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On Design of Two-Stage CIC Compensation Filter

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Abstract- This paper presents the compensation filter design for the two-stage CIC decimation filter. The goal is twofold: to avoid the integrator section at high input rate and obtain a low wideband passband droop of the overall filter. To this end the decimation is split into two stages with the cascaded less order RRS filters at each stage. The first stage can be implemented either in non recursive form or using the polyphase decomposition. The simple compensation filter and the sharpening are applied to the second section where RRS filter is implemented as a CIC filter. The resulting structure is a multiplierless and with no integrators at high input rate. Additionally, the structure exhibits a low passband droop and a high stopband attenuation.

I. INTRODUCTION

Sigma-delta analog to digital converter (ADC) is the most widely used ADC. It consists of two main building blocks: analog sigma-delta modulator and a digital decimator which decrease the sampling rate of the oversampled signal to the Nyquist rate. The key component of the decimator is the digital filter which is responsible for the aliasing elimination introduced during the decreasing of sampling rate [1-2].

A commonly used decimation filter is the recursive running sum (RRS) or comb filter with the transfer function

$$H(z)=\left[\frac{1}{M}\left(\frac{1-z^{-M}}{1-z^{-1}}\right)\right]^{K}, \tag{1}$$

where M is the decimation ratio, and K is the number of the stages. Efficient realisation of the RRS filter proposed by Hogenauer [3] is the cascaded-integrator-comb (CIC) filter, which consists of two main sections: an integrator and a comb, separated by a down-sampler, [3].

The comb section operates at the lower data rate, while the integrator section works at the higher input data rate thereby resulting in an overflow and higher chip area and higher power dissipation for this section. In order to resolve this problem the non-recursive structure of (1) can be used [4]-[6].

Magnitude response of the filter (1) exhibits a linear-phase, lowpass sinMx/sin$x$ characteristic which can be expressed as

$$\left|H(e^{j\omega})\right|=\left|\frac{\sin(\omega M/2)}{M\sin(\omega/2)}\right|^{K}, \tag{2}$$

The characteristic has a low attenuation and a high droop in the desired passband that is dependent upon the decimation factor M and the cascade size K.

A second decimator block with a decimation ratio which is usually significantly smaller than that of the CIC usually follows the CIC filter [7]. The decimation factor of the second stage determines the frequency at which the worst-case aliasing occurs as well as the passband edge frequency where the worst-case passband distortion occurs [7]. For example for the case of a factor-of-N second decimation, the passband edge of interest normalized with respect to the high sampling rate is at [7]

$$\frac{\omega_c}{\pi}=\frac{\pi}{NM}\frac{1}{\pi}=\frac{1}{NM}, \tag{3}$$

where M is the decimation factor of the RRS stage. Similarly the worst-case aliasing will be at the frequency

$$\frac{\omega_A}{\pi}=\frac{2}{M}-\frac{1}{NM}=\frac{2N-1}{NM}. \tag{4}$$

Several schemes have been proposed to improve the pass band characteristic of the RRS filter as for example [7]-[12]. However the majority of methods compensate the passband droop in the narrow passband. In our previous work [12] we proposed the wide-band compensation (N of Eq (3) is 2). However the structure has the integrator section which work at high input rate.

In this paper we propose the modification of the method [12]. To this end we split the decimation in two stages to avoid the integrator section at the high input rate. The passband droop is decreased by introducing simple compensator filter and sharpening technique in the second stage. Next section introduces the two-stage structure followed by sin-based compensator filter and sharpening technique. Section 4 describes the proposed structure. Comparisons of the method with some recent results are given in Section 5.

