Performances of the Novel Modified CIC FIR Filter Functions

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***Abstract* — Design of novel selective multiplerless Cascaded-Integrator-Comb (CIC) finite impulse response (FIR) filter functions with applications in modern communication systems is presented here. Performance comparisons of the modified CIC filter functions and the classical CIC filter functions through a few design examples are also presented. Novel designed filter functions show substantial performance improvements compared to classical CIC filters. Depending on the chosen filter order, the achieved improvements are from 6.70 % up to 43.44 %.**

***Keywords* — CIC filters, FIR filters, linear phase, multiplierless structure, selective filters.**

I. Introduction

SOFTWARE defined radio has find important place in modern communication systems where majority of signal processing is performed in the digital domain. In software radio systems, a requirement of the very large sample rate changes leads to large order digital filters, which can easily become a bottleneck in the overall system design. In order to reduce the computational demands, one can use a CIC finite impulse response (FIR) filter that uses only additions/subtractions. The term "Cascaded-Integrator-Comb (CIC)" filters was first reported in the early 1980s by E.B. Hogenauer [1]. Many papers have addressed the problem of some modifications of the classical CIC filters for sample rate conversion in software defined radio systems [2]-[4]. Modified CIC filters are ideal for applications in which high sampling rates make the use of multipliers a computationally expensive option. Other solutions proposed in the literature give some modified filter structures for sigma-delta analog-to-digital converters [4], [5].

Because of the disadvantages of a CIC FIR filter, some solutions given in the literature try to improve the classical CIC magnitude response characteristic. In [6]-[11], the authors use a compensation filter in the cascade with the original filter. Then, some other authors use sharpening technique [12] or design new classes of filter functions as shown in [13]-[17] till now.

This paper is focused on novel CIC FIR filter functions based on cascading CIC sections of different lengths (non-identical CIC sections). The novel CIC FIR filter functions are designed based on the classical CIC filters, by spreading the delays in the CIC filter comb stages. The delays are set around one specific value of filter order N. The performances of the proposed CIC filter functions are compared with those of the classical CIC filters under fair conditions: the same number of cascaded sections and the same group delay. The results show substantial performance improvement of the proposed filter class compared to the classical CIC filters.

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II. Classical CIC FIR Filter

The normalized CIC FIR filter function of one section (i.e. CIC filter section) in z-domain is defined with

$$H(N,z)=\frac{1}{N}\cdot\left(1+z^{-1}+\cdots+z^{-(N-1)}\right)=\frac{1-z^{-N}}{N\cdot(1-z^{-1})}. \quad (1)$$

There are two basic building blocks: a comb with the filter function in z-domain $H_c(N,z)=(1-z^{-N})$, and an integrator which function in z-domain is $H_i(z)=1/(1-z^{-1})$.

A poor magnitude characteristic of the CIC filter composed of one section, Eq. (1), is improved by cascading several identical CIC filters. The classical CIC FIR filter function of normalized amplitude response characteristic, represented in the z-domain, is defined as

$$H(N,K,z)=\left(\frac{1-z^{-N}}{N\cdot(1-z^{-1})}\right)^K, \quad (2)$$

where N is the decimation factor, and K is the number of CIC filter sections [1].

The frequency response characteristic of CIC FIR filter function can be written in the form

$$H(N,K,z=e^{j\omega})=e^{-jK(N-1)\omega/2}\cdot\left(\frac{\sin(N\omega/2)}{N\cdot\sin(\omega/2)}\right)^K. \quad (3)$$

III. Proposed Novel CIC FIR Filter Functions

The modified CIC FIR filter functions are designed as cascade of four non-identical CIC filter sections: one section $H(N-3,z)$, two identical sections $H(N,z)$, and one section $H(N+3,z)$, of the form given by (1), as well as seven cascaded non-identical CIC FIR filter sections which are repeated an integer number of times.

