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Requiem for the Net Migrant

Despite recent research that has demonstrated the clear superiority of a multiregional perspective in measuring and projecting the dynamics of internal migration flows, many scholars continue to adopt the uniregional perspective that is forced to focus on net *migrants, a nonexistent category of individuals. Net migration models are misspecified because the rates that they use confound changing migration propensities with changing population stocks. Moreover, they obscure regularities in age profiles of migration and thereby further misspecify the spatial dynamics generating observed settlement patterns. Thus, the use of the net migration rate as the dependent variable in explanatory models of migration can produce a misspecification of the fundamental relationships that are the subject of inquiry. This paper considers deficiencies of the net migration concept and illustrates them with numerical examples.*

Scores of papers published over the past two decades have reported on efforts to identify the determinants and project the consequences of interregional migration within various nations. Generally, such studies have sought to associate data on migration streams between pairs of regions and such variables as wage rates, unemployment rates, income levels, education, degrees of urbanization, and the distances separating origin and destination regions. Among the several ways of adjusting for relative differences in regional population levels, the division of stream totals by population size to define rates has been especially popular. When the stream is directional and divided by the origin "at risk" population, it is called a gross migration rate or probability. When the *difference* between an in-flow and an out-flow stream is divided by the population experiencing both, the resulting measure is called a net migration rate, even though its interpretation as a rate is ambiguous.[1]

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[1]Throughout this paper, I focus on analyses carried out using *rates* rather than *counts* of the absolute numbers of events, even though some published studies of net migration have used the latter.

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Several scholars have identified the weakness of net migration as a measure of geographical mobility (Lieberson 1979; Morrison 1977; Rogers 1976); nevertheless, one continues to find numerous recent articles in respected journals that incorporate this flawed measure in sophisticated econometric modeling exercises seeking to identify the socioeconomic determinants of interregional migration (Dunlevy and Bellante 1983; Foot and Milne 1984; Greenwood, Hunt, and McDowell 1986; Izraeli and Lin 1984; Milne, Foot, and Dungan 1985; Tabuchi 1988).

Twenty-five years ago, frustrated with the inadequacies of traditional land-use control instruments, city planner John Reps offered a vigorous critique of zoning, calling his presentation to the American Society of Planning Officials "Requiem for Zoning" (Reps 1964). Nine years later, another city planner