Michael Einreinhof

Homework #08

Convective Boundary Conditions

Problem 1: Derive the one-dimensional equation

$$\rho c_p V \frac{dT}{dt} = kA\frac{dT}{dx} + kA\frac{dT}{dy} \qquad F_o = \frac{\alpha \Delta t}{\Delta x^2} \qquad B_i = \frac{h\Delta t}{k}$$

middle

$$\frac{dT}{dt} = \frac{kA}{\rho c_p V}\frac{dT}{dx} + \frac{kA}{\rho c_p V}\frac{dT}{dy} \qquad \alpha = \frac{k}{\rho c_p}$$

$$\frac{dT}{dt} = \frac{k}{\rho c_p \Delta x^2} dT + \frac{k}{\rho c_p \Delta y^2} dT$$

$$dT = \frac{k\Delta t}{\rho c_p \Delta x^2} dT + \frac{k\Delta t}{\rho c_p \Delta x^2} dT$$

$$dT = F_o dT + F_o dT$$

$$T - T_{i-1} = F_o\left(T_{i-1,j-1} - 2T_{i-1} + T_{i-1,j+1}\right) + F_o\left(T_{i-1,k-1} - 2T_{i-1} + T_{i-1,k+1}\right)$$

$$T = T_{i-1} + F_o T_{i-1,j-1} - 2F_o T_{i-1} + F_o T_{i-1,j+1} + F_o T_{i-1,k-1} - 2F_o T_{i-1} + F_o T_{i-1,k+1}$$

$$T = (1-4F_o)T_{i-1} + F_o\left(T_{i-1,j-1} + T_{i-1,j+1} + T_{i-1,k-1} + T_{i-1,k+1}\right)$$

Problem 2: Derive the equation for a 2-D plane surface with convection

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$$\rho c_p V \frac{dT}{dt} = kA\frac{dT}{dx} + kA\frac{dT}{dy} + hA(T_a - T)$$

sides

$$V = \frac{\Delta x \Delta y \Delta z}{2} \qquad A = \frac{\Delta x \Delta z}{2} \qquad F_o = \frac{\alpha \Delta t}{\Delta x} \qquad \alpha = \frac{k}{\rho c_p}$$

$$A = \Delta y \Delta z \qquad Bi = \frac{h\Delta x}{k}$$

$$\frac{dT}{dt} = \frac{2k\,\Delta y\Delta z}{\rho c_p\,\Delta x\Delta y\Delta z}\frac{dT}{dx} + \frac{k\,\frac{\Delta x\Delta z}{2}}{\rho c_p\,\frac{\Delta x\Delta y\Delta z}{2}}\frac{dT}{dy} + hA(T_a - T)$$

$$\frac{dT}{dt} = \frac{2k}{\rho c_p \Delta x^2}dT + \frac{k}{\rho c_p \Delta y^2}dT + 2B_iF_o(T_a - T)$$

$$dT = \frac{2k\,\Delta t}{\rho c_p \Delta x^2}dT + \frac{k\Delta t}{\rho c_p \Delta y^2}dT + 2B_iF_o(T_a - T)$$

$$dT = 2F_o dT + F_o dT + 2B_iF_o(T_a - T)$$

$$T - T_{i-1} = 2F_o(T_{i-1,j-1} - T_{i-1,j}) + F_o(T_{i-1,k-1} - 2T_{i-1} + T_{i-1,k+1}) + 2B_iF_oT_a - 2B_iF_oT_{i-1}$$

$$T = T_{i-1} + 2F_oT_{i-1,j-1} - 2F_oT_{i-1} + F_oT_{i-1,k-1} - 2F_oT_{i-1} + F_oT_{i-1,k+1} + 2B_iF_oT_a - 2B_iF_oT_{i-1}$$

$$T = (1 - 4F_o - 2B_iF_o)T_{i-1} + 2F\left(T_{i-1,j-1} + \tfrac{1}{2}(T_{i-1,k-1} + T_{i-1,k+1}) + B_iT_a\right)$$

