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Omni-directional Walking of a Quadruped Robot with Optimal Body Postures on a Slope

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***Abstract*— In this paper, we discuss the optimal body postures of a quadruped robot to perform omni-directional static walking on a slope. The optimal body posture is the posture with the maximum possible moving speed w.r.t. slope and moving direction. The proposed method based on dynamically changing body posture during gait-transitions, is used to maintain high robot motion velocity on slope. The timing of changing body posture is designed by considering the stability during gait-transition. Using the proposed method, the robot can walk into any direction with the fastest moving speed on a slope. Through walking experiments by computer simulation, the validity of the proposed method has been verified.**

***Index Terms*— Quadruped Robot; Omni-directional Walking; Optimal Body Posture; Successive Gait-transition**

I. Introduction

Quadruped robots have advantages over wheeled robots in walking on an irregular terrain and walking into any direction. Researches on quadruped robots have been widely carried out [1]–[3]. Fukuoka et al. realized dynamic walking of a quadruped robot on irregular terrain using a neural system method [4]. However, the robot should rather aim for high stability than achieving high speed when the task is conveyance. This is the reason focusing on the static walking in our study. For increasing stability, Tsukakoshi et al. proposed the intermittent crawl gait [5] and Konno et al. proposed an adaptive intermittent crawl gait [6]. However, when a quadruped robot walks on a slope with these gaits, the moving speed will become very slow. This is why the zigzag trajectory or the 4-leg supporting period is used.

We have discussed the successive gait-transition method [7] for a quadruped robot to realize omni-directional static walking. Furthermore, we extended it in order that the robot may walk into any direction on a slope [8]. Using this method, the robot successively performs gait-transition among crawl gaits and rotation gaits with the least number of steps using the *common foot position* (*CFP*: a leg position common to two gaits before and after gait-transition). When we think of walking on a slope, it is required to keep the body posture horizontal when the task for the robot is conveyance. But in such case, there will occur a problem that the leg movable region becomes narrow and the moving speed will become slow. In order to improve the moving speed of robot, we discussed the body posture with the largest leg movable region [8]. That is the body posture paralleled to the slope. However in this case, it is necessary to shift the CFP to perform the successive gait-transition.

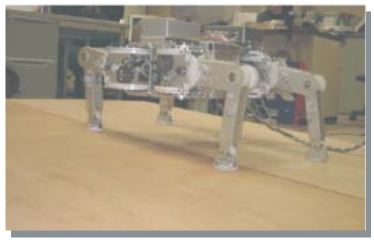


Fig. 1. A quadruped robot: TITAN-VIII

The available leg movable region will become narrow, and therefore, moving speed also decreases. According to this reason, it can be stated that it is possible to derive an optimal body posture that gives the fastest moving speed. Igarashi et al. proposed the *trajectory following gait (TFG)* [9] [10] in which a robot changes its body posture to avoid obstacles. However, optimality with regard to moving speed was not discussed and omni-direction walking with optimal body posture on a slope was not realized.

In this paper, we propose a method for quadruped robots integrating (1) static walking with high environmental adaptability, (2) omni-directional movement and (3) high moving speed. In Section II, each setting for realizing omni-directional walking on a slope with specified body posture is described. In Section III, a search method of the optimal body posture that gives the fastest moving speed on a slope is introduced. In Section IV, a method for successive gait-transition on a slope with the optimal body posture is discussed. In Section V, the validity of the proposed method is demonstrated by computer simulations.

II. Omni-directional Walking on a Slope

In this study, we utilize the model of a quadruped robot: TITAN-VIII [11] (Fig.1). We assume the mechanical leg movable area to be an octahedron for ease of calculation. However, the method proposed in this paper is applicable for any leg movable area. As shown in Fig.2, the leg movable region R_i on a slope is a polygon resulted from intersection of leg movable areas and the slope.

