314653006
hw4

P9 $f(-1)=1,\ f'(-1)=1,\ f'(1)=2,\ f(2)=1.$

let $H_3(x)= a_3x^3+a_2x^2+a_1x+a_0$

then $H_3'(x) = 3a_3x^2+2a_2x+a_1$, fit the give data, and write as matrix

let $M = \begin{bmatrix} -1 & 1 & -1 & 1 \\ 3 & -2 & 1 & 0 \\ 3 & 2 & 1 & 0 \\ 8 & 4 & 2 & 1 \end{bmatrix}$, $M \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}$,

Since $\det(M)=0$, inverse of M not exist,

therefore, there is no solution for $\begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$, and. H_3 not exist. #

P12 By long division.

$$r(x) = \frac{a_0+a_2x^2+a_4x^4}{1+b_2x^2} = a_0 + (a_2-b_2)x^2 + \left(a_4+\tfrac{1}{2}b_2\right)x^4 - \tfrac{1}{24}b_2x^6 + r_8x^8 + \cdots$$

compare the coefficients with $f(x)$,

Solve $\begin{cases} 1 = a_0 \\ -\frac{1}{2} = a_2-b_2 \\ \frac{1}{24} = a_4+\frac{1}{2}b_2 \\ -\frac{1}{6!} = -\frac{1}{24}b_2 \end{cases} \Rightarrow \begin{cases} a_0 = 1 \\ b_2 = \frac{1}{30} \\ a_2 = -\frac{7}{15} \\ a_4 = \frac{1}{40} \end{cases}$ #