Designed novel CIC FIR filter functions of normalized amplitude response characteristic can be written in non-recursive form as follows

$$H(N,K,L,z)=\left(\frac{1}{N-3}\sum_{r=0}^{N-4}z^{-r}\right)\left(\frac{1}{N}\sum_{r=0}^{N-1}z^{-r}\right)\left(\frac{1}{N}\sum_{r=0}^{N-1}z^{-r}\right)\cdot$$
$$\cdot\left(\frac{1}{N+3}\sum_{r=0}^{N+2}z^{-r}\right)\cdot\left[\left(\frac{1}{N-3}\sum_{r=0}^{N-4}z^{-r}\right)\left(\frac{1}{N-2}\sum_{r=0}^{N-3}z^{-r}\right)\left(\frac{1}{N-1}\sum_{r=0}^{N-2}z^{-r}\right)\right]^{L}\cdot \quad (4)$$
$$\cdot\left[\left(\frac{1}{N}\sum_{r=0}^{N-1}z^{-r}\right)\left(\frac{1}{N+1}\sum_{r=0}^{N}z^{-r}\right)\left(\frac{1}{N+2}\sum_{r=0}^{N+1}z^{-r}\right)\left(\frac{1}{N+3}\sum_{r=0}^{N+2}z^{-r}\right)\right]^{L}$$

where N and L are free integer parameters, and $K=7L+4$. This form is suitable for hardware realization, because it is unconditionally stable.

The recursive form of novel CIC FIR filter functions with normalized amplitude response characteristic is

$$H(N,K,L,z)=$$
$$=\frac{1-z^{-(N-3)}}{(N-3)\cdot(1-z^{-1})}\frac{1-z^{-N}}{N\cdot(1-z^{-1})}\frac{1-z^{-N}}{N\cdot(1-z^{-1})}\frac{1-z^{-(N+3)}}{(N+3)\cdot(1-z^{-1})}\cdot$$
$$\left(\frac{1-z^{-(N-3)}}{(N-3)\cdot(1-z^{-1})}\frac{1-z^{-(N-2)}}{(N-2)\cdot(1-z^{-1})}\frac{1-z^{-(N-1)}}{(N-1)\cdot(1-z^{-1})}\right)^{L}\cdot$$
$$\left(\frac{1-z^{-N}}{N\cdot(1-z^{-1})}\frac{1-z^{-(N+1)}}{(N+1)\cdot(1-z^{-1})}\frac{1-z^{-(N+2)}}{(N+2)\cdot(1-z^{-1})}\frac{1-z^{-(N+3)}}{(N+3)\cdot(1-z^{-1})}\right)^{L}$$

and $K=7L+4$. (5)

The frequency response of designed novel FIR filter functions is obtained by evaluating the filter function in the z-plane at the sample points defined by setting $z=e^{j\omega}$, where $\omega=2\pi\cdot f$ has units of radians per second. Using Euler's identity, it can be separated into a real-valued magnitude $A(N,K,L,\omega)$ and a real-valued phase $\varphi(N,K,L,\omega)$ for each frequency ω,

$$H(N,K,L,z=e^{j\omega})=e^{j\cdot\varphi(N,K,L,\omega)}\cdot A(N,K,L,\omega)$$

where the parameter $K=7L+4$. (6)

The normalized amplitude response characteristic of the proposed filter functions is defined in the form

$$A(N,K,L,\omega)=$$
$$=\frac{\sin((N-3)\omega/2)}{(N-3)\cdot\sin(\omega/2)}\frac{\sin(N\omega/2)}{N\cdot\sin(\omega/2)}\frac{\sin(N\omega/2)}{N\cdot\sin(\omega/2)}\frac{\sin((N+3)\omega/2)}{(N+3)\cdot\sin(\omega/2)}\cdot$$
$$\left(\frac{\sin((N-3)\omega/2)}{(N-3)\cdot\sin(\omega/2)}\frac{\sin((N-2)\omega/2)}{(N-2)\cdot\sin(\omega/2)}\frac{\sin((N-1)\omega/2)}{(N-1)\cdot\sin(\omega/2)}\right)^{L}\cdot$$
$$\left(\frac{\sin(N\omega/2)}{N\cdot\sin(\omega/2)}\frac{\sin((N+1)\omega/2)}{(N+1)\cdot\sin(\omega/2)}\frac{\sin((N+2)\omega/2)}{(N+2)\cdot\sin(\omega/2)}\frac{\sin((N+3)\omega/2)}{(N+3)\cdot\sin(\omega/2)}\right)^{L}$$

and $K=7L+4$. (7)

The normalized magnitude response characteristic $\left|H(N,K,L,e^{j\omega})\right|$ is obtained as absolute value of the normalized amplitude response characteristic $A(N,K,L,\omega)$ given by (7).