A. Coordinate systems

In order to consider the omni-directional walking of a quadruped robot on a slope, we set coordinate systems as shown in Fig.3. All coordinate systems are right-hand coordinate systems. We assume the slope with a fixed inclination angle ψ, and that the height from the center of

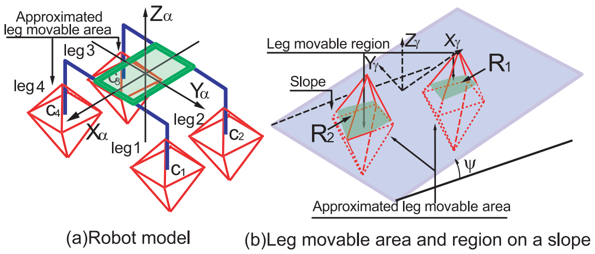


Fig. 2. Robot model, leg movable area (a) and region (b)

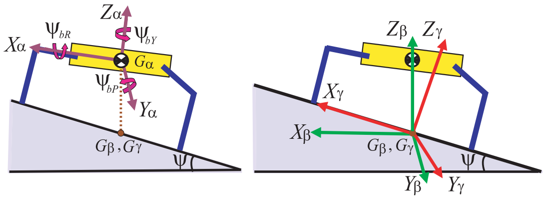


Fig. 3. Coordinate systems Σ_α, Σ_β and Σ_γ

gravity (COG) to slope is a fixed value h. The body posture is determined by the rotations around 3 axes, *Roll*, *Pitch* and *Yaw* (denoted as ψ_{bR}, ψ_{bP}, ψ_{bY}). In this study, gaits are first planned on slope coordinate, and then transformed into ones on body coordinate, to control the robot.

Body coordinate Σ_α : The origin G_α is located at COG, its X-axis is in the front direction of the body, its Y-axis is in the left direction and Z-axis is in the top direction.

Horizontal surface coordinate Σ_β : The origin G_β is an intersection of the gravity direction from COG and slope. Its Z-axis is normal to the horizontal plane, and its X-axis is in the direction of orthogonal projection of the X_α-axis to horizontal plane.

Slope coordinate Σ_γ : The origin G_γ is common to G_β. The directions of Σ_γ's axes are the one resulted after rotating Σ_β around Y_β-axis in the angle ψ and around Z_β-axis in the angle ψ_{bY}.

The homogeneous transformation matrices between co-ordinates can be described as

$$ {}^{\alpha}\mathbf{T}_{\beta} = \left[\begin{array}{c|c} \mathbf{E}^{\mathbf{j}\psi_{bP}}\mathbf{E}^{\mathbf{i}\psi_{bR}} & \mathbf{E}^{\mathbf{j}\psi_{bP}}\mathbf{E}^{\mathbf{i}\psi_{bR}}\mathbf{p} \\ \hline 0 \quad 0 \quad 0 & 1 \end{array}\right], \tag{1} $$

$$ {}^{\beta}\mathbf{T}_{\gamma} = \left[\begin{array}{c|c} \mathbf{E}^{\mathbf{j}\psi}\mathbf{E}^{\mathbf{k}\psi_{bY}} & 0 \\ \hline 0 \quad 0 \quad 0 & 1 \end{array}\right], \tag{2} $$

$$ {}^{\alpha}\mathbf{T}_{\gamma} = {}^{\alpha}\mathbf{T}_{\beta}{}^{\beta}\mathbf{T}_{\gamma} \tag{3} $$

where $\mathbf{p}$ ($\mathbf{p} = \left[\begin{array}{ccc} 0 & 0 & -h \end{array}\right]^T$) is the vector from the origin of Σ_α to that of Σ_β.

B. Common foot position (CFP) on a slope

In order to perform the gait-transition with the least number of steps, the feet trajectory before and after gait-transition are designed to have common position as shown in Fig.4. When such CFP is set, robot can walk into any direction from a stop state with non negative stability, no matter which leg swings first.

