

MovieLens Recommendation System

HarvardX Data Science Professional Certificate: PH125.9x Capstone 1

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06 August, 2020

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1 Introduction

Competing in the era of artificial intelligence requires companies to rethink both their business and operating models (Iansiti and Lakhani 2020), placing greater emphasis on the application of data science principles to exploit the availability of big data and the power of machine learning to create value (Mohr and Hürtgen 2018).

Recommendation systems are among the most important applications of machine learning deployed by digital companies today (Schrage 2017). Companies such as Amazon and Netflix use these systems to understand their customers better and to target them with products more effectively (Schrage 2018). In 2009, Netflix awarded a \$1M prize to the team of data scientists who had successfully met the challenge of improving their movie recommendation algorithm by 10% (Lohr 2009; Koren 2009).

The [MovieLens](#) datasets have provided a popular environment for experimentation with machine learning since their launch in 1997 (Harper and Konstan 2015).

The goal of this project was to develop a recommendation system using one of the MovieLens datasets which consists of 10 million movie ratings. To facilitate this work, the dataset was split into a training set (edx) and a final hold-out test set (validation) using code provided by the course organisers. The objective was for the final algorithm to predict ratings with a root mean square error (RMSE) of less than 0.86490 versus the actual ratings included in the validation set.

This report sets out the exploratory analysis of the data using common visualisation techniques followed by the methods used to develop, train and test the algorithm before providing and discussing the results from each iteration of the algorithm and concluding on the outcome of the final model, its limitations and potential for future work.

The report was compiled using R Markdown in [RStudio](#), an integrated development environment for programming in R, a language and software environment for statistical computing.

2 Exploratory Analysis

The edx dataset is a data.table, data.frame consisting of 9,000,055 rows and 6 columns, with ratings provided by a total of 69,878 unique users for a total of 10,677 unique movies. If each unique user had provided a rating for each unique rating the dataset would include a total of approximately 746 million ratings. Clearly, therefore, this dataset includes many missing values, i.e. every user has not rated every movie.

Table 1: edx dataset: variable class and first 5 rows

userId	movieId	rating	timestamp	title	genres
integer	numeric	numeric	integer	character	character
1	122	5	838985046	Boomerang (1992)	Comedy Romance
1	185	5	838983525	Net, The (1995)	Action Crime Thriller
1	292	5	838983421	Outbreak (1995)	Action Drama Sci-Fi Thriller
1	316	5	838983392	Stargate (1994)	Action Adventure Sci-Fi
1	329	5	838983392	Star Trek: Generations (1994)	Action Adventure Drama Sci-Fi
1	355	5	838984474	Flintstones, The (1994)	Children Comedy Fantasy

2.1 Ratings (\$rating)

The overall average rating in the edx dataset was 3.51. The minimum rating awarded to any movie was 0.5 and the maximum rating awarded was 5. The distribution of total ratings included in the dataset (Figure 1) shows that the most common rating across all movies was 4, and that, overall, whole star ratings (7,156,885; 79.5%) were used more than half star ratings (1,843,170; 20.5%).

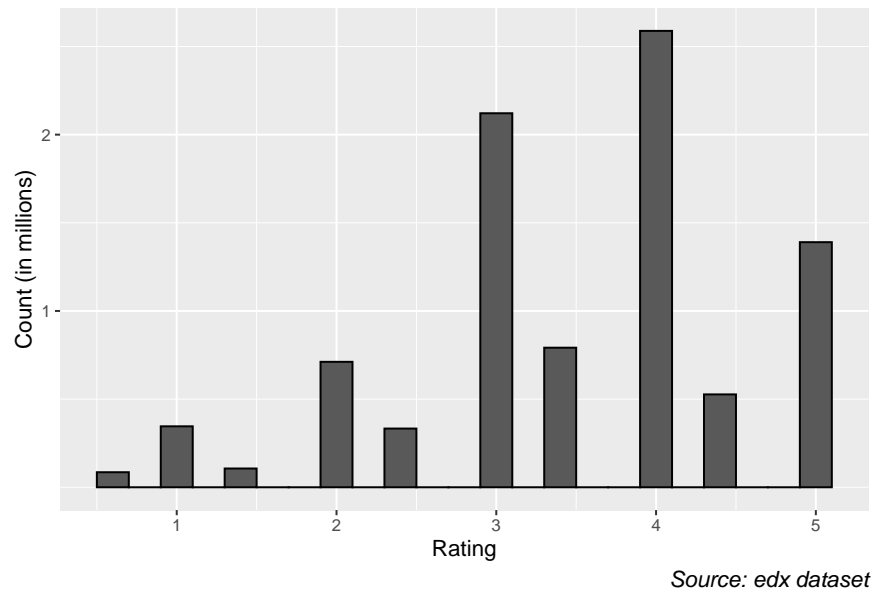


Figure 1: Overall ratings distribution

2.2 Movies (\$movieId)

Unsurprisingly, some movies are more highly rated than others (see Figure 2). Further analysis reveals significant variation in the number of ratings received by each movie (see Figure 3), with the movie with the most ratings, Pulp Fiction (1994), receiving a total of 31362 ratings whereas as many as 126 movies were only rated once. There is clearly a movie effect on the rating awarded and, as such, adjusting for this effect (or bias) was considered worthwhile for inclusion in the training algorithm.