The linear phase response characteristic of the proposed modified CIC FIR filter has the form

$$\varphi(N,K,L,\omega)=-(N-1)\cdot K\cdot\omega/2+2\cdot\nu\cdot\pi,\ \nu=0,1,2,...$$

and $K=7L+4$. (8)

The constant group delay response characteristic of the modified CIC FIR filter functions is expressed as

$$\tau(N,K,L,\omega)=-d\varphi(N,K,L,\omega)/d\omega=(N-1)\cdot K/2,$$

and $K=7L+4$. (9)

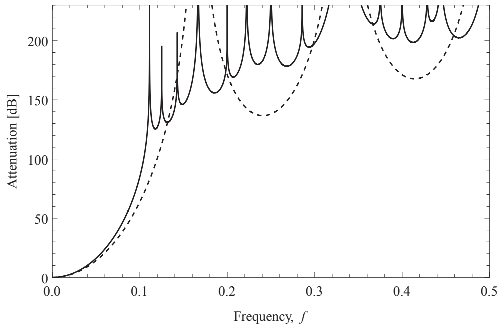
It is independent of the frequency. For N being odd number, the group delay is an integer multiple of $K/2$. If N is even number, the group delay is equal to an integer plus half multiple of $K/2$.

IV. PERFORMANCES OF THE FREQUENCY RESPONSES

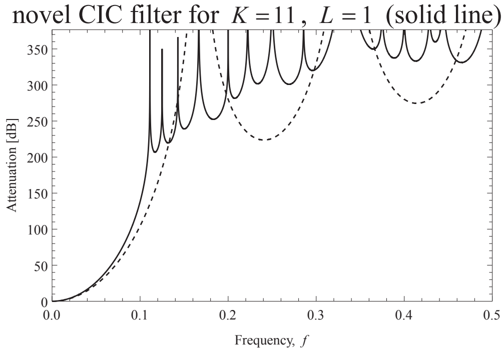
A few illustrative examples are designed and simulations are conducted in order to compare frequency response characteristics of the classical CIC filter functions and the novel designed class of selective CIC FIR filter functions. The novel filter functions $H(N,K,L,z)$ from Eq. (5) and the classical CIC filters $H(N,K,z)$ from Eq. (2) have the same number of cascaded sections K, with the difference that the CIC filters have an identical structure in all cascades, and the proposed functions have a cascade connections of non-identical CIC sections. The choice of free parameters N and L is done in the same way as for CIC filters, there are the same restrictions on the group delay response. The parameter K can take different values $K=7L+4$. Generaly, the main task of the comparison was to vary the free parameters and to compare obtained filter characteristics.

Detailed analyses of the normalized magnitude response characteristics in dB, defined for the classical CIC filters as $\alpha_{CIC}(f)=-20\cdot\log\left|H(N,K,e^{j2\pi f})\right|$ and for the novel class of CIC FIR filter functions as $\alpha(f)=-20\cdot\log\left|H(N,K,L,e^{j2\pi f})\right|$, versus normalized frequency $f=\omega/(2\pi)$, are depicted in Fig. 1. The maximum attenuation in the passband is $\alpha_{\max}=0.28\,\text{dB}$. The filter functions have the same level of constant group delay, as well as number of delay elements, but the novel designed class gives higher insertion losses in stopband, as well as it has higher selectivity.

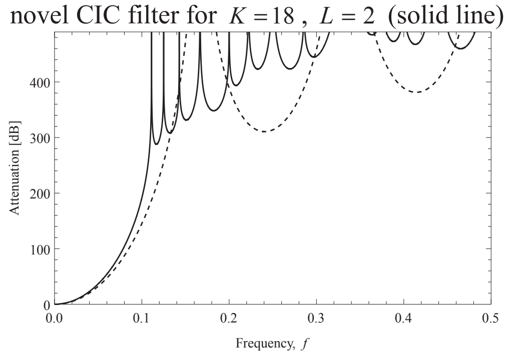
Zooms of the normalized magnitude response characteristics of classical CIC filter and proposed CIC FIR filter functions are given in Figs. 2 and 3, for $N=6$ and $N=11$, respectively. The novel CIC filter functions have two peaks in the transition area of the classical filter (on frequency between the passband f_{cp} and stopband f_{cs} cut-off frequencies). Note that obtained attenuations of the novel functions in the stopband area are higher then attenuations of the classical CIC filter. For filter order $N=6$, in case of $K=11$ (Fig. 2*a*), classical CIC filter has attenuation of 136.68 dB and novel function 146.11 dB. It is achieved improvement of 9.43 dB or approximately about 6.89 %. In case of $K=18$ (Fig. 2*b*), it is achieved improvement of 15.45 dB (6.90 %). In case of $K=25$ (Fig. 2*c*), achieved improvement is 20.80 dB (6.70 %). For filter order $N=11$, in case of $K=11$ (Fig. 3*a*), it is achieved improvement of 56.55 dB (43.44 %). In case of $K=18$ (Fig. 3*b*), it is achieved improvement of 87.58 dB (39.57 %). For $K=25$ (Fig. 3*c*), it is achieved significant improvement of 117.77 dB (37.69 %).



