09.08.2019

Digital Image Processing (CSE/ECE 478)

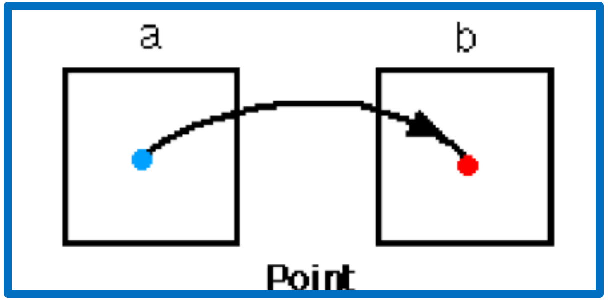
Lecture 4 : Histogram Processing & Intro. to Spatial Filters

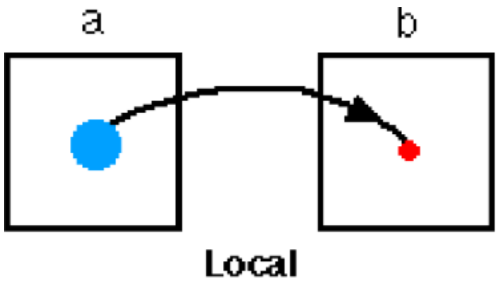
Ravi Kiran

Recap …

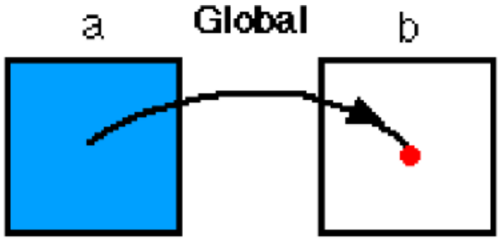
Spatial Domain Processing

- Manipulating Pixels Directly in Spatial Domain
- **Point to Point**





- Neighborhood to Point
- Global Attribute to Point

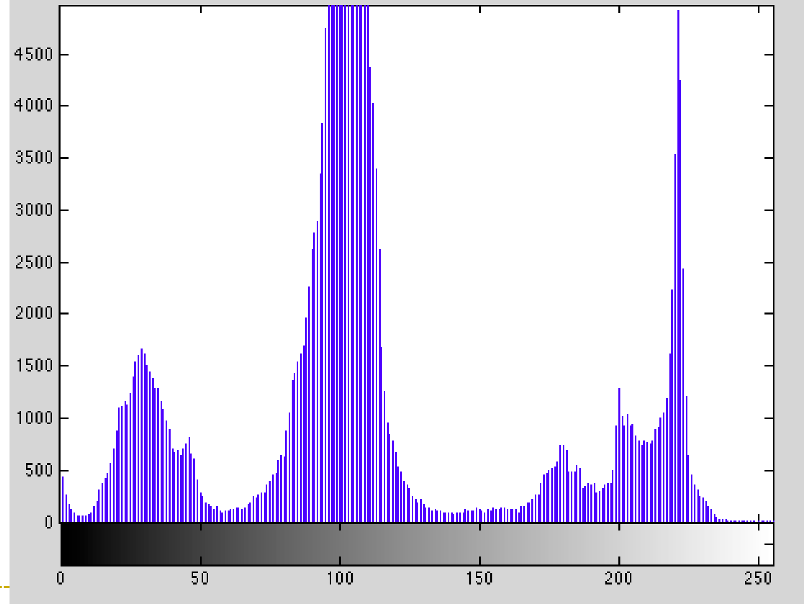


Histogram

$$h_r(i) = n_i$$

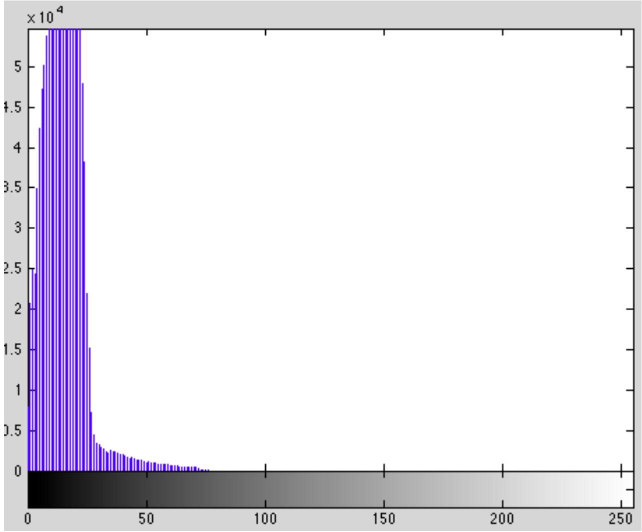
i → intensity value, range [0 L-1]

