

Type-checking Linearity in Core: Semantic Linearity for a Lazy Optimising Compiler

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Linear Haskell

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bad :: Ptr  $\multimap$  IO ()
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bad x = do
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    free x
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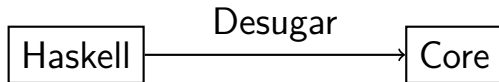
```
bad :: Ptr  $\multimap$  IO ()  
bad x = do  
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    free x
```

```
ok :: Ptr  $\multimap$  IO ()  
ok x = free x
```

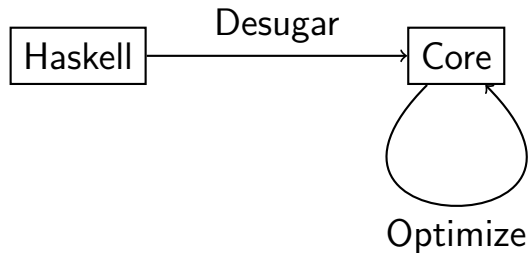
Linearity in the Glasgow Haskell Compiler (GHC)

Haskell

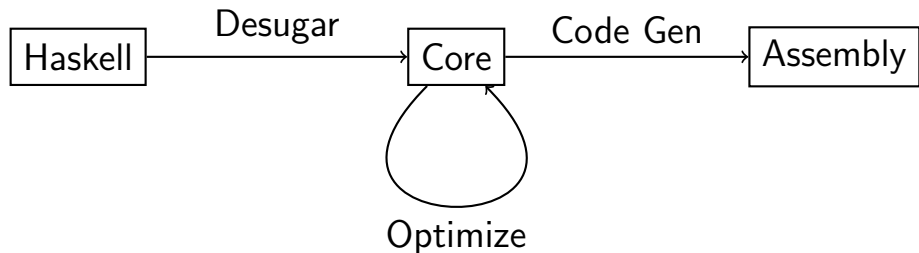
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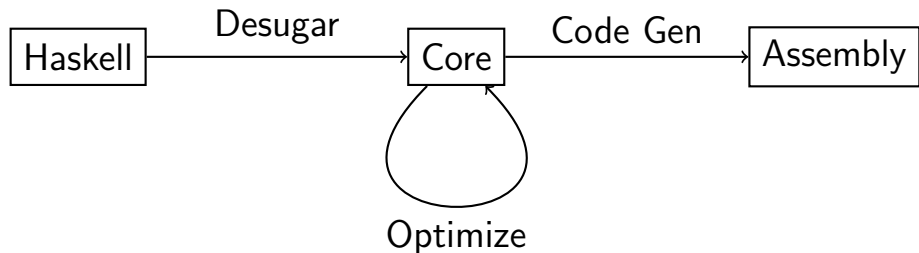
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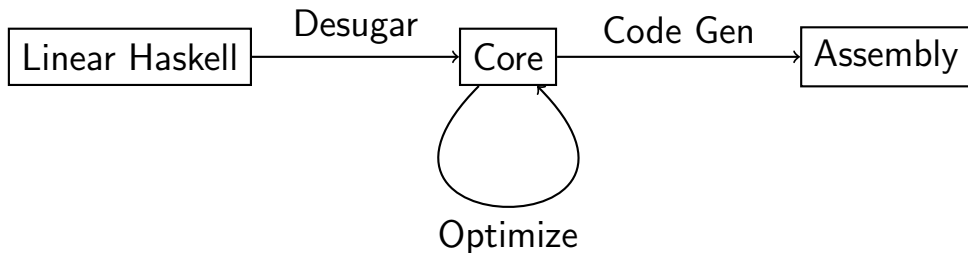


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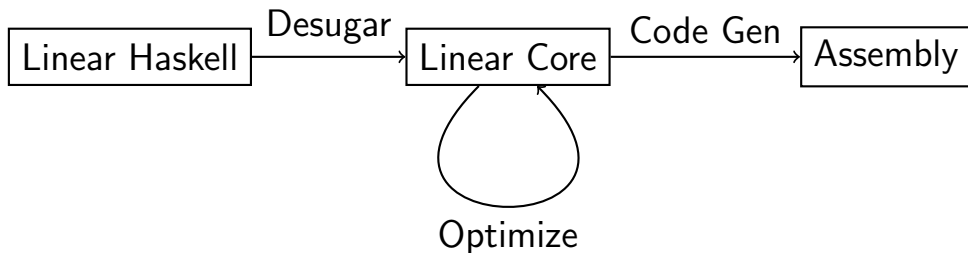
Core is both lazy and typed