(a) Classical CIC filter for $K = 11$ (dashed line), novel CIC filter for $K = 11$, $L = 1$ (solid line)



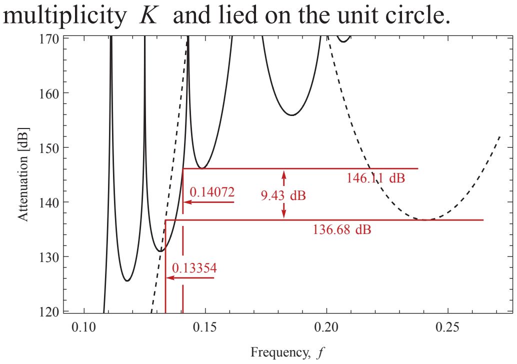
(b) Classical CIC filter for $K = 18$ (dashed line), novel CIC filter for $K = 18$, $L = 2$ (solid line)



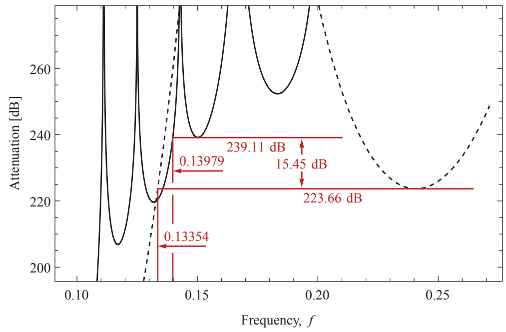
(c) Classical CIC filter for $K = 25$ (dashed line), novel CIC filter for $K = 25$, $L = 3$ (solid line)

Fig. 1. Comparison of normalized magnitude response characteristics in dB of classical CIC filter (dashed lines), and novel CIC FIR filter functions (solid lines), for $N = 6$.

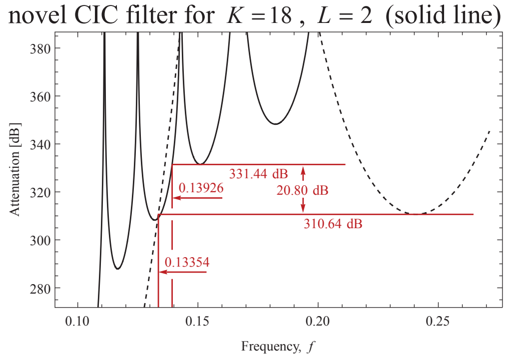
The locations of zeros in z-plane along with their multiplicities for classical and proposed filter functions are shown in Fig. 4, for parameter values $N = 6$, and $K = 18$. The CIC filter function has $N-1$ different zeros, $z_r = e^{j \cdot 2\pi \cdot r / N}$, $r = 1, 2, ..., N-1$. All zeros are multiple with multiplicity K and lied on the unit circle.



(a) Classical CIC filter for $K = 11$ (dashed line), novel CIC filter for $K = 11$, $L = 1$ (solid line)



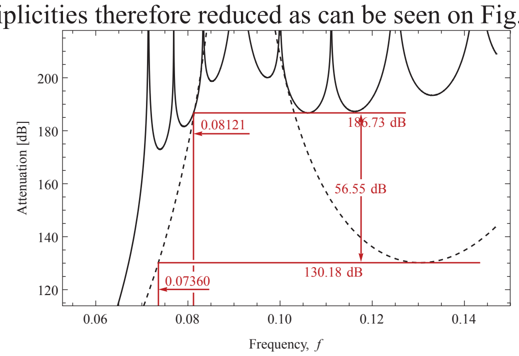
(b) Classical CIC filter for $K = 18$ (dashed line), novel CIC filter for $K = 18$, $L = 2$ (solid line)