n_i → number of pixels with intensity i



Histograms

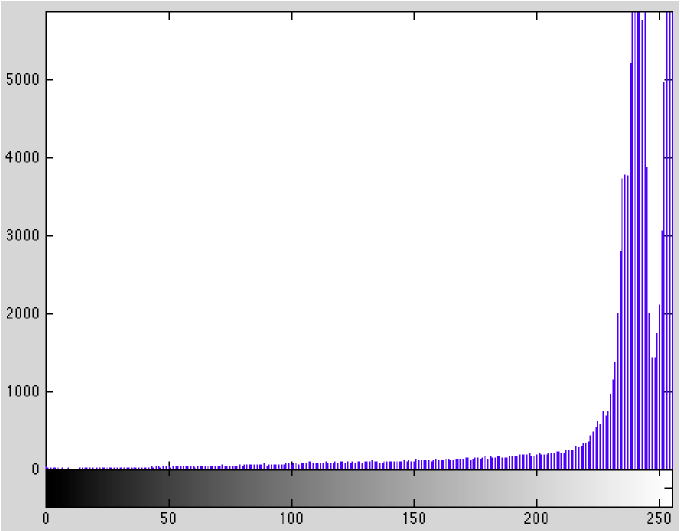
- Histograms and brightness



Under exposure

Histograms

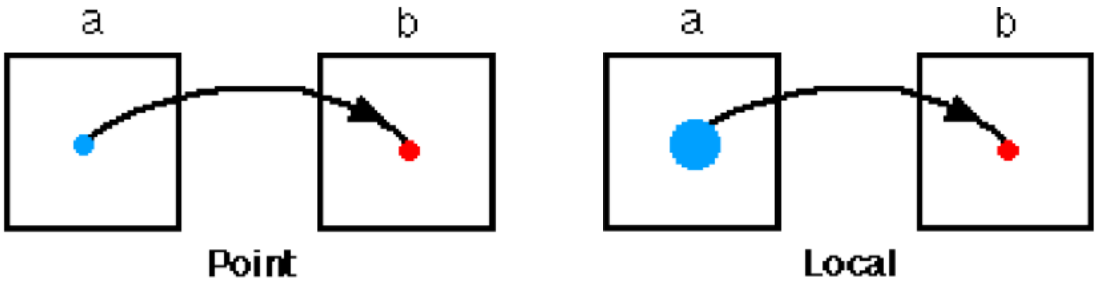
- Histograms and brightness



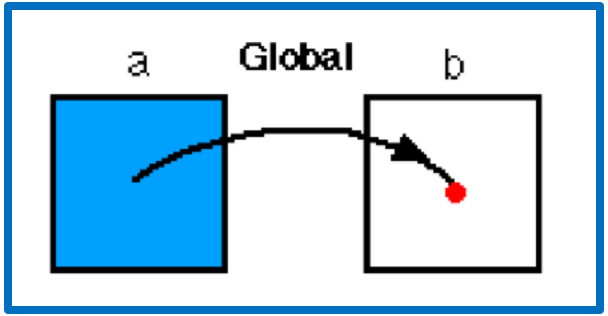
Over exposure

Spatial Domain Processing

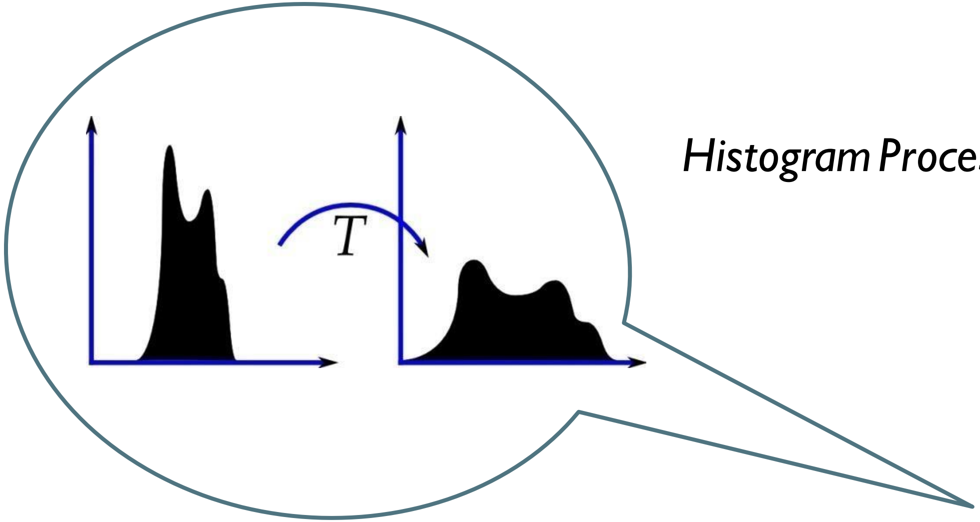
- Manipulating Pixels Directly in Spatial Domain
- Point to Point



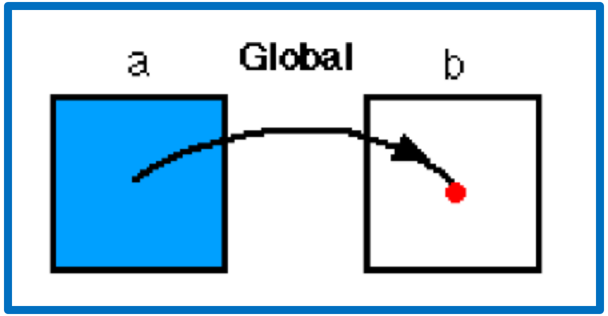
- Neighborhood to Point
- **Global Attribute to Point**



Spatial Domain Processing



Histogram Processing

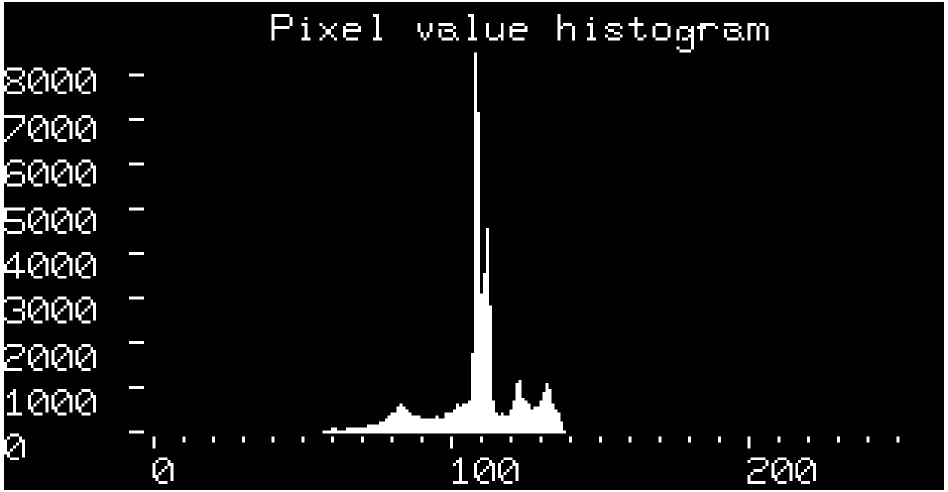


- **Global Attribute to Point**

Histogram Processing

- Histogram Stretching/ Contrast Stretching
- Histogram Equalization
- Histogram Specification
- Local Histogram Equalization