In the case walking on a horizontal plane, the point of vertical projecting of COG to the ground is same as the intersection of two diagonal lines of supporting legs. But

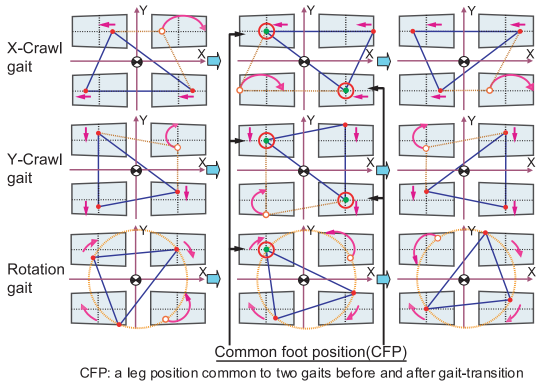


Fig. 4. CFP for each walking gait on a slope

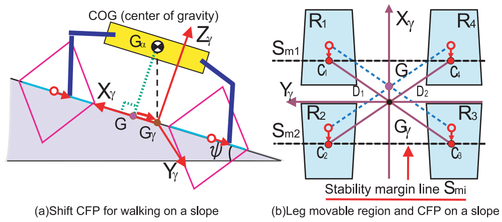


Fig. 5. Calculation of CFP on a slope

for walking on a slope, as shown in Fig.5, it is necessary to shift CFP according to the following condition.

1) The point G_γ must be on the intersection of the supporting leg diagonal lines for CFP.
2) The pre-set minimum stability margin $S_{\rm min}$, must be maintained and the stability margin $S_{\rm M}$ should be set as S_{hor} as possible. S_{hor} is the ordinary stability margin for walking on horizontal plane.
3) The pre-set minimum stroke length must be maintained.

The algorithm for deciding the CFP on a slope is shown in Fig.6. Here, symbols are defined as follows:

S_M : The value of stability margin.

S_{mi} : Stability margin lines on a slope. It is made to pull two lines to the positive/negative side of X_γ-axis as shown in Fig.5(b). ($S_{m1} : X_\gamma = S_M$ and $S_{m2} : X_\gamma = -S_M$).

R_i : The movable region of each leg considering the minimum stroke.

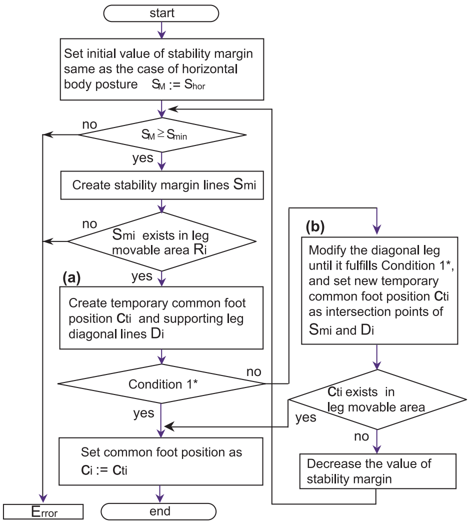
$\mathbf{c}_{ti}$: Temporary CFP. As shown in Fig.6 it can be classified into two case (a) and (b). At first calculation, it is a middle point of a line resulted from intersection of R_i and S_{mi}.

Error : With specified body posture, the omni-directional walking on a slope is impossible.

C. Omni-directional walking with optimal body posture on a slope

Depending on the position of the center of rotation, a gait for walking is selected from X-crawl, RX-crawl, Y-crawl, RY-crawl, O-rotation and RO-rotation [7]. The omni-directional walking with optimal body posture on a slope is realizable with following procedure.

