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FA Test 3

1. Closed bounded linear Map.

2. False

3. True, Y

4. Surjective

5.

6.

7. False, No.

8.

9. l^2

10.

2. $||x||_1 \leq \alpha \, ||x||_2$

But For Equivalent Norms this condition should be both ways

i.e. $||x||_1 \leq \alpha ||x||_2$ & $||x_2|| \leq \beta ||x||_1$

$\therefore$ for only given condition, statement is <u>false</u>

4. Since A is a open linear Map,

we know $A(X) \subset Y$ must be a open linear spa

Only such possibility of subspace is Y itself

Hence <u>surjective.</u>

We can see this by

let $y_0 \notin A(X)$ Then $y_0 \neq 0$

Consider $a_n \in \mathbb{R}$ with $a_n \to 0$

As $A(X)$ is a subspace & $y_0 \notin T(X)$

$a_n y_0 \notin A(x) \ \forall n$

But $a_n y_0 \to 0$ (0 vector)

So $Y \setminus A(X)$ is not closed which contradicts fact that $A(X)$ is open

$\therefore$ $A(X)$ is surjective map on <u>Y</u>

1. We can show this using Closed Graph theorem as F^{-1} is closed & F is Injective, F^{-1} is bounded

Also we can show this using Bounded Inverse Theorem

3. $A: X \to Y$ → bounded Linear map

& $\exists \gamma$ s.t. $\gamma \|x\| \leq \|Ax\|$

⇒ A is open map also [Using Theorem

So, $A^{-1}: R(A) \to X$ is continuous and bounded

R(A) will be Y as A is open Map.

[Using similar reasoning as Q.4]

9. We can show this by creating a linear Map

$T: H \to l^2$

which is 1-1 onto & $(Tu, Tv)_{l^2} = (u, v)_H$

& $\|Tu\|_{l^2} = \|u\|_H$

i.e. using Fundamental Theorem of Infinite Dimensional Vector spaces.

7. Using Pythagoras Theorem, $\langle x_1, x_2 \rangle = 0$

$\|x_1 + x_2\|^2 = \langle x_1 + x_2, x_1 + x_2 \rangle = \langle x_1, x_1 \rangle + \langle x_2, x_2 \rangle$

$= \|x_1\|^2 + \|x_2\|^2$

But the converse is not always true