Stepped Triangular CIC Filter for Rational Sample Rate Conversion

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***Abstract*— The modification of the conventional CIC (cascaded–integrator-comb) filter for rational sample rate conversion (SRC) is presented here, where the conversion factor is a ratio of two mutually prime numbers. Specifically, we consider the case where the decimation factor *M* can be expressed as a product of two integers. The overall filter realization is based on a stepped triangular form of the CIC impulse response and the corresponding expanded cosine filter. This filter performs sampling rate conversion efficiently by using only additions/subtractions making it attractive for software radio (SWR) applications.**

***Keywords*—CIC filter, rational sample rate conversion, stepped triangular form, cosine filter, software radio.**

I. Introduction

The transfer function of the cascaded-integrator-comb (CIC) filter for sampling rate conversion (SRC) by a factor M/L (Fig.1.) is given by [1]

$$H(z)=\frac{(1-z^{-DM})^{K_1}(1-z^{-DL})^{K_2}}{(1-z^{-1})^{K}}, \qquad (1)$$

where D is the delay of each comb stage and $K=K_1+K_2$. This filter is very simple and uses only additions/subtractions. However, it has a limited number of tuning parameters, and does not provide enough attenuation in the region of interest in the stopband. Various methods have been proposed to improve the characteristics of the conventional CIC filter to make them suitable for software radio (SWR) applications, [2]-[7]. In [2] is proposed a modified CIC filter of the form

$$H_m(z)=\frac{(1-z^{-D_1})\,(1-z^{-D_2})\cdots(1-z^{-D_N})}{(1-z^{-1})^{K}}, \qquad (2)$$

where $D_1, D_2, \ldots, D_N$ is a set of comb delays. It has been shown that the modified CIC filter provides higher image attenuation than the conventional CIC filter. Additionally, the modified

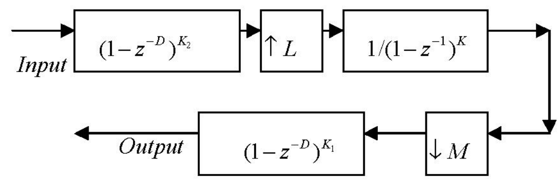

Figure 1. Conventional CIC filter for STC by *L/M*.

CIC filter provides higher SNR, where the SNR is defined as the power ratio after lowpass filtering of the lowest power level in the desired signal to the highest power level in the images, [2]. The objective of this work is to propose an alternate modification of the conventional CIC filter in order to achieve a higher attenuation of the images and an improved SNR than in [2] and perform sampling rate conversion by using only additions/subtractions. Specially, we consider the case where the decimation factor M is a product of two integers, i.e.,

$$M=M_1M_2. \qquad (3)$$

The proposed structure is based on a stepped triangular (ST) CIC form of the CIC impulse response and the corresponding expanded cosine filter. Section 2 introduces stepped triangular form of CIC filter and the expanded cosine filter. Section 3 describes the proposed filter and the corresponding efficient structure.

II. Stepped Triangular CIC Filter

We develop next the relation between the stepped triangular impulse response filter and the CIC filter of the order M

$$G(z)=\frac{1}{M}\frac{(1-z^{-M})}{1-z^{-1}}. \qquad (4)$$

In general, the transfer function of ST CIC filter is given by

$$H_{ST}(z)=\frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}}\;\frac{1}{N_2}\frac{1-z^{-N_1N_2}}{1-z^{-N_1}};\quad N_1N_2=N_{1,2}. \qquad (5)$$

Note that $N_{1,2}$ can be either equal to or different than M. Considering (3), Eq. (5) can be rewritten as

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$$H_{ST}(z)=\frac{1}{M_2}\frac{1\text{-}z^{-M}}{1-z^{-M_1}}\;\frac{1}{M_1}\frac{1\text{-}z^{-M_1}}{1-z^{-1}}\;\frac{1}{N_2}\frac{1-z^{-N_1N_2}}{1-z^{-N_1}}\quad . \tag{6}$$