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II. TWO-STAGE STRUCTURE

We consider here the case where $M = M_1 M_2$. We rewrite the transfer function of RRS filter as

$$H(z) = \left[H_1(z)H_2(z^{M_1})\right]^K, \qquad (5)$$

where

$$H_1(z) = \frac{1}{M_1}\frac{1-z^{-M_1}}{1-z^{-1}} \qquad (6.a)$$

$$H_2(z^{M_1}) = \frac{1}{M_2}\frac{1-z^{-M_1M_2}}{1-z^{-M_1}}. \qquad (6.b)$$

An efficient implementation of the above using multirate identity is shown in Fig. 1, where H_1 and H_2 are both RRS filters with lenghts M_1 and M_2, respectively. In more general case we consider that the number of the cascaded combs can be different as explained in [11]. Denote the number of cascaded RRS filters at first and second stages as K_1 and K_2, respectively, as shown in Fig.1a. In a general case using a polyphase decomposition, the filters of the first section can be moved to operate at a lower rate. In the special case where the decimation factor M_1 can be presented in terms of power-of-two, the first stage can be implemented in nonrecursive form [4-6]. As an example Fig.2 shows the nonrecursive form of the first stage for $M_I=2^3$. Recently in [6], it has been demonstrated that the polyphase realization of the nonrecursive structure has the advantages of low power consumption and high speed operation compared to the original nonrecursive implementation.

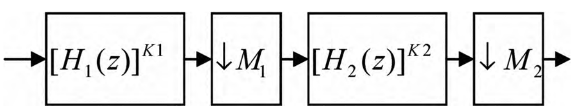


Fig.1. Two-stage structure.

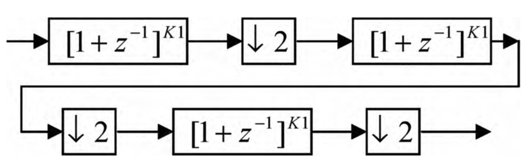


Fig.2. Nonrecursive structure of the first stage for $M_I=2^3$.

Next issue is how to improve the passband characteristic of the overall structure.

III. COMPENSATION FILTER AND SHARPENING

Consider the filter with the magnitude response [8]

$$\left|G(e^{j\omega M})\right| = \left|1 + 2^{-b}\sin^2(\omega M/2)\right| \qquad (7)$$

Using the well known relation

$$\sin^2\alpha = (1-\cos 2\alpha)/2 \quad , \qquad (8)$$

the corresponding transfer function is given as

$$G(z^M) = B[1 + Az^{-M} + z^{-2M}], \qquad (9)$$

where B is the scaling factor,

$$B = -2^{-(b+2)}, \qquad (10)$$

and

$$A = -[2^{(b+2)} + 2], \qquad (11)$$

where b is integer and the values of b are in the interval [-1, 2].

To improve the passband characteristics, we propose to use the sharpening technique which can be used for simultaneous improvements of both the pass band and stop band characteristics of a linear-phase FIR digital filter [13]. The technique uses the amplitude change function (ACF) which is a polynomial relationship of the form $H_0 = f(H)$ between the amplitudes of the overall and the prototype filters, H_0 and H, respectively. The improvement in the gain response near the pass band edge $H=1$, or near the stop band edge $H=0$, depends on the order of tangencies m and. n of the ACF at $H=1$ or at $H=0$.

The expressions for the m^{th} and n^{th} order tangencies of the ACF at H=1 and H=0, respectively, are given as,

$$H_0 = H^{n+1}\sum_{s=0}^{m}\frac{(n+s)!}{n!s!}(1-H)^s$$

$$= H^{n+1}\sum_{s=0}^{m}C(n+s,s)(1-H)^s, \qquad (12)$$

where $C(n+s,s)$ is the binomial coefficient. The values of the ACF for some typical values of m and n are given in Table II.

Better improving of the original magnitude response is obtained using more complex polynomials.

TABLE I.

ACF POLYNOMIALS

m	*n*	H_0
1	0	$2H-H^2$
1	1	$3H^2-2H^3$
2	0	H^3-3H^2+3H
3	3	$-20H^7+70H^6-84H^5+35H^4$

IV. PROPOSED STRUCTURE

We apply the sharpening to the second stage of the Structure from Fig.1, i.e. to the cascaded filters $H_2^{K_2}(z)$ and the compensator (9) as shown in Fig.3. a.

