Indian Institute of Technology Kharagpur
Department of Mathematics
MA41007 - Functional Analysis
Test - 2, AUTUMN 2021

NAME:

ROLL NO:

Instructions: Answers all the questions. No queries will be entertained during examination.

1. Let $X = C_{00}$ with norm $\|.\|_p,\ 1 \leq p \leq \infty$ and let $f : X \to K$ be defined by $f(x) = \sum_{j=0}^{\infty} x(j),\ x = (x(1), x(2), \ldots) \in X$. Then f is continuous for $p = ----------$.

2. Let $X = C^1[a,b]$ and $Y = C[a,b]$ both with norm $\|.\|_\infty$ and $A : X \to Y$ be defined by $Ax = x',\ x \in X$. Then the null space $N(A) = ----------$ and $dimN(A) = ---------$, and A is $---------------$ operator.

3. Let X a normed linear space and $f : X \to K$ be a linear functional. "Then the null space $N(f)$ is a colosed subspace of X if and only if f is a continuous". Is this (true/false)= $-----------$

4. Let X be a finite dimensional normed linear space and $\{A_n\}$ be a sequence of linear operators on X. If $\{A_n x\}$ converges for every $x \in X$, let $Ax = \lim_{n\to\infty} A_n x,\ x \in X$. Then $\|A_n - A\| \to 0$ as $n \to \infty$(True/False): $------------$

5. Let X and Y be Banach spaces and X_0 be a subspace of X and $A : X_0 \subset X \to Y$ be a injective closed operator. If $A^{-1} : R(A) \to X$ is continuous, then $R(A)$ is $--------------$

****