Problem 3: derive the equation for the exterior corner

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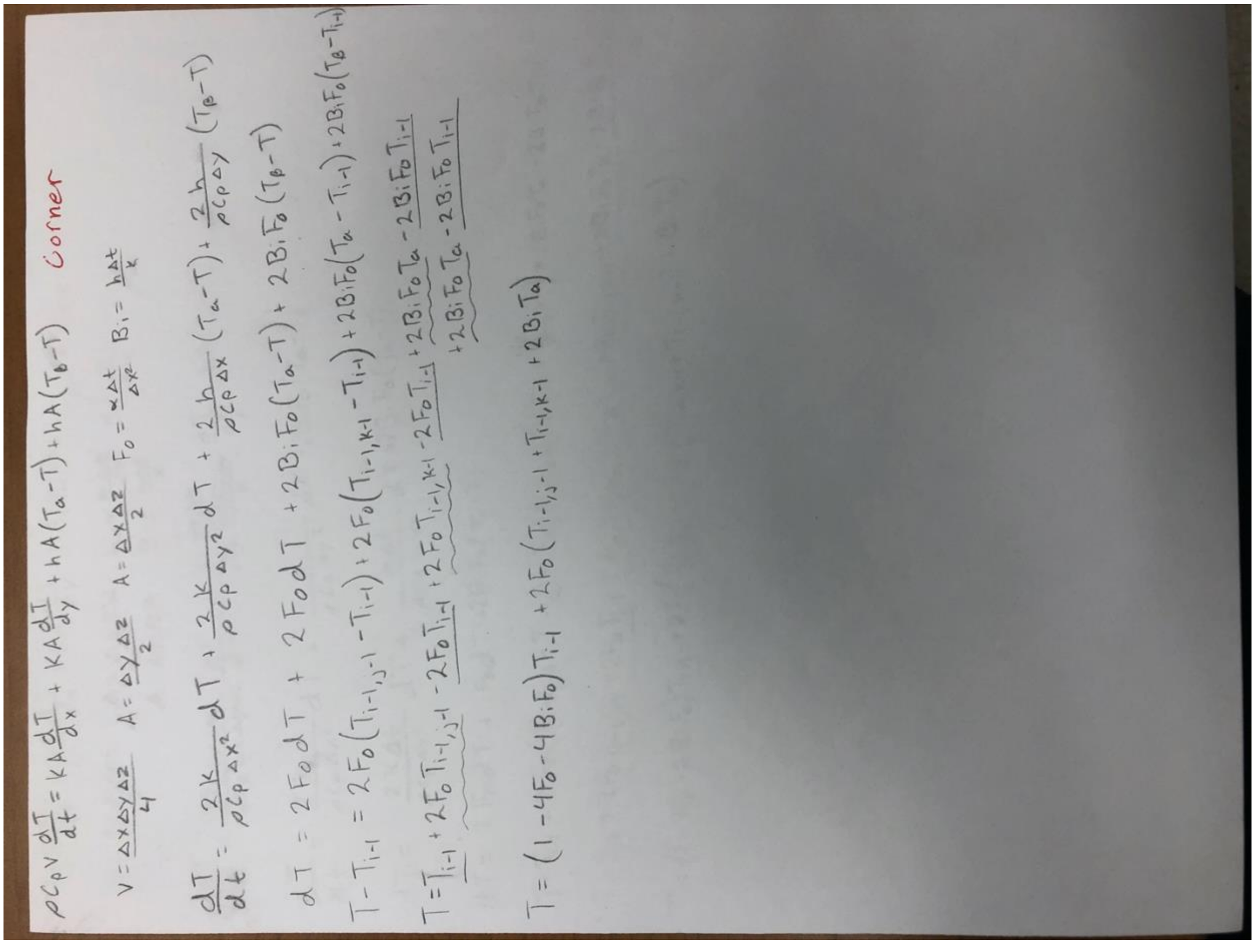

$$\rho c_p V \frac{dT}{dt} = kA\frac{dT}{dx} + kA\frac{dT}{dy} + hA(T_a - T) + hA(T_B - T)$$ Corner

$$V = \frac{\Delta x \Delta y \Delta z}{4} \quad A = \frac{\Delta y \Delta z}{2} \quad A = \frac{\Delta x \Delta z}{2} \quad F_o = \frac{\alpha \Delta t}{\Delta x^2} \quad B_i = \frac{h \Delta t}{k}$$