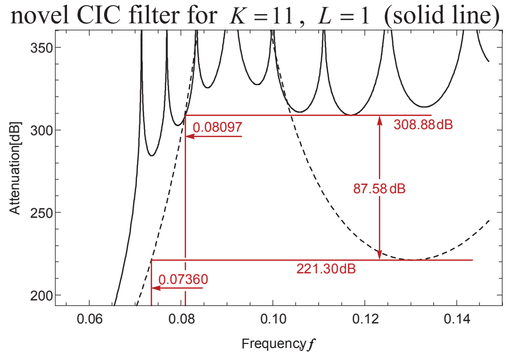
(c) Classical CIC filter for $K = 25$ (dashed line), novel CIC filter for $K = 25$, $L = 3$ (solid line)

Fig. 2. Zooms of normalized magnitude response characteristics in dB of classical CIC filter (dashed lines), and novel CIC FIR filter functions (solid lines), for $N = 6$.

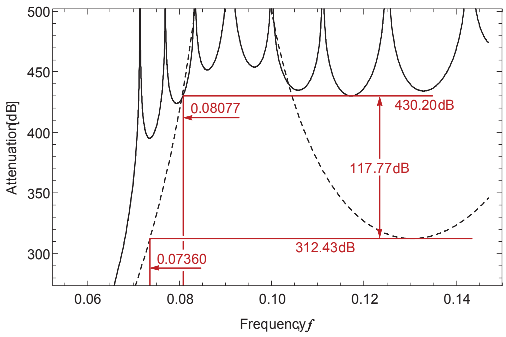
The total number of zeros is $N \cdot K$. The locations of zeros of the classical CIC filters for different values of parameter N, are given in [15]. The zeros of the proposed filter functions are more evenly distributed with their multiplicities therefore reduced as can be seen on Fig. 4b.



(a) Classical CIC filter for $K = 11$ (dashed line), novel CIC filter for $K = 11$, $L = 1$ (solid line)

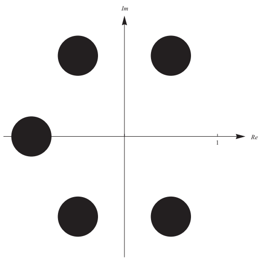


(b) Classical CIC filter for $K = 18$ (dashed line), novel CIC filter for $K = 18$, $L = 2$ (solid line)

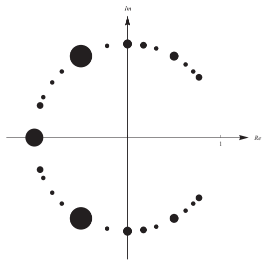


(c) Classical CIC filter for $K = 25$ (dashed line), novel CIC filter for $K = 25$, $L = 3$ (solid line)

Fig. 3. Zooms of normalized magnitude response characteristics in dB of classical CIC filter (dashed lines) and novel CIC FIR filter functions (solid lines), for $N = 11$.



(a) Classical CIC filter



(b) Proposed filter, $L = 2$

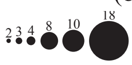
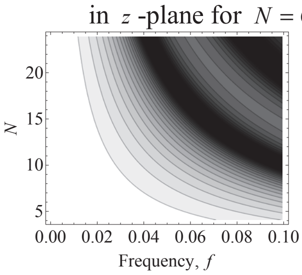
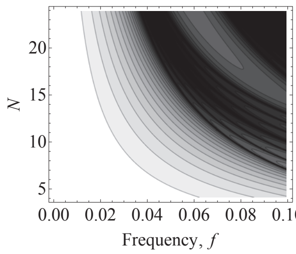


Fig. 4. Locations and multiplicities of filter function zeros in z -plane for $N = 6$, and $K = 18$ cascades.



(a) Classical CIC filter



(b) Novel filter class, $L = 2$

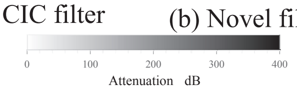


Fig. 5. 2D contour plots of magnitude frequency response characteristics for classical and proposed CIC FIR filters for $N \in \{4-24\}$ and $K = 18$ (lower frequency parts).

Fig. 5 presents two-dimensional (2D) contour plots of normalized magnitude response characteristics of the classical CIC filters and the proposed novel CIC FIR filter functions. As the value of the parameter N increases, as well as the normalized frequency increases, the benefits of the proposed filter class become less apparent, and the characteristics closely resemble that of the classical CIC filters. Therefore, it can be concluded that the proposed filter class is more efficient in lower part of frequency range and for smaller values of parameter N .

V. CONCLUSION

This paper deals with the novel CIC FIR filter functions designed by spreading the delays in the CIC filter comb stages. The delays are set around one specific value of filter order N .

