Forecasting Housing Prices in Oslo, Norway

A Comparative Analysis of ARIMA and ETS Models

Exam Paper

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1. Introduction

Homeownership holds a significant position in the financial portfolio of many, often representing their most substantial investment and largest financial asset of people, in Norway. Reflecting this importance, nearly four out of every five households own their dwelling, indicating a deep-rooted inclination towards homeownership (Statistics Norway, September 14, 2017). This widespread ownership is not unique to a particular region; in fact, Norway's homeownership rate is above the average of OECD countries (OECD, June 10, 2021). Given its relevance, the future trajectory of the housing market is of interest, not just to existing homeowners but to a wide spectrum of stakeholders, ranging from prospective buyers and sellers to policymakers, constructors, investors, and beyond. Oslo, being the priciest city in Norway, sits at the center of this interest, especially concerning row housing — a prevalent form of urban living that attracts significant attention from a vast audience due to financial feasibility.

As such, estimating the future state of the row house market in Norway's capital five years ahead is of great interest. To achieve this, quarterly data from the house price index for row houses, spanning from 1992 to 2023, from Norway's official statistics institute, Statistics Norway (SN) is analyzed. Employing the forecasting techniques of ETS and ARIMA models and comparing their predictions against a simple drift model as a benchmark, this paper intends to identify which, ARIMA or ETS, performs best in forecasting the market's future state.

2. Dataset Characteristics

The time series data is retrieved as subset from a large and comprehensive dataset on housing information in Norway from the SN on the house market in Oslo and Bærum. While these are two municipalities, they are combined by standard in official statistics, due to proximity and interconnection, and will be referred to as Oslo. SN provides statistics across a wide field of topics and present both raw and seasonally adjusted data. However, for this dataset, information on how seasonal effects have been adjusted were not accessible, thus the raw data was preferred for this investigation. It consists of quarterly data, calculated at the end of each period and adjusted for inflation, on the house price index (HPI) values, representing the average value change of row houses sold on the free market. Thus, this dataset enables analysis of how the housing market may develop going forward and is therefore suitable for the purpose of this paper. There are 126 observations, with no missing values, covering the time period from first quarter of 1992 to the second quarter in 2023. The index uses 2015 as the base year, index value equal 100, the lowest value of the market was recorded in the first quarter of 1993, 13.60, and the highest was in the first quarter of 2022, at 164.50. Between 1992 and 2023, the annual growth rate was 7.88% and a compounded increase in mean prices of 949% over the whole time period. Figure 1 displays a strong upward trend for the development of HPI values, with a noticeable fluctuations around 2006-2008, 2016-2018 and from 20121, but otherwise a generally stable development at an accelerating rate. From the initial inspection, one observes some cyclical tendencies and no distinct seasonal pattern (although small potential seasonality can be observed) in Figure 1.

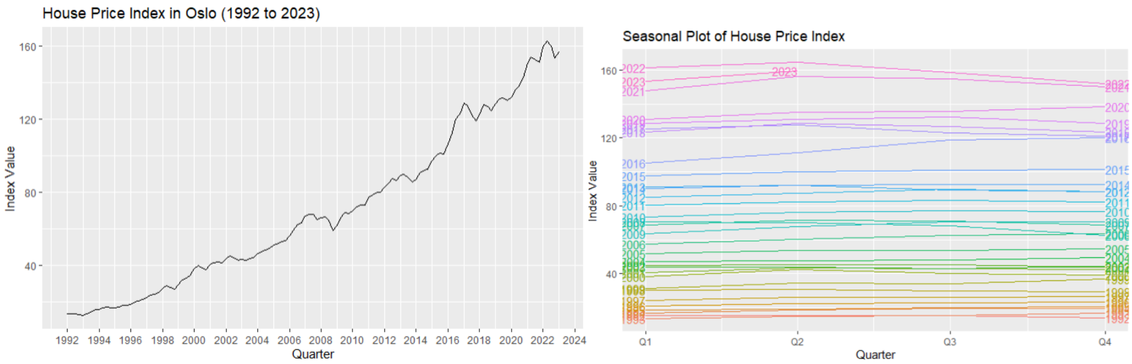

Figure 1: *Graph of Raw Data and Seasonal Plot for House Price Index Values*

3. Time Series Analysis

This section describes the analysis of the time series and considerations for transformation, decomposing, stationarity, structural breaks and elaborating on the corresponding measures that have been applied before proceeding to constructing models and forecasting.