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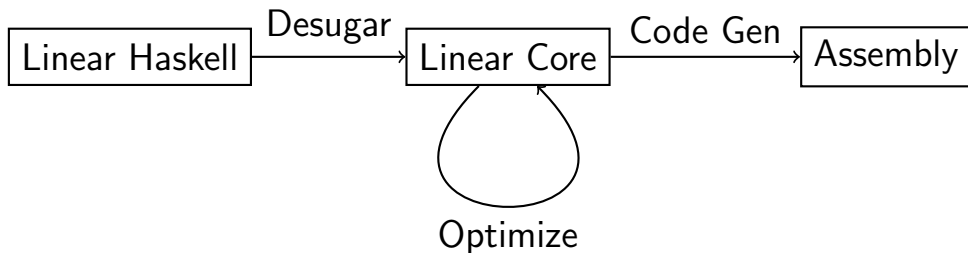
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Core *should be* both lazy and *linearly* typed

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$$\begin{array}{l} \text{let } y = \text{free } x \text{ in } y \\ \implies \\ \text{let } y = \text{free } x \text{ in } \text{free } x \end{array}$$

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Linearity is ignored in Core, or most programs would be rejected

Semantic vs Syntactic Linearity

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- **Key insight:** Under lazy evaluation,
syntactic occurrence \nRightarrow *consuming* a resource
syntactic linearity \neq *semantic* linearity
- We type *syntactic* linearity in Core, but that is not enough
- Optimisations push laziness x linearity to the limit

Our Contributions

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- We implemented Linear Core as a GHC plugin

Semantic Linearity, by example

Semantic Linearity: Lets

```
let  $y = \text{free ptr}$   
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Resources in lets are only consumed if the binder is evaluated

Semantic Linearity: Case

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 $(a, b) \rightarrow \textit{something } a \ b$

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Resources are *kind of* consumed if the expression is evaluated

Linear Core

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- Using a *let*-var equates to using its annotated context (Δ)

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Key idea: We need to branch on *WHNF-ness*

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Metatheory: Linear Core

- Not obvious whether these rules make sense together
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- We proved the system is type safe via preservation + progress
 - *Irrelevance* lemma
 - Linear-var substitution lemma
 - + substitution on case alternatives
 - Δ -var substitution lemma
 - + substitution on case alternatives
 - Unr-var substitution lemma
 - + substitution on case alternatives

Metatheory: Optimising Transformations

- Inlining
- β -reduction
- β -reduction with sharing
- β -reduction for multiplicity abstractions
- Case-of-known-constructor
- Full laziness
- Local transformations (three of them)
- η -expansion
- η -reduction
- Binder swap
- Reverse binder swap (contentious!)
- Case-of-case

GHC Plugin: Linear Core Implementation

We implemented Linear Core as a GHC plugin

Library	Total Accepted	Total Rejected	Unique Rejected	Linear modulo Call-by-name	Linear Rejected	\neg Linear Rejected	Unknown Rejected
linear-smc	19438	4	1	1	0	0	0
priority-sesh	6781	19	1	0	0	0	1
linear-base	112311	538	87	10	8	2	67

Figure: Linear Core Plugin on Linear Libraries

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- Builds on the shoulders of Linear Haskell and Linear Mini-Core
- There's much more in the thesis!

Fim

Semantic Linearity: Case of Var

$(\lambda x. \text{ case } x \text{ of } _ \rightarrow x)$

Semantic Linearity: Case of Var

$$(\lambda x. \mathbf{case} \ x \ \mathbf{of} \ _ \rightarrow x)$$

$\Longrightarrow_{\text{call by name}}$

Semantic Linearity: Case of Var

$$\begin{aligned} & (\lambda x. \mathbf{case} \ x \ \mathbf{of} \ _ \rightarrow x) \\ & \quad \Longrightarrow_{\text{call by name}} \\ & \mathbf{case} \ free \ x \ \mathbf{of} \ _ \rightarrow free \ x \end{aligned}$$

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Semantic Linearity: Case of Var

$(\lambda x. \text{ **case** } x \text{ **of** } _ \rightarrow x)$

$\implies_{\text{call by name}}$

$\text{**case** } free\ x \text{ **of** } _ \rightarrow free\ x$

$\implies_{\text{call by need}}$

$\text{let } y = free\ x \text{ in } \text{**case** } y \text{ **of** } _ \rightarrow y$

System FC

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- A coercion $\sigma_1 \sim \sigma_2$ can be used to safely *cast* an expression e of type σ_1 to type σ_2 , written $e \blacktriangleright \sigma_1 \sim \sigma_2$.

System FC

Definition (Syntax)

$u ::= x \mid K$	Variables and data constructors
$e ::= u$	Term atoms
$\quad \mid \Lambda a:\kappa. e \mid e \varphi$	Type abstraction/application
$\quad \mid \lambda x:\sigma. e \mid e_1 e_2$	Term abstraction/application
$\quad \mid \mathbf{let} \ x:\sigma = e_1 \mathbf{in} \ e_2$	
$\quad \mid \mathbf{case} \ e_1 \mathbf{of} \ \overline{p \rightarrow e_2}$	
$\quad \mid e \blacktriangleright \gamma$	Cast
$p ::= K \ \overline{b:\kappa} \ \overline{x:\sigma}$	Pattern