Contrast Stretching

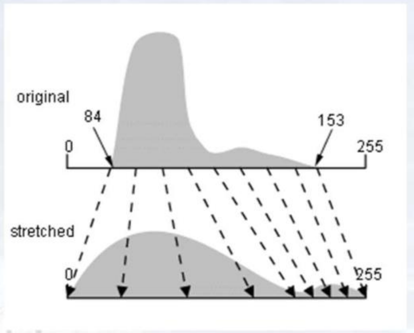


Contrast Stretching

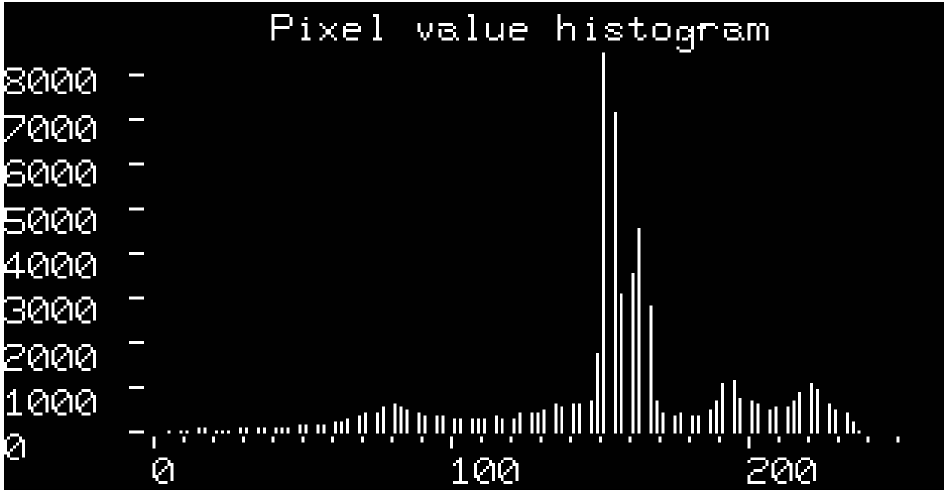
$$f_{ac}(a) = a_{min} + (a - a_{low}) \cdot \frac{a_{max} - a_{min}}{a_{high} - a_{low}}$$

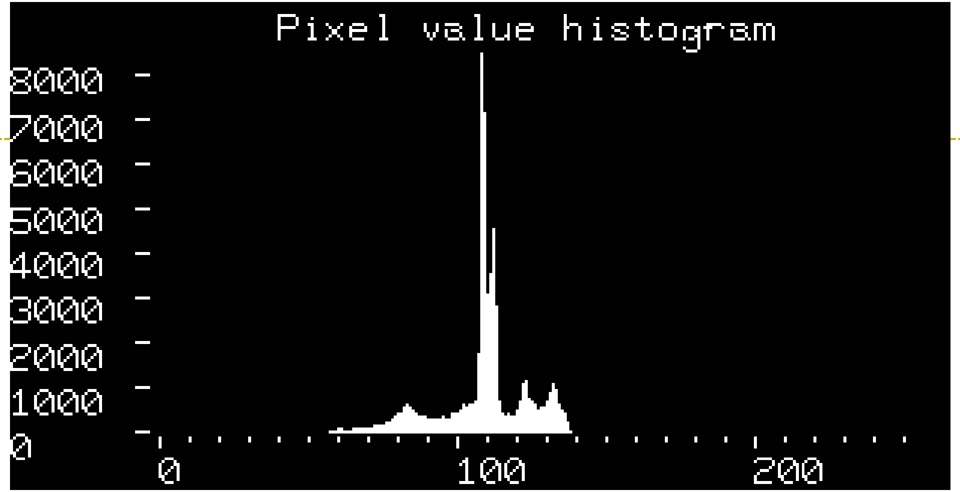
If a_{min} = 0 and a_{max} = 255

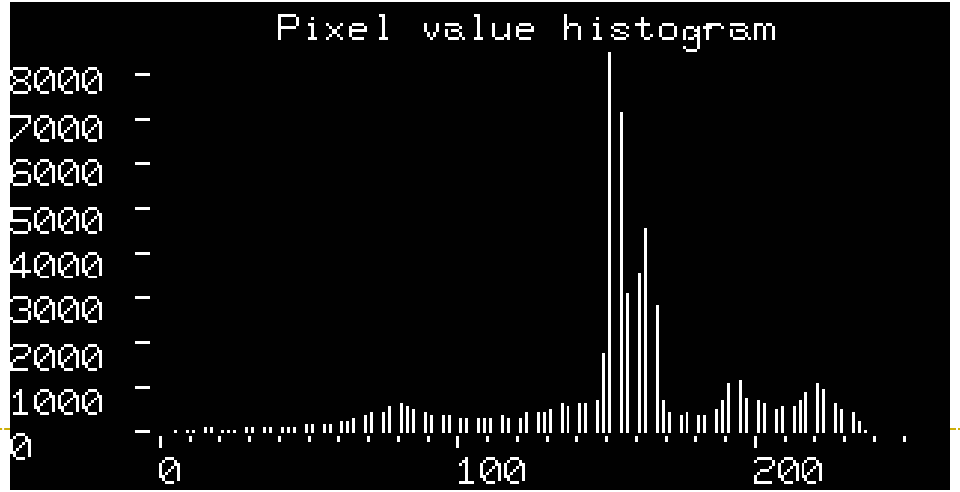
$$f_{ac}(a) = (a - a_{low}) \cdot \frac{255}{a_{high} - a_{low}}$$



Contrast Stretching




Pixel value histogram
8000
7000
6000
5000
4000
3000
2000
1000
0
0
100
200

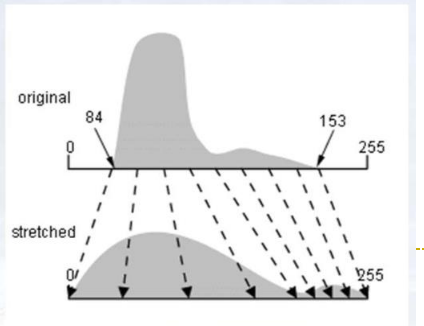

Pixel value histogram
8000
7000
6000
5000
4000
3000
2000
1000
0
0
100
200

Contrast Stretching

$$f_{ac}(a) = a_{\min} + (a - a_{\text{low}}) \cdot \frac{a_{\max} - a_{\min}}{a_{\text{high}} - a_{\text{low}}}$$