3.1 Data Transformation

To achieve more accurate forecasts, two separate mathematical transformations of the data was tested in search for a simpler pattern to forecast, logarithmic transformation and Box-Cox transformation. Both transformations help in stabilizing variance in the data and can reduce the necessity of more intricate procedures, where a logarithmic is considered easier to interpret in contrast to the combination of that with power transformations, which can handle more complex patterns at the cost of less interpretability, as in Box-Cox (Hyndman & Athanasopoulos, 2021). Transforming the data with Box-Cox, using a lambda value of 0.316669, resulted in an almost linear curve, graph on the right in Figure 2, in contrast to the transition from a convex to a concave curve from using only logarithms, graph on the left. Despite theoretically less interpretable, the Box-Cox transformed data portrays a simpler and more linear pattern and is therefore used in subsequently processes.

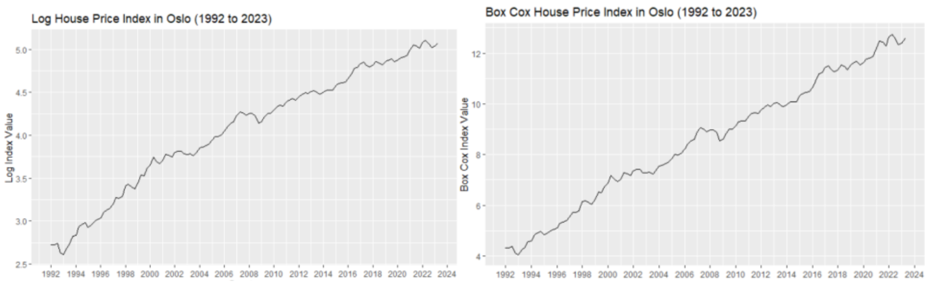

Figure 2: *Left Graph: Logarithmic Transformation; Right Graph: Box-Cox Transformation*

3.2 Data Decomposition

The time series was decomposed into its latent components, trend, season and remainder using classical additive decomposition, on the background of the conversion of multiplicative relationships to additive following the data transformation. Figure 3, left plot, displays a strong, almost linear, positive trend which increases with time and a magnitude of 4 to 12 for the trend component. The seasonal component appears to maintain a consistent pattern, shape, and size. However, one must consider the seasonal assumptions of the method, but, when examining the relative magnitude of the seasonal component, ranging between -0.04 and 0.04, along with the pattern in the seasonal plot, Figure 1, seasonality may likely only represent a minor component. Similarly, the remainder component is on a similar small scale, -0.10 to 0.15, and stable variance over time indicates homoskedasticity.

The ACF Plot to the right in Figure 3 displays a strong autocorrelation, over 0.5, up to 20 lags and significant up 36 lags with a slow decay rate, which confirms a strong trend and could imply potential non-stationary data. An interpretation strengthened by a highly significant first lag in the partial autocorrelation as seen in the bottom PACF plot. Finally, the lack of seasonal spikes in the ACF Plot strengthens an assumption of minimal impact of seasonality.

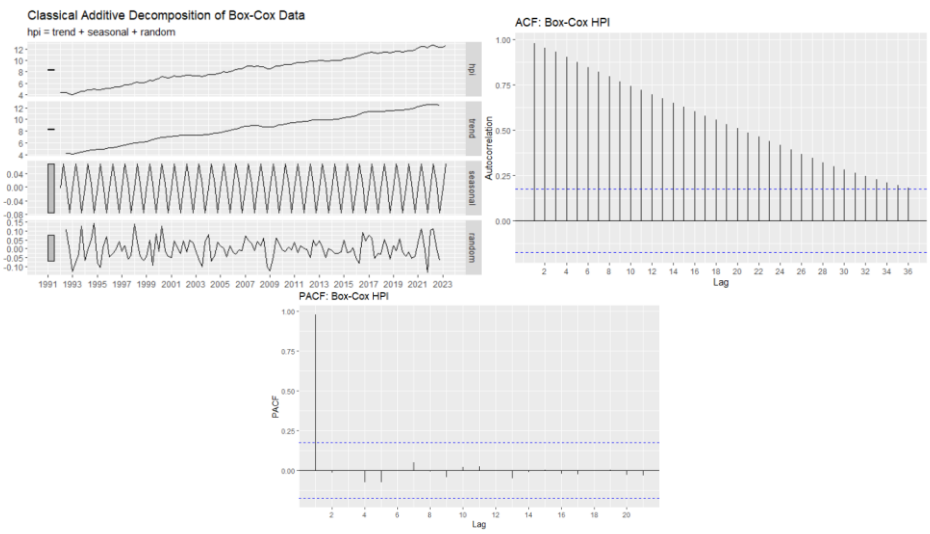

Figure 3: *Upper Left: Trend, Season, and Remained Components. Upper Right: ACF Plot. Bottm: PACF Plot*

