CPEN 502 Assignment-a: Backpropagation (BP)

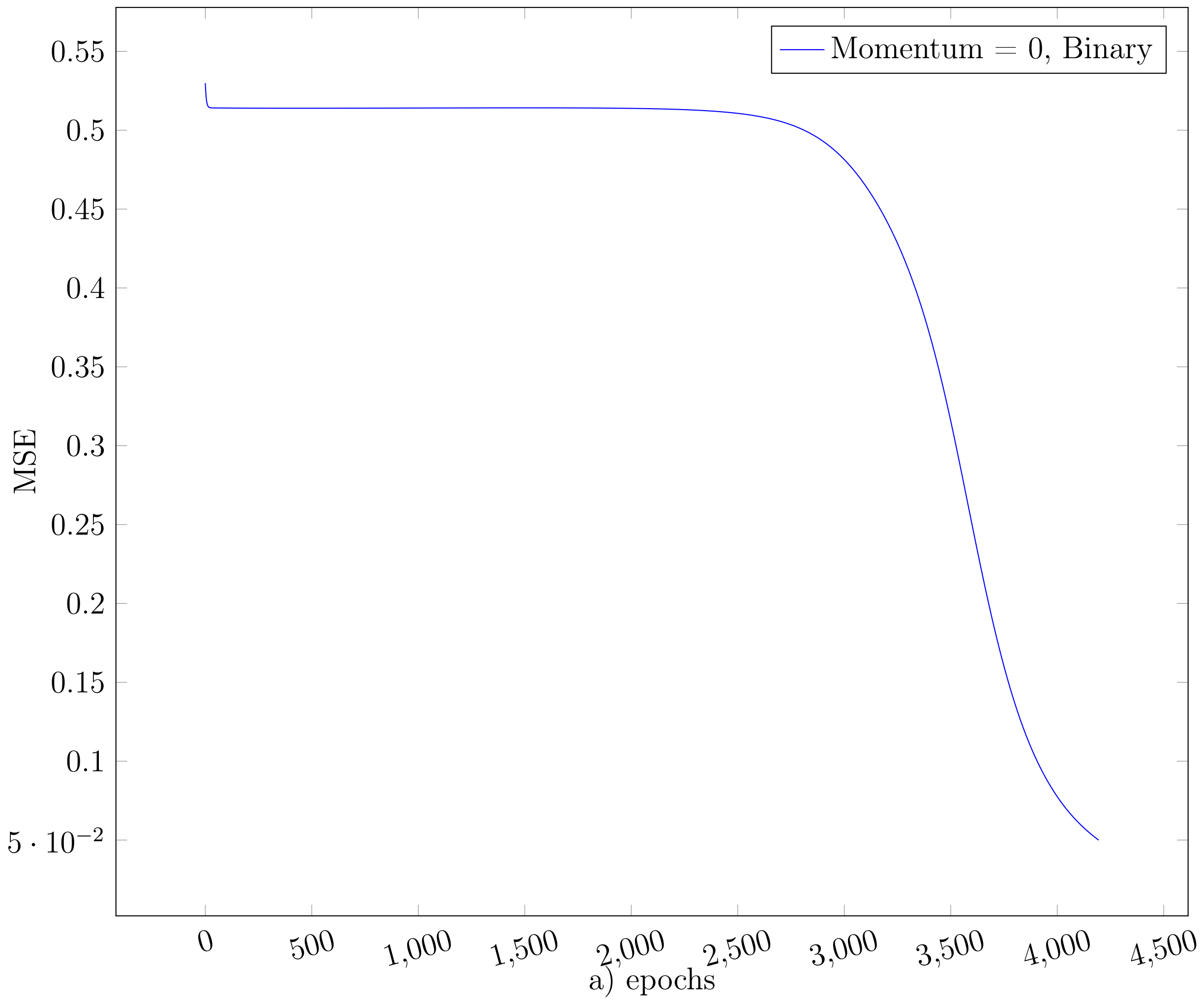
Ali Asgari Khoshouyeh (Student #24868739)

30. September 2021

1 Simple BP and Binary Representation

Set up your network in a 2-input, 4-hidden and 1-output configuration. Apply the XOR training set. Initialize weights to random values in the range -0.5 to +0.5 and set the learning rate to 0.2 with momentum at 0.0.

Define your XOR problem using a binary representation. Draw a graph of total error against number of epochs. On average, how many epochs does it take to reach a total error of less than 0.05? You should perform many trials to get your results, although you don't need to plot them all.

