Problem 5.2

a) Brute force multiplication means each bit of one of the integers has to be multiplied by each bit of the other integer. So the time complexity of brute force multiplication is $O(n^2)$

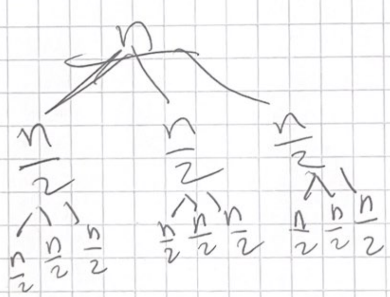
b) $x = x_l \cdot 2^{\frac{n}{2}} + x_r$ l - left $\frac{n}{2}$ bits of x and y.
$y = y_l \cdot 2^{\frac{n}{2}} + y_r$ r - right

$$xy = (x_l \cdot 2^{\frac{n}{2}} + x_r)(y_l \cdot 2^{\frac{n}{2}} + y_r)$$
$$= 2^n x_l y_l + 2^{\frac{n}{2}}(x_l y_r + x_r y_l) + x_r y_r$$

c) Recurrence for this

$$T(n) = 3T\left(\frac{n}{2}\right) + O(n)$$

d) $T(n) = 3T\left(\frac{n}{2}\right) + O(n)$



$i = 0$ $n$

$i = 1$ $3 \cdot \frac{n}{2}$

$i = 2$ $9 \cdot \frac{n}{4}$

$i = 3$ $27 \cdot \frac{n}{8}$

$\Big\} \; 3^i \cdot \frac{n}{2^i}$

$$\frac{n}{2^i} = 1 \Rightarrow \log_2 n$$

$$T(n) = \sum_{i=1}^{\log_2 n} n \cdot \left(\frac{3}{2}\right)^i$$

$$2 \cdot 3 \cdot (3^{\frac{\log n}{\log 2}} - n) = 3 \cdot 3^{\log_2 n} - 3n$$

$3^{\log_2 n} \Rightarrow n^{\log_2 3}$

e) master theorem

$n^{\log_2 3} = n^{1.585}$

$\rightarrow$ so $\boxed{n^{\log_2 3}}$

$n^{1.585} > n$