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Optimal turning gait of a quadruped walking robot

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SUMMARY

The analysis of the gait mode is an important step for constructing a walking robot. In this study an optimal turning gait of a four-legged walking robot is proposed, which maximizes walking speed keeping walking stability larger than a certain required value. To obtain an optimal gait algorithm, for body trajectory of the robot, maximum walking speed, feasible gait modes and stability margin are analyzed for a quadruped walking robot designed in this study. The proposed gait is sufficiently flexible, as to permit crab walking, turning walking and a pure rotational walking about a robot's geometric center. The validity of the proposed gait algorithm is confirmed by many simulations.

KEYWORDS: Walking robot; Turning gait; Quadruped; Gait algorithm.

1. INTRODUCTION

Since the advent of industrial robots at the end of 1950s, robots working in a fixed position have taken a leading role in automation. In order that robots can be used successfully under such working environment as under water, in nuclear power plants and in other hostile environments, mobility should be added to them, hence the need for a mobile robot. Mobile robots can be divided into two types according to driving mechanism: wheeled type and legged type. Although the wheeled type robot is more energy efficient, easier to control and faster than the legged one, it cannot overcome obstacles like stairs. While the legged type robot (walking robot) has faults in terms of a complex structure and poor controllability, it can operate well, even in hostile environments.[1] Our research is concerned with the development of a four-legged walking robot. For the mobility of walking robot, one must devise an optimal gait mode and control the movement of each leg in order to be stable during locomotion. For many years several gait modes have been developed by many researchers. Among them Hirose's circular gait[2] and Bien's turning gait[3] are well known. The circular gait maximized the walking speed using a critical stability margin of zero. But it has some drawbacks that one cannot control the translation and the rotation of walking robot simultaneously and independently, and the motion of the robot can easily become unstable at some small disturbances. Using the turning gait by Bien, the translation and the rotation of the robot can be controlled, simultaneously and independently keeping the maximum stability margin. But in this turning gait, walking speed was not considered as an important factor. In this paper, an optimal turning gait is proposed. The new gain complements the drawbacks of above two gaits, that is, it maximizes walking speed while keeping the stability margin larger than a certain required threshold value, and the translation and the rotation can be controlled simultaneously and independently. The validity of the proposed gait algorithm is proved by simulations.

2. CONFIGURATION OF ROBOT

The insect-like walking robot used in this research is shown in Figure 1, and the photograph of one leg is shown in Figure 2. The detail specification of the robot is as follows:

—motion mechanism of each leg: parallelogram structure with 5 links
—Degree of freedom of each leg: 3 D.O.F.
—No. of legs: 4
—Power source: D.C. servo motor with 20 W
—length of each leg: 50 cm at full stretch
—body plate: 30 cm × 30 cm
—speed reduction: warm gear reduction (20:1)

3. DEFINITION AND CONDITION

3.1 *Definition of terminology*

The terminology used in this paper is as follows:[4]

(i) Transfer phase of a leg is a period during which the leg is lifted. Support phase is a period during which the leg is on the ground.
(ii) Cycle time T_c is the time in which the robot returns to it's initial posture after all its four legs have been lifted once, one after another.
(iii) Duty factor is the time fraction of support phase in one cycle time.
(iv) A gait is regular if all the legs have same duty factor.
(v) Support polygon of a gait is a polygon made of all tip points of legs in support phase.
(vi) Stability margin is the shortest distance from the projection of body center on ground plane to the boundaries of the support polygon (the shortest front or rear stability margin in this paper).
(vii) Front stability margin and rear stability margins are the distances from the projection of body center on ground plane to the front and the rear boundary of

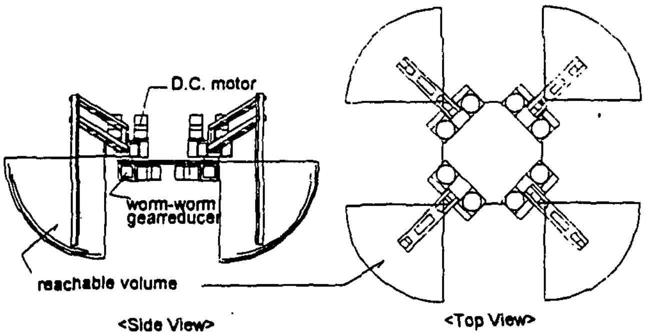

Fig. 1. Model of walking robot.

support polygon, respectively, as measured in the direction of motion.