Considering that we would like to compensate the passband droop, and to not increase to much the complexity of the filter, we propose to use sharpening polynomial with m=1 and n=0 from Table I. Denoting

$$H_{2c}(z) = H_2^{K_2}(z)G(z^{M_2}), \qquad (13)$$

we have

$$\begin{aligned} Sh\{H_{2c}(z)\} &= 2H_{2c}(z) - H_{2c}^2(z) \\ &= H_{2c}(z)[2z^{-\tau} - H_{2c}(z)] \end{aligned}, \qquad (14)$$

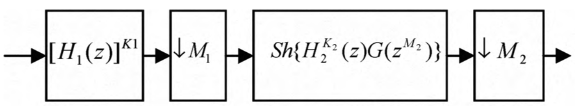
where $Sh\{x\}$ means sharpening of x.

The delay τ is introduced to keep the linear phase of the filter and is defined as

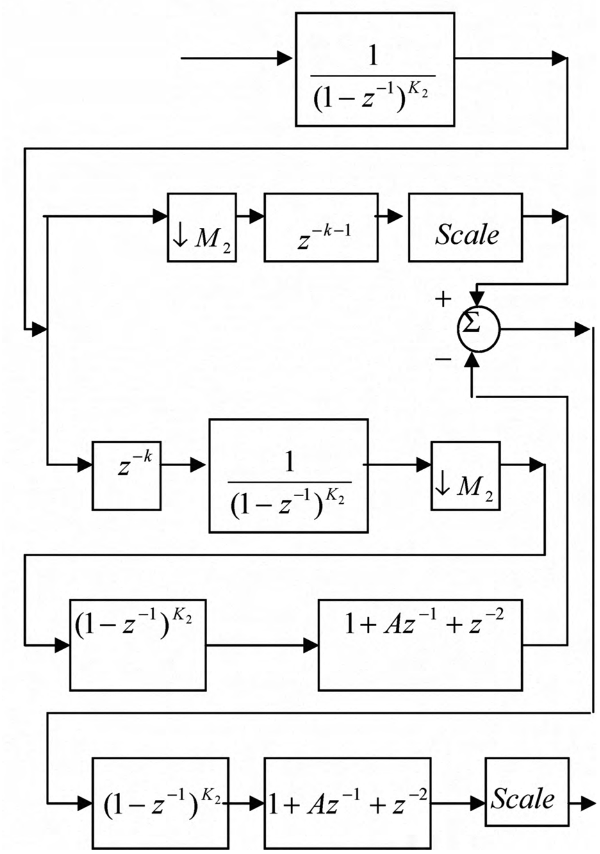
$$\tau = (M_2 - 1)K_2/2 + M_2. \qquad (15)$$

Note that K_2 has to be even to avoid fractional delay, i.e. $K_2=2k$, where k is an integer.

Using multirate identities [1], [2], and applying the delay z^{-k} at the input to avoid advance z^k we obtain the more efficient structure of the second stage, shown in Fig.3b.



a. Overall structure.



b. Second stage.

Fig.3. Proposed structure.

The value of the parameter b depends on the number of the cascaded filters K_2. Simple MATLAB simulation determines the values of b as shown in Table II.

TABLE II.

TYPICAL VALUES FOR b

PARAMETER K_2	Parameter b
2	2
3	1
4	0
5	0
6	0, -1

From (6), and (14) we have the corresponding magnitude response

$$\begin{aligned} &\left|H_p(e^{j\omega})\right| = \\ &\left|H_1^{K_1}(e^{j\omega})\right| \times \{2\left|H_{2c}(e^{j\omega})\right| - \left|H_{2c}(e^{j\omega})\right|^2\} \end{aligned}, \qquad (16)$$