Standard gait planning on a slope [8]:



Condition 1*: the point G_γ on the intersection of the supporting leg diagonal lines for common foot positions

Fig. 6. Algorithm to decide CFP on a slope

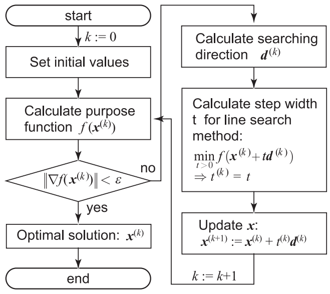


Fig. 7. Algorithm of gradient method for optimal body posture

TABLE I
SPECIFICATIONS OF ROBOT MODEL AND PARAMETERS

Robot Size (standard posture) [mm]	400(W)×600(L)×250(H)
Leg movable area (basal plane) [mm]	300(W)×200(L)
Leg movable area (height) [mm]	160(Top), -140(Bottom)
Height of COG [m]	0.243
Inclination angle of slope [deg]	10.0
Stability margin on horizontal (S_{hor}) [m]	0.2

1) Select a gait according to the position of the turning center.
2) Derive the leg movable region on a slope.
3) Derive the landing position of the swinging leg and the lifting position of the supporting leg based on the leg movable region.
4) Plan the leg trajectories that connect corresponding points on a slope.
5) Transform the above-mentioned result to body coordinate system to control the robot.

Successive gait-transition with optimal body posture:

1) Select next gait.
2) Derive the optimal body posture for next gait.
3) Derive the legs position with new leg movable region of optimal body posture for next gait.
4) Move the legs to new leg positions.
5) Change body posture to optimal body posture for next gait.

From next section, we introduce the detail of a search method of optimal body posture and successive gait-transition with optimal body posture.

III. OPTIMAL BODY POSTURE ACCORDING TO WALK ENVIRONMENT

In this section, we discuss the optimization problem of body posture, where the moving speed is maximized. Since the leg movable regions, represented as polygons, will change according to body posture, the optimization problem is difficult to formulate. Therefore, in this study, we use the gradient method, which is a typical method for a nonlinear optimization problem.

A. The optimization problem of body posture

A gradient method is the method that uses the gradient vector $\nabla f(\boldsymbol{x})$ of the object function $f(\boldsymbol{x})$ in point $\boldsymbol{x}$ to minimize or maximize the object function. In this study, we set the object function as largest stroke length for crawl gait, and maximum rotation angle for rotation gait, because these values have dominant influence to the motion speed of two gaits. Here the inputs are inclination angle and direction of a slope, and the output is optimal body posture $\boldsymbol{x}$. Variable $\boldsymbol{x}$ is a 2-dimensional vector $\boldsymbol{x} = (\psi_{bR}, \psi_{bP})^T$, where they are robot's *Roll* and *Pitch* angle. We use the algorithm of gradient method as shown in Fig.7 and set stability margin as $S_{\rm M} = S_{\min} = S_{\rm hor}$ to decide the CFP on the slope.

Step1: Initialization. The random initial point $\boldsymbol{x}^{(0)} \in R^2$ is chosen and set with $k := 0$.

Step2: Calculation of the search direction. Calculate the search direction vector $\boldsymbol{d}^{(k)}$ by (4). Calculation will be ended if an equation (5) is realized to a sufficiently small positive constant ε. Resultant $\boldsymbol{x}$ is the obtained optimal value. The gradient of the object function is expressed with an equation (6).

$$\boldsymbol{d}^{(k)} := -\nabla f(\boldsymbol{x}^{(k)}) \tag{4}$$

$$\|\nabla f(\boldsymbol{x}^{(k)})\| < \varepsilon \tag{5}$$

$$\nabla f(\boldsymbol{x}^{(k)}) = (\partial f(\boldsymbol{x}^{(k)})/\partial\psi_{bR}, \partial f(\boldsymbol{x}^{(k)})/\partial\psi_{bP})^T \tag{6}$$

Step3: Line-search. We use the line search method to solve 1-dimensional optimization problem.

$$\min_{t>0} f(\boldsymbol{x}^{(k)} + t\boldsymbol{d}^{(k)}) \tag{7}$$

$$\begin{cases} f(\boldsymbol{x}^{(k)} + t\boldsymbol{d}^{(k)}) \leq f(\boldsymbol{x}^{(k)}) + \beta t \nabla f(\boldsymbol{x}^{(k)})^T \boldsymbol{d}^{(k)} \\ (\nabla f(\boldsymbol{x}^{(k)} + t\boldsymbol{d}^{(k)})^T)\boldsymbol{d}^{(k)} \geq \gamma \nabla f(\boldsymbol{x}^{(k)})^T \boldsymbol{d}^{(k)} \end{cases} \tag{8}$$