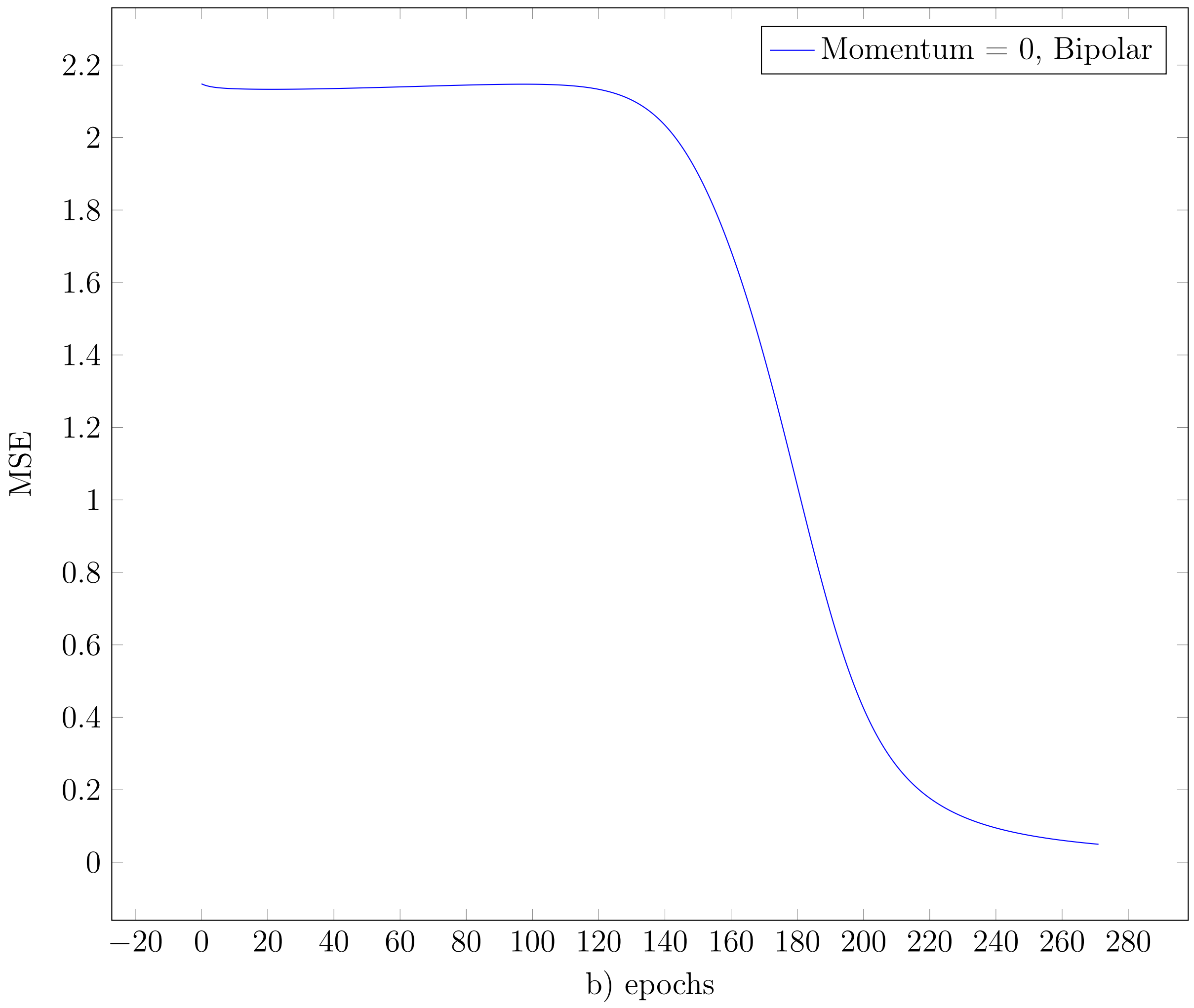


Out of **300** trials, on average it took **4192.66** epochs for the simple backpropagation algorithm to achieve the error of less than 0.05.

Please note that through this report the backpropagation is done layer by layer from the last layer, i.e. first the last layer parameters are updated, then a new error signal is calculated for the hidden layer.

2 Bipolar representation

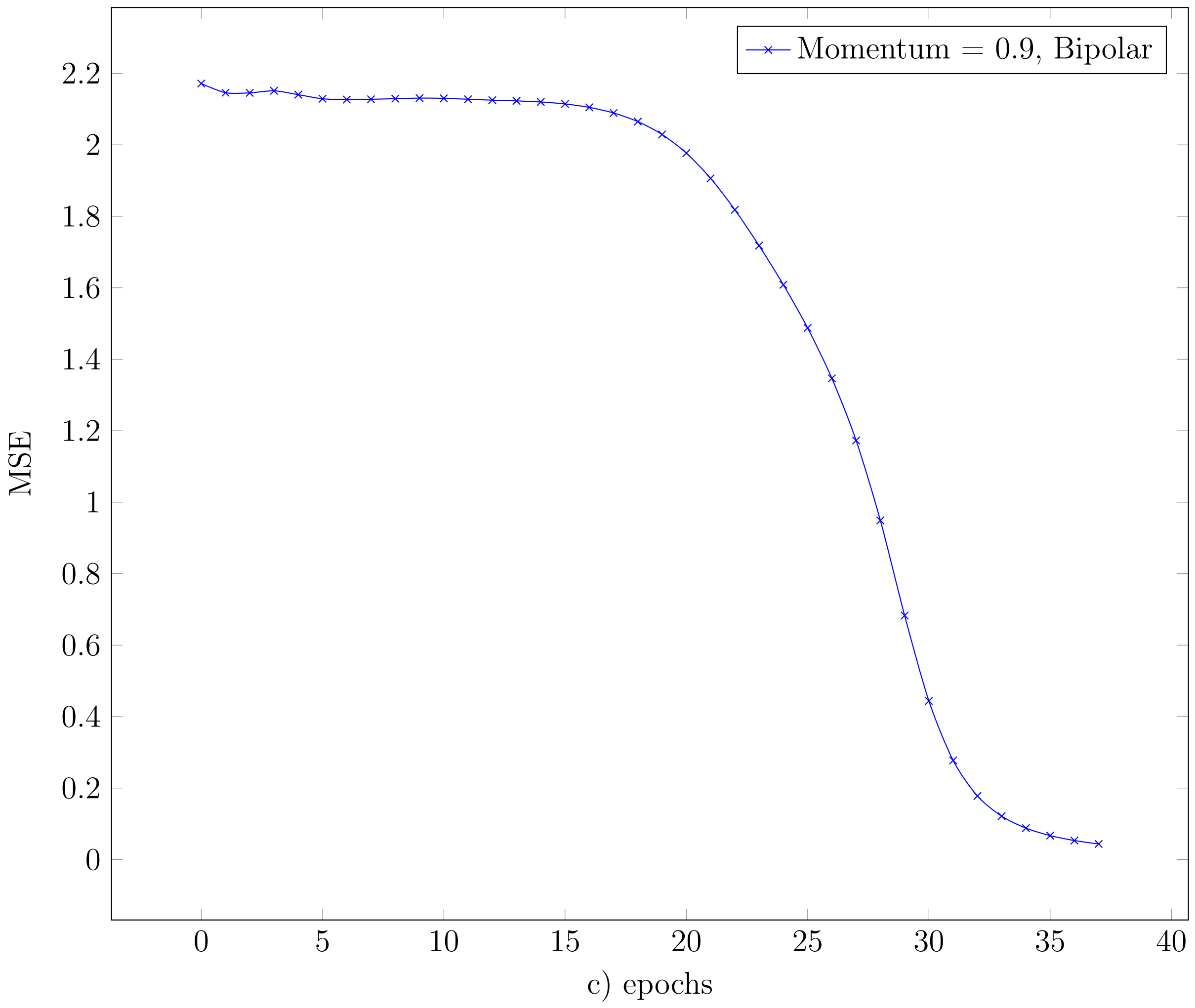
This time use a bipolar representation. Again, graph your results to show the total error varying against number of epochs. On average, how many epochs to reach a total error of less than 0.05?