We relate N_1 and N_2 with M_1 and M_2 so that a better magnitude characteristics of the overall filter are obtained while at the same time the subfilters in Eq. (6) can be moved to a lower rate part using multirate identity [8].
Consequently, from Eq. (6) we arrive at

$$H_{ST}(z)=\begin{cases}\dfrac{1}{M_2}\dfrac{1\text{-}z^{-M}}{1-z^{-M_1}}\left[\dfrac{1}{M_1}\dfrac{1-z^{-M1}}{1-z^{-1}}\right]^2 & for\ N_2=M_1 \\ & N_1=1 \\ \left[\dfrac{1}{M_2}\dfrac{1\text{-}z^{-M}}{1-z^{-M_1}}\right]^2\dfrac{1}{M_1}\dfrac{1-z^{-M1}}{1-z^{-1}} & for\ N_2=M_2 \\ & N_1=M_1\end{cases} \quad . \tag{7}$$

N_2 in Eq. (7) is the number of levels in ST impulse response. More levels result in better magnitude characteristics. To this end we choose

$$H_{ST}(z)=\begin{cases}\dfrac{1}{M_2}\dfrac{1\text{-}z^{-M}}{1-z^{-M_1}}\left[\dfrac{1}{M_1}\dfrac{1-z^{-M1}}{1-z^{-1}}\right]^2 & for\ M_1>M_2 \\ \left[\dfrac{1}{M_2}\dfrac{1\text{-}z^{-M}}{1-z^{-M_1}}\right]^2\dfrac{1}{M_1}\dfrac{1-z^{-M1}}{1-z^{-1}} & for\ M_1<M_2\end{cases} \quad . \tag{8}$$

Example 1:

We consider ST CIC filter for M=12.

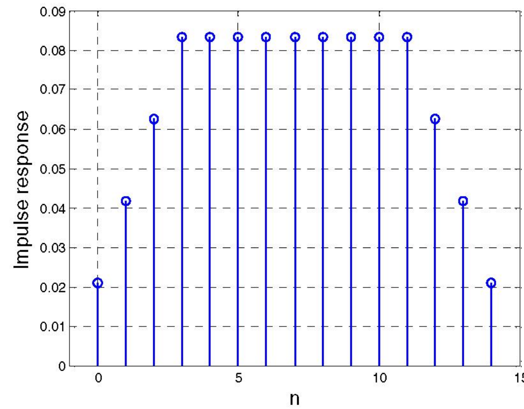
From Eq. (8) we have

$$H(z)=\begin{cases}\dfrac{1}{3}\cdot\dfrac{1-z^{-12}}{1-z^{-4}}\left[\dfrac{1}{4}\cdot\dfrac{1-z^{-4}}{1-z^{-1}}\right]^2 & \text{for } M_1=4, M_2=3, \\ \left[\dfrac{1}{4}\cdot\dfrac{1-z^{-12}}{1-z^{-4}}\right]\cdot\dfrac{1}{3}\cdot\dfrac{1-z^{-3}}{1-z^{-1}} & \text{for } M_1=3, M_2=4.\end{cases}$$

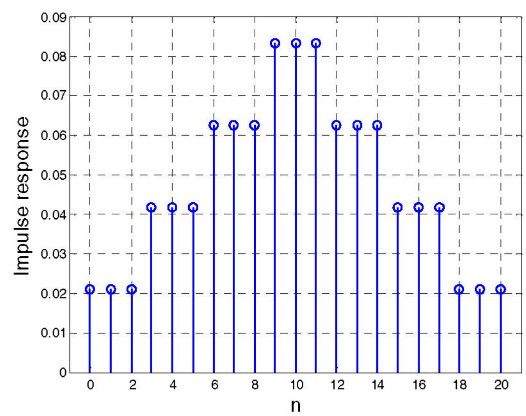
The corresponding impulse responses are shown in Fig. 2(a) and (b), while the associated gain responses are presented in Fig. 2 (c), along with the gain response of the overall CIC filter. It can been seen that the stopband attenuation has increase in the level N_2 and a decrease in N_1.