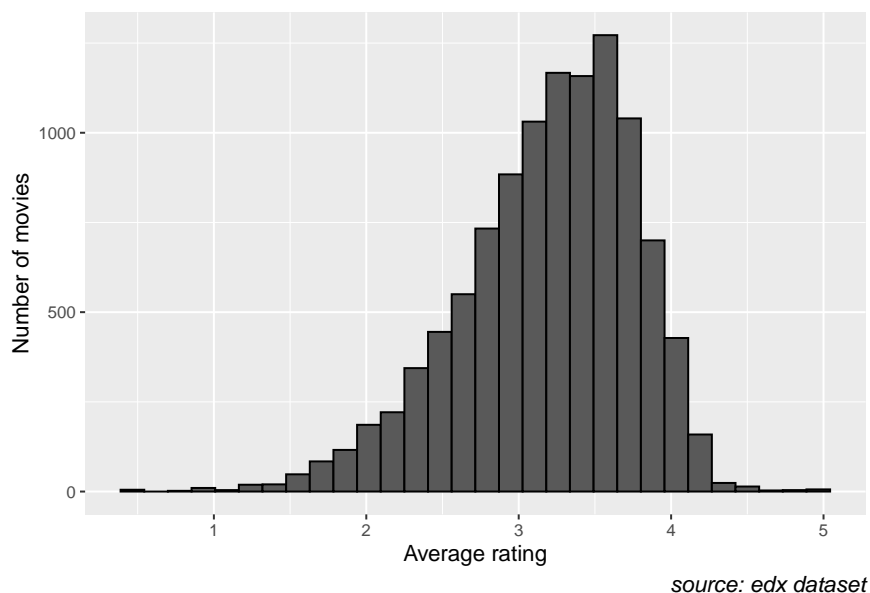


Figure 2: Movie distribution by average rating

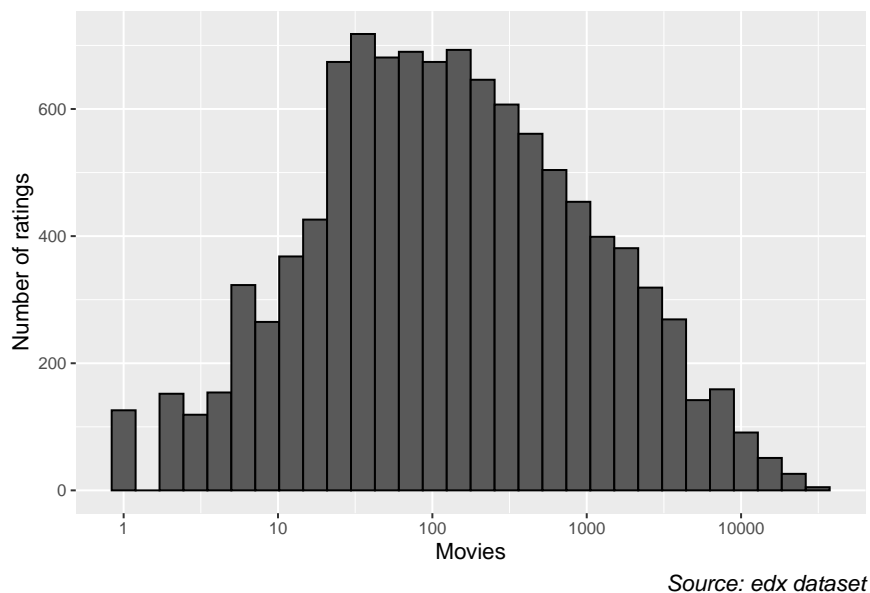


Figure 3: Number of ratings by movie

2.3 Users (\$userId)

Exploration of user data revealed a similar pattern to that observed for movies, with some users appearing more generous in the way they assessed movies, having provided higher ratings than others (see Figure 4). Some users contributed many more ratings than other users (Figure 5). For example, one user provided a total of 6616 ratings whereas as many as 1059 provided fewer than 10 movie ratings each. This analysis identifies a clear user effect (or bias) which, if adjusted for, may further improve the accuracy of a movie recommendation system.

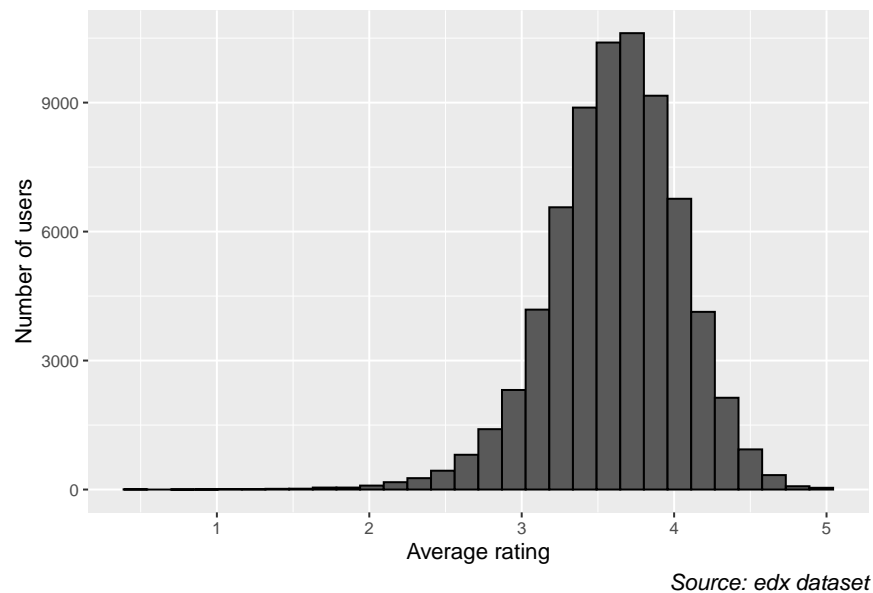


Figure 4: User distribution by average rating

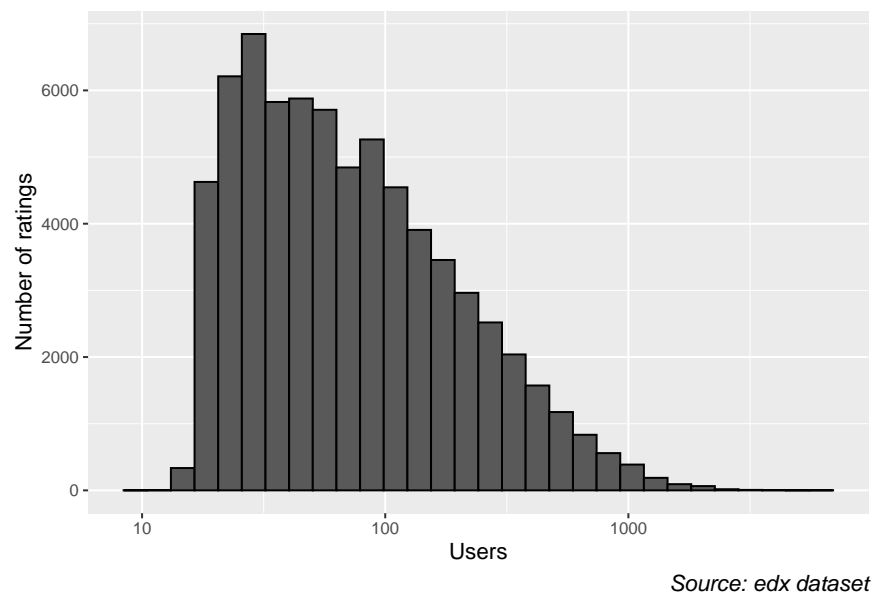


Figure 5: Number of ratings by user

2.4 Movie Genre (\$genres)

As was shown in Table 1 above, the genres variable provides the names of the genres which each movie is identified with. For example, Boomerang (1992) in the first row of the edx dataset includes Comedy|Romance in the genre variable. Some movies were assigned to more than one genre and there were a total of 797 unique genre combinations included in the dataset. Separating these combinations into rows with single genres, it was possible to identify 20 different genre categories (including “no genre listed”) and to rank these by the number of ratings (Table 2).