One t ($t > 0$) is found to fulfill the equation (8) and set it as $t^{(k)}$. Set step width as $t^{(k)}$, and set $\boldsymbol{x}^{(k+1)} := \boldsymbol{x}^{(k)} + t^{(k)}\boldsymbol{d}^{(k)}$. Then set $k := k+1$ and return to Step2. Here, β and γ are constants which fills $\beta \in (0, 1/2)$ or $\gamma \in (\beta, 1)$.

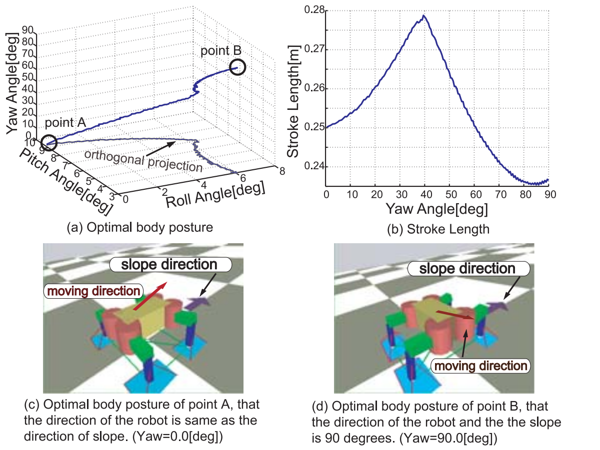


Fig. 8. The optimal body posture of the specified moving direction (30[deg] with the X_α-axis) on the slope

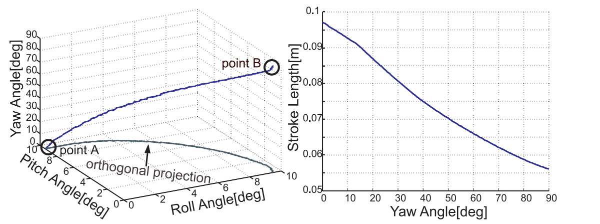


Fig. 9. The optimal body posture of the arbitrary moving direction on the slope

B. Determination of the optimal body posture

For crawl gait, we derive the optimal body posture for specified moving direction and the arbitrary direction. For rotation gait, we derive it from rotation on the spot. By combining them, the omni-directional walking with the optimal posture can be realized.

The specification of the robot and parameters shown in Tab.I are used. We use simulations to determine the optimal body posture. Simulations are performed with changing *Yaw* angle of the robot body on the slope. Yaw angle is examined within the range of $0 \leq Yaw \leq 90$[deg] since the robot is symmetrical.

1) Crawl gait for specified moving direction: With specified moving direction, the posture that gives the maximum moving speed (maximum stroke length) is searched. As an example, we describe the case of X-crawl gait with the specified moving direction that is set as an angle of 30[deg] with the X_α-axis (forward left of the robot) as shown in Fig.8(c)(d).

2) Crawl gait for arbitrary direction: When the moving direction is not specified, the posture in which the maximum moving speed can be taken from a stop state to arbitrary moving directions is derived.

3) Rotation gait: The posture, in which the maximum rotation angle is taken in rotation gait, is derived.

Simulation results are shown in Fig.8-Fig.10, and the optimal body posture of special point A and B are shown in Tab.II.

