Type-checking Linearity in Core: Semantic Linearity for a Lazy Optimising Compiler

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Linear Haskell

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\mathit{bad} :: \mathit{Ptr} \multimap \mathit{IO}\ () \\
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A linear function $\multimap$ consumes its argument *exactly once*

Linearity in the Glasgow Haskell Compiler (GHC)

$\boxed{\text{Haskell}}$

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$$\boxed{\text{Haskell}} \xrightarrow{\text{Desugar}} \boxed{\text{Core}}$$

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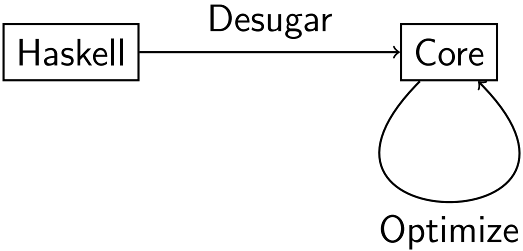

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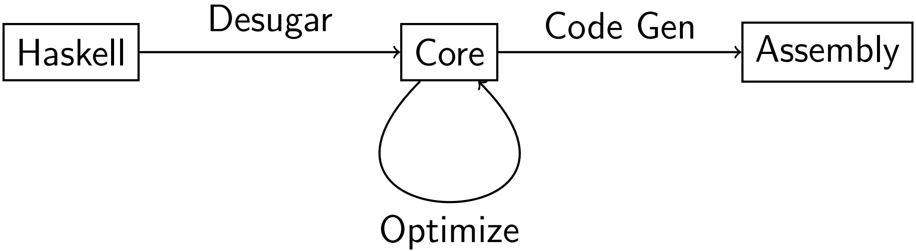

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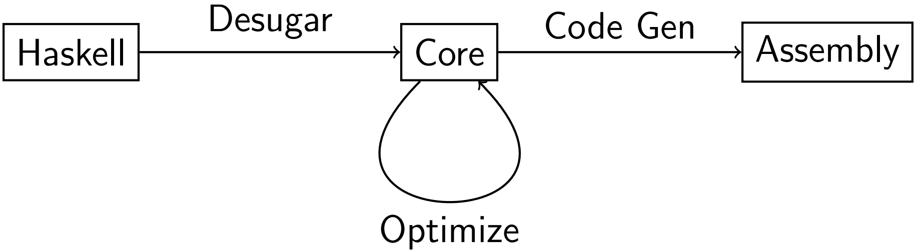

Core is both lazy and typed

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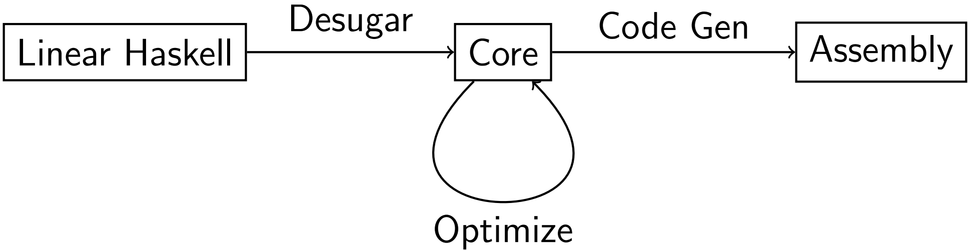

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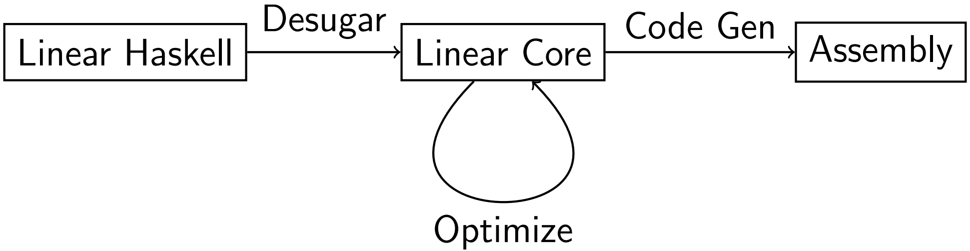

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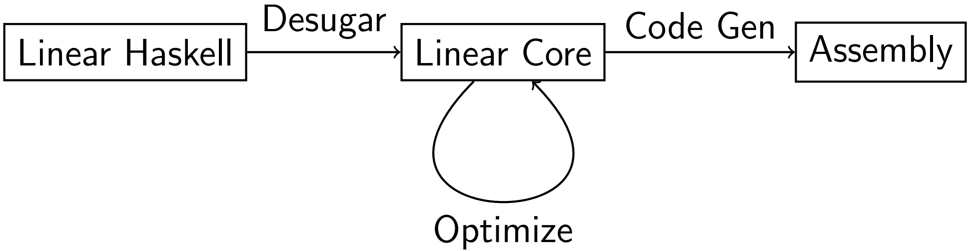