$$\frac{dT}{dt} = \frac{2k}{\rho c_p \Delta x^2} dT + \frac{2k}{\rho c_p \Delta y^2} dT + \frac{2h}{\rho c_p \Delta x}(T_a - T) + \frac{2h}{\rho c_p \Delta y}(T_B - T)$$

$$dT = 2F_o dT + 2F_o dT + 2B_iF_o(T_a - T) + 2B_iF_o(T_B - T)$$

$$T - T_{i-1} = 2F_o(T_{i-1,j-1} - T_{i-1}) + 2F_o(T_{i-1,k-1} - T_{i-1}) + 2B_iF_o(T_a - T_{i-1}) + 2B_iF_o(T_B - T_{i-1})$$

$$T = T_{i-1} + 2F_oT_{i-1,j-1} - 2F_oT_{i-1} + 2F_oT_{i-1,k-1} - 2F_oT_{i-1} + 2B_iF_oT_a - 2B_iF_oT_{i-1} + 2B_iF_oT_a - 2B_iF_oT_{i-1}$$

$$T = (1 - 4F_o - 4B_iF_o)T_{i-1} + 2F_o(T_{i-1,j-1} + T_{i-1,k-1} + 2B_iT_a)$$

Problem 4: Snickers Bar

Equations Used:

eqTSide(snickerBlock,fo,bi,j,k,ambientTemp)

eqBSide(snickerBlock,fo,bi,j,k,ambientTemp)

eqLSide(snickerBlock,fo,bi,j,k,ambientTemp)

eqLCorner(snickerBlock,fo,bi,j,k,ambientTemp)

eqRCorner(snickerBlock,fo,bi,j,k,ambientTemp)

eqRSide(snickerBlock,fo,bi,j,k,ambientTemp)

eqLBCorner(snickerBlock,fo,bi,j,k,ambientTemp)

eqRBCorner(snickerBlock,fo,bi,j,k,ambientTemp)

eqCenter(snickerBlock,fo,j,k,ambientTemp)

Temperature Profile Plots:

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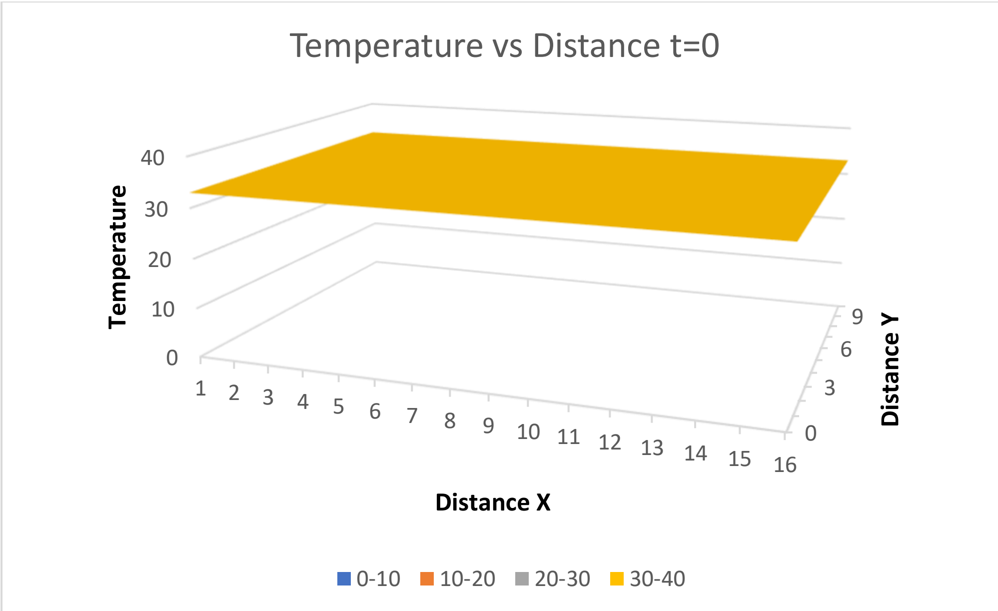