Table 2: Individual genres ranked by number of ratings

Genre	No. of Ratings	Ave. Rating
Drama	3910127	3.67
Comedy	3540930	3.44
Action	2560545	3.42
Thriller	2325899	3.51
Adventure	1908892	3.49
Romance	1712100	3.55
Sci-Fi	1341183	3.40
Crime	1327715	3.67
Fantasy	925637	3.50
Children	737994	3.42
Horror	691485	3.27
Mystery	568332	3.68
War	511147	3.78
Animation	467168	3.60
Musical	433080	3.56
Western	189394	3.56
Film-Noir	118541	4.01
Documentary	93066	3.78
IMAX	8181	3.77
(no genres listed)	7	3.64

Drama and comedy movies had the most ratings whereas Documentary and IMAX movies had the fewest ratings. Seven ratings were provided for movies for which no genre was listed. Table 2 also shows a variation in average rating by genre. Grouping the data by unique genre combinations and filtering only those genre combinations with at least 100,000 ratings in order to simplify the analysis for the purposes of visualisation, shows a clear effect of genre with ‘Comedy’ movies achieving the lowest average rating whereas ‘Crime|Drama’ and ‘Drama|War’ films achieving the highest rating (Figure 6). Clearly, there is merit in seeking to address this effect in training the algorithm for the movie recommendation system.

2.5 Movie Title (\$title)

The title variable includes both the title of the movie and the year of release, in brackets. Table 3 shows the top 10 movie titles by the number of ratings.

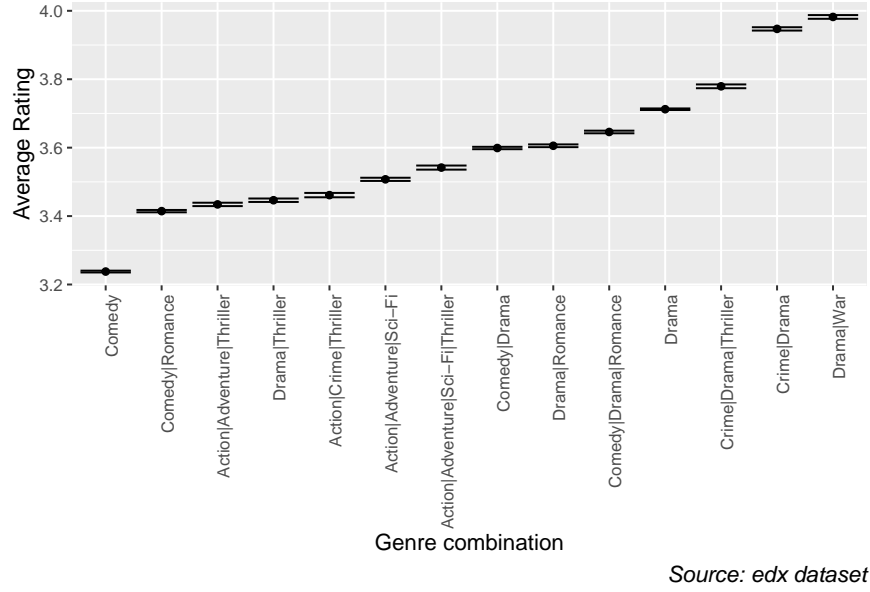


Figure 6: Average rating by genre

Table 3: Top 10 Movies by Number of Ratings

title	n
Pulp Fiction (1994)	31362
Forrest Gump (1994)	31079
Silence of the Lambs, The (1991)	30382
Jurassic Park (1993)	29360
Shawshank Redemption, The (1994)	28015
Braveheart (1995)	26212
Fugitive, The (1993)	25998
Terminator 2: Judgment Day (1991)	25984
Star Wars: Episode IV - A New Hope (a.k.a. Star Wars) (1977)	25672
Apollo 13 (1995)	24284

In order to explore the effect, if any, of the year of release on average rating, the title string was split into two separate columns, one for the title and the other for the year of release. The mutated dataset was then used to explore the effect of release year on rating and, as can be seen in Figure 7, average rating varied by year of release. Interestingly, the average rating peaked for movies released between 1940 and 1950 and declined for movies released since that period.

However, as shown in Figure 8, there are very few ratings within the dataset assigned to movies released prior to 1970. The movies with the most ratings were released during the 1990s, peaking in 1995 with approximately 9% of the total number of ratings included in the edx dataset. Thus, as with other variables adjusting for the effect of release year should improve the accuracy of the training algorithm but taking account of the greater uncertainty in point estimates created by small sample sizes in some years would also be important.