III. PROPOSED FILTER

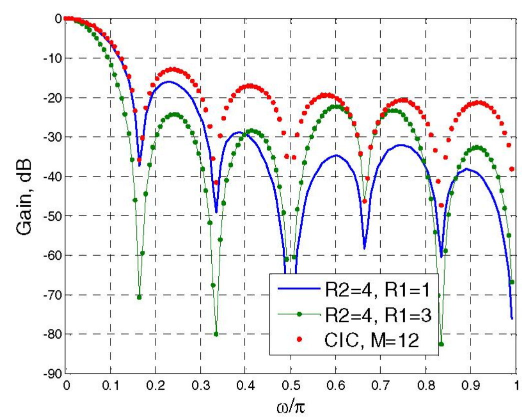
Consider sampling rate conversion by a factor L/M where $M > L$ and the decimation factor can be expressed as in Eq. (3). We consider a decimation filter which provides enough attenuation of the aliasing caused by down-sampling and also enough attenuation of images introduced by interpolation. It is assumed that the signal occupies ¾ of the available band.



(a) R_2= M_1=4, R_1=1, M_2=3.



(b) N_2=M_2=4, N_1=M_1=3.



(c) Gain responses.

Figure 2. Example 1.

Additionally, we propose to cascade an expanded cosine filter $H_{COS}(z)$ with ST CIC filter of Eq. (8) to introduce an additional zero in the frequency band where the worst case of aliasing occurs so as to increase the SNR. The expanded filter is given by

$$H_{\cos}(z) = (1 + z^{-R})/2 \tag{9}$$

where

$$R_{\min} \le R \le M\ . \tag{10}$$

The value of R which is equal to R_{min} from Eq. (10) usually results in higher aliasing attenuation. This value is computed as using

$$R_{\min} = \mathrm{int}\left[1/\omega_1\right] \tag{11}$$

where int[.] means integer part of [.] and

$$\omega_1 = \frac{2}{M} - \omega_p;\ \omega_p = \frac{3}{4}\frac{1}{M}\ . \tag{12}$$

The magnitude characteristic of $H_{COS}(z)$ is given as

$$H_{\cos}(\omega) = \cos(R\omega/2)\ . \tag{13}$$

Using Eqs. (8) and (9) we arrive at the transfer function of the proposed decimation filter

$$H_p(z) = \left[\frac{1}{M_2}\frac{1-z^{-M}}{1-z^{-M_1}}\right]^{k_1}\left[\frac{1}{M_1}\frac{1-z^{-M_1}}{1-z^{-1}}\right]^{k_2}\left[\frac{(1+z^{-R})}{2}\right]^{k_3}, \tag{14}$$

where k_1, k_2 and k_3 denote the corresponding stages and

$$\begin{aligned} k_2 &\ge 2k_1 \quad for\ M_1 > M_2 \\ k_1 &\ge 2k_2 \quad for\ M_1 < M_2 \end{aligned}\ . \tag{15}$$

Denoting

$$H_{p1}(z) = \left[1 + z^{-R}\right]^{k_3}\left[\frac{1}{1-z^{-1}}\right]^{k_2} \tag{16}$$

$$H_{p2}(z) = \left[1 - z^{-1}\right]^{k_2-k_1};H_{p3}(z) = \left[1 - z^{-1}\right]^{k_1} \tag{17}$$

and making use of the multirate identity [8] we have the final structure shown in Fig. 3 (a).

The structure consists of three stages. In general, the first stage is a cascade of k_3 combs with a delay R and k_2 integrators. The next stage is a cascade of (k_2-k_1) combs or integrators as indicated in Eq. (16). Finally, the last stage is a cascade of k_1 combs.

Note that for R=M, k_3 combs from first stage can be moved to the last stage with an unity delay. Similarly for R=R_1M_1, where R_1 is an integer, the cascade of combs with delay R can be moved from the first stage to the second stage with a delay of R_1. The complexity of the proposed filter, presented in terms of their memory requirements and number of additions (or subtractions) per output sample (APOS), is given in Tables I and II.

TABLE I. M_1>M_2

R	Memory req.	APOS
R=$R_{\min}$	$4k_1$+Rk_3	k_1+k_1M_2+(k_3+$2k_1$)M
R=R_1M_1	$4k_1$+R_1k_3	k_1+(k_1+k_3)M_2+$2k_1M$
R=M	$4k_1$+k_3	k_1+k_3+k_1M_2+$2k_1M$

TABLE II. M_1<M_2

R	Memory req.	APOS
R=$R_{\min}$	$4k_2$+Rk_3	$2k_2$+k_2M_2+(k_3+k_2)M
R=R_1M_1	$4k_2$+R_1k_3	$2k_2$+(k_2+k_3)M_2+k_2M
R=M	$4k_2$+k_3	$2k_2$+k_3+k_2M_2+k_2M

In the following example we compare the proposed structure with the one proposed in [2].