3.2 Walking condition

The following walking conditions are assumed for the analysis of gait modes:

(i) The body plate is held at constant height and parallel to the ground plane during walk.
(ii) Translational and/or rotational walking speeds are maintained constant for one cycle movement.
(iii) The swing speed of foot tip relative to body cannot be larger than the maximum swing speed V_m.
(iv) Walking is statically stable while keeping the duty factor larger than 0.75.
(v) In one cycle walking, the final posture of robot coincides with the initial one.
(vi) The trajectory of a foot tip in body coordinate is identical in transfer and support phases. But the directions of the trajectories are opposite in the two phases.

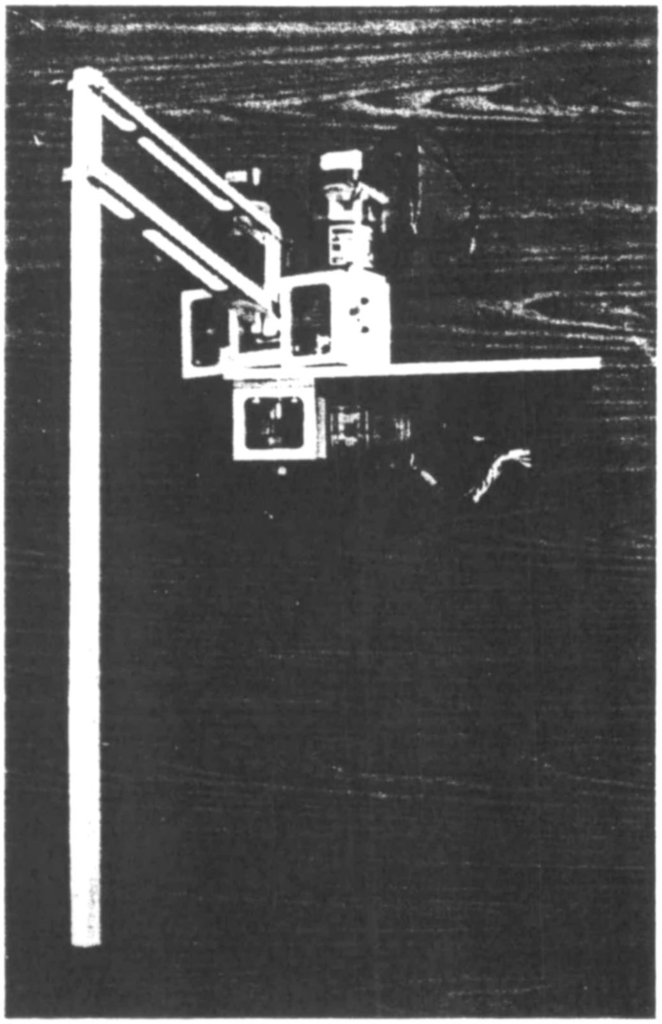

Fig. 2. Photograph of one leg.

4. BODY TRAJECTORY

A body trajectory is that along which the body center of robot passes. The body trajectory is determined in the path planning process and divided into several walking cycles, as shown in Figure 3. The magnitudes λ and ϕ (Figure 4), in which the Robot moves and rotates in one walking cycle, are determined from the body trajectory under the kinematic constraint that each leg tip should be positioned within its reachable area during the walking cycle. In Figure 4 λ, ϕ and α are not independent variables. For example, if λ is selected suitably from a given body trajectory, ϕ and α are calculated automatically. The λ, ϕ and α values are used as input data in determining an optimal gait mode.

5. MAXIMUM WALKING SPEED

If λ, ϕ and α are determined from a given body trajectory and if an appropriate duty factor β is selected, the maximum walking speed can be calculated as follow:

$$V_{max} = \lambda / T_{c\,min} \qquad \omega_{max} = \pi / T_{c\,min} \tag{1}$$