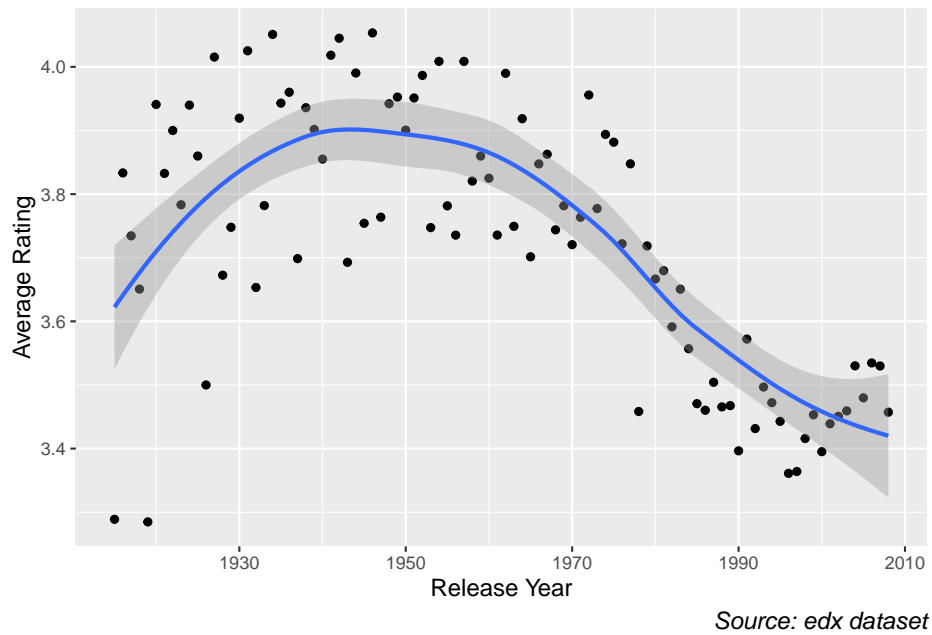


Figure 7: Average rating by year of release

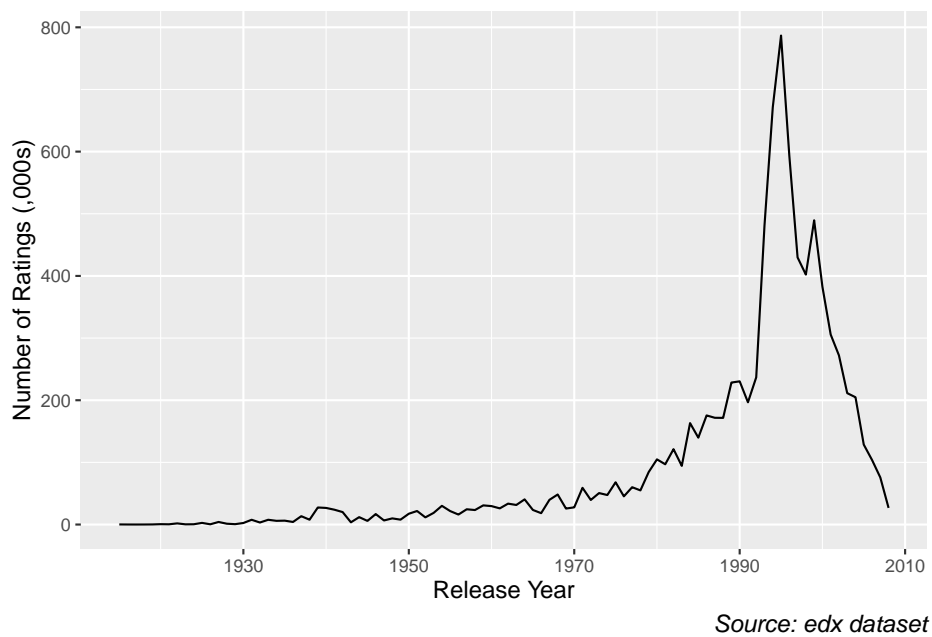


Figure 8: Number of ratings by year of release

2.6 Date of review (\$timestamp)

The `timestamp` is a convenient way of digitally recording both date (yymmdd) and time (hhmmss) information, based on an epoch (time zero) of midnight on 1 January 1970. To aid analysis of the effect of review date on ratings, the timestamp data was mutated into date format, omitting time data and rounding to the nearest week (in order to effectively smooth the data).

The earliest review included in the dataset was completed in 1995. This was also when the average rating was highest, with a gradual decline in ratings observed until around 2005 after which average ratings began to increase. The effect of review date on the average rating was modest relative to that observed for movies and users but there was still some variation over time (see Figure 9), justifying its inclusion in the development of the recommendation algorithm.

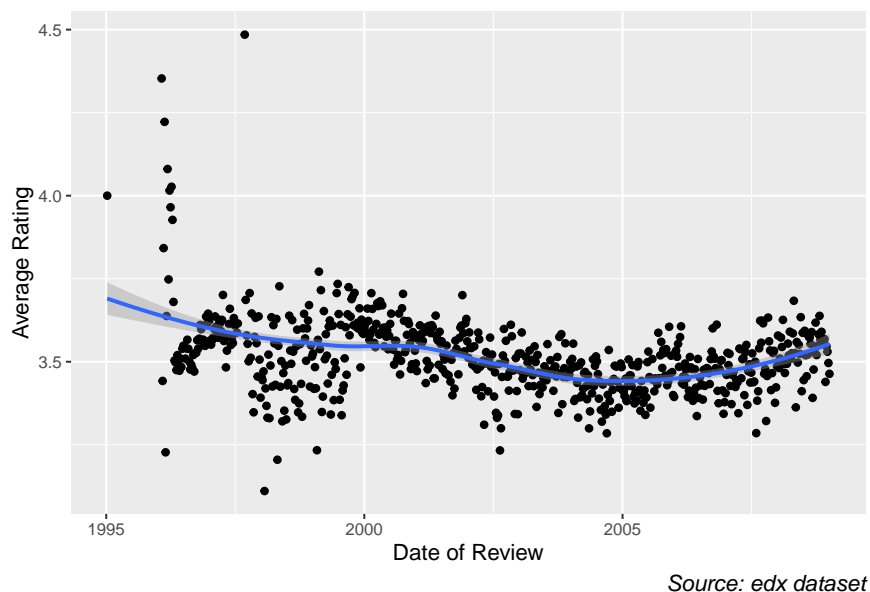


Figure 9: Average rating by date of review

3 Methods

3.1 Splitting edx out into train and test sets

As the validation dataset was reserved for the final hold-out test, the edx dataset needed to be used both to train and test the algorithm in development. This is important to allow for cross-validation and refinement of the final model without the risk of over-training. Other methods for cross-validation include K-fold cross validation and bootstrapping but were not utilised here (see Irizarry (2020) for further information).

Here, the same technique was applied as with the original movielens dataset, using the caret function ‘createDataPartition’ to divide the edx dataset into train (80%) and test (20%) sets. As before, the dplyr functions ‘semi_join’ and ‘anti_join’ were used, firstly to ensure that the test set only included users and movies that are present in the train set and, secondly to add the removed data to the train set in order to maximise the data available for training purposes.