where

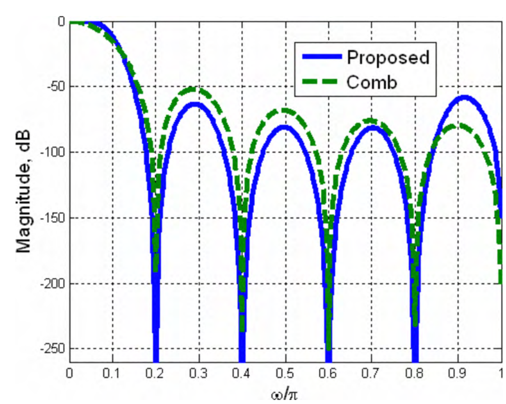
$$H_1^{K_1}(e^{j\omega}) = \left[\frac{\sin(M_1\omega/2)}{M_1\sin(\omega/2)}\right]^{K_1} \qquad (17)$$

$$\begin{aligned} &H_{2c}(e^{j\omega}) = \\ &\left[\frac{\sin(M\omega/2)}{M_2\sin(\omega M_1/2)}\right]^{K_2}[1+2^{-b}\sin^2(\omega M/2)] \end{aligned}. \qquad (18)$$

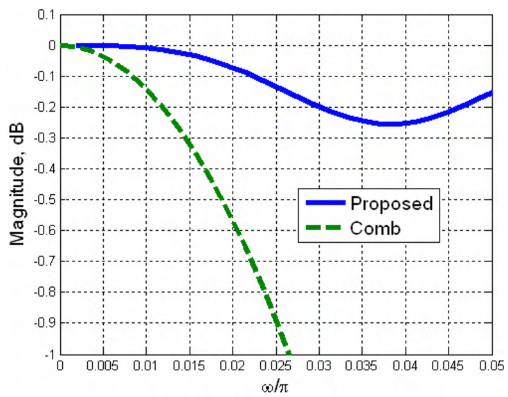
Next examples illustrate the performances of the structure.

Example 1:

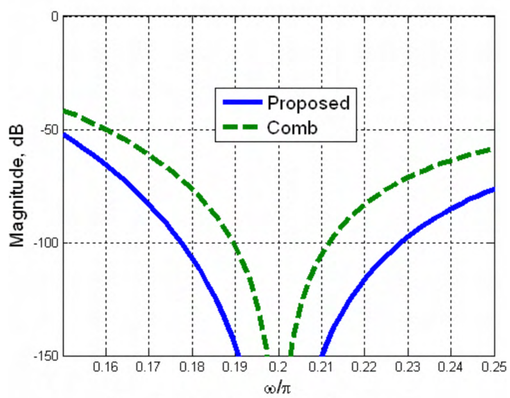
We consider M=10 and M_1=2, M_2=5, K_1=3, K_2=6. We consider N=2.(See Eq (3)). From Table 1 we have value of b=0. Figures 4a, b and c show the corresponding magnitude responses along with that of comb filter with K=4.



a. Overall magnitude responses.

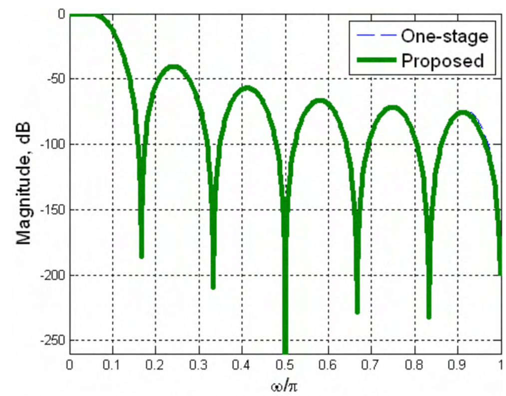


b. Passband zoom.