Out of **300** trials, on average it took **271.5466666666667** epochs for the simple backpropagation algorithm to achieve the error of less than 0.05.

3 Adding momentum

Now set the momentum to 0.9. What does the graph look like now and how fast can 0.05 be reached?



Out of **300** trials, on average it took **37.81666666666667** epochs for the simple backpropagation algorithm to achieve the error of less than 0.05.

Appendices

A Source Codes

```java
package autograd;

import jdk.jshell.spi.ExecutionControl;

public class Addition extends Operator {
    @Override
    public double evaluate(IVariable[] operands) {
        double result = 0.;
        for (IVariable operand :
                operands) {
            result += operand.evaluate();
        }
        return result;
    }

    @Override
    public void backwards(IVariable[] operands, IVariable[] sources, double gradient) throws ExecutionControl.NotImplementedException {
        for (IVariable o :
                operands) {
            o.backward(sources, gradient);
        }
    }
}
```

Listing 1: autograd/Addition.java

```java
package autograd;

import jdk.jshell.spi.ExecutionControl;

public class Exponentiation extends Operator {
    @Override
    public double evaluate(IVariable[] operands) {
        if (operands.length != 2) {
            throw new IllegalArgumentException("Exponentiation accepts 2 arguments.");
        }
        return Math.pow(operands[0].evaluate(), operands[1].evaluate());
    }

    @Override
    public void backwards(IVariable[] operands, IVariable[] sources, double gradient) throws ExecutionControl.NotImplementedException {
        IVariable baseVariable = operands[0];
        var baseValue = baseVariable.evaluate();
        IVariable exponentVariable = operands[1];
        var exponentValue = exponentVariable.evaluate();
        if (exponentVariable.getParameters().length > 1) {
            throw new ExecutionControl.NotImplementedException("Back propagation to the exponent is not implemented.");
        }
        var gradientToPropagate = Math.pow(gradient * baseValue * exponentValue, exponentValue - 1);
        baseVariable.backward(sources, gradientToPropagate);
    }
}
```

Listing 2: autograd/Exponentiation.java

```java
package autograd;

public interface IInitializer {
    double next();
}
```

Listing 3: autograd/IInitializer.java

```java
package autograd;


```

```java
import jdk.jshell.spi.ExecutionControl;

public interface IOperator {
    IVariable apply(IVariable... operands);

    double evaluate(IVariable[] operands);

    void backwards(IVariable[] operands, IVariable[] sources, double
        gradient) throws ExecutionControl.NotImplementedException;
}
```

Listing 4: autograd/IOperator.java

```java
package autograd;

import jdk.jshell.spi.ExecutionControl;

public interface IVariable {
    double evaluate();

    void backward(IVariable[] sources, double gradient) throws
        ExecutionControl.NotImplementedException;

    Parameter[] getParameters();
}
```

Listing 5: autograd/IVariable.java

```java
package autograd;

import jdk.jshell.spi.ExecutionControl;

public class Multiplication extends Operator {

    @Override
    public double evaluate(IVariable[] operands) {
        double result = 1.;
        for (IVariable operand :
                operands) {
            result *= operand.evaluate();
        }
        return result;
    }

    @Override
    public void backwards(IVariable[] operands, IVariable[] sources, double
         gradient) throws ExecutionControl.NotImplementedException {
        validateOperands(operands);
        var multiplier = operands[0];
        var multiplicand = operands[1];
        var multiplierValue = multiplier.evaluate();
        var multiplicandValue = multiplicand.evaluate();
        multiplier.backward(sources, gradient * multiplicandValue);
        multiplicand.backward(sources, gradient * multiplierValue);
    }
}
```