According to the walking condition (6) in section 3.2,

$$V_{ti} \cdot T_c \cdot (1-\beta) = V_{si} \cdot T_c \cdot \beta$$

or

$$V_{ti} = V_{si} \cdot \beta/(1-\beta) \tag{2}$$

Because the swing speed of leg with respect to body

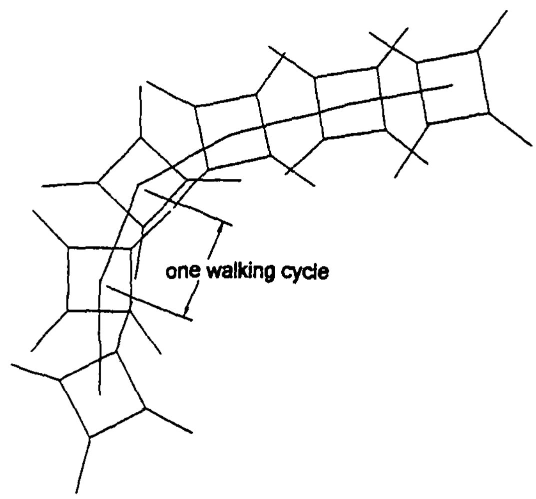

Fig. 3. Body trajectory divided into several walking cycles.

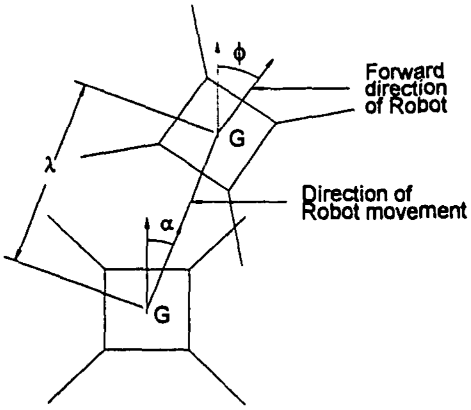

Fig. 4. Definition of λ, ϕ and α in a walking cycle.

coordinate is the same as the speed of hip with respect to world coordinate in the support phase,

$$V_{st} = V_{ht} \tag{3}$$

Substituting V_{ht} into V_{st} in (2),

$$V_{lt} = V_{ht}\beta/(1-\beta) \tag{4}$$

On the other hand, if infinitesimal movements of body plate during δT are $\delta\lambda$ and $\delta\phi$, the infinitesimal movement δS of hip is calculated according to Figure 5.

$$\delta S^2 = \delta\lambda^2 + d^2\,\delta\phi^2 + 2d\,\delta\lambda\,\delta\phi\sin a_i \tag{5}$$

where α_i is the angle between the line linking lip i and body center and the direction of body center movement. If the body plate is square,

$$\begin{aligned} a_1 &= 45^\circ + \alpha \\ a_2 &= 45^\circ - a \\ a_3 &= 135^\circ + \alpha \\ a_4 &= 135^\circ - \alpha \end{aligned} \tag{6}$$

Letting $c = \delta\lambda/\delta\phi = \lambda/\phi$ $(\phi \neq 0)$ and rearranging (5),

$$\delta S^2 = \delta\phi^2(1 + d^2c^2 + 2dc\sin a_i)$$

or

$$\delta S = \delta\phi(1 + d^2c^2 + 2dc\sin a_i)^{1/2} \tag{7}$$

Using $V_{ht} = \delta S/\delta T$ and $\delta\phi/\delta T = \phi/T_c$ (according to walking condition (2)) and dividing both sides of (7) by δT_3,

$$V_{ht} = (\phi/T_c)(1 + d^2c^2 + 2dc\sin a_i)^{1/2}$$

or

$$V_{ht} = X_i/T_c \tag{8}$$

where $V_{ht} = \phi(1 + d^2c^2 + 2dc\sin a_i)^{1/2}$ and X_i is a function of λ or ϕ. Substituting (8) into (4),

$$V_{lt} = X_i\beta/T_c(1-\beta) \tag{9}$$

According to the walking condition (3), V_{lt} should be smaller than V_m, i.e.,

$$\max\{V_{lt} \mid i = 1 \text{ to } 4\} \leq V_m \tag{10}$$

Letting $X_{\max} = \max\{X_i \mid i = 1 \text{ to } 4\}$ and from (9) and (10),

$$\beta \cdot X_{\max}/V_m \cdot (1-\beta) \leq T_c \tag{11}$$

Thus

$$T_{c\,\min} = \beta \cdot X_{\max}/V_m(1-\beta) \tag{12}$$

Substituting (12) into (1)

$$\begin{aligned} V_{\max} &= \lambda V_m(1-\beta)/\beta X_{\max} \\ \omega_{\max} &= \phi V_m(1-\beta)/\beta X_{\max} \end{aligned} \tag{13}$$

Equation (13) means that the maximum walking speed can be determined if λ or ϕ is obtained from the body trajectory and if β is selected suitably.