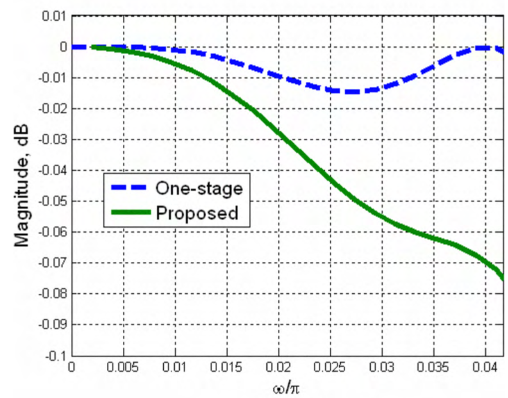


b. Stopband around the first zero.

Fig. 4. Example 1: M_1=2, M_2=5, K_1=3, K_2=6, b=0, K=4, N=2.



a. Overall magnitude responses.



c. Passband zoom.

d.

Fig.5. Comparison with [12].

Note that the proposed filter exhibits better passband and the stopband characteristics than the corresponding comb filter. The polyphase decomposition of the filter in the first stage is

$$H_1^3(z) = E_0(z^2) + z^{-1}E_1(z^2)\,, \qquad (19)$$

where $E_0(z^2)$ and $E_1(z^2)$ are the polyphase components

$$E_0(z^2) = 1 + 3z^{-2}\,, \qquad (20\text{ a})$$

$$E_1(z^2) = 3 + z^{-2}\,. \qquad (20\text{ b})$$

Figure 5 compares the magnitude responses of the proposed structure (M_1=2, M_2=6, K_1= K_2=4, b=0) and the method [12] with M=12, K=4, b=0. Note that is a very small difference in the magnitude responses. However as we can see in the Passband zoom, the proposed structure exhibits slightly higher passband droop.

In next section we compare the proposed method with method [9] which provides a wideband compensation of CIC filter and with method [11] which consists of two stage CIC structure without integrators at high input rate.

V. Comparisons

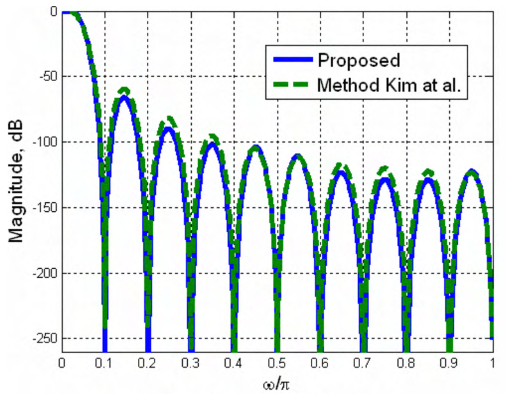
In this section we compare the performance of the proposed method with that of some recent methods with similar complexity.

Method Kim et al [9]

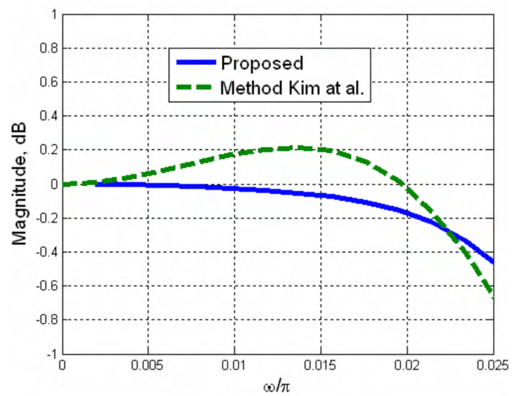
Here the authors proposed the use of a CIC roll-off compensation filter in a W-CDMA digital IF receiver. The coefficients of the compensation filter are given by