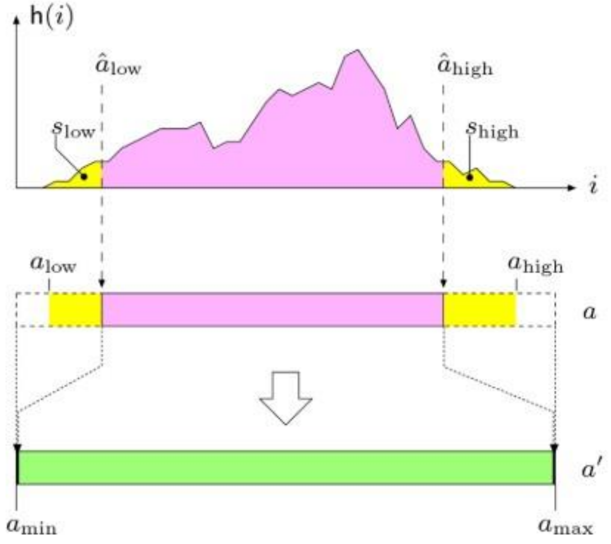
If a_{min} = 0 and a_{max} = 255

$$f_{ac}(a) = (a - a_{\text{low}}) \cdot \frac{255}{a_{\text{high}} - a_{\text{low}}}$$



Single pixel with intensity 0 or 255. What happens ?

Contrast Stretching ver. 2

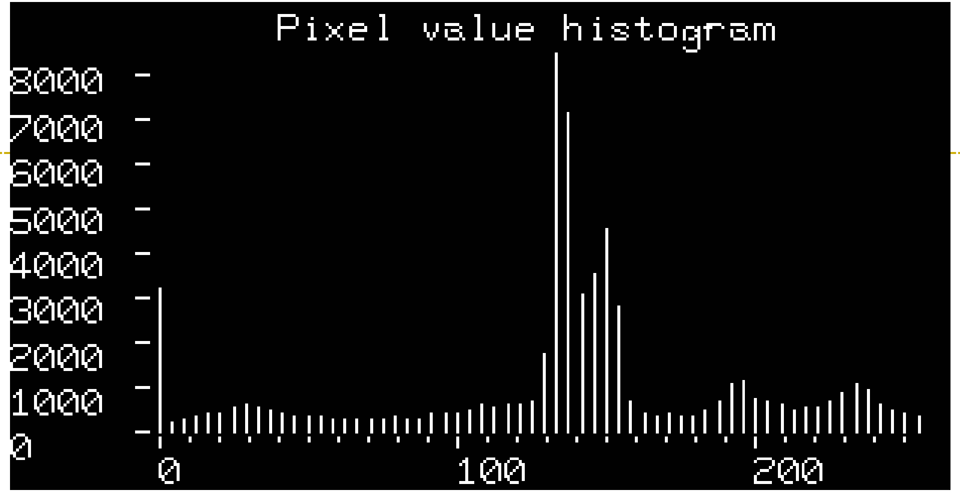


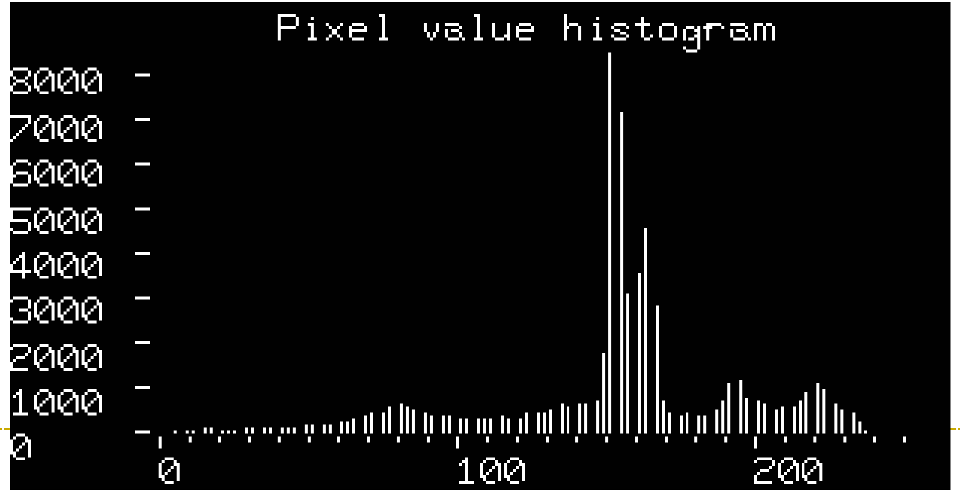
$$\hat{a}_{\text{low}} = \min\{\, i \mid \mathsf{H}(i) \geq M \cdot N \cdot s_{\text{low}} \}$$

$$\hat{a}_{\text{high}} = \max\{\, i \mid \mathsf{H}(i) \leq M \cdot N \cdot (1 - s_{\text{high}}) \}$$

$$f_{\text{mac}}(a) = \begin{cases} a_{\min} & \text{for } a \leq \hat{a}_{\text{low}} \\ a_{\min} + (a - \hat{a}_{\text{low}}) \cdot \dfrac{a_{\max} - a_{\min}}{\hat{a}_{\text{high}} - \hat{a}_{\text{low}}} & \text{for } \hat{a}_{\text{low}} < a < \hat{a}_{\text{high}} \\ a_{\max} & \text{for } a \geq \hat{a}_{\text{high}} \end{cases}$$