3.3 Stationarity

Following the observations in the ACF plot in Figure 3, more formal tests of presence of unit root are conducted to investigate whether the time series has statistical properties that does not change with time, stationarity (Hyndman & Athanasopoulos, 2021). The tests used were the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) and Augmented Dickey–Fuller (ADF) and for all tests the values 0.05 level of significance are considered. Table 1 presents a summary of the outcomes of the KPSS and ADF tests for examining the stationarity of the time series data, see Appendix, Figure 1 and 2, for details. The KPSS test has two main types: "tau" and "mu". The "tau" type inspects if the data is stationary around a deterministic trend. Conversely, the "mu" type checks for the presence of a unit root and a stochastic trend. Initially, the test values for "tau" and "mu" were 0.2110 and 2.5803, respectively. With test values of 0.2110 > 0.146 and 2.5803 > 0.463, there is compelling evidence of non-stationarity, either in terms of a trend or drift. Following the guidelines of Hyndman and Athanasopoulos (2021), a first order differencing was undertaken. The post-differencing values were 0.0271 (tau) and 0.0345 (mu), both of which suggest that the series became stationary around a deterministic trend and drift.

Concerning the ADF tests: For the trend component, both the initial and post-differencing values for "tau3", "phi2", and "phi3" denote the absence of a unit root, suggesting the series is stationary around a trend. On the other hand, for the parameters "tau2", "phi1", and "tau1", the pre-differencing results display ambiguity concerning drift. However, test values after applying a first-order differencing, provides strong evidence of stationarity, see Appendix, Figure 3, for visualization of data.

	KPSS		ADF					
			Trend			Drift		None
Confidence, 5%	tau	mu	tau3	phi2	phi3	tau2	phi1	tau1
	0,146	0,463	-3,45	4,88	6,49	-2,89	4,71	-1,95
Initial t-values	0,2110	2,5803	-3,7789	11,8905	7,1751	-0,5853	9,7998	3,9607
After 1 lag diff	0,0271	0,0345	-9,8857	32,5844	48,8700	-9,9147	49,1570	-7,3164

Table 1: *KPSS and ADF Test Values Before and After Differencing*

3.4 Structural Breaks

The final step of the time series analysis pertains to check for structural breaks, to identify potential sources for instability and worsened performance of the forecasting models. To identify any, if multiple, breakpoints in the series, the Quandt Likelihood Ratio was applied, and with p-value = 0.979, there is no strong evidence of structural breaks and thus no further action were undertaken. See Appendix, Figure 4, for visualization.

4. Methodology

After the preceding measures, the data is ready for application of predictive models such as ARIMA and ETS. These models are selected due their complementary nature in addressing forecasting of time series. Both manual and automatically selected models are compared, where the parameters are derived from time series analysis. To measure performance, the data is split into training and test sets, with the training set comprising 80%, 101 samples, of the observations and the test set consisting of 20%, 25 samples. For each class of models, one manual and one automatic model is constructed, measured in performance in terms of the information criteria AIC, AICc, BIC, and the behavior of residuals inspected to choose one model for each class, and finally the classes are compared to each other and a benchmark.

4.1 ETS Model Specifications

An ETS model does not require stationary data and produces forecast by decomposing a time series into the latent components trend, season, and error. It finds the best coefficients using maximum likelihood estimation, and produces a forecast by relying on exponentially smoothed averages of previous observations. The parameters of an ETS model then corresponds to whether how each component should be combined in an additive or multiplicative fashion. For instance, the trend may be damped or additive, the season may be multiplicative if it is variance increases with time, and so on. Given the logarithmic transformation of the data, an additive approach for the manual model is therefore an appropriate choice. From examining the decomposed components in Figure 3, there is a strong upward going trend. The seasonal component is relatively small in magnitude, as is the remainder component. Thus, two manual models are assessed, an ETS(A,A,A), which means that all components are added, and an ETS(A,A,N), to check the effect of setting the seasonal component to none.