6. FEASIBLE GAIT MODE

When the legs of robot are numbered as shown in Figure 6, it may be proved that there are only four feasible gait modes, as follows:

4 – 1 – 3 – 2: mode 1
3 – 2 – 4 – 1: mode 2
4 – 3 – 2 – 1: mode 3
3 – 4 – 1 – 2: mode 4

where the serial arrangement of numbers means the sequence of the legs which are lifted. For the implementation of each gait mode, the lifting and the placing time, $T_{l_{ij}}$ and $T_{p_{ij}}$, for leg i in gait mode j should be

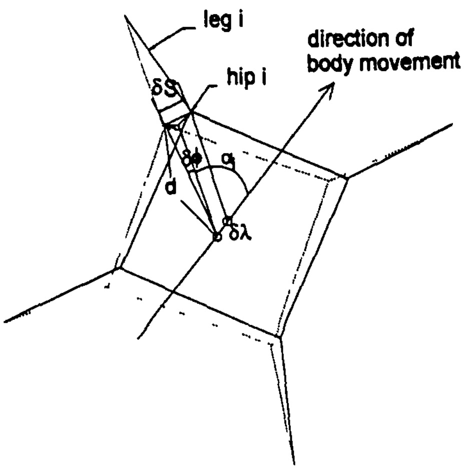

Fig. 5. Infinitesimal movement of body and hip.

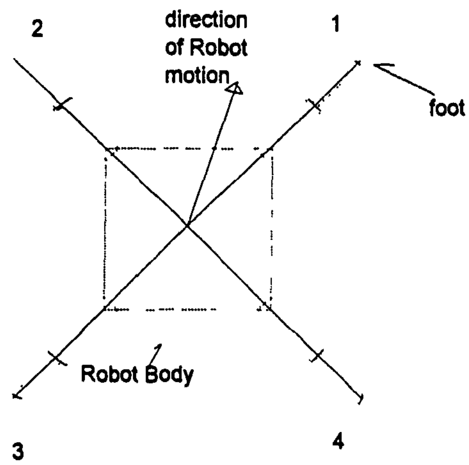

Fig. 6. Leg numbers and direction of Robot movement.

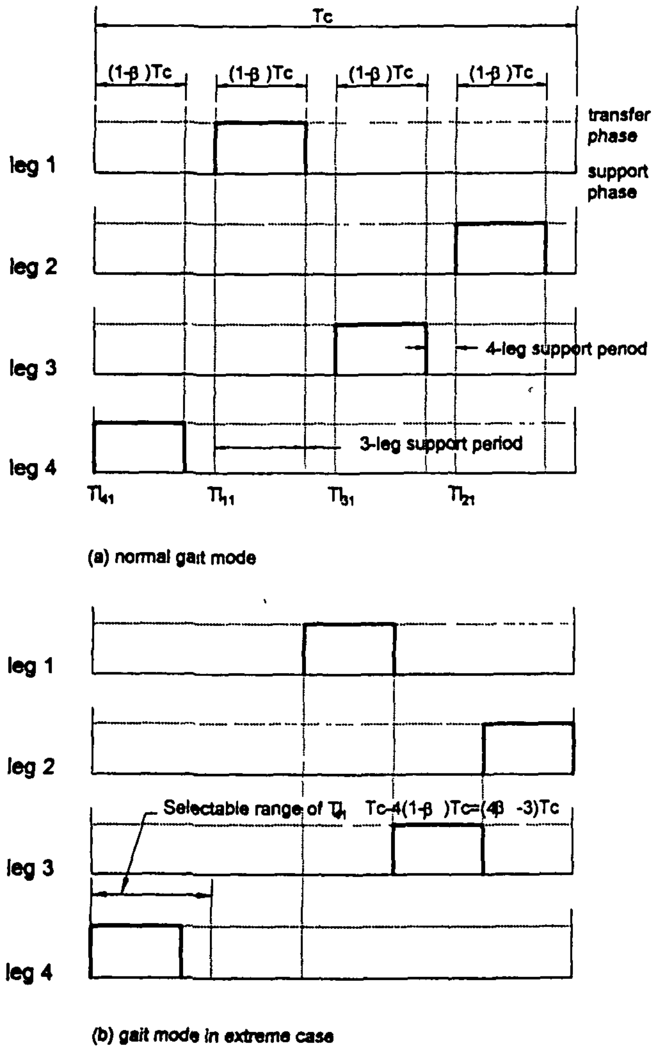

Fig. 7. Leg phase diagram for gait mode 1.