3.2 Calculating the error loss

The residual mean square error (RMSE) is defined as the standard deviation of the residuals (prediction errors) where residuals are a measure of spread of data points from the regression line (Glen 2020). The RMSE was calculated to represent the error loss between the predicted ratings derived from applying the algorithm and actual ratings in the test set. In the formula shown below, $y_{u,i}$ is defined as the actual rating provided by user i for movie u , $\hat{y}_{u,i}$ is the predicted rating for the same, and N is the total number of user/movie combinations.

$$RMSE = \sqrt{\frac{1}{N} \sum_{u,i} (\hat{y}_{u,i} - y_{u,i})^2}$$

The objective of the project was to develop an algorithm that achieved an RMSE below 0.86490 as set out below. A simple table was created to capture the project objective as well as the results obtained during development within the edx dataset and in the final hold-out test in the validation dataset (see Section 4: Results).

Method	RMSE	Difference
Project objective	0.86490	-

3.3 Developing the algorithm

The simplest algorithm for predicting ratings is to apply the same rating to all movies. Here, the actual rating for movie i by user u , $Y_{u,i}$, is the sum of this “true” rating, μ , plus $\epsilon_{u,i}$, the independent errors sampled for the same distribution.

$$Y_{u,i} = \mu + \epsilon_{u,i}$$

The average of all ratings is the estimate of μ that minimises the RMSE. Thus, $\hat{\mu} = \text{mean}(\text{train_set\$rating})$ was the simple formula used to train the first algorithm.

The exploratory analysis detailed in the previous section showed that ratings were not assigned equally across all movies. That is, some movies achieved a higher average rating than others and, accounting for this effect (or bias) will therefore improve the accuracy of the prediction. Thus, the training algorithm was further refined by taking into account the effect of movie on rating, b_i .

$$Y_{u,i} = \mu + b_i + \epsilon_{u,i}$$

A linear regression model would take some time to run given the large dataset involved. Instead, the least squares estimate of the movie effects, \hat{b}_i , can be derived from the average of $Y_{u,i} - \hat{\mu}$ for each movie i and, thus, the following formula was used to take account of movie effects within the training algorithm.

$$\hat{y}_{u,i} = \hat{\mu} + \hat{b}_i$$

The exploratory analysis also showed that different users rated movies differently so further refinements were made to the algorithm to adjust for user effects (b_u). As previously, rather than fitting linear regression models, the least square estimates of the user effect, \hat{b}_u was calculated using the formulas shown below.

$$Y_{u,i} = \mu + b_i + b_u + \epsilon_{u,i}$$

$$\hat{b}_u = \text{mean}(\hat{y}_{u,i} - \hat{\mu} - \hat{b}_i)$$

Movie ratings were also dependent on genre, with some genres achieving higher average ratings than others. This effect was observed even when movies were allocated to multiple genres, as in the original dataset. Therefore, the rating for each movie and user was further refined by adjusting for genre effect, b_g , and the least squares estimate of the genre effect, \hat{b}_g calculated using the formula shown below.

$$Y_{u,i} = \mu + b_i + b_u + b_g + \epsilon_{u,i}$$

$$\hat{b}_g = \text{mean}(\hat{y}_{u,i} - \hat{\mu} - \hat{b}_i - \hat{b}_u)$$

The fourth bias to adjust for within the model was the release year of the movie. The exploratory analysis in the previous section showed an effect of the release year, b_y , and the least squares estimate of the year effect, \hat{b}_y calculated using the formula shown below, building on the algorithm developed already.

$$Y_{u,i} = \mu + b_i + b_u + b_g + b_y + \epsilon_{u,i}$$

$$\hat{b}_y = \text{mean}(\hat{y}_{u,i} - \hat{\mu} - \hat{b}_i - \hat{b}_u - \hat{b}_g)$$

There was also a small effect of the date of review (b_r) on the average rating awarded for each movie and user. The most appropriate way to incorporate this into the model would be to apply a smooth

function to the day of release for each rating by movie and user. Rounding the date of review to the nearest week served to effectively smooth the data. The least squares estimate, taking into account the date of review effect, \hat{b}_r was calculated using the formula shown below.

$$Y_{u,i} = \mu + b_i + b_u + b_g + b_y + b_r + \epsilon_{u,i}$$

$$\hat{b}_r = \text{mean}(\hat{y}_{u,i} - \hat{\mu} - \hat{b}_i - \hat{b}_u - \hat{b}_g - \hat{b}_y)$$

3.4 Regularising of the algorithm

Finally, the exploratory analysis showed that not only is the average rating affected by the movie, user, genre, year of release and date of review, but that the number of ratings also varies. Thus, for example, some movies and genres of movie received fewer ratings than others while some users provided fewer ratings than others. Similarly, the number of ratings varied by year of release and date of review. In each of these cases, the consequence of this variation is that the estimates of the effect (b) will have been subject to greater uncertainty when based on a smaller number of ratings.

Regularisation is an effective method for penalising large effect estimates that are based on small sample sizes (Irizarry 2020). The penalty term, λ , is a tuning parameter chosen using cross-validation within the edx dataset. Thus, the movie effect, b_i can be regularised to penalise these large effects as follows.

$$\frac{1}{N} \sum_{u,i} (y_{u,i} - \mu - b_i)^2 + \lambda \sum_i b_i^2$$

Based on the above, the least squares estimate for the regularised effect of movies can be calculated as below, where n_i is the number of ratings made for movie i . The effect of $\frac{1}{\lambda + n_i}$ is such that when the sample size is large, i.e. n_i is a big number, λ has little impact on the estimate, $\hat{b}_i(\lambda)$. On the other hand, where the sample size is small, i.e. n_i is small, the impact of λ increases and the estimate shrinks towards zero.

$$\hat{b}_i(\lambda) = \frac{1}{\lambda + n_i} \sum_{u=1}^{n_i} (Y_{u,i} - \hat{\mu})$$

Here, the regularisation model was developed to adjust for all of the effects previously described, as shown below. A range of values for λ (range: 4-6, with increments of 0.1) was applied in order to tune the model to minimise the RMSE value. As before, all tuning was completed within the edx dataset, using the train and test sets, so as to avoid over-training the model in the validation set.

$$\frac{1}{N} \sum_{u,i} (y_{u,i} - \mu - b_i - b_u - b_g - b_y - b_r)^2 + \lambda \left(\sum_i b_i^2 + \sum_u b_u^2 + \sum_g b_g^2 + \sum_y b_y^2 + \sum_r b_r^2 \right)$$

3.5 Validating the final model

Having refined the model algorithm within the train and test sets created by partitioning edx, the final stage of the project was to train the algorithm using the full edx dataset and then to predict ratings within the validation dataset. Prior to doing this, it was necessary to incorporate the date of review and year of release variables in the validation set using the mutate function from the dplyr package.