Example 2:

We apply the design of a sampling rate converter for a conversion factor of 9/10 [2].

From Eqs. (11) and (12) we have

$$\omega_1 = \frac{2}{10} - \frac{3}{4} \times \frac{1}{10} = 0.125$$

and R_{min}=8.

We choose M_1=2 and M_2=5, and R=8.

Using k_3=1, k_1=8 and, k_2=4 from Eqs. (13) and (14) we have

$$H_p(z) = \left[\frac{1}{5}\frac{1-z^{-10}}{1-z^{-2}}\right]^8\left[\frac{1}{2}\frac{1-z^{-2}}{1-z^{-1}}\right]^4\left[\frac{(1+z^{-8})}{2}\right]$$

The corresponding gain responses are shown in Fig.4.

The conventional CIC has two interpolation combs, two interpolation integrators, two decimation combs, and two decimation integrators.

The modified CIC filter [2] of the same order N=4 has four combs with the delays 16, 14, 12 and 10, and four integrators, all working at the high input rate.

The corresponding SNRs are:

15 dB (Conventional CIC),

50 dB (Modified CIC [2]), and

68.9 dB (proposed), respectively.

The modified CIC filter requires:

16+14+12+10+4=56

memory elements, while the proposed filter requires 20 memory elements (Table II).

The proposed and, the modified CIC filters require 73 and 80 APOS, respectively. Therefore the proposed filter exhibits better performance while having a lower complexity.

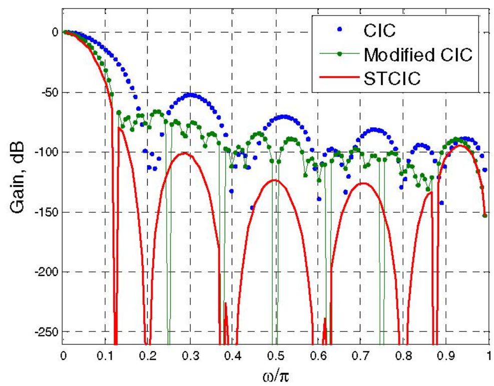


Figure 4. Example 2: M_1=2, M_2=5, k_1=8, k_2=4, k_3=1.

IV. CONCLUDING REMARKS

This paper has presented a new sampling rate conversion filter for a rational conversion factor where interpolation and decimation factors are mutually prime numbers. The proposed filter is based on the stepped triangular CIC filter and the expanded cosine filter. The expansion factor is typically chosen to decrease the aliasing in the band where the worst case of aliasing occurs. We consider the case that the decimation factor M is product of factors M_1 and M_2. This enables the decimation to take place in two stages: decimation by M_1 followed by decimation of M_2. Better performances are obtained for $M_1 < M_2$. The resulting structure has three stages of integrators and/or combs and exhibits lower complexity and better performances than the modified CIC filter introduced in [2] and, consequently it is a good candidate for software radio (SWR) applications. Passband droop in the output signals can be corrected using simple IIR filter with two multiplications per output sample [2].

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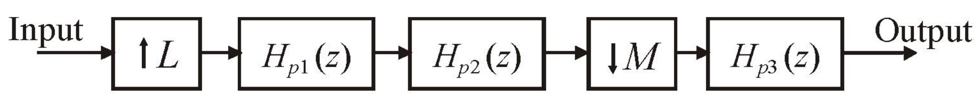
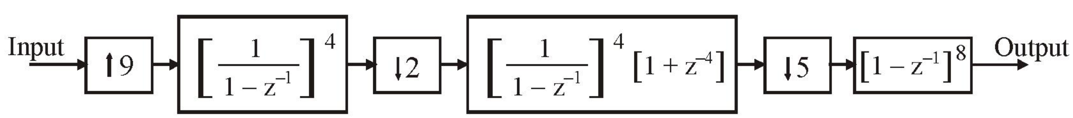


(a) General structure.



(b) Example 1: SRC 9/10, M_1=2, M_2=5.

Figure 3. .Proposed structure.