#P1. $u'' = f$, $u(0)=0$, $u(1)=0$, $f(x) = \begin{cases} 1, & 0.4 \le x \le 0.6 \\ 0, & \text{o.w.} \end{cases}$

Since f is constant, guess a like.

$$u = \begin{cases} ax+b, & x < 0.4 \\ cx^2+dx+e, & 0.4 \le x \le 0.6 \\ fx+g, & x > 0.6 \end{cases}$$

since $u(0)=u(1)=0$, $b=0$, $g=-f$.

$$u' = \begin{cases} a, & x < 0.4 \\ 2cx+d, & 0.4 \le x \le 0.6 \\ f, & x > 0.6 \end{cases}$$

check $u'(0.4) \Rightarrow a = 0.8c + d$.

$u(0.4) \Rightarrow 0.16c + 0.4d + e = a$.

$u'(0.6) \Rightarrow 1.2c + d = f$.

$u(0.6) \Rightarrow 0.36c + 0.6d + e = -0.4f$.

$u''(0.4) \Rightarrow 2c = 1 \Rightarrow c = 0.5$

Solve $\begin{cases} a = -0.1, b = 0 \\ c = 0.5, d = -0.5, e = 0.08 \\ f = 0.1, g = -0.1 \end{cases}$

$$\Rightarrow u = \begin{cases} -0.1x, & x < 0.4 \\ 0.5x^2 - 0.5x + 0.08, & 0.4 \le x \le 0.6 \\ 0.1x - 0.1, & x > 0.6 \end{cases}$$