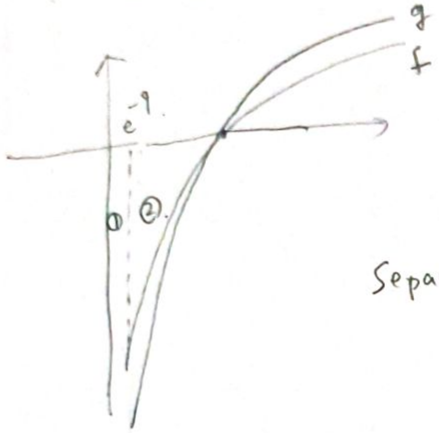
Programming

P1 there is a closed-form solution.

$$\int_0^\infty \frac{1}{1+25x^2}\,dx = \frac{1}{5}\tan^{-1}(5x) + C \Big|_0^\infty = \frac{\pi}{10}.$$

By computer, $\boxed{\text{val} = 0.3141\ldots,\ \text{err.} = -4.4408\times 10^{-16}}$

P2 There isn't a closed-form for $\displaystyle\int_0^1 \frac{\ln(x)}{1+25x^2}\,dx$.



let $f(x) = \dfrac{\ln(x)}{1+25x^2}$, $g(x) = \ln(x)$.

Separate $\displaystyle\int_0^1 \frac{\ln(x)}{1+25x^2}\,dx = \int_0^{e^{-10}} \frac{\ln(x)}{1+25x^2}\,dx + \int_{e^{-10}}^1 \frac{\ln(x)}{1+25x^2}\,dx$

① left. Since $\displaystyle\lim_{x\to 0} \frac{\ln(x)}{\left(\frac{\ln(x)}{1+25x^2}\right)} = 1$, use $\ln(x)$ when close to 0^+.

Since $\displaystyle |\ln(x)| \geq \left|\frac{\ln(x)}{1+25x^2}\right| \geq \left|\frac{\ln(x)}{1+25e^{-20}}\right|$ $\forall x \in (0, e^{-10}]$,

And both function are negative in the domain,

$$\left|\ln(x) - \frac{\ln(x)}{1+25x^2}\right| \leq \frac{25e^{-20}}{1+25e^{-20}}|\ln(x)|$$

Val of left : $-0.0004\ldots$, err : 2.5733×10^{-11}

② Use trapezoid to calculate $\displaystyle\int_{e^{-10}}^1 \frac{\ln(x)}{1+25x^2}\,dx$, with 10^7 equally spaced steps.

Consider each step as single interval $\left(a, a+\frac{1-e^{-10}}{10^7}\right)$ to calculate the error,

And since f is strictly increasing domain of f_i is $\left(a, a+\frac{1-e^{-10}}{10^7}\right)$

$$|E(f_i)| \leq \left|-\frac{1}{12}\cdot(10^{-7})^3 f''(k_i)\right|,$$ where $k_i = \min\left(a, a+\frac{(1-e^{-10})}{10^7}\right)$, $a = e^{-10} + \frac{1-e^{-10}}{10^7}\cdot i$

$$\Rightarrow |E(f)| \leq \left|-\frac{10^{-4}}{12}\sum_{i=0}^{10^7-1} f''\left(e^{-10} + \frac{(1-e^{-10})}{10^7}\cdot i\right)\right|.$$

Val of right : $-0.5449\ldots$
err : 1.8376×10^{-11}

Total val : $-0.5454\ldots$
err : 4.4109×10^{-11}