A few design examples of the novel proposed CIC FIR filter functions are used to validate their characteristics by comparing them with those of the classical CIC filters under fair conditions. A way to do fair comparison is to design the filter functions for the same value of the group delay, and with the same number of cascaded sections.

Possible applications of this filter functions are in medicine and pharmacy, as well as in some special researches with high-resolution demands. The future research will be on real platforms of signal processing.

REFERENCES

[1] E. B. Hogenauer, “An economical class of digital filters for decimation and interpolation”, *IEEE Transactions on Acoustics, Speech and Signal Processing*, pp. 155-162, April 1981.

[2] W. A. Abu-Al-Saud, G. L. Stuber, “Modified CIC filter for sample rate conversion in software radio systems”, *IEEE Signal Processing Letters*, May 2003, vol.10, no.5, pp. 152-154.

[3] G. Jovanović Doleček, “Modified CIC filter for rational sample rate conversion”, *ECTI Transactions on Computer and Information Technology*, May 2010, vol. 4, no. 1, pp. 15-20.

[4] L. L. Presti, “Efficient modified-sinc filters for sigma-delta A/D converters”, *IEEE Transactions on Circuits and Systems II: Analog to Digital Signal Processing*, 2000, vol. 47, no. 11, pp. 1204-1213.

[5] M. Laddomada, “Comb-based decimation filters for ΣΔ A/D converters: Novel schemes and comparisons”, *IEEE Transactions on Signal Processing*, 2007, vol. 55, no. 5, pp. 1769-1779.

[6] G. Jovanović Doleček, S. K. Mitra, “A simple method for the compensation of CIC decimation filter”, *Electronics Letters*, vol. 44, no. 19. pp. 1162–1163, 2008.

[7] G. Jovanović Doleček, “A simple wideband CIC compensator”, *Electronics Letters*, vol. 45, no. 19, pp. 1270–1272, 2009.

[8] G. Jovanović Doleček, F. Harris, “Design of wideband CIC compensator filter for a digital IF receiver”, *Digital Signal Processing*, vol. 19, pp. 827-837, 2009.

[9] A. Fernandez-Vazquez, G. Jovanović Doleček, “Maximally flat CIC compensation filter: Design and multiplierless implementation”, *IEEE Transactions on Circuits and Systems II*, vol. 59, no. 2, pp. 113–117, 2012.

[10] G. Jovanović Dolecek, A. Fernandez-Vazquez, “Novel droop-compensated comb decimation filter with improved alias rejections”, *AEU - International Journal of Electronics and Communications*, vol. 67, pp. 387–396, 2013.

[11] G. Jovanović Doleček, A. Fernandez-Vazquez, “Trigonometrical approach to design a simple wideband comb compensator”, *AEU - International Journal of Electronics and Communications*, vol. 68, no. 5, pp. 437-441, May 2014.

[12] J. O. Coleman, “Chebyshev stopbands for CIC decimation filters and CIC-implemented array tapers in 1D and 2D”, *IEEE Transactions on Circuits and Systems I*, vol. 59, no. 12, pp. 2956–2968, 2012.

[13] B. P. Stošić, V. D. Pavlović, D. N. Milić, “Comparison of novel designed class of CIC FIR filter functions with classical CIC filters”, *XLIX International Scientific Conference on Information, Communication and Energy Systems and Technologies - ICEST 2014*, Serbia, Niš, June 25- July 27, 2014.

[14] V. D. Pavlović, D. N. Milić, B. P. Stošić, “Characteristics of novel designed class of CIC FIR filter functions over classical CIC filters”, *1st International Conference on Electrical, Electronic and Computing Engineering - IcETRAN 2014*, Serbia, Vrnjačka Banja, June 2-5, 2014.

[15] B. P. Stošić, V. D. Pavlović, “On design of a novel class of selective CIC FIR filter functions with improved response”, *AEU – International Journal of Electronics and Communications*, vol. 68, no. 8, pp. 720-729, 2014.

[16] B. P. Stošić, V. D. Pavlović, “Design of selective CIC filter functions”, *AEU – International Journal of Electronics and Communications*, vol. 68, no. 12, pp. 1231-1233, 2014.

[17] D. N. Milić, V. D. Pavlović, “A New Class of Low Complexity Low-Pass Multiplierless Linear-Phase Special CIC FIR Filters”, *IEEE Signal Processing Letters*, vol. 21, no. 12, pp. 1511-1515, 2014.