Ver. 2


Pixel value histogram
8000
7000
6000
5000
4000
3000
2000
1000
0
0
100
200


Pixel value histogram
8000
7000
6000
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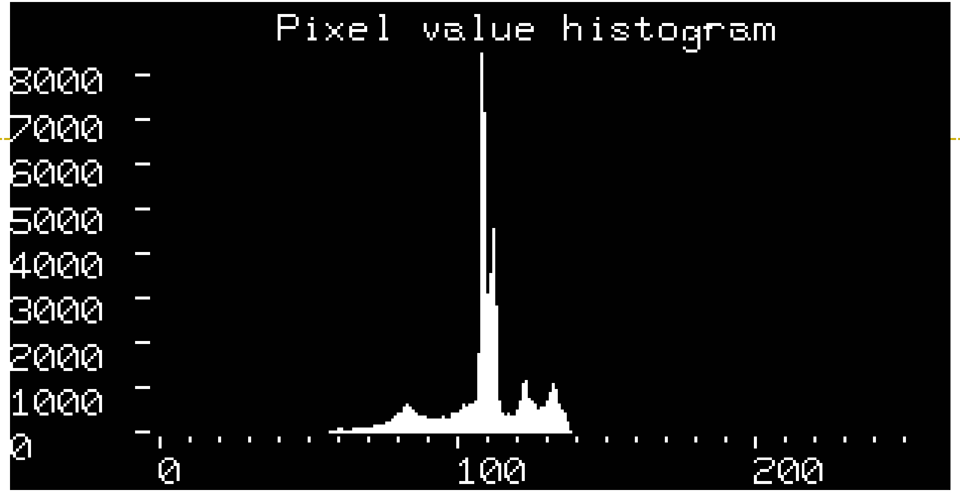
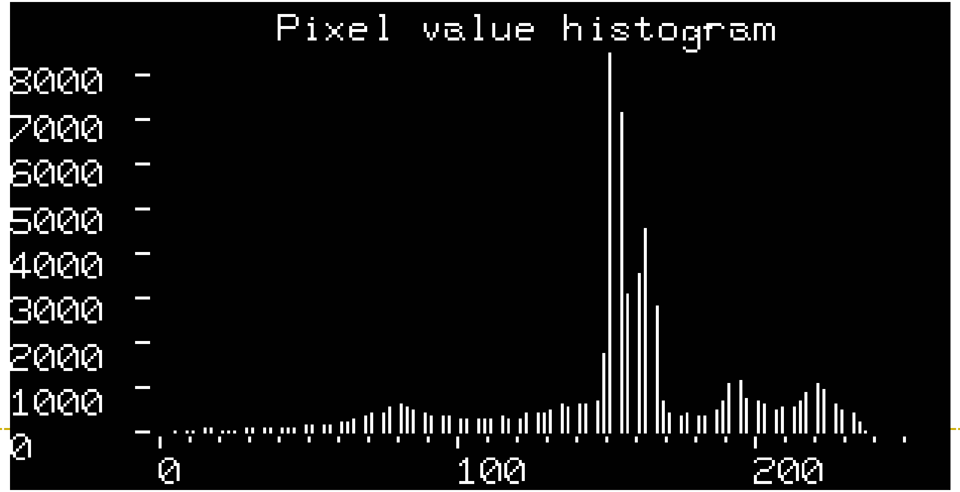
4000
3000
2000
1000
0
0
100
200



Pixel value histogram
8000
7000
6000
5000
4000
3000
2000
1000
0
0
100
200



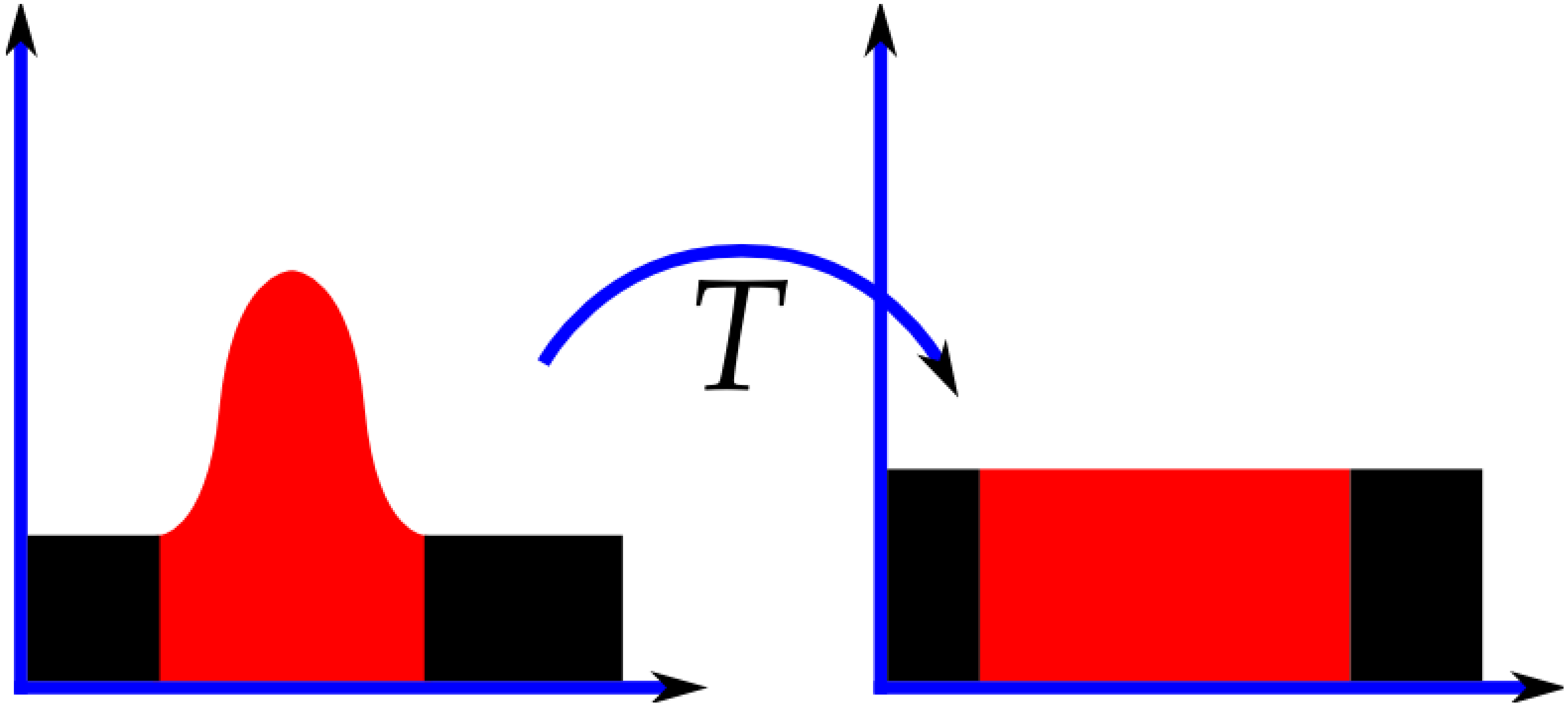
Pixel value histogram
8000
7000
6000
5000
4000
3000
2000
1000
0
0
100
200

Contrast Stretching – Enough?

