Politecnico di Milano

Ingegneria Matematica, MTM

A.Y. 2024/2025

FINTECH

FINAL PROJECT, GROUP 11

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1 Introduction

This work aims to faithfully reproduce the dynamics of the reference benchmark portfolio. The Statistical Arbitrage Risk (SAR) analysis quantifies how easily an instrument can be replicated by measuring the variability between its theoretical behavior and observed returns and, despite generally indicating that the index should be straightforward to track, there are specific periods in which significant tracking errors emerge due to factor selection bias and nonlinear market dynamics.

2 Methodology

2.1 Model fitting and selection

We fitted a suite of candidate models (Ridge, Lasso, OLS, PCA, Elastic Net and the Kalman Filter) to historical return series of the benchmark and its underlying risk factors. For each model, we performed an exhaustive grid search over its regularization and state noise hyperparameters, selecting the configuration that maximized the correlation between the model's predicted portfolio returns and those of the target index.

2.2 Cross-Validation Buffering

To prevent any look ahead bias in our time series framework, we employ a forward chain rolling window CV with an explicit temporal buffer of B weeks. At each iteration t:

1. **Training set:** weeks 1 through $t - B$.
2. **Buffer zone:** weeks $t - B + 1$ through $t - 1$, which are excluded from both training and validation.
3. **Validation set:** week t (or a window of k successive weeks starting at t).

Out of fold predictions $\hat{y}_t^{(i)}$ generated in this manner feed into the level-2 learner.

2.3 Bias reduction and smoothing techniques

Beta scaling: once raw factor exposures were obtained, we rescaled them to align the model's aggregate beta to that of the benchmark, thus correcting for systematic under or over exposure.

2.4 Transaction cost analysis and rebalancing policy

To realistically evaluate portfolio replication performance, we incorporated a cost aware Kalman Filter framework enhanced with adaptive rebalancing logic. Transaction costs were explicitly modeled using per trade fixed charges (e.g., 5 basis points per asset traded), calibrated to approximate real world execution frictions. Rather than rebalancing every week, we enforced a buffering threshold on the absolute weight change, allowing updates only when the portfolio allocation shifted meaningfully. This avoided unnecessary turnover while retaining model responsiveness to market changes. The resulting returns incorporate both transaction costs and a dynamic rebalancing rule.

2.5 Ensemble estimation and leverage strategies

To enhance portfolio robustness, we implement an ensemble model that aggregates the weights produced by multiple ML algorithms. in order to improve our portfolio replication dynamics, we apply James Stein shrinkage to the adaptive ensemble weights, blending them with an equally weighted benchmark (30% shrinkage intensity). This stabilizes the model by reducing sensitivity to estimation noise and extreme exposures. Any residual capital not allocated to risky assets due to the shrinkage induced dampening is automatically held in cash, earning a risk-free rate. We further account for trading frictions by deducting transaction costs proportional to weekly turnover (5 basis points). The resulting portfolio combines shrinkage adjusted asset returns, passive cash returns, and trading costs into a single stream of net performance, which is then compounded over time.

2.6 Backtested trading strategy

Finally, we implemented a pairs-trading strategy on the spread between the replicated portfolio and the benchmark. Whenever the deviation crossed a statistical threshold, we simulated trades designed to profit from its reversion, measuring profitability, Sharpe ratio and drawdown characteristics.

3 Main results and comments, Ensemblig vs Kalman filter model

3.1 Adaptivity versus conservatism

The Kalman filter treats factor exposures as latent states that evolve gradually under a tight state noise prior. This strong "inertia" makes it exceptionally cautious it barely adjusts weights in response to extreme moves like those in early 2020. As a result, it preserves exceptionally high correlation with the index (0.78) and limits large drawdowns, but it also forgoes the rebound rallies, yielding almost zero annualized active return.

3.2 Signal aggregation and risk return trade off

The ensemble, by pooling forecasts from Lasso, OLS, Elastic Net and Kalman captures a richer tapestry of predictive signals. Shrinkage then tempers any single model's overcommitments. This breadth of inputs injects incremental capture of upside swings hence the roughly 3.13% annualized active return but it necessarily "blurs" some idiosyncratic day to day tracking, lowering correlation to 0.51.

3.3 Impact on risk adjusted performance

Although both methods register negative Information Ratios (IR), the ensemble's IR (–0.07) is markedly better than the Kalman's (–0.38). By diversifying across orthogonal error patterns, the ensemble reduces the variance of the tracking error more than it dilutes returns. In effect, the shrinkage enforced consensus leans into signals that consistently add value out of sample, boosting return per unit of error even at the cost of some "tightness" of fit.

3.4 Sensitivity to extreme events

The Kalman filter's design inherently dampens the impact of outliers treating the COVID-19 crash as "noise" to be partly ignored whereas the ensemble, having models that are less averse to sudden shifts, incorporates more of that dislocation into its weights. Post crash, those opportunistic allocations pay off, lifting realized returns but also contributing to lower instantaneous correlation until the positions mean revert.

4 Pair Trading Strategy Based on Kalman Filter Replicas

We develop a market neutral pair trading strategy leveraging two independently trained Kalman filter portfolios, both replicating the same target index. Despite sharing the same objective, the two models occasionally diverge due to differences in parameterization and sensitivity to market conditions. These transient divergences generate opportunities for statistical arbitrage.

4.1 How it works

The strategy tracks the rolling correlation and spread between the replicas' returns. A position is opened when the correlation falls below a predefined threshold going long on the underperformer and short on the outperformer and closed once correlation reverts above the exit level.

Returns are benchmarked against the individual replicas and the target index, with performance assessed through the annualized Sharpe ratio. The method is market neutral, does not rely on absolute forecasts, and repurposes model disagreement into a profitable trading signal offering a robust enhancement to the overall replication framework.