The final model, adjusting for biases introduced by movie, user, genre, release year, review date, and collectively regularised using the optimal value for λ , was used to predict ratings in the validation dataset, and to calculate the final validation RMSE.

4 Results

4.1 Simple average

Predicting the average rating from the train set (3.51) for every entry in the test set resulted in a RMSE of 1.06, substantially above the project objective. Moreover, an RMSE of 1.06 means that predicted ratings are more than 1 star away from the actual rating, an unacceptable error loss for a movie recommendation system.

Method	RMSE	Difference
Project objective	0.86490	-
Simple average	1.06057	0.19567

4.2 Adjusting for movie effects

Figure 10 shows that the estimate of movie effect (b_i) varies considerably across all of the movies included in the train set. Adding this effect into the algorithm, in order to adjust for the movie effect, improved the accuracy of the model by 11.02%, yielding an RMSE of 0.94, albeit still well above the target.

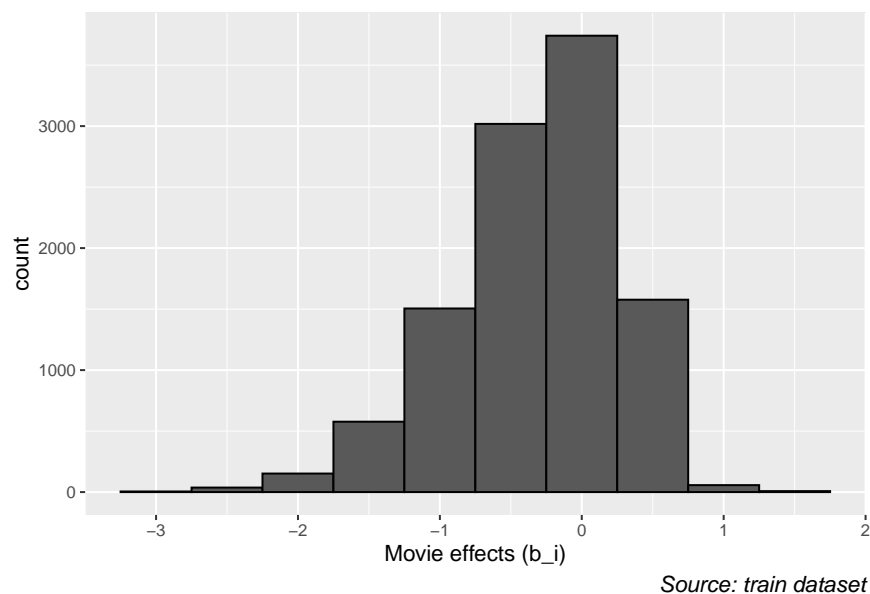


Figure 10: Distribution of movie effects

Method	RMSE	Difference
Project objective	0.86490	-
Simple average	1.06057	0.19567
Movie effects (b_i)	0.94371	0.07881

4.3 Adjusting for user effects

Figure 11 shows the estimated effect of user (b_u) building on the movie effects model above. Whilst b_u showed less variability than was observed with b_i , it was evident that adjusting for user effects enhanced the accuracy of the algorithm. Indeed, adjusting for user effects resulted in an RMSE of 0.86617. Thus, adjusting for both movie and user effects improved the RMSE by 18.33% versus the simple model, demonstrating the strong bias introduced by each of these variables on ratings.

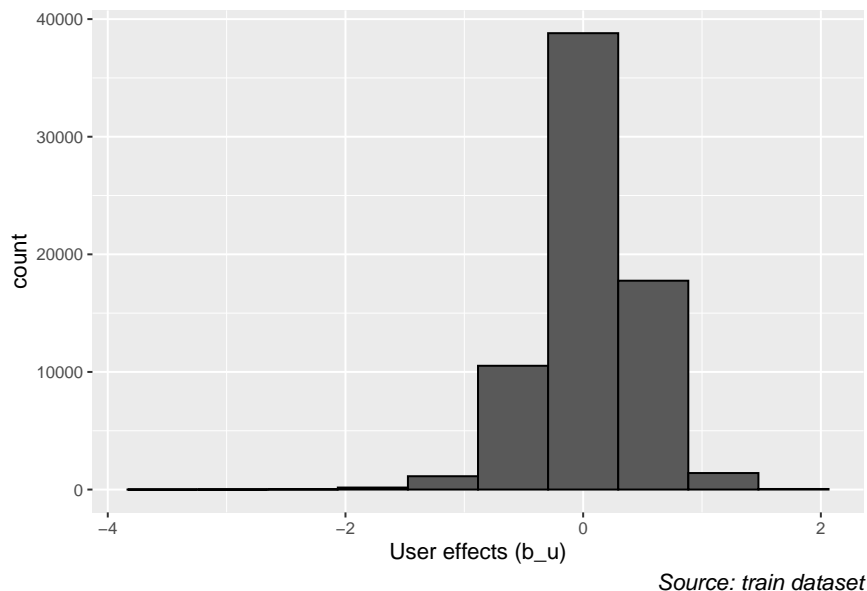


Figure 11: Distribution of user effects

Method	RMSE	Difference
Project objective	0.86490	-
Simple average	1.06057	0.19567
Movie effects (b_i)	0.94371	0.07881
Movie + User effects (b_u)	0.86617	0.00127

4.4 Adjusting for genre effects

Figure 12 shows the distribution of estimate genre effects, b_g in the train set, once again showing some variation across different genre combinations.

The output from the model when adjusting for genre, in addition to movie and user bias, was an RMSE of 0.86582. Thus adding genre effects into the model only provided a modest improvement in the accuracy of the algorithm, reducing the RMSE by 0.04% versus the previous model and 18.36% versus the original model. This improvement did bring the model very close to meeting the project objective, reducing the difference to only 0.00092.

