MEAM 620 Homework 2

Due: February 18, 2015, 11:59pm

1. List two advantages and two disadvantages each for using (a) rotation matrices;(b) axis angle representation; (c) exponential coordinates; (d) Euler angles; and (e) quaternions to describe rotations?

2. Consider the problem of fitting a smooth curve to the following waypoints in 2D:

$$\begin{aligned} t_0 &= 0, (x_0, y_0) = (-1, 0) \\ t_1 &= 5, (x_1, y_1) = (0, 2) \\ t_2 &= 6, (x_2, y_2) = (1, 0) \\ t_0 &= 0, (\dot{x}_0, \dot{y}_0) = (-1, -5) \end{aligned}$$

Note that the first three constraints are position constraints, while the last is a velocity constraint. Any other necessary derivative constraints at $t_0 = 0$ and $t_2 = 6$ should be set to $(0, 0)$. To minimize the functional:

$$\int_{t=0}^{T} \|x^{(n)}\|^2 dt, \tag{1}$$

the endpoints need to be constrained in position, velocity, and up to and including the $(n-1)$st derivative. All derivatives (velocity, acceleration, etc.) at $t_1 = 5$ should be left unspecified, and you will need to add the appropriate number of continuity constraints at that point. Also, note you will need to find $x(t)$ and $y(t)$.

a. A minimum acceleration trajectory can be constructed by fitting a cubic spline. Construct this trajectory for the waypoint constraints above. Explicitly write down the solution you find (ie. write down $x(t) = c_0 + c_1 t + c_2 t^2 \ldots$, where you fill in $c_0, c_1, c_2, \ldots$ with the coefficient values you found) and create a plot illustrating each trajectory and the waypoints. Include your Matlab code with your submission.

b. What is the minimum order polynomial you need to construct a minimum jerk trajectory? Construct this trajectory, write down your solution, and create a plot as in part a.

c. What is the minimum order polynomial you need to construct a minimum snap trajectory? Construct this trajectory, write down your solution, and create a plot.

d. Name one advantage and one disadvantage of choosing a lower order polynomial over a higher one.

3. Calculate the angular velocities ω^s and ω^b for the rotation:

$$R = e^{\hat{\omega}_1 t} e^{\hat{\omega}_2 t} \tag{2}$$

4. What skew-symmetric matrix $\hat{\omega} \in so(3)$ corresponds to the rotation

$$R = \begin{bmatrix} -0.3038 & -0.6313 & -0.7135 \\ -0.9332 & 0.3481 & 0.0893 \\ 0.1920 & 0.6930 & -0.6949 \end{bmatrix}$$

Is it unique?