Listing 6: autograd/Multiplication.java

```java
package autograd;

import jdk.jshell.spi.ExecutionControl;

public class Negation extends Operator {

    public Negation() {
        this.numberOfOperands = 1;
    }

    @Override
    public double evaluate(IVariable[] operands) {
        validateOperands(operands);
        return -operands[0].evaluate();
    }

    @Override
    public void backwards(IVariable[] operands, IVariable[] sources, double
         gradient) throws ExecutionControl.NotImplementedException {
        operands[0].backward(sources, -gradient);
    }
}
```

Listing 7: autograd/Negation.java

```java
package autograd;

import jdk.jshell.spi.ExecutionControl;

import java.util.Arrays;
import java.util.HashSet;

public class Operation implements IVariable {
    private final IOperator operator;
    private final IVariable[] operands;

    public Operation(IOperator operator, IVariable... operands) {
        this.operator = operator;
        this.operands = operands;
    }

    @Override
    public double evaluate() {
        return operator.evaluate(operands);
    }

    @Override
    public void backward(IVariable[] sources, double gradient) throws
        ExecutionControl.NotImplementedException {
        operator.backwards(operands, sources, gradient);
    }


    @Override
    public Parameter[] getParameters() {
        HashSet<Parameter> result = new HashSet<>();
        for (IVariable o :
                this.operands) {
```

```java
                result.addAll(Arrays.asList(o.getParameters()));
            }
            return result.toArray(new Parameter[0]);
        }

        public IVariable[] getOperands() {
            return operands;
        }
}
```

Listing 8: autograd/Operation.java

```java
package autograd;

public abstract class Operator implements IOperator {
    protected Integer numberOfOperands;

    public Operator() {
        this.numberOfOperands = null;
    }

    @Override
    public IVariable apply(IVariable... operands) {
        return new Operation(this, operands);
    }

    protected void validateOperands(IVariable[] operands) {
        if (this.numberOfOperands == null) {
            return;
        }
        if (operands.length != this.numberOfOperands) {
            throw new IllegalArgumentException(String.format("%s accepts
                only one operand.", this.getClass().getName()));
        }
    }
}
```

Listing 9: autograd/Operator.java

```java
package autograd;

import java.util.Arrays;

public class Parameter implements IVariable {
    private double value;
    private double gradient;
    private boolean trainable;
    private int layer;

    public Parameter() {

    }

    public Parameter(double value) {
        this.value = value;
        trainable = true;
    }

    public Parameter(double value, boolean trainable) {
        this.value = value;
        this.trainable = trainable;
    }

    public static IVariable[] createTensor(double[] desired) {
        var result = new Parameter[desired.length];
        for (int i = 0; i < result.length; i++) {
            result[i] = new Parameter(desired[i]);
        }
        return result;
    }

    @Override
    public double evaluate() {
        return value;
    }

    @Override
    public void backward(IVariable[] sources, double gradient) {
        if (Arrays.stream(sources).anyMatch(x -> x == this)) {
            setGradient(gradient + getGradient());
        }
    }

    @Override
    public Parameter[] getParameters() {
        return new Parameter[]{this};
    }

    public double getValue() {
        return this.value;
    }

    public void setValue(double value) {
        this.value = value;
    }

    public double getGradient() {
        return gradient;
    }

    private void setGradient(double gradient) {
        this.gradient = gradient;
    }

    public boolean isTrainable() {
        return this.trainable;
    }

    public void zeroGradient() {
        this.setGradient(0);
    }

    public int getLayer() {
        return layer;
    }

    public void setLayer(int layer) {
        this.layer = layer;
    }
}
```

Listing 10: autograd/Parameter.java

```java
package autograd;

import jdk.jshell.spi.ExecutionControl;

public class Sigmoid extends Operator {

    public Sigmoid() {
        this.numberOfOperands = 1;
    }

    @Override
    public double evaluate(IVariable[] operands) {
        if (operands.length != 1) {
            throw new IllegalArgumentException("Sigmoid operator only
                accepts one operand");
        }
        return 1. / (1 + Math.exp(-operands[0].evaluate()));
    }

    @Override
    public void backwards(IVariable[] operands, IVariable[] sources, double
         gradient) throws ExecutionControl.NotImplementedException {
        validateOperands(operands);
        var x = operands[0];
        var y = evaluate(operands);
        x.backward(sources, gradient * y * (1 - y));
    }
}
```

Listing 11: autograd/Sigmoid.java

```java
package autograd;

import java.util.Random;

public class UniformInitializer implements IInitializer {

    double a;
    double b;
    Random random;

    public UniformInitializer(double a, double b) {
        this.a = a;
        this.b = b;
        this.random = new Random();
    }

    @Override
    public double next() {
        return random.nextDouble(a, b);
    }
}
```

Listing 12: autograd/UniformInitializer.java

```java
package dataset;

public class BinaryToBipolarWrapper implements IDataSet {

    IDataSet binaryDataSet;

    public BinaryToBipolarWrapper(IDataSet binaryDataSet) {
        this.binaryDataSet = binaryDataSet;
    }

    @Override
    public DataPoint next() {
        DataPoint result = binaryDataSet.next();
        if (result == null) return null;
        double[] x = result.getX().clone();
        double[] y = result.getY().clone();
        for (int i = 0; i < x.length; i++) {
            x[i] = 2 * x[i] - 1;
        }
        for (int i = 0; i < y.length; i++) {
            y[i] = 2 * y[i] - 1;
        }
        return new DataPoint(x, y);
    }

    @Override
    public void reset() {
        binaryDataSet.reset();
    }
}
```