Linear Core is both lazy and *linearly* typed

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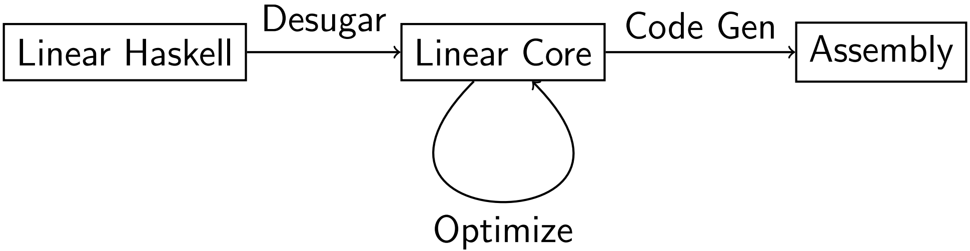

Linear Core *should be* both lazy and *linearly* typed

So, why isn't Core linear?

Optimised programs stop *looking* linear, but are linear *semantically*

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Linearity is ignored in Core, or most programs would be rejected

Semantic vs Syntactic Linearity

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- We type *syntactic* linearity in Core, but that is not enough
- Optimisations push laziness x linearity to the limit

Our Contributions

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- We implemented Linear Core as a GHC plugin

Semantic Linearity, by example

Semantic Linearity: Lets

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in if *condition*
  then y
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Resources in lets are only consumed if the binder is evaluated

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$$\begin{array}{l}\textbf{case } \textit{use } x \textbf{ of}\\ \quad \textit{Result } v \to ()\end{array}$$

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Resources are *kind of* consumed if the expression is evaluated

Linear Core

Linear Core: *Let*-vars

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Let-binder bodies don't consume resources

- Annotate Let-vars with linear resources Δ used in its body
- Using a Let-var entails using all of its Δ

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(Let)

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Resources used in the binder are still available in the body:

- Can consume them using the let-var
- Or directly, if the let-var is unused

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Key idea: We need to branch on *WHNF-ness*

Linear Core: Case WHNF

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  - *Irrelevance* lemma
  - Linear-var substitution lemma
    - + substitution on case alternatives
  - Δ-var substitution lemma
    - + substitution on case alternatives
  - Unr-var substitution lemma
    - + substitution on case alternatives

Metatheory: Optimising Transformations

- Inlining
- β-reduction
- β-reduction with sharing
- β-reduction for multiplicity abstractions
- Case-of-known-constructor
- Full laziness
- Local transformations (three of them)
- η-expansion
- η-reduction
- Binder swap
- Reverse binder swap (contentious!)
- Case-of-case

GHC Plugin: Linear Core Implementation

We implemented Linear Core as a GHC plugin

Library	Total Accepted	Total Rejected	Unique Rejected	Linear modulo Call-by-name	Linear Rejected	¬ Linear Rejected	Unknown Rejected
linear-smc	19438	4	1	1	0	0	0
priority-sesh	6781	19	1	0	0	0	1
linear-base	112311	538	87	10	8	2	67

Figure: Linear Core Plugin on Linear Libraries

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- There's much more in the thesis!

Fim

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System FC

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- A coercion $\sigma_1 \sim \sigma_2$ can be used to safely *cast* an expression e of type σ_1 to type σ_2, written $e \blacktriangleright \sigma_1 \sim \sigma_2$.

System FC

Definition (Syntax)

$$
\begin{array}{lcll}
u & ::= & x \mid K & \text{Variables and data constructors} \\
e & ::= & u & \text{Term atoms} \\
 & \mid & \Lambda a{:}\kappa.\ e \ \mid\ e\ \varphi & \text{Type abstraction/application} \\
 & \mid & \lambda x{:}\sigma.\ e \ \mid\ e_1\ e_2 & \text{Term abstraction/application} \\
 & \mid & \mathbf{let}\ x{:}\sigma = e_1\ \mathbf{in}\ e_2 & \\
 & \mid & \mathbf{case}\ e_1\ \mathbf{of}\ \overline{p \to e_2} & \\
 & \mid & e \blacktriangleright \gamma & \text{Cast} \\
 & & & \\
p & ::= & K\ \overline{b{:}\kappa}\ \overline{x{:}\sigma} & \text{Pattern}
\end{array}
$$