determined. For this purpose, the leg phase diagram can be used. Figure 7 shows the leg phase diagram for mode 1. For the case of gait mode 1, the selectable range of the lifting time of each leg is determined as follows:

$$\begin{aligned} &0 \leq Tl_{41} \leq T_c(4\beta - 3) \\ &Tl_{41} + (1-\beta)T_c \leq Tl_{11} \leq T_c(3\beta - 2) \\ &Tl_{11} + (1-\beta)T_c \leq Tl_{31} \leq T_c(2\beta - 1) \\ &Tl_{31} + (1-\beta)T_c \leq Tl_{21} \leq T_c\beta \end{aligned} \tag{14}$$

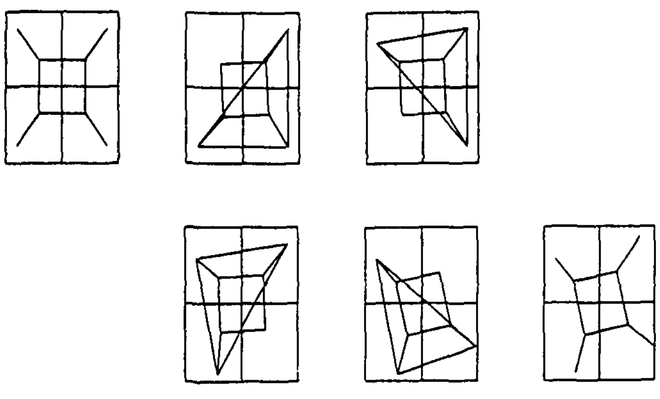

Fig. 8. Support triangles during a walking cycles.

7. STABILITY MARGIN

The stability margin of each gait mode can be estimated from the relations between the support polygon (triangle) and the vertical projection of body center on ground plane which includes a support triangle. That is, the projection of body center should be positioned within the boundary of the support triangle in order that the walking process be stable. There are 8 positions (${}^W\mathbb{P}_i$'s) of foot tips in one walking cycle, i.e., 4 points in the initial posture and 4 points in the final posture. The support triangles are made from three foot tip points selected among them. Support triangles made during the walking cycle are shown in Figure 8. The position ${}^W\mathbb{P}_i$ of foot tip i, in world coordinate is calculated as follows:

$${}^W\mathbb{P}_i = {}^W\mathbb{T}_B\, {}^B\mathbb{P}_i \tag{15}$$

where ${}^B\mathbb{P}_i$ is the position of foot tip i in body coordinate and ${}^W\mathbb{T}_B$ is a homogeneous transformation matrix from body coordinate to world coordinate, i.e.,

$${}^W\mathbb{T}_B = \begin{bmatrix} \cos\phi & -\sin\phi & 0 & \lambda\cos\alpha \\ \sin\phi & \cos\phi & 0 & \lambda\sin\alpha \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{16}$$

where ϕ and λ are variables which are zeros at initial state of a walking cycle. The stability margin is the shortest distance from the vertical projection of body center on the ground plane to the boundary of the support polygon. In Figure 9, several kinds of stability margin are defined on a support triangle.

According to the definition, the stability margin is $\overline{GD}$ in Figure 9. But only the rear stability margin S_r ($\overline{GF}$ in Figure) and the front stability margin S_f ($\overline{\geq}$ in Figure) are used for convenience of analysis in this paper. S_r and S_f are the function of lifting and placing time, Tl_{ij} and Tp_{ij} of each leg. The stability margin S_{ij} of the gait mode j can be determined as follows, when leg i is in transfer phase:

$$S_{ij} = \min\{S_r(Tl_{ij}), S_f(Tl_{ij}), S_r(Tp_{ij}), S_f(Tp_{ij})\} \tag{17}$$