Listing 13: dataset/BinaryToBipolarWrapper.java

```java
package dataset;

public class DataPoint {
    private final double[] x;
    private final double[] y;

    public DataPoint(double[] x, double[] y) {
        this.x = x;
        this.y = y;
    }

    public double[] getY() {
        return y;
    }

    public double[] getX() {
        return x;
    }
}
```

Listing 14: dataset/DataPoint.java

```java
package dataset;

public interface IDataSet {
    DataPoint next();

    void reset();
}
```

Listing 15: dataset/IDataSet.java

```java
package dataset;

public class XORBinaryDataSet implements IDataSet {

    protected double[][] x;
    protected double[] y;
    private int index;

    public XORBinaryDataSet() {
        index = 0;
        x = new double[][]{
                {0., 0.},
                {0., 1.},
                {1., 0.},
                {1., 1.},
        };
        y = new double[]{
                0.,
                1.,
                1.,
                0.,
        };
    }

    @Override
    public DataPoint next() {
        if (index < x.length) {
            var result = new DataPoint(x[index], new double[]{y[index]});
            index++;
            return result;
        }
        return null;
    }

    @Override
    public void reset() {
        index = 0;
    }
}
```

Listing 16: dataset/XORBinaryDataSet.java

```java
package nn;

import autograd.IVariable;
import autograd.Parameter;

public class BipolarSigmoid implements ILayer {

    @Override
    public IVariable[] apply(IVariable[] input) {
        var sigmoid = new autograd.Sigmoid();
        var scalar = new Parameter(2, false);
        var constant = new Parameter(-1, false);
        var addition = new autograd.Addition();
        var multiplication = new autograd.Multiplication();
        var result = new IVariable[input.length];
        for (int i = 0; i < input.length; i++) {
            result[i] = addition.apply(
                    multiplication.apply(
                            scalar,
                            sigmoid.apply(input[i])),
                    constant
            );
        }
        return result;
    }
}
```

Listing 17: nn/BipolarSigmoid.java

```
package nn;

import java.util.ArrayList;

public class ConvergenceCollector implements IFitCallback {
    ArrayList<Double> loss;

    public ConvergenceCollector() {
        this.loss = new ArrayList<>();
    }

    @Override
    public void collect(int epoch, double loss) {
        this.loss.add(loss);
    }

    public int getEpochs() {
        return loss.size();
    }

    @Override
    public String toString() {
        StringBuilder sb = new StringBuilder();
        for (int i = 0; i < loss.size(); i++) {
            sb.append(+i + " " + loss.get(i) + "\n");
        }
        return sb.toString();
    }
}
```

Listing 18: nn/ConvergenceCollector.java

```
package nn;

import autograd.IInitializer;
```

```java
import autograd.IVariable;
import autograd.Parameter;
import autograd.UniformInitializer;

public class Factory {
    public static Model createNeuralNetwork(int[] sizes, ILayer activation,
         IInitializer initializer) {
        if (sizes.length < 2) {
            throw new IllegalArgumentException("Sizes must at least contain
                 2 integers for the first and the second layer.");
        }
        var inputs = new Parameter[sizes[0]];
        for (int i = 0; i < inputs.length; i++) {
            inputs[i] = new Parameter(initializer.next());
        }
        IVariable[] lastLayerOutput = inputs;
        for (int i = 1; i < sizes.length; i++) {
            lastLayerOutput = new Linear(sizes[i - 1], sizes[i],
                initializer).apply(lastLayerOutput);
            lastLayerOutput = activation.apply(lastLayerOutput);
        }
        return new Model(inputs, lastLayerOutput);
    }

    public static Model createNeuralNetwork(int[] sizes, ILayer activation)
         {
        return createNeuralNetwork(sizes, activation, new
           UniformInitializer(-0.5, 0.5));
    }
}
```

Listing 19: nn/Factory.java

```java
package nn;

public interface IFitCallback {
    void collect(int epoch, double loss);
}
```

Listing 20: nn/IFitCallback.java

```java
package nn;

import autograd.IVariable;

public interface ILayer {
    IVariable[] apply(IVariable[] input);
}
```

Listing 21: nn/ILayer.java

```java
package nn;

import autograd.*;

public class Linear implements ILayer {
    private final IVariable[][] weight;
    private final IVariable[] bias;

    public Linear(int inFeatures, int outFeatures, IInitializer initializer
        ) {
        this.weight = new Parameter[outFeatures][inFeatures];
        this.bias = new Parameter[outFeatures];
        for (int i = 0; i < outFeatures; i++) {
            for (int j = 0; j < inFeatures; j++) {
                this.weight[i][j] = new Parameter(initializer.next());
            }
            this.bias[i] = new Parameter(initializer.next());
        }
    }

    @Override
    public IVariable[] apply(IVariable[] input) {
        var result = new IVariable[this.weight.length];
        for (int i = 0; i < this.weight.length; i++) {
            int inputSize = this.weight[i].length;
            IVariable[] muls = new IVariable[inputSize + 1];
            for (int j = 0; j < inputSize; j++) {
                muls[j] = new Multiplication().apply(this.weight[i][j],
                    input[j]);
            }
            muls[inputSize] = this.bias[i];
            result[i] = new Addition().apply(muls);
        }
        return result;
    }


    private int getWidth() {
        return this.weight.length;
    }
}
```