Model Parameters	AIC	AICc	BIC
A, A, A	32,85678	34,8348	56,39286
A, A, N	48,08305	48,71463	61,15865
Automatic (A, A, A)	32,85678	34,8348	56,39286

Table 2: *Performance Comparison of ETS Models*

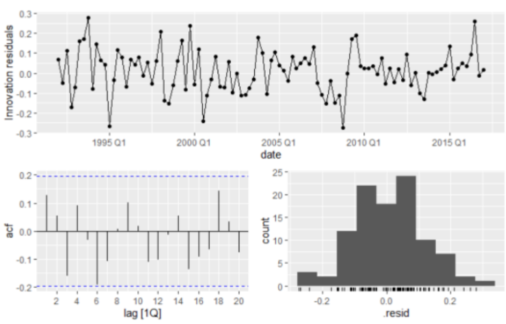

Figure 4: *Residual of Best ETS Model*

Table 2 displays the performance of the models, and it reveals that the automatically derived model arrives at the same parameters as the initial guess; a model that incorporates all decomposed components in an additive manner, which also performs best by all information criteria. Examining the residuals visually, no significant correlation is spotted in the ACF plot in Figure 4, which is supported by a Ljung-Box test value of 0.0590894 at 10 lags. Moreover, the distribution has an approximately normal shape, supported by a Shapiro-Wilk normality test of the residuals and p-value of 0.8077, and over time they resemble white noise with a relatively stable variance, Figure 4.

4.2 ARIMA Model Specifications

ARIMA models are adept at handling a variety of data patterns and can tackle both seasonal and non-seasonal data by utilizing autocorrelations and moving averages, but in contrast to ETS models, they require stationary data. The non-seasonal components, given by the parameters (p,d,q), constitute the order of an autoregressive model, order of differencing necessary, and the moving average model of previous forecasting errors, respectively. The seasonality is captured by the parameters (P,D,Q) and M.

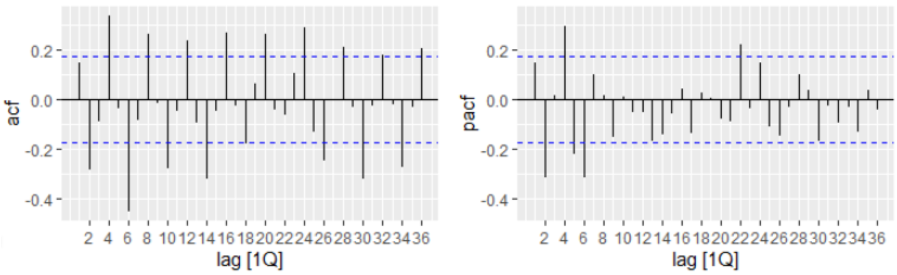

Figure 5: *ACF and PACF Plots of Stationary Data*

Hyndman and Athanasopoulos (2021) suggest that the data could align with an ARIMA(p, d, 0) model if the ACF plot displays an exponentially decaying or sinusoidal pattern and the PACF plot presents a significant spike at a specific lag without substantial spikes thereafter. The reverse patterns in ACF and PACF are indicative of an ARIMA(0, d, q) model. However, when both p and q are positive, ACF and PACF plots may not provide explicit directions. The following experimentation of parameters is inspired by their general guidelines. The ACF plot in Figure 5 resembles a sinusoidal pattern, potentially an AR(1) components or higher, with noticeable spikes at lags such as 2, 4, and 6 indicating seasonality. The PACF plot show pronounced negative and positive spikes before a drop off after lag 6, implying a relationship between the current and that datapoint. Moreover, the seasonality is further substantiated by a prominent seasonal lag, 4, in the PACF. Based on this, an ARIMA model with combining non-seasonal parameters (6,1,0) and seasonal parameters (1,1,1)4 is utilized due to a single spike at PACF lag 4, since there is repeated and exponential changes in the ACF, and seasonal differencing is evidently necessary.

The automatically derived model performs better on all information criteria metrics, see Table 3. However, when examining its residuals (Appendix, Figure 5), autocorrelation is present which is supported by a Ljung-Box test value of 0.0409 at 10 lags. Moreover, the distribution does not have a normal shape, supported by Shapiro-Wilk normality test p-value of 0.002, see the. In contrast, the manual model's residuals yield p-values of 0.2333 and 0.3259, respectively. Therefore, although the manual model has greater complexity and lower information criteria scores, but with residuals without significant correlation, a better distribution, and resembling white noise, this model is preferred, see Figure 6.