The stability margin of the gait mode j is dominated by the smallest stability margin among four S_{ij} ($i = 1$ to 4), i.e.,

$$S_j = \min\{S_{ij} \mid i = 1 \text{ to } 4\} \tag{18}$$

As an optimal gait mode, the gait mode with the largest

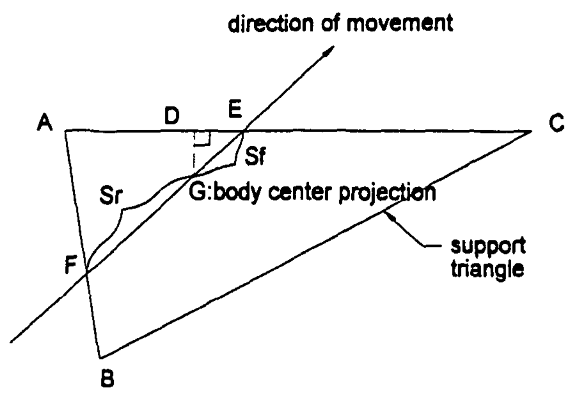

Fig. 9. Stability margins and support triangles.

stability margin is selected among four gait modes, and the stability margin S of the optimal gait mode selected as such is the stability margin of the robot

$$S = \max\{S_j \mid j = 1 \text{ to } 4\} \tag{19}$$

8. ALGORITHM FOR OPTIMAL GAIT

The main objective of this paper is to develop an optimal gait mode which maximizes walking speed while maintaining a walking stability larger than a certain required value. Such a goal can be described as follows:

Maximizing

λ / T_c

under the constraint of

$S \geqq S_{th}$

For solving the optimization problem described above, and algorithm is constructed as shown in Figure 10. First of all, λ is initialized with a maximum value from the body trajectory, considering the kinematic constraint, and the duty factor is set to 0.75 which is the maximum value to guarantee a statically stable walking. Then ϕ, α,

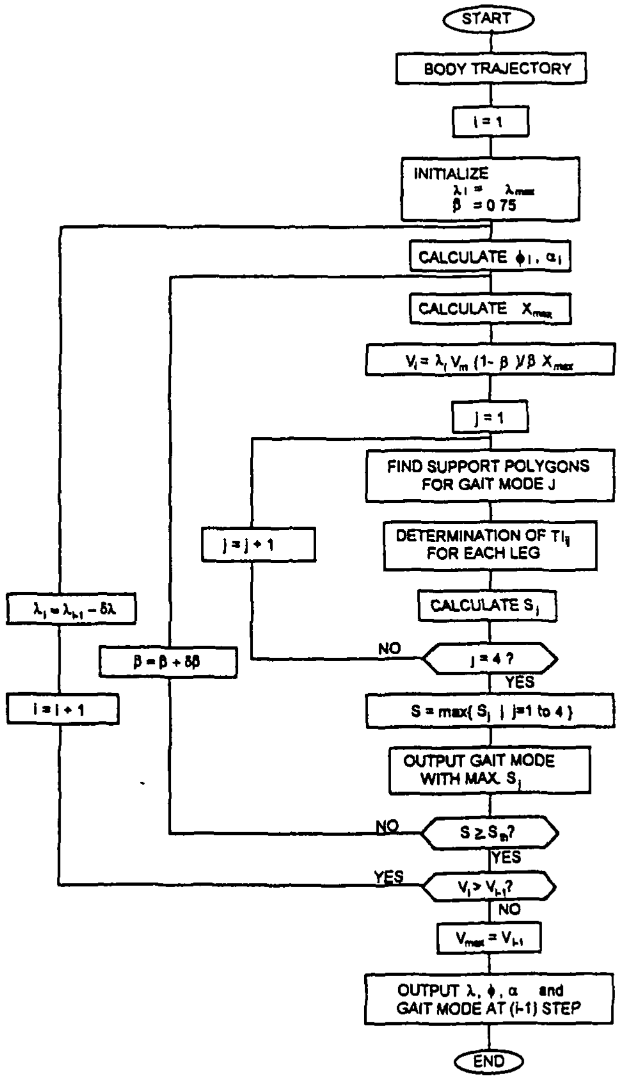

Fig. 10. Algorithm for optimal gait mode.