Listing 22: nn/Linear.java

```java
package nn;

import autograd.*;
import jdk.jshell.spi.ExecutionControl;
import optimization.ILoss;

public class MinimumSquaredError implements IVariable, ILoss {

    private final IVariable operation;
    private final Parameter[] desired;

    public MinimumSquaredError(IVariable[] output) {
        var negation = new Negation();
        var addition = new Addition();
        var multiplication = new Multiplication();
        var exponentiation = new Exponentiation();
        Parameter two = new Parameter(2, false);
        Parameter half = new Parameter(0.5, false);
        int length = output.length;
        desired = new Parameter[output.length];
        var summationTerms = new IVariable[length];
        for (int i = 0; i < length; i++) {
            desired[i] = new Parameter();
            summationTerms[i] = exponentiation.apply(
                    addition.apply(output[i], negation.apply(desired[i])),
                    two
            );
        }
        this.operation = multiplication.apply(addition.apply(summationTerms
            ), half);
    }

    @Override
    public double evaluate() {
        return operation.evaluate();
    }

    @Override
    public void backward(IVariable[] sources, double gradient) throws
        ExecutionControl.NotImplementedException {
        operation.backward(sources, gradient);
    }

    @Override
    public Parameter[] getParameters() {
        return this.operation.getParameters();
    }

    @Override
    public void setDesired(double[] desired) {
        for (int i = 0; i < this.desired.length; i++) {
            this.desired[i].setValue(desired[i]);
        }
    }
}
```

Listing 23: nn/MinimumSquaredError.java

```java
package nn;

import autograd.IVariable;
import autograd.Operation;
import autograd.Parameter;
import dataset.DataPoint;
import dataset.IDataSet;
import jdk.jshell.spi.ExecutionControl;
import optimization.ILoss;
import optimization.IOptimizer;

import java.util.Arrays;
import java.util.HashSet;
import java.util.List;
import java.util.Map;
import java.util.stream.Collectors;

public class Model {
    private final Parameter[] input;
    private final IVariable[] output;

    public Model(Parameter[] input, IVariable[] output) {
        this.input = input;
        this.output = output;
    }


    public double[] evaluate(double[] input) {
        var result = new double[output.length];
        for (int i = 0; i < input.length; i++) {
            this.input[i].setValue(input[i]);
        }
        for (int i = 0; i < output.length; i++) {
            result[i] = output[i].evaluate();
        }
        return result;
    }

    public Parameter[] getParameters() {
        HashSet<Parameter> result = new HashSet<>();
        for (IVariable o :
                this.output) {
            result.addAll(Arrays.asList(o.getParameters()));
        }
        return result.toArray(new Parameter[0]);
    }

    public Parameter[] getTrainableParameters() {
        var results = new HashSet<Parameter>();
        for (Parameter p :
                getParameters()) {
            if (p.isTrainable()) {
                results.add(p);
            }
        }
        for (Parameter p : input) {
            results.remove(p);
        }

        return results.toArray(new Parameter[0]);
    }

    public IVariable[] getOutput() {
        return output;
    }

    public double fit(IDataSet dataSet, IOptimizer optimizer, ILoss loss,
       int epochs, double lossLimit) throws ExecutionControl.
       NotImplementedException {
        return fit(dataSet, optimizer, loss, epochs, lossLimit, (epoch, l)
           -> {
        });
    }

    public double fit(IDataSet dataSet, IOptimizer optimizer, ILoss loss,
       int epochs, double lossLimit, IFitCallback callback) throws
       ExecutionControl.NotImplementedException {
        var parameters = getTrainableParameters();
        Map<Integer, List<Parameter>> layeredParameters = layerParameters(
            parameters);
        if (epochs < 1) {
            throw new IllegalArgumentException("At least one epochs
               required.");
        }
        double totalLoss = 0;
        for (int i = 0; i < epochs; i++) {
            totalLoss = 0;
            dataSet.reset();
            DataPoint dataPoint;
            while ((dataPoint = dataSet.next()) != null) {
                setInput(dataPoint.getX());
                loss.setDesired(dataPoint.getY());
                totalLoss += loss.evaluate();
                for (Integer j : layeredParameters.keySet().stream().sorted
                    ().toList()) {
                    Parameter[] layerParameters = layeredParameters.get(j).
                        toArray(new Parameter[0]);
                    loss.backward(layerParameters, 1.);
                    optimizer.update(layerParameters);
                }
            }
            callback.collect(i, totalLoss);
            if (totalLoss < lossLimit) {
                break;
            }
        }
        return totalLoss;
    }

    private Map<Integer, List<Parameter>> layerParameters(Parameter[]
        parameters) {
        setLayers(getOutput(), 0);
        return Arrays.stream(parameters).collect(Collectors.groupingBy(
            Parameter::getLayer));

    }

    private void setLayers(IVariable[] outputs, int layer) {
        if (outputs.length == 0) return;
        HashSet<IVariable> nextOutput = new HashSet<>();
        for (IVariable i : outputs) {
            if (i instanceof Parameter) {
                ((Parameter) i).setLayer(layer);
            }
            if (i instanceof Operation) {
                nextOutput.addAll(Arrays.asList(((Operation) i).getOperands
                    ()));
            }
        }
        setLayers(nextOutput.toArray(new IVariable[0]), layer + 1);
    }

    private void setInput(double[] x) {
        for (int i = 0; i < input.length; i++) {
            input[i].setValue(x[i]);
        }
    }
```

```java
}
```