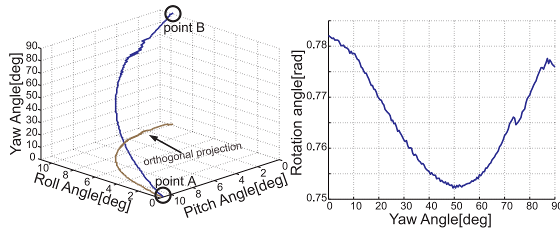


Fig. 10. The optimal body posture of the rotation gait on the slope

TABLE II

OPTIMAL BODY POSTURE OF SPECIAL POINT A AND B

Gait Pattern	OBP Point	Yaw [deg]	Roll [deg]	Pitch [deg]
Crawl gait for arbitrary direction	A	0	0.0	9.4
	B	90	9.5	0.0
Crawl gait for specified moving direction	A	0	0.1	9.3
	B	90	6.0	2.9
Rotation gait	A	0	0.0	10.7
	B	90	10.0	-0.1

IV. SUCCESSIVE GAIT-TRANSITION WITH OPTIMAL BODY POSTURE

The successive gait-transition with the optimal body posture on a slope can be realized with the procedure introduced in section II.C. The gait-transition between crawl gaits and rotation gaits can be classified as follows.

- Gait-transition from crawl gait to crawl gait
- Gait-transition from crawl gait to rotation gait
- Gait-transition from rotation gait to crawl gait
- Gait-transition from rotation gait to rotation gait

In order to make the optimal body posture for new gait, it is necessary to change the body posture during gait-transition. Here, we set the timing of changing body posture to a swing-leg period where stability margin is kept more than 0, and all leg positions are within new leg movable region for the next step. The timing of changing body posture and the order of moving legs to new positions depend on the above-mentioned gait-transition patterns.

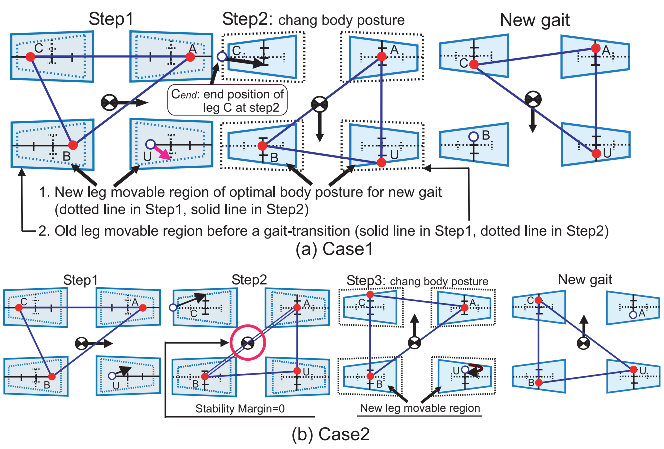


Fig. 11. Gait-transition case1 from crawl gait to crawl gait (Case1 and Case2)

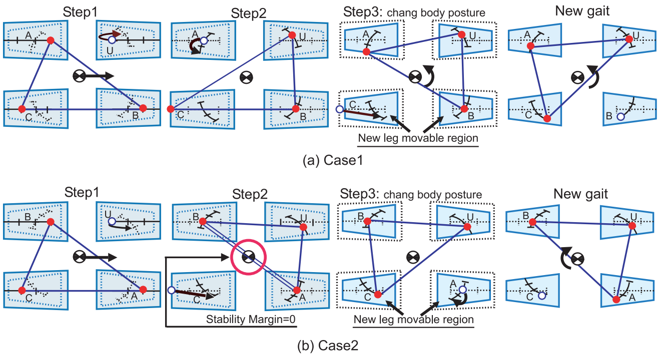


Fig. 12. Gait-transition from crawl gait to rotation gait (Case1 and Case2)

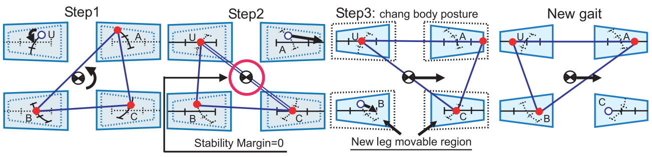


Fig. 13. Gait-transition from rotation gait to crawl gait

A. Gait-transition from crawl gait to crawl gait

The gait-transition from crawl to crawl can be classified into the following two cases based on which leg is first swung at start of gait-transition. **Case 1:** the first swinging leg is front to the next walking direction and **Case 2:** the leg is back. The gait-transitions for these two cases are shown in Fig.11, where 'U' describes the swinging leg at step1, 'C' is the leg opposite to the leg U, 'A' describes the front leg in the walking direction of old gait except the leg U and the leg C, and 'B' is the opposite leg of the leg A, respectively.