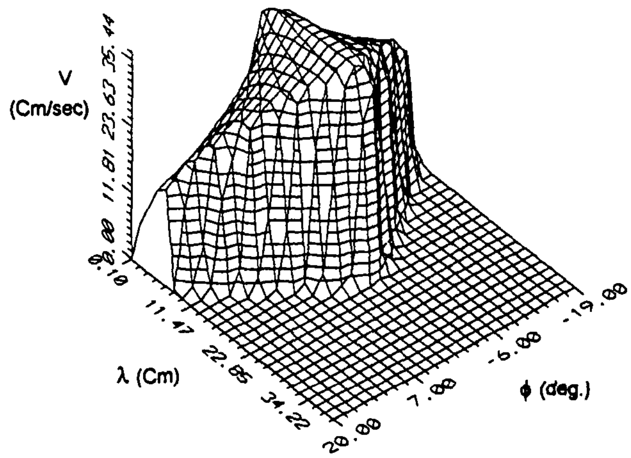

Fig. 11. λ, ϕ and walking speed.

X_{max} and V_i (V_{max} in section 5) are calculated according to the body trajectory and the equations in section 5. Under this condition the gait mode with the maximum stability margin is selected among four feasible gait modes. If the stability margin of the selected gait mode is smaller than a threshold value, the duty factor is increased to decrease the maximum speed of the robot. These processes are repeated until $S \geqq S_{th}$; they are also repeated with other values of λ until the maximum walking speed is obtained.

9. SIMULATION

The relations between λ, ϕ and V (walking speed) are shown in Figure 11. The kinematic constraint of the robot is satisfied in the region in which V is equal to or larger than zero in Figure 11. It is shown in the figure that the range of λ, in which the kinematic constraint is satisfied, decreases with the increase of the magnitude of ϕ, and that the walking speed can be faster as the rotation angle per cycle gets smaller. In an extreme case of $\phi = 0$, a pure linear movement like a crab gait is obtained and the speed reaches peak value. Figure 12 shows the relations between stability, duty factor and walking speed. It is shown in the figure that the walking speed decreases with the increase of the stability and the duty factor. This means that the duty factor should be increased for stable walking and the increase of the duty factor makes the robot slower.

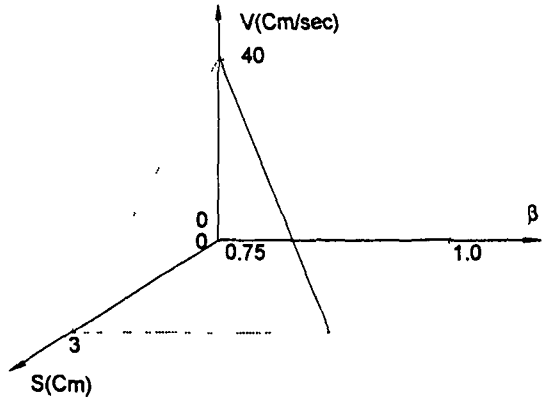

Fig. 12. Stability, duty factor and walking speed.

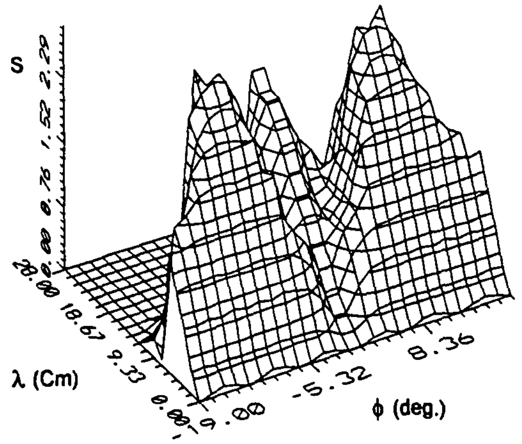

Fig. 13. Effect of λ, ϕ and S on gait mode.

The effects of λ, ϕ and s on gait modes are shown in Figure 13. Gait mode 1 and 2 are similar and they show the highest stability at $\phi = 0$, because they are identical to a crab gait. The turning motion in positive direction has the highest stability in the gait mode 3. This means that the gait sequence of gait mode 3 is designed favorably to rotate the body in a positive direction. In the gait mode 4, the turning motion in a negative direction is favorable. According to these facts we can imagine which gait mode can be selected in each case.

10. CONCLUSION

In this paper a new gait mode for a quadruped walking robot is proposed, which maximizes the walking speed while keeping the walking stability greater than a certain required value. In order to obtain an optimal gait algorithm, the body trajectory, maximum walking speed, feasible gait modes and stability margins are analyzed for the model of a quadruped walking robot designed in this study. The availability and performance of the proposed gait algorithm were confirmed in simulations.

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