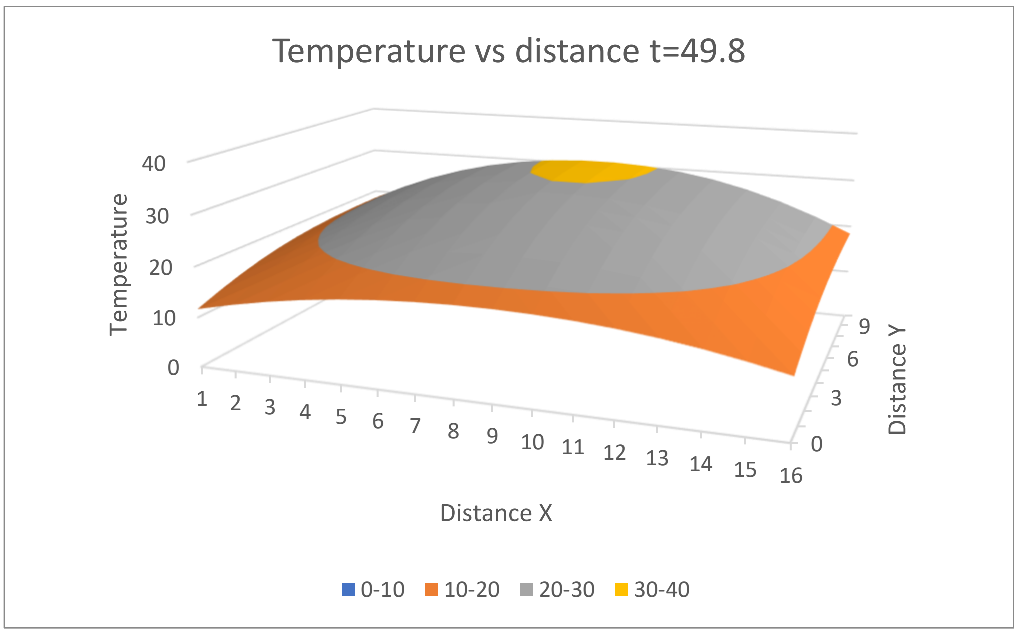

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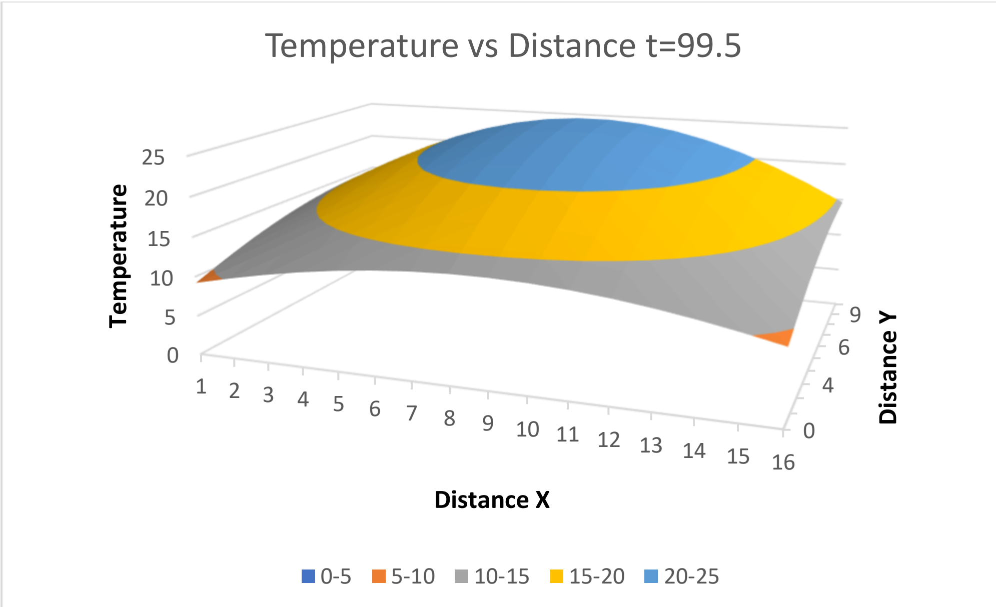

