{4}$ to $\frac{n}{2}$.

➔ The first half of the columns in the first quadrant should be replaced with the first half of the columns in the third quadrant. The central box of the quadrants should also be changed and the box in the middle row of each quadrant and in the leftiest column should not be changed. (Image 2).

➔ If the square has a side greater than 6, the last half minus one of the columns in quadrant 2 should be replaced with those in quadrant 4. (Image 2).

30	39	48	1	10	19	28	128	137	146	99	108	117	126
38	47	7	9	18	27	29	136	145	105	107	116	125	127
46	6	8	17	26	35	37	144	104	106	115	124	133	135
5	14	16	25	34	36	45	103	112	114	123	132	134	143
13	15	24	33	42	44	4	111	113	122	131	140	142	102
21	23	32	41	43	3	12	119	121	130	139	141	101	110
22	31	40	49	2	11	20	120	129	138	147	100	109	118
177	186	195	148	157	166	175	79	88	97	50	59	68	77
185	194	154	156	165	174	176	87	96	56	58	67	76	78
193	153	155	164	173	182	184	95	55	57	66	75	84	86
152	161	163	172	181	183	192	54	63	65	74	83	85	94
160	162	171	180	189	191	151	62	64	73	82	91	93	53
168	170	179	188	190	150	159	70	72	81	90	92	52	61
169	178	187	196	149	158	167	71	80	89	98	51	60	69

Image 2 - Changes to be made.

3. **Double pair squares:**

➔ First, four quadrants are selected at the corners of the square. Each quadrant has side $\frac{n}{4}$.

➔ After that, a fifth quadrant is selected, in the exact middle of the square, with side $\frac{n}{2}$ (Image 3).

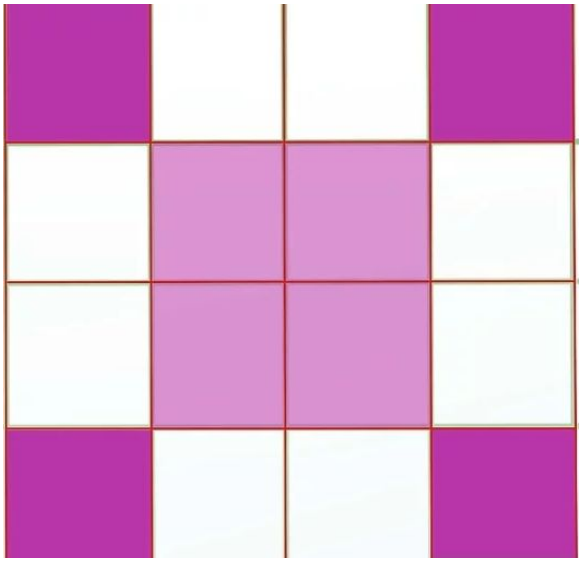

Image 3 - 4x4 square example.

➔ The square is filled in order from 1 to n², however, all positions not in one of the 5 quadrants should be ignored.

➔ Then the remainder of the square is filled in reverse order (highest to lowest), but only in the missing positions (Image 4).

1	2	62	61	60	59	7	8
9	10	54	53	52	51	15	16
48	47	19	20	21	22	42	41
40	39	27	28	29	30	34	33
32	31	35	36	37	38	26	25
24	23	43	44	45	46	18	17
49	50	14	13	12	11	55	56
57	58	6	5	4	3	63	64

Image 4 - 8x8 square example.

In all cases, the algorithm complexity is $O(n^2)$.

References

WIKIHOW. **How to Solve a Magic Square.** <https://www.wikihow.com/Solve-a-Magic-Square>. Access in Oct 31, 2019.

GEEKS FOR GEEKS. **Magic Square.** <https://www.geeksforgeeks.org/magic-square/>. Access in Oct 31, 2019.