	Model Parameters	AIC	AICc	BIC
ARIMA	(6, 1, 0)(1, 1, 1)[4]	-144,97	-142,88	-121,9
	(1, 1, 0)(1, 0, 1)[4] w/drift	**-152,92**	**-152,29**	**-139,9**

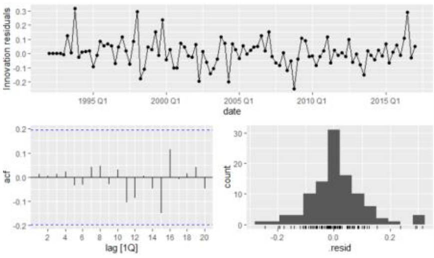

Table 3: *Performance Comparison of ARIMA Models* **Figure 6:** *Residuals of Manual ARIMA Model*

5. Results

Prior to examining the models forecasting capabilities, the Box-Cox transformed data is converted back to the original scale. In order to compare the models' forecasting performance a test set is utilized, out of sample data, to gauge their performance in an unbiased manner. A common benchmark to employ is a simple Drift model, which make a simple assumption of a linear change over time based on average change in historical data and can indicate whether other models are too complex.

Model	MAE	MASE	MAPE	RMSE
Drift	0,1856201	0,5743329	1,55582	0,2160033
ETS	1,2137009	3,7553491	10,067034	1,3220561
ARIMA	0,3030396	0,9376441	2,550836	0,3422543

Table 4: *Measures of Error on Test Data*

The accuracy metrics considered are; Mean Absolute Error (MAE), gives the average absolute errors between predicted and actual observations; Mean Absolute Scaled Error (MASE), the MAE of the model relative to the MAE of a naïve forecast (Values < 1 = the model forecasts better than a simple naïve forecast; Mean Absolute Percentage Error (MAPE), the average percentage error in the model's predictions; Root Mean Square Error (RMSE), how much error the system typically makes in its predictions. As seen in Table 4, the benchmark model outperforms all models across all metrics, the ETS model performs consistently worst, and the ARIMA model lies between. This makes sense when considering the shape of Box-Cox transformed the time series in Figure 2, the relative magnitudes of the seasonal and remained component. However, the fluctuations in the time series, even after logarithmic transformation is not entirely linear, as seen, for instance, in the variation in the time periods 2006-2008 and 2016-2018.

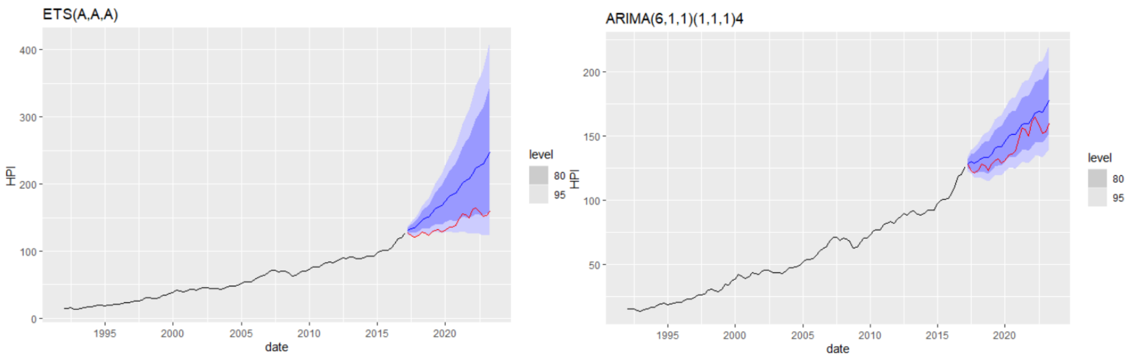

Figure 7: Comparison of *ETS and ARIMA Forecasts on Test Data (2017 – 2023)*

Both models overshoot in their forecasting relative to the test data, as seen in Figure 7, although the ARIMA model do so to a much smaller degree. The ETS model produces a more exponential development in its future values and accelerate further away from the test data, with a much larger confidence interval that ends up diverging with the 95% interval outside the test data. The ARIMA model forecasts a trajectory that follows the actual development much closer to actual values and stay within the 95% confidence interval. Both models appear capable of incorporating the fluctuations that are persistent, and to be expected, throughout the time series and seem realistic, in contrast to the Drift model (by its linear nature). Considering the overall bad accuracy performances of ETS model in Table 4, and particularly the indications of it being outperformed by a simple naïve forecasting method (MASE value 3.75 > 1), the ARIMA model is used to create a five-year forecast. According to its prediction, the mean house price index for apartments in Oslo will increase between the second quarter of 2023 and second quarter of 2028 from 159,4 to 205,9, which decompounds into an annual growth rate of 5.25%, in comparison to the annual growth rate of 7.88% between 1992 and 2023.