- Do all intensities have equal distribution ?

Histogram Equalization

Histogram Equalization



Histogram Equalization

Assumptions

- $s = T(r)$ is a monotonically increasing function in $0 <= r <= L - 1$
- $0 <= T(r) <= L - 1$

Histogram Equalization

- Histogram as PDF
  - r and s as continuous random variables
  - $p_r(r)$ and $p_s(s)$ as probability distribution functions of r and s
  - $p_r(r)\,.dr = p_s(s)\,.ds$

Histogram Equalization

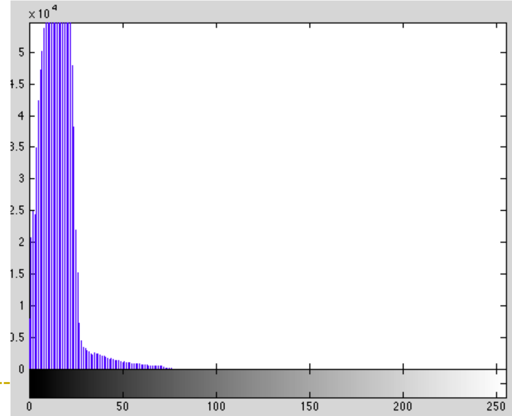
Derivation (on blackboard)
[Section 3.3.1 in GW]