$$[-a/(1-2a), 1/(1-2a), -a/(1-2a)]\,. \qquad (21)$$

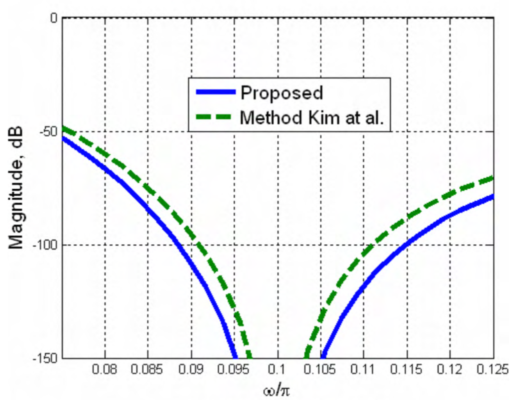
The performance of the compensation filter depends on the value of a, which is obtained by minimizing the corresponding error function. Figure 6 compares the performance of our proposed filter with that of a decimation filter designed using a CIC roll-off compensation filter when the decimation rate and output data rate are 20 and 2 times, respectively and the order of CIC filter was 5. The corresponding value of a is 0.1806. In the proposed design we have M_1=4, M_2=5, K_1=5, K_2=6, b=0.



a. Overall magnitude responses.



b. Passband zoom.

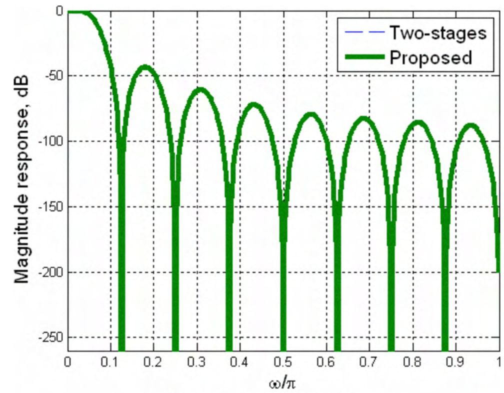


c. Stopband near first zero.

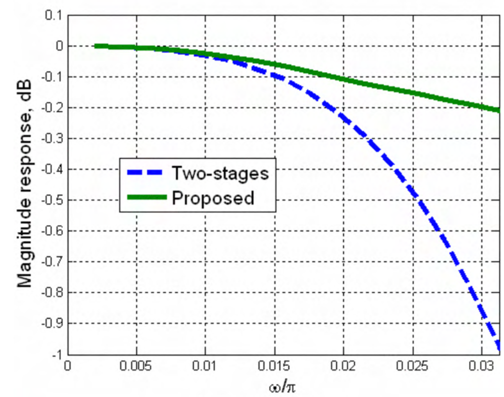
Fig.. 6. Comparison with [9].

Method [11]

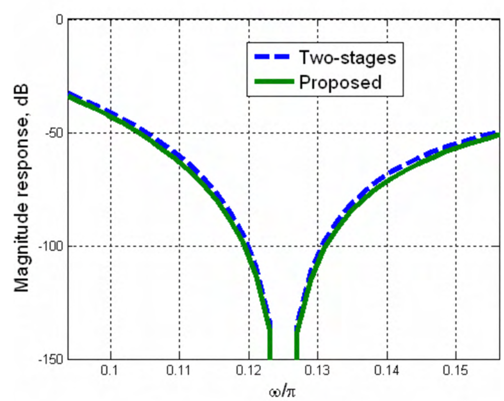
In this method is proposed a two-stage structure with no filtering at high input rate. In the second stage is applied sharpening with polynomial n=m=1 from Table I. This method results in a high attenuation around first zero. However the pass band droop is low only for the narrow pass band. Figure 7 compares the proposed structure with the method [11], with parameters: M_1=M_2=4, K_1=4, K_2=2. The parameters of the proposed structure are: M_1=M_2=4, K_1=K_2=4, b=0.



a. Overall magnitude responses.



b. Passband zoom.



c. Stopband near first zero.

Fig.. 7. Comparison with [11].

Note that the magnitude responses are very similar. However, the proposed structure exhibits a lower passband droop.

VI. Conclusions

This paper presents a new multiplierless two-stage decimation filter. Unlike to CIC filter the proposed structure does not have the integrator section at the high input rate. Using the polyphase decomposition of the filter at the first stage all filtering is moved to the lower rate which is M_1 times less than the input rate.

Additionally the structure exhibits a low wideband passband droop and a high attenuation. The price is the slightly increased complexity.

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