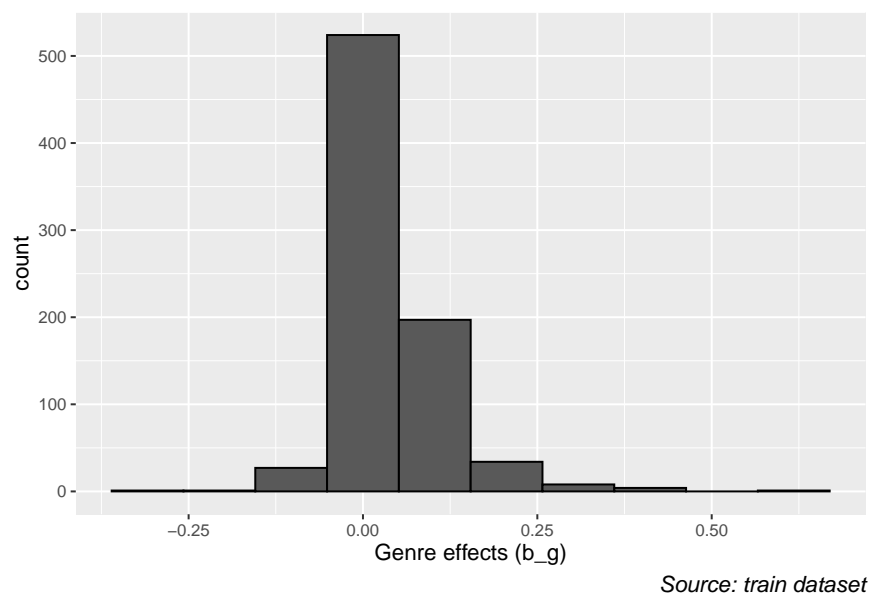


Figure 12: Distribution of genre effects

Method	RMSE	Difference
Project objective	0.86490	-
Simple average	1.06057	0.19567
Movie effects (b_i)	0.94371	0.07881
Movie + User effects (b_u)	0.86617	0.00127
Movie, User and Genre effects (b_g)	0.86582	0.00092

4.5 Adjusting for release year effects

The year of movie release adds modest additional variability to the average rating in the train set as shown in Figure 13. Indeed, incorporating this into the training algorithm yielded a modest incremental improvement of 0.02% in the accuracy of ratings prediction bringing the RMSE slightly closer to meeting the project objective at 0.86567.

Method	RMSE	Difference
Project objective	0.86490	-
Simple average	1.06057	0.19567
Movie effects (b_i)	0.94371	0.07881
Movie + User effects (b_u)	0.86617	0.00127
Movie, User and Genre effects (b_g)	0.86582	0.00092
Movie, User, Genre and Year effects (b_y)	0.86567	0.00077

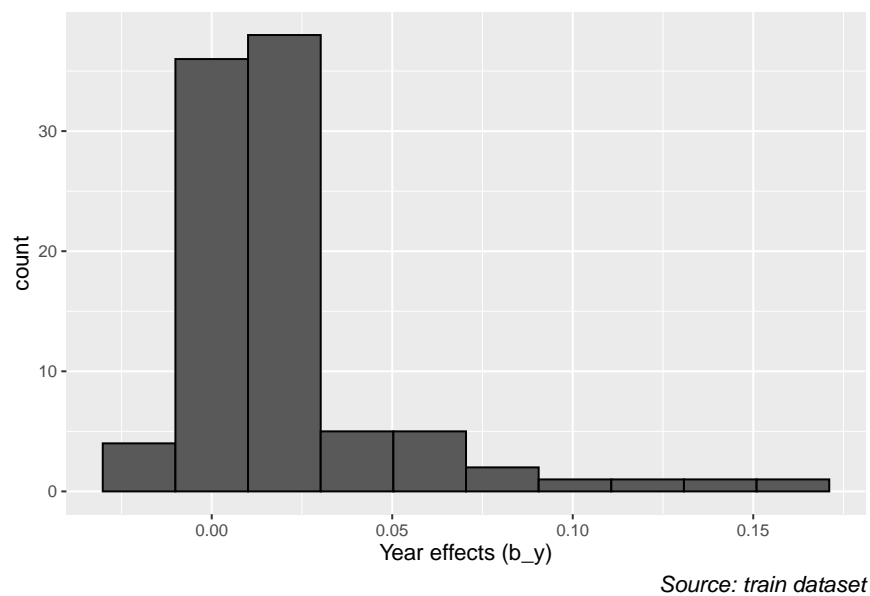


Figure 13: Distribution of release year effects

4.6 Adjusting for review date effects

The final bias to be adjusted for was the date of review. The exploratory analysis had shown that this had a small impact on ratings and this was confirmed by visualising the distribution of b_r in Figure 14.

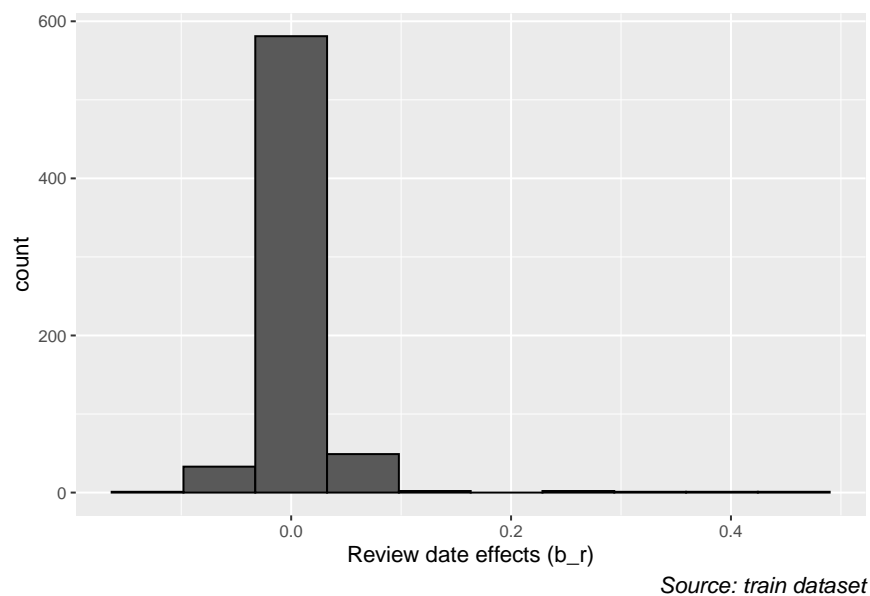


Figure 14: Distribution of review date effects

Adding the effect of review date into the algorithm delivered an RMSE of 0.86549, an improvement of 18.39% versus the original model but still not quite enough to meet the project objective.