6. Conclusions

This study undertook a comparative analysis between the ETS and ARIMA forecasting models to forecast the house price index in Oslo, Norway. The findings revealed that, when benchmarked against a range of accuracy metrics, the ARIMA model consistently surpassed the ETS model in predictive performance. Interestingly, both these sophisticated models were outperformed by the benchmark model, a simple drift model, in terms of sheer accuracy. However, it's crucial to highlight that while the drift model's linear approach yielded higher accuracy, it failed to account for the natural oscillations inherent to the housing market. This limitation could lead to misinterpretations when extrapolating future market behavior.

The ARIMA model, in contrast, offers a more nuanced estimation of the housing market's future trajectory. This is of interest for a broad spectrum of stakeholders - including homebuyers, sellers, builders, and policymakers - as it paints a comprehensive picture of market trends. Notably, if we use the historical annual growth rates as a reference point, the projected growth from our ARIMA model, being relatively lower, suggests potential profitable avenues (with historical growth at 7.88% compared to the forecasted 5.25%).

Concerning limitations, the dataset analyzed primarily captures the mean cumulative shifts in apartment values. Without a detailed breakdown between types of row houses, and the various characteristics of these, and their locations/regions in Oslo, there is unaccounted for variance. As such, the predictions derived are a holistic picture of one of several market sectors concerning housing. Future research could benefit from a broader exploration of forecasting models, for instance, dynamic regression models, and a more exhaustive parameter grid search to further refine the predictive accuracy and provide more actionable insights into the housing market's future.

7. References

Hyndman, R.J., & Athanasopoulos, G. (2021). *Forecasting: principles and practice* (3rd edition), OTexts: Melbourne, Australia. OTexts.com/fpp3.

OECD. (June 10, 2021). *Housing Sector Country Snapshot: NORWAY*. https://housingpolicytoolkit.oecd.org/www/CountryFiches/housing-policy-Norway.pdf

Statistics Norway. (September 14, 2017). *Large majority own their dwelling*. https://www.ssb.no/en/bygg-bolig-og-eiendom/artikler-og-publikasjoner/large-majority-own-their-dwelling

8. Appendix

```
#######################
# KPSS Unit Root Test #
#######################

Test is of type: tau with 4 lags.

Value of test-statistic is: 0.211

Critical value for a significance level of:
                10pct  5pct 2.5pct  1pct
critical values 0.119 0.146  0.176 0.216
```

```
#######################
# KPSS Unit Root Test #
#######################

Test is of type: mu with 4 lags.

Value of test-statistic is: 2.5803

Critical value for a significance level of:
                10pct  5pct 2.5pct  1pct
critical values 0.347 0.463  0.574 0.739
```

```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression trend


Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.33573 -0.07734 -0.00938  0.07447  0.33352 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.832059   0.205034   4.058 8.84e-05 ***
z.lag.1     -0.183246   0.048492  -3.779 0.000247 ***
tt           0.012346   0.003303   3.738 0.000286 ***
z.diff.lag   0.242557   0.089364   2.714 0.007622 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1228 on 120 degrees of freedom
Multiple R-squared:  0.126,	Adjusted R-squared:  0.1041 
F-statistic: 5.765 on 3 and 120 DF,  p-value: 0.001021


Value of test-statistic is: -3.7789 11.8905 7.1751 

Critical values for test statistics: 
      1pct  5pct 10pct
tau3 -3.99 -3.43 -3.13
phi2  6.22  4.75  4.07
phi3  8.43  6.49  5.47
```

```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression drift


Call:
lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.35640 -0.07872  0.00124  0.08784  0.33559 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)  
(Intercept)  0.080886   0.042746   1.892   0.0608 .
z.lag.1     -0.002788   0.004763  -0.585   0.5594  
z.diff.lag   0.146535   0.090063   1.627   0.1063  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1292 on 121 degrees of freedom
Multiple R-squared:  0.02421,	Adjusted R-squared:  0.008081 
F-statistic: 1.501 on 2 and 121 DF,  p-value: 0.227


Value of test-statistic is: -0.5853 9.7998 

Critical values for test statistics: 
      1pct  5pct 10pct
tau2 -3.46 -2.88 -2.57
phi1  6.52  4.63  3.81
```

```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression none


Call:
lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.34914 -0.07782  0.00979  0.08488  0.31492 

Coefficients:
           Estimate Std. Error t value Pr(>|t|)    
z.lag.1    0.005798   0.001464   3.961 0.000126 ***
z.diff.lag 0.170921   0.090074   1.898 0.060117 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1306 on 122 degrees of freedom
Multiple R-squared:  0.2062,	Adjusted R-squared:  0.1931 
F-statistic: 15.84 on 2 and 122 DF,  p-value: 7.649e-07


Value of test-statistic is: 3.9607 

Critical values for test statistics: 
      1pct  5pct 10pct
tau1 -2.58 -1.95 -1.62
```