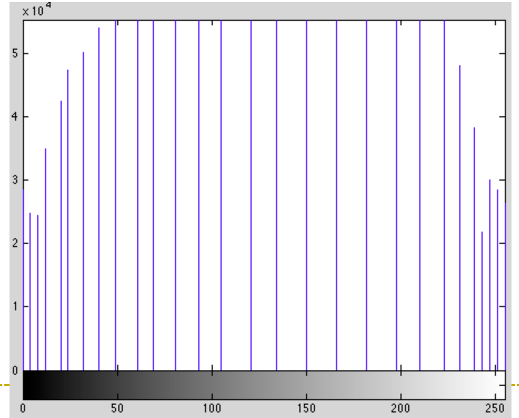
Histogram Equalization

Intensity transformation

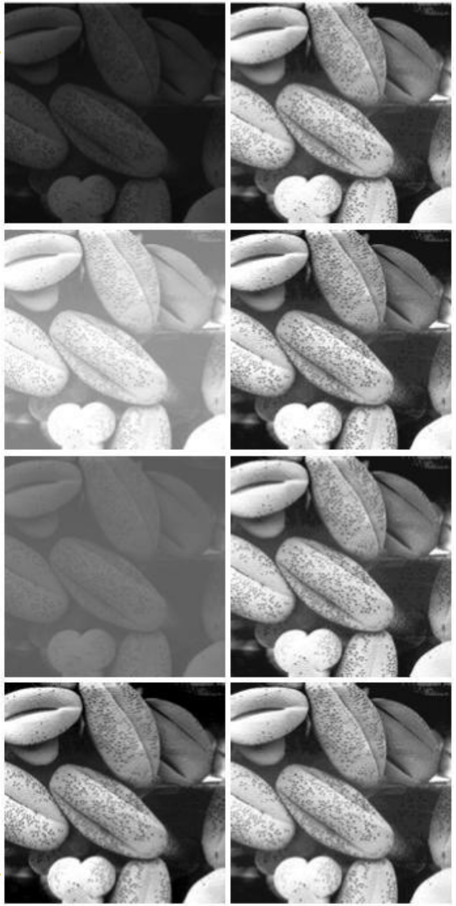
Histogram Equalization

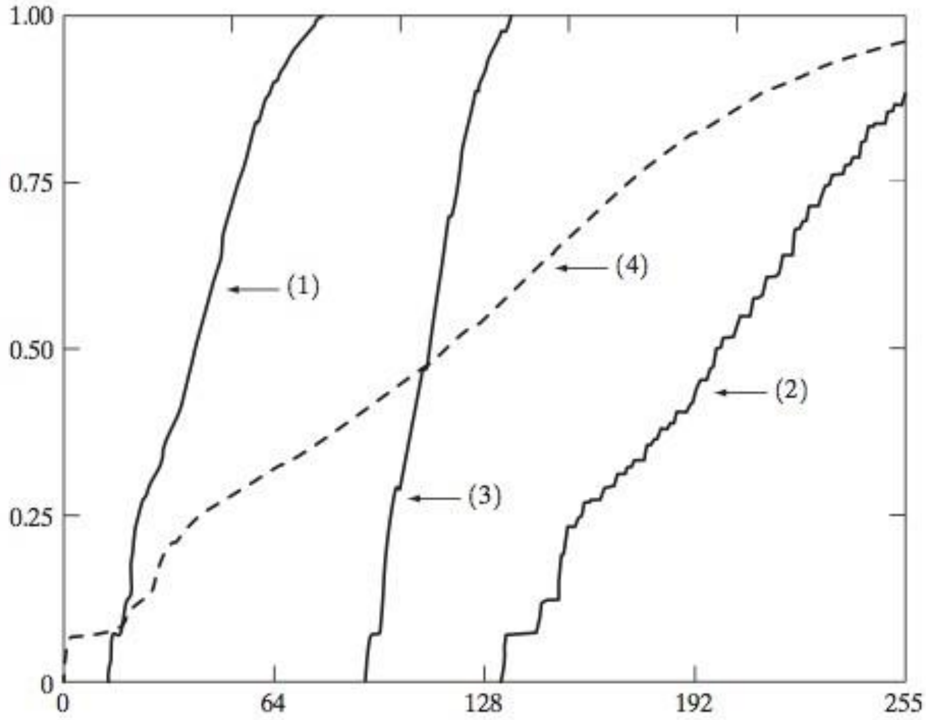
Histogram Equalization





Histogram Equalization





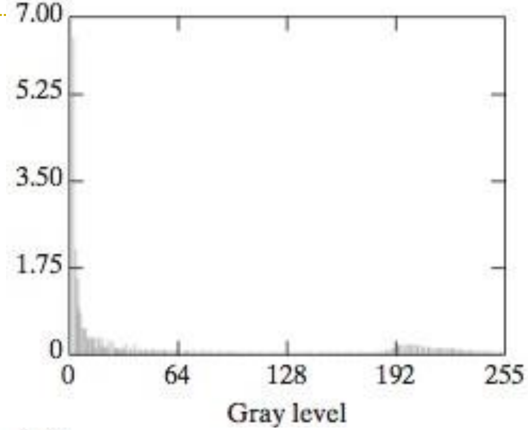
1 through 4 : top to bottom

Image Courtesy: Gonzalez andWoods

Histogram Equalization



Histogram Equalization



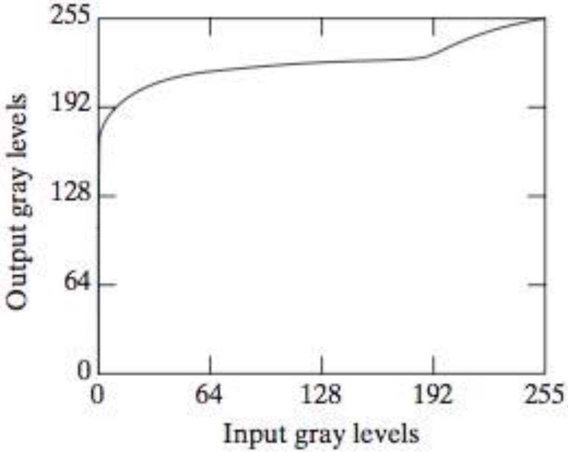


Image Courtesy: Gonzalez andWoods

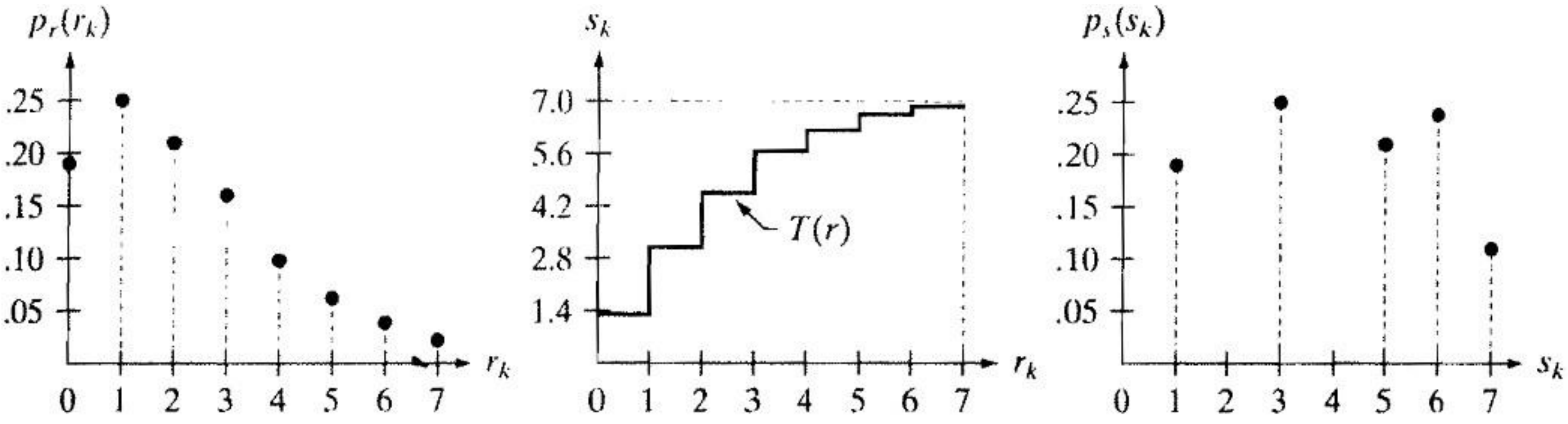
Histogram Equalization - Example

64 x 64 image

3-bits / pixel

r_k	n_k	$p_r(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02

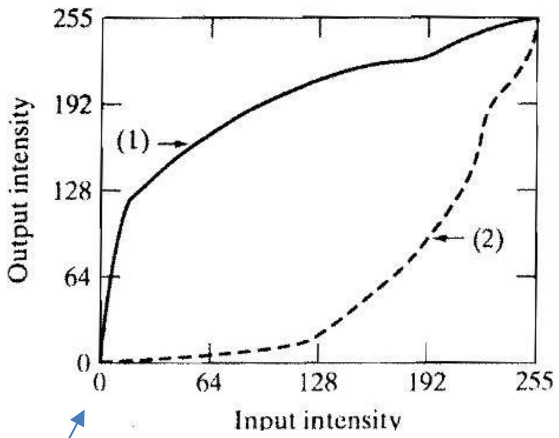
Histogram Equalization - Example



a b c

FIGURE 3.19 Illustration of histogram equalization of a 3-bit (8 intensity levels) image. (a) Original histogram. (b) Transformation function. (c) Equalized histogram.

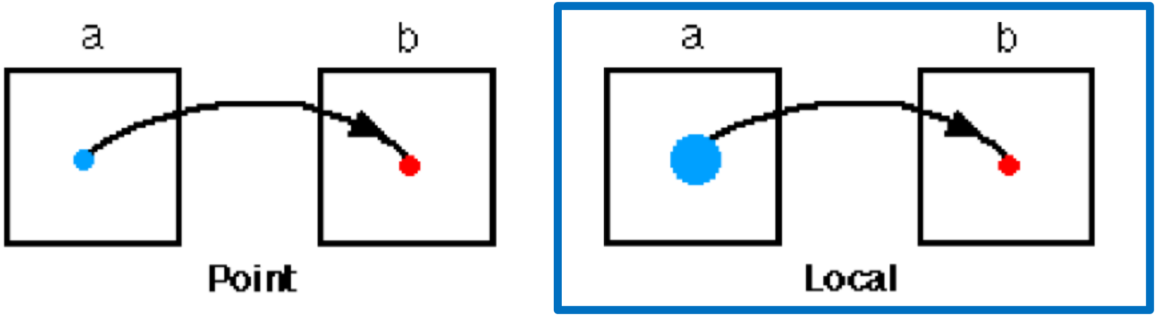
Histogram Specification / Matching [Section 3.3.2]



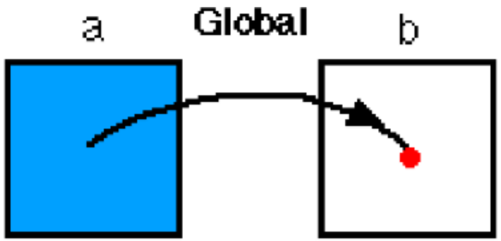
Compare with the curves we saw for contrast enhancement. What's the difference ?