In both Case1 and Case2, the body posture cannot be changed at step1. Because the existence of the end position of leg C at step2 (C_{end} as shown in Fig.11(a)) in the new leg movable region is not assured. The timing of changing body posture is set as follows.

In **Case1**, body posture will be changed at **step2**. Because the legs A and B are on the CFP, the leg U is thus already at the correct positions of new gait.

In **Case2**, the body must stop at step2 in order for the polygon of supporting legs to be generated in the walking direction. At step2, COG is on the supporting leg diagonal line and therefore stability margin becomes 0. From this reason, the body posture cannot be changed at step2. The body posture can be changed at **step3**, because the legs A and B are on the CFP, the leg C is thus already at the correct positions of new gait.

B. Gait-transition from crawl gait to rotation gait

Depending on the leg's position at step2 of gait-transition, we can classify the gait-transition into two cases as shown in Fig.12. The body posture will be changed at **step3**, because the leg B is on the CFP, leg U and A are thus already at the correct positions of new gait.

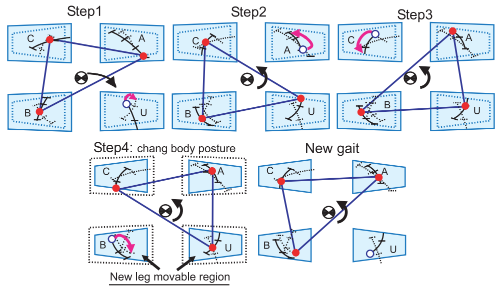


Fig. 14. Gait-transition from rotation gait to rotation gait

C. Gait-transition from rotation gait to crawl gait

The gait-transition from rotation to crawl can be performed from any state of the rotation gait. Its procedure is shown in Fig.13. The body posture will be changed at **step3**, because the legs U and C are on the CFP, the leg A is thus already at the correct positions of new gait.

D. Gait-transition from rotation gait to rotation gait

Same as the gait-transition from rotation to crawl, the gait transition from rotation to rotation can be performed from any state of the rotation gait. Its procedure is shown in Fig.14. The body posture will be changed at **step4**, because the leg U is on the CFP, the legs A and C are thus already at the correct positions of new gait.

V. SIMULATION

In order to verify the validity of the proposed method, the stability margin in slope walking by the method was measured using simulations. In the simulation, the robot performs omni-directional walking with given walking speed and rotation angular velocity using the full size model of a real robot TITAN-VIII. The time period of swinging leg is fixed to 1.0[s].

We performed the entire gait-transition pattern using simulations on a slope with inclination angle ψ=10[deg]. The simulation results are shown in Fig.15-Fig.18, and the changing of body posture during the period of gait-transition are shown in (C) of each figure. Here, Fig.15(A) shows an example of the gait-transition from crawl to crawl (case1) on a slope. In the simulation, the direction of the robot is set to Yaw=60[deg]. The robot kept the optimal body posture throughout the simulation for fastest moving speed. Concretely, during gait-transition the optimal body posture is changed from Roll=8.7[deg], Pitch=4.3[deg] to Roll=7.8[deg], Pitch=5.6[deg]. The numbers in the figures (1,2,$\cdots$,8) denote the moments of robot motion in the simulation. From the moment 1 to 3 the robot is walking forward by X-crawl gait. At moment 4, the leg 1 starts swinging toward left and the gait-transition starts. At moment 5 the body posture is changed (Roll:8.6$\rightarrow$7.8[deg], Pitch:4.3$\rightarrow$5.6[deg]), and the leg 3 swings to left. At moment 6 the gait-transition is finished. The robot walks left by Y-crawl gait from 6 to 8. In the simulations, the stability margin was kept larger than 0 during the gait-transition. From these results, it can be stated that the gait-