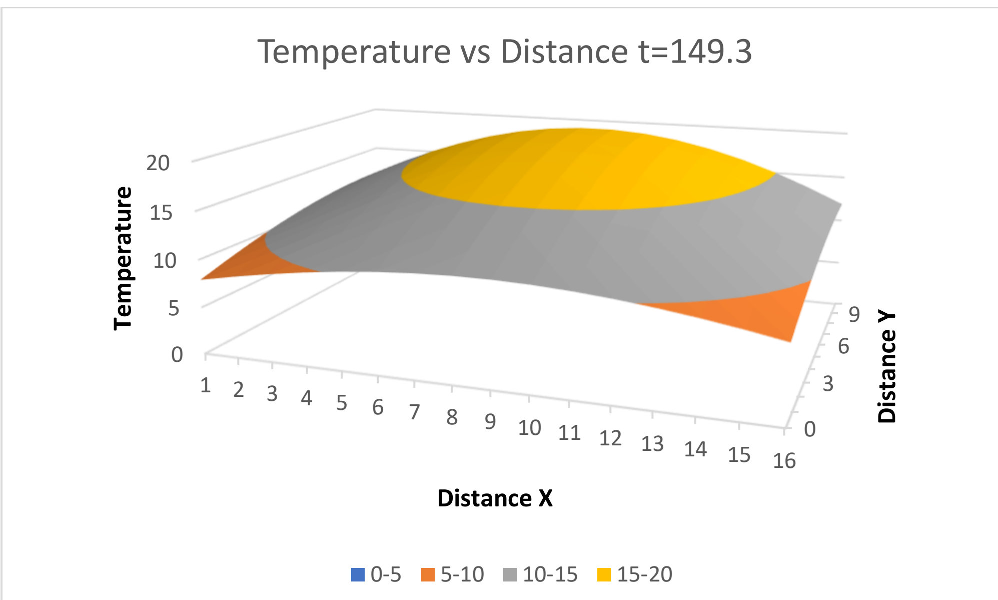

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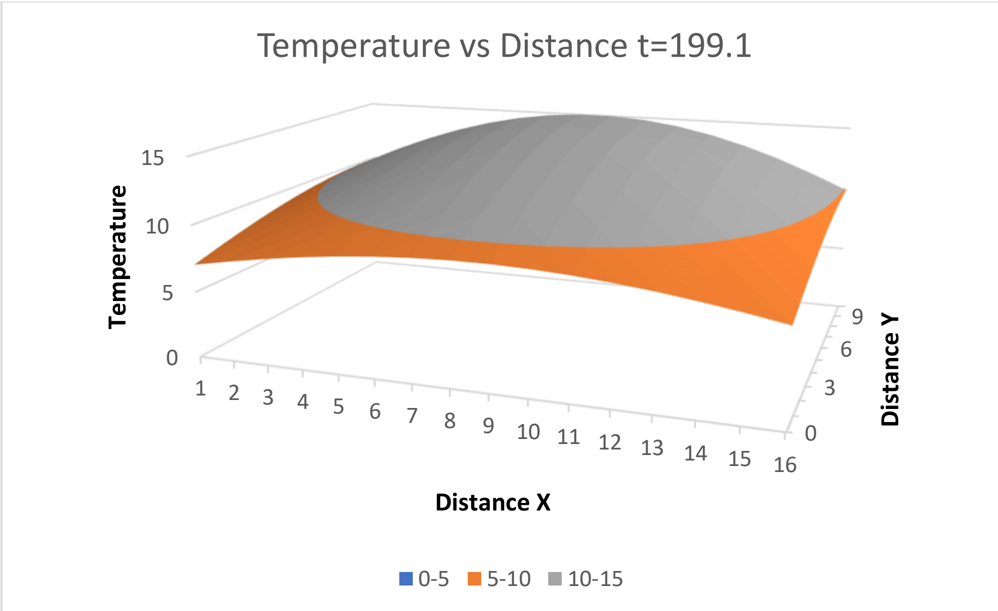

<u>How long does the tunnel need to be?</u>

Time=199.1s*(1min/60s)=3.318min

Length = 1m/min*(3.318min) = <u>3.318meters</u>