Question 3.

This query cannot be written since there is no primary key for the professor (there is no unique identifier). Thus, there is no way to distinguish a professor from another professor with the same name. We cannot create a new field in relational algebra and thus, in the hypothetical context that the ProfName is a unique ID, we can do this.

First, we do a joint of the Enrollment and the Course Table where the csid of the enrollment and the cid of the course is the same. Followed by another joint relation between the resulting table and the Teach table where the Course.cid equals the Teach.cid. The resulting table is all the Enrollment entities with the specific course and professor.

$$R1 := ((Enrollment \rhd\lhd_{(Enrollment.csid = Course.cid)} (Course)) \rhd\lhd_{(Course.cid = Teach.cid)} (Teach))$$

Then, we joint the relation R1 with itself where the sid of one relation is equal to the sid of the other, the "Unique" ProfName is the same but the course csid is different. It results in all of the Enrollement entities who have taken 2 or more classes with a specific professor. It is important to note that there will be multiple entries per student depending on the number of class he took.

$$R2 := R1 \rhd\lhd_{(R1.sid = R1sid) \wedge (R1_A.ProfName = R1_B.ProfName) \wedge (R1.csid \neq R1.csid)} R1$$

Since we just want the student, the teacher and the student name we do a projection of R2 to extract those and eliminate duplicates. Therefore, the R3 is a table with students who have taken more than one class with a specific teacher.

$$R3 := \prod_{sid,\ teacher,\ sname} (R2 \rhd\lhd_{(R2.sid = Student.sid)} (Student)$$

Then, we want to make sure that each pair is included only once so we join the relation with itself where the ProfName is the same but the sid is low than the other. Therefore, it eliminates the duplicates pairs. The result of R4 is all the pairs of students who took more than one class with the same prof without duplicates but with also the ProfName field. The fields are the name of each students, their sid and the ProfName they shared.

$$R4 := R3_A \rhd\lhd_{(R3_A.sid < R3_B.sid) \wedge (R3_A.ProfName = R3_B.ProfName)} R3_B$$

Since we only want to extract the student names and don't care about the ProfName, we do a projection on the two names to extract them. The resulting R4 results in the pairs of students who took more than one class with the same prof without duplicates.

$$R5 := \prod_{R4.sname1,\ R4.sname2} R4$$