Figure 1: KPSS and ADF Test Values Before Differencing

```
#######################
# KPSS Unit Root Test #
#######################

Test is of type: tau with 4 lags.

Value of test-statistic is: 0.0271

Critical value for a significance level of:
                10pct  5pct 2.5pct  1pct
critical values 0.119 0.146  0.176 0.216
```

```
#######################
# KPSS Unit Root Test #
#######################

Test is of type: mu with 4 lags.

Value of test-statistic is: 0.0345

Critical value for a significance level of:
                10pct  5pct 2.5pct  1pct
critical values 0.347 0.463  0.574 0.739
```

```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression trend


Call:
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

Residuals:
     Min       1Q   Median       3Q      Max
-0.37351 -0.07179 -0.01049  0.06463  0.30801

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.0824173  0.0242284   3.402 0.000913 ***
z.lag.1     -1.1265476  0.1139567  -9.886  < 2e-16 ***
tt          -0.0001185  0.0003145  -0.377 0.706926
z.diff.lag   0.3190665  0.0871968   3.659 0.000378 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1237 on 119 degrees of freedom
Multiple R-squared:  0.4833,	Adjusted R-squared:  0.4703
F-statistic:  37.1 on 3 and 119 DF,  p-value: < 2.2e-16


Value of test-statistic is: -9.8857 32.5844 48.87

Critical values for test statistics:
      1pct  5pct 10pct
tau3 -3.99 -3.43 -3.13
phi2  6.22  4.75  4.07
phi3  8.43  6.49  5.47
```

```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression drift


Call:
lm(formula = z.diff ~ z.lag.1 + 1 + z.diff.lag)

Residuals:
     Min       1Q   Median       3Q      Max
-0.37359 -0.06888 -0.00886  0.06410  0.30174

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.07481    0.01336   5.600 1.39e-07 ***
z.lag.1     -1.12444    0.11341  -9.915  < 2e-16 ***
z.diff.lag   0.31819    0.08685   3.664 0.000372 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1232 on 120 degrees of freedom
Multiple R-squared:  0.4827,	Adjusted R-squared:  0.4741
F-statistic: 55.98 on 2 and 120 DF,  p-value: < 2.2e-16


Value of test-statistic is: -9.9147 49.157

Critical values for test statistics:
      1pct  5pct 10pct
tau2 -3.46 -2.88 -2.57
phi1  6.52  4.63  3.81
```

```
###############################################
# Augmented Dickey-Fuller Test Unit Root Test #
###############################################

Test regression none


Call:
lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)

Residuals:
     Min       1Q   Median       3Q      Max
-0.27826 -0.03304  0.04407  0.14414  0.41177

Coefficients:
           Estimate Std. Error t value Pr(>|t|)
z.lag.1     -0.7719     0.1055  -7.316 3.05e-11 ***
z.diff.lag   0.1428     0.0906   1.576    0.118
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1378 on 121 degrees of freedom
Multiple R-squared:  0.3475,	Adjusted R-squared:  0.3368
F-statistic: 32.23 on 2 and 121 DF,  p-value: 6.028e-12


Value of test-statistic is: -7.3164

Critical values for test statistics:
      1pct  5pct 10pct
tau1 -2.58 -1.95 -1.62
```