Method	RMSE	Difference
Project objective	0.86490	-
Simple average	1.06057	0.19567
Movie effects (b_i)	0.94371	0.07881
Movie + User effects (b_u)	0.86617	0.00127
Movie, User and Genre effects (b_g)	0.86582	0.00092
Movie, User, Genre and Year effects (b_y)	0.86567	0.00077
Movie, User, Genre, Year and Review Date effects (b_r)	0.86549	0.00059

4.7 Regularising the algorithm

The final step in developing the algorithm was to apply regularisation. Figure 15 shows the RMSE delivered across each of the values for λ tested. The optimum value for λ was 5.1 which minimised RMSE to 0.86483, which was just sufficient to surpass the target RMSE set as the project objective. This represented a total improvement of 18.46% in the accuracy of the model by adjusting for movie, user, genre, year of release and review date effects and applying regularisation to the combination of these effects.

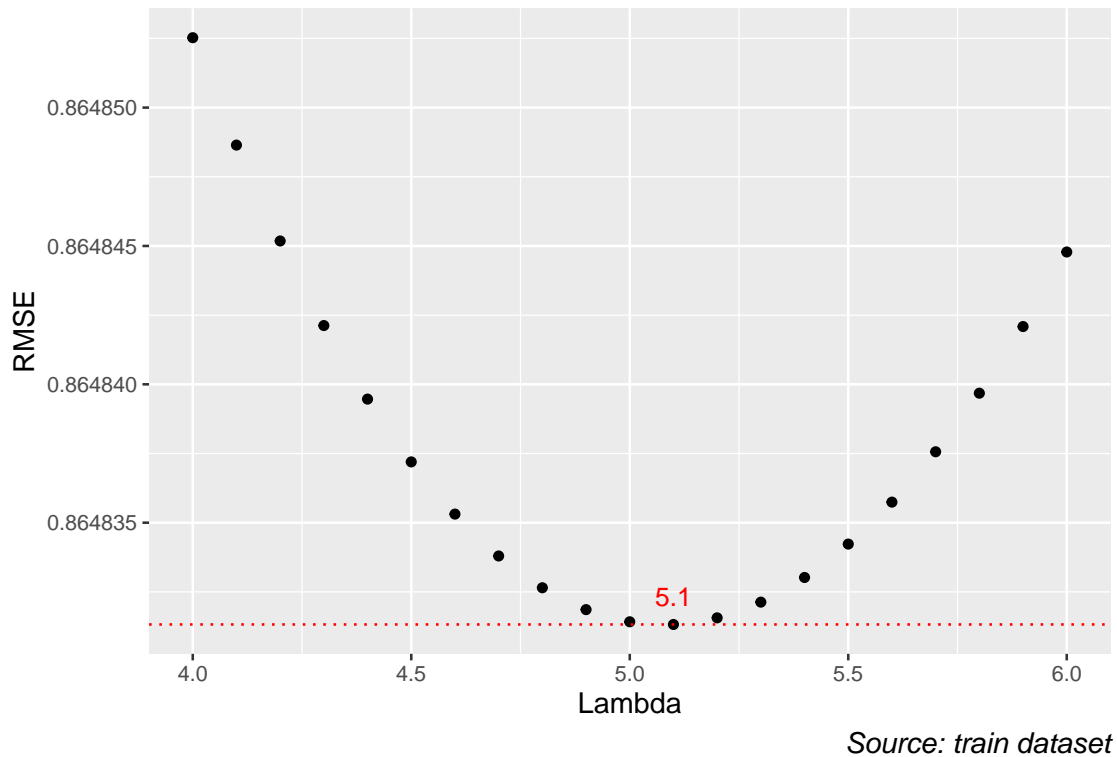


Figure 15: Selecting the tuning parameter

Method	RMSE	Difference
Project objective	0.86490	-
Simple average	1.06057	0.19567
Movie effects (b_i)	0.94371	0.07881
Movie + User effects (b_u)	0.86617	0.00127
Movie, User and Genre effects (b_g)	0.86582	0.00092
Movie, User, Genre and Year effects (b_y)	0.86567	0.00077
Movie, User, Genre, Year and Review Date effects (b_r)	0.86549	0.00059
Regularised Movie, User, Genre, Year and Review Date effects	0.86483	-0.00007

4.8 Final hold-out test in validation dataset

The final hold-out test in the validation dataset achieved an RMSE of 0.86405, an improvement of 18.53% versus the simple model based on the overall average rating and 0.00085 below the target RMSE set as project objective.

Method	RMSE	Difference
Project objective	0.86490	-
Validation of Final Model	0.86405	-0.00085

5 Conclusions

The objective of this project was to develop a recommendation system using the MovieLens 10M dataset that predicted ratings with a residual mean square error of less than 0.86490. Adjusting for a number of estimated biases introduced by the movie, user, genre, release year and review date, and then regularising these in order to constrain the variability of effect sizes, met the project objective yielding a model with an RMSE of 0.86483. This was confirmed in a final test using the previously unused validation dataset, with an RMSE of 0.86405.

Although the algorithm developed here met the project objective it still includes a sizeable error loss, not all of which may be considered truly independent, which suggests that there is still scope to improve the accuracy of the recommendation system with techniques that can account for some of this non-independent error. One such approach is matrix factorisation, a powerful technique for user or item-based collaborative filtering based machine learning which can be used to quantify residuals within this error loss based on patterns observed between groups of movies or groups of users such that the residual error in predictions can be further reduced (Irizarry 2020; Koren, Bell, and Volinsky 2009).

The techniques used in this project were limited due to the impracticality of using some powerful tools to train such a large dataset on a personal computer. One of the key advantages of matrix factorisation which has contributed to its popularity in recommendation systems is that it is both scalable and compact which makes it memory efficient and compatible with use on personal computers (Koren, Bell, and Volinsky 2009). Thus, further work on the recommendation system developed here would focus on the use of matrix factorisation.

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