Spatial Domain Processing

- Manipulating Pixels Directly in Spatial Domain

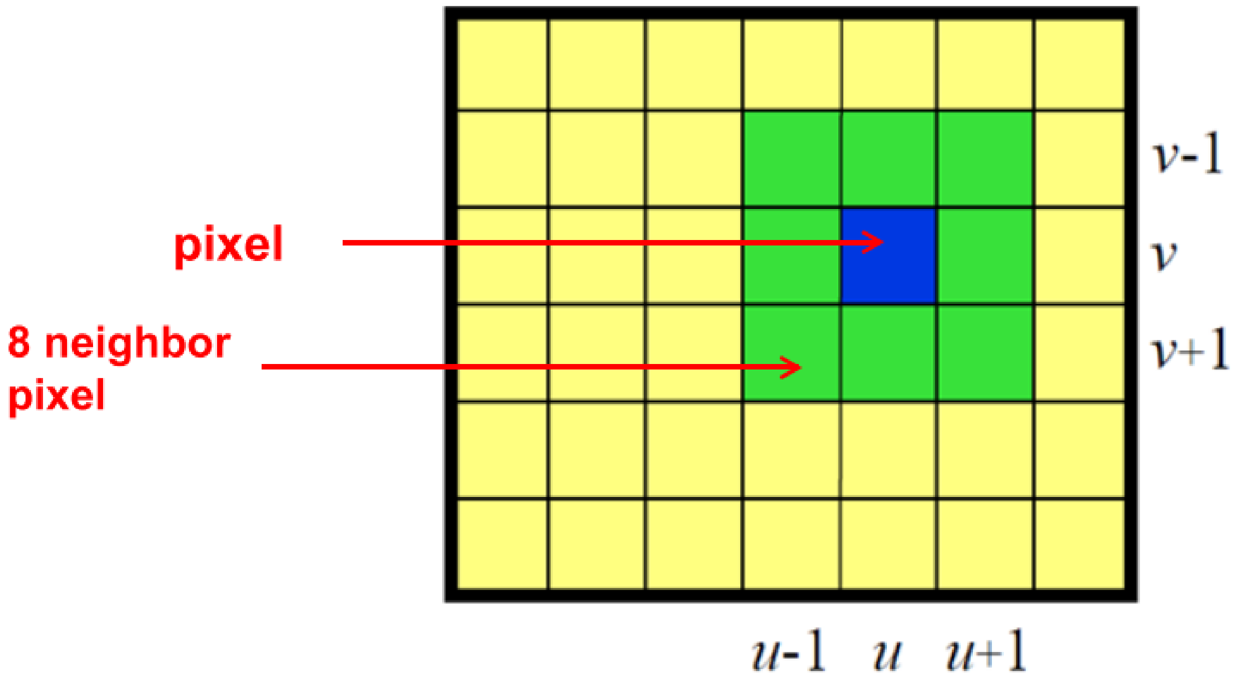


- Point to Point
- **Neighborhood to Point**



- Global Attribute to Point

Neighborhood



Local Histogram Processing

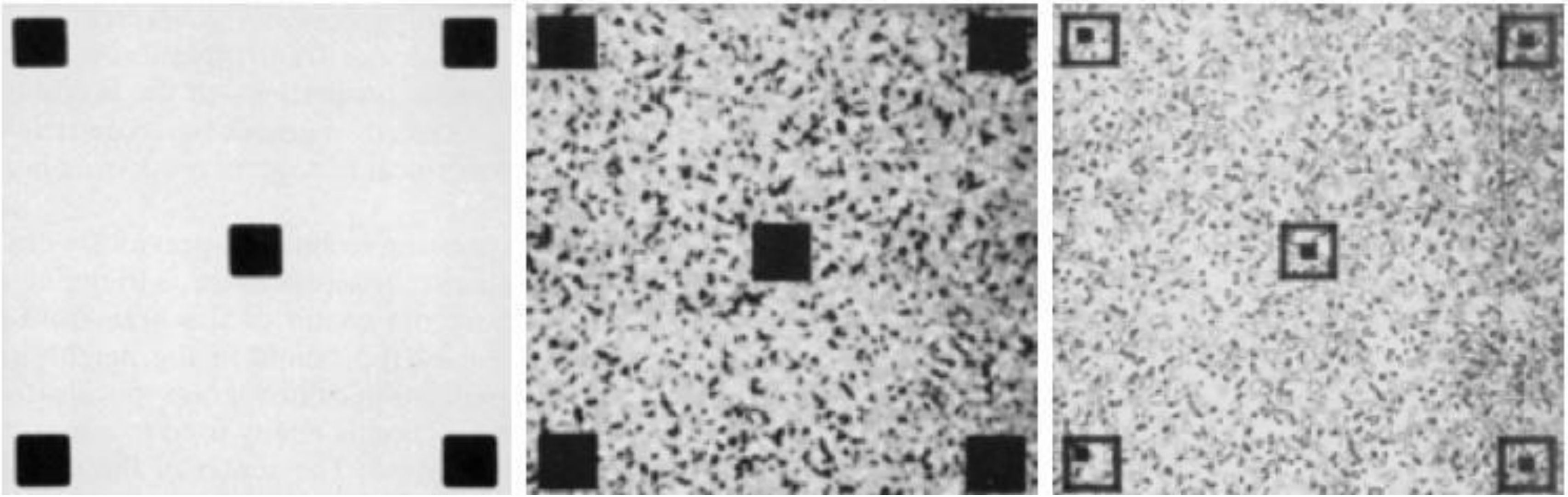


Image Courtesy: Gonzalez andWoods

References

- GW Chapter – 3.3.1 to 3.3.3

- Transformations of Random Variables
  - http://www.randomservices.org/random/dist/Transformations.html
  - Section 1 of http://www.cs.cmu.edu/~minx/transform.pdf
  - Leibnitz Integration Rule : https://en.wikipedia.org/wiki/Leibniz_integral_rule#Alternative_derivation