Figure 2: KPSS and ADF Test Values

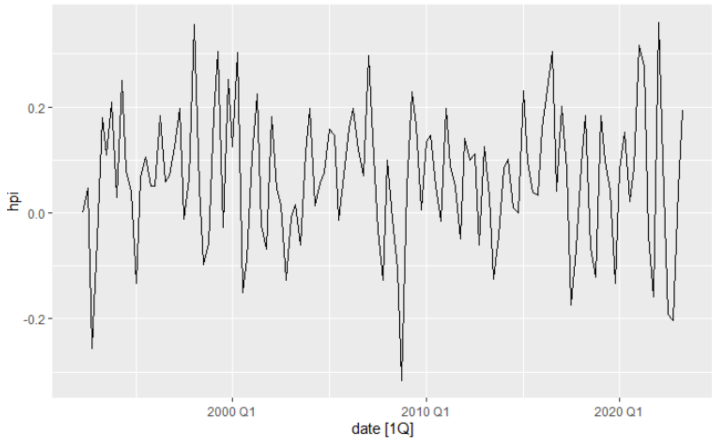

Figure 3: Differenced Data Plot

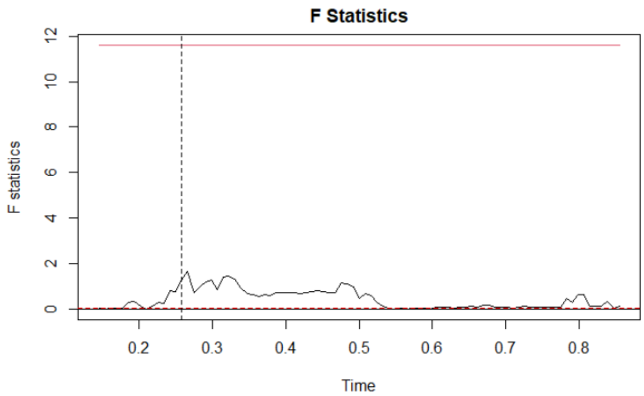

Figure 4: QLR Test for Structural Breaks

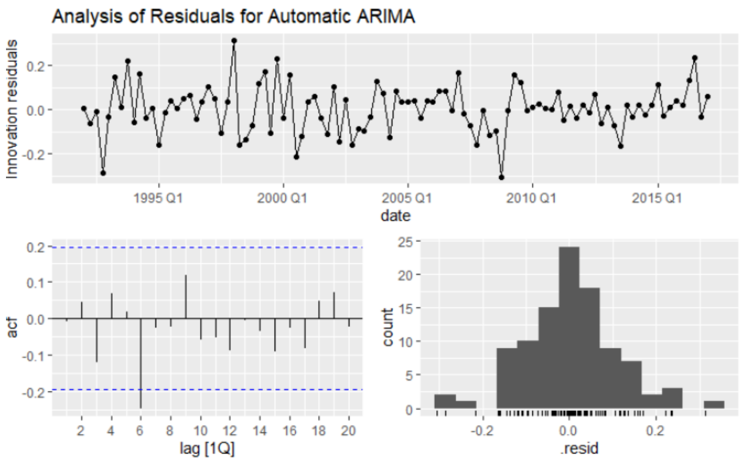

Figure 5: Residuals of Automatic ARIMA Model

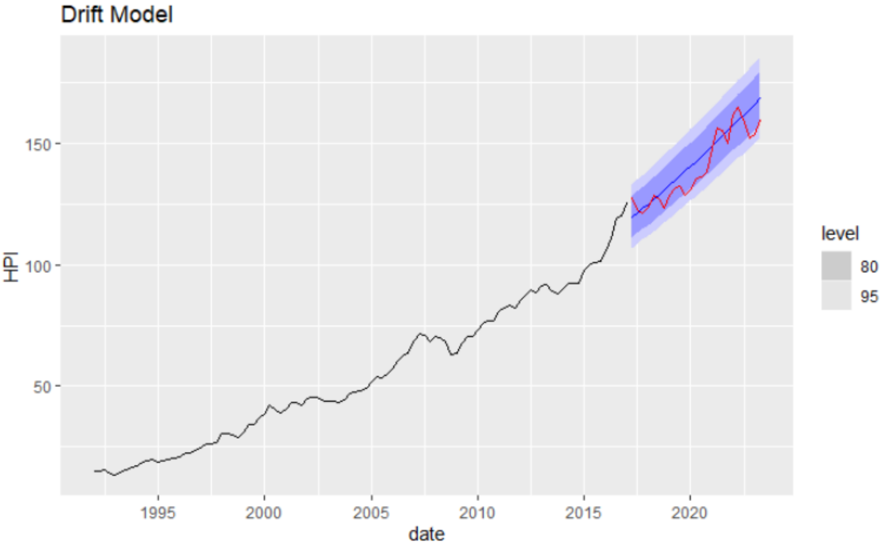

Figure 6: Forecast of Drift Model on Test Data