Listing 24: nn/Model.java

```java
package nn;

import autograd.IVariable;

public class Sigmoid implements ILayer {

    @Override
    public IVariable[] apply(IVariable[] input) {
        var operator = new autograd.Sigmoid();
        var result = new IVariable[input.length];
        for (int i = 0; i < input.length; i++) {
            result[i] = operator.apply(input[i]);
        }
        return result;
    }
}
```

Listing 25: nn/Sigmoid.java

```java
package optimization;

import autograd.Parameter;

import java.util.HashMap;

public class GradientDescent implements IOptimizer {

    private final HashMap<Parameter, Double> lastDelta;
    private final double learningRate;
    private final double momentum;

    public GradientDescent(double learningRate, double momentum) {
        this.lastDelta = new HashMap<>();
        this.learningRate = learningRate;
        this.momentum = momentum;
    }

    @Override
    public void update(Parameter[] parameters) {
        for (Parameter p :
                parameters) {
            double delta = -p.getGradient() * learningRate + momentum *
                lastDelta.getOrDefault(p, 0.);
            p.setValue(p.getValue() + delta);
            p.zeroGradient();
            lastDelta.put(p, delta);
        }
    }
}
```

Listing 26: optimization/GradientDescent.java

```java
package optimization;

```

```java
import autograd.IVariable;

public interface ILoss extends IVariable {
    void setDesired(double[] desired);
}
```

Listing 27: optimization/ILoss.java

```java
package optimization;

import autograd.Parameter;

public interface IOptimizer {
    void update(Parameter[] parameters);
}
```

Listing 28: optimization/IOptimizer.java

```java
package autograd;


import org.junit.Assert;
import org.junit.Test;

public class VariableTest {

    @Test
    public void testAddition() {
        Assert.assertEquals(new Addition().apply(new Parameter(12), new
            Parameter(2.)).evaluate(), 14., 0.);
    }

    @Test
    public void testVariableEvaluation() {
        Assert.assertEquals(new Parameter(250).evaluate(), 250., 0);
    }

}
```

Listing 29: autograd/VariableTest.java

```java
package nn;

import autograd.Parameter;
import jdk.jshell.spi.ExecutionControl;
import org.junit.Assert;
import org.junit.Test;

import java.util.Arrays;

public class NeuralNetworkTest {

    @Test
    public void testNeuralNetworkFactory() {
        var model = Factory.createNeuralNetwork(new int[]{2, 4, 1}, new
            Sigmoid());
        var result = model.evaluate(new double[]{0, 0});
        Assert.assertEquals(result.length, 1);
    }

    @Test
    public void testNeuralNetworkGradient() throws ExecutionControl.
        NotImplementedException {
        var model = Factory.createNeuralNetwork(new int[]{2, 4, 1}, new
            Sigmoid());
        Parameter[] parameters = model.getTrainableParameters();
        for (Parameter parameter :
                parameters) {
            parameter.setValue(1);
        }
        var result = model.evaluate(new double[]{1, 0});
        double[] desired = new double[]{1};
        Assert.assertEquals(result.length, 1);
        double delta = 1e-5;
        double expected = 0.9892621636390686; // obtained by pytorch
        Assert.assertEquals(result[0], expected, delta);
        var loss = new MinimumSquaredError(model.getOutput());
        loss.setDesired(desired);
        loss.backward(parameters, 1);
        var gradients = new double[parameters.length];
        for (int i = 0; i < parameters.length; i++) {
            gradients[i] = parameters[i].getGradient();
        }

        Assert.assertArrayEquals(Arrays.stream(new double[]{ // calculated
            by pytorch
                -0.000114063048386015, -0.00010046639363281429,
                    -0.00010046639363281429, -0.00010046639363281429,
                    -0.00010046639363281429, -1.197589335788507e-05,
                    -1.197589335788507e-05, -1.197589335788507e-05,
                    -1.197589335788507e-05, -1.197589335788507e-05,
                    -1.197589335788507e-05, -1.197589335788507e-05,
                    -1.197589335788507e-05, -0.0, -0.0, -0.0, -0.0
        }).sorted().toArray(), Arrays.stream(gradients).sorted().toArray(),
             delta);
    }

    @Test
    public void testNeuralNetworkGradientBipolar() throws ExecutionControl.
        NotImplementedException {
        var model = Factory.createNeuralNetwork(new int[]{2, 4, 1}, new
            BipolarSigmoid());
        Parameter[] parameters = model.getTrainableParameters();
        for (Parameter parameter :
                parameters) {
            parameter.setValue(1);
        }
        var result = model.evaluate(new double[]{1, -1});
        double[] desired = new double[]{1};
        Assert.assertEquals(result.length, 1);
        double delta = 1e-5;
        double expected = 0.8904789686203003; // obtained by pytorch
        Assert.assertEquals(expected, result[0], delta);
        var loss = new MinimumSquaredError(model.getOutput());
        loss.setDesired(desired);
        loss.backward(parameters, 1);
        var gradients = new double[parameters.length];
        for (int i = 0; i < parameters.length; i++) {
            gradients[i] = parameters[i].getGradient();
        }

        Assert.assertArrayEquals(Arrays.stream(new double[]{ // calculated
            by pytorch
                -0.011338012292981148, -0.005239490419626236,
                    -0.005239490419626236, -0.005239490419626236,
                    -0.005239490419626236, -0.004458376672118902,
                    -0.004458376672118902, -0.004458376672118902,
                    -0.004458376672118902, -0.004458376672118902,
                    -0.004458376672118902, -0.004458376672118902,
                    -0.004458376672118902, 0.004458376672118902,
                    0.004458376672118902, 0.004458376672118902,
                    0.004458376672118902
        }).sorted().toArray(), Arrays.stream(gradients).sorted().toArray(),
             delta);
    }
}
```