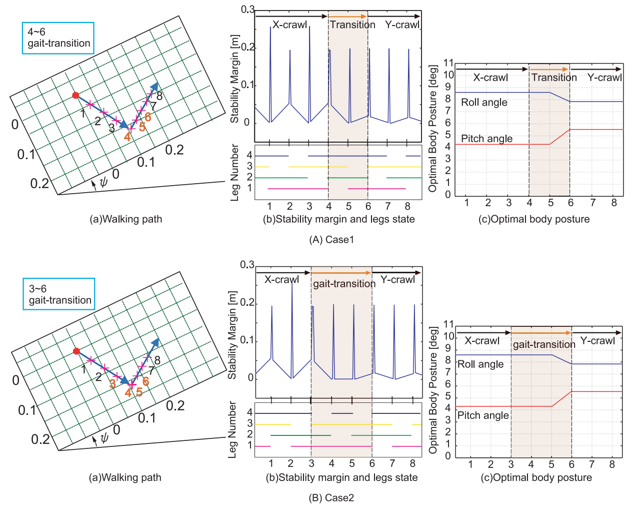


Fig. 15. Walking path of COG, corresponding stability margin, legs state and optimal body posture for the Gait-transition from Crawl gait to Crawl gait (Case1 and Case2)

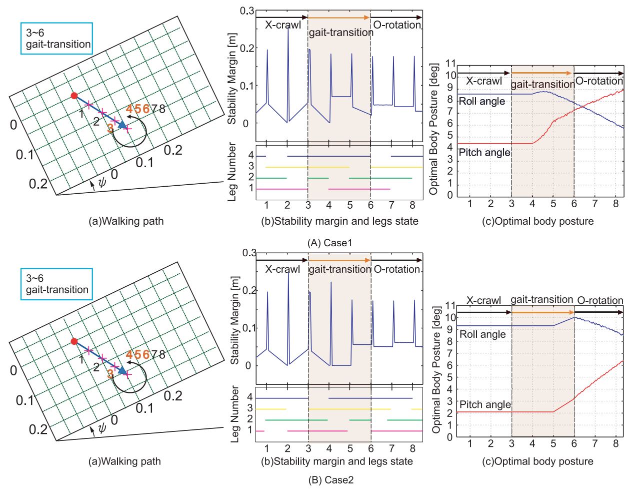


Fig. 16. Simulation result of Gait-transition from Crawl gait to Rotation gait (Case1 and Case2)

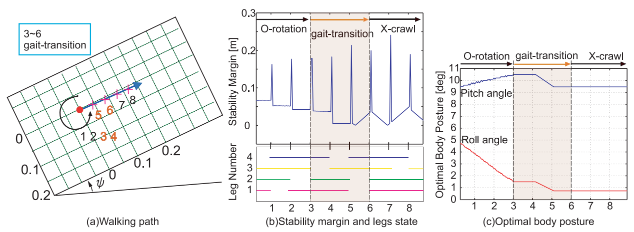


Fig. 17. Simulation result of Gait-transition from Rotation gait to Crawl gait

transition with preserved optimal body posture is performed stably using the proposed method.

VI. Conclusion

In order to realize omni-directional static walking on a slope with high environmental adaptability and highest moving speed, we proposed a method of successive gait-transition with optimal body posture. By the proposed method, the quadruped robot could move without stopping

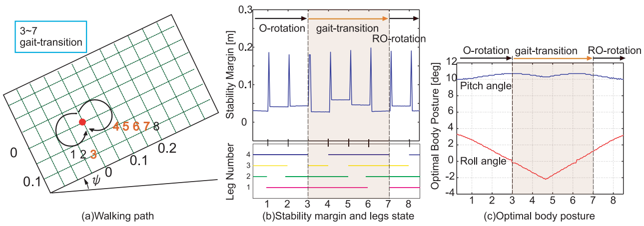


Fig. 18. Simulation result of Gait-transition from Rotation gait to Rotation gait

in all directions with optimal body postures on a slope. Through walking experiments by computer simulation, the validity of the proposed approach has been verified.

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