Listing 30: nn/NeuralNetworkTest.java

```java
package optimization;

import autograd.UniformInitializer;
import dataset.BinaryToBipolarWrapper;
import dataset.XORBinaryDataSet;
import jdk.jshell.spi.ExecutionControl;
import nn.*;
import org.junit.Assert;
import org.junit.Test;

import java.io.FileWriter;
import java.io.IOException;
import java.util.ArrayList;
import java.util.Comparator;
import java.util.Optional;

public class GradientDescentTest {

    private final static int trials = 300;

    @Test
    public void TestSimpleGD() throws ExecutionControl.
        NotImplementedException {
        var model = Factory.createNeuralNetwork(
                new int[]{2, 4, 1},
                new Sigmoid(),
                new UniformInitializer(-0.5, 0.5)
        );
        var dataSet = new XORBinaryDataSet();
        var optimizer = new GradientDescent(0.2, 0.);
        var loss = new MinimumSquaredError(model.getOutput());
        double finalLoss = model.fit(dataSet, optimizer, loss, 40000, 0.05)
            ;
        Assert.assertTrue("Big loss " + finalLoss, finalLoss < 0.05);
    }

    @Test
    public void TestConvergence() throws ExecutionControl.NotImplementedException, IOException {
        int diverged = 0;
        ArrayList<ConvergenceCollector> stats = new ArrayList<>();
        for (int i = 0; i < GradientDescentTest.trials; i++) {
            var model = Factory.createNeuralNetwork(
                    new int[]{2, 4, 1},
                    new Sigmoid(),
                    new UniformInitializer(-0.5, 0.5)
            );
            var dataSet = new XORBinaryDataSet();
            var optimizer = new GradientDescent(0.2, 0.);
            var loss = new MinimumSquaredError(model.getOutput());
            var collector = new ConvergenceCollector();
            double finalLoss = model.fit(dataSet, optimizer, loss, 40000, 0.05, collector);
            stats.add(collector);
            if (finalLoss > 0.05) {
                diverged += 1;
            }
        }
        outputGraphData("a", stats);
        Assert.assertTrue("Convergence with high probability busted!", diverged < 6);
    }

    private void outputGraphData(String assignmentPart, ArrayList<ConvergenceCollector> stats) throws IOException {
        FileWriter of = new FileWriter("doc/" + assignmentPart + "_avg.tex");
        double average = stats.stream().mapToInt(ConvergenceCollector::getEpochs).average().getAsDouble();
        of.write(String.valueOf(average));
        of.close();

        Optional<ConvergenceCollector> representative = stats.stream().min(Comparator.comparingDouble(c -> Math.abs(c.getEpochs() - average)));
        of = new FileWriter("doc/" + assignmentPart + ".tex");
        of.write(representative.get().toString());
        of.close();
    }

    @Test
    public void TestBipolarGD() throws ExecutionControl.NotImplementedException, IOException {
        int diverged = 0;
        int trials = GradientDescentTest.trials;
        ArrayList<ConvergenceCollector> stats = new ArrayList<>();
        for (int i = 0; i < trials; i++) {
            var model = Factory.createNeuralNetwork(
                    new int[]{2, 4, 1},
                    new BipolarSigmoid(),
                    new UniformInitializer(-0.5, 0.5)
            );
            var dataSet = new BinaryToBipolarWrapper(new XORBinaryDataSet()
);
            var optimizer = new GradientDescent(0.2, 0.);
            var loss = new MinimumSquaredError(model.getOutput());
            var collector = new ConvergenceCollector();
            double finalLoss = model.fit(dataSet, optimizer, loss, 3500, 0.05, collector);
            if (finalLoss > 0.05) {
                diverged += 1;
            }
            stats.add(collector);
        }
        outputGraphData("b", stats);
        Assert.assertTrue("Convergence with high probability busted! " + diverged + " failure out of " + trials, diverged < 6);
    }

    @Test
    public void TestBipolarMomentumGD() throws ExecutionControl.NotImplementedException, IOException {

        int diverged = 0;
        int trials = GradientDescentTest.trials;
        ArrayList<ConvergenceCollector> stats = new ArrayList<>();
        for (int i = 0; i < trials; i++) {
            var model = Factory.createNeuralNetwork(
                    new int[]{2, 4, 1},
                    new BipolarSigmoid(),
                    new UniformInitializer(-0.5, 0.5)
            );
            var dataSet = new BinaryToBipolarWrapper(new XORBinaryDataSet());
            var optimizer = new GradientDescent(0.2, 0.9);
            var loss = new MinimumSquaredError(model.getOutput());
            var collector = new ConvergenceCollector();
            double finalLoss = model.fit(dataSet, optimizer, loss, 1000, 0.05, collector);
            if (finalLoss > 0.05) {
                diverged += 1;
            }
            stats.add(collector);
        }
        outputGraphData("c", stats);
        Assert.assertTrue("Convergence with high probability busted! " + diverged + " failure out of " + trials, diverged < 6);
    }
}
```

Listing 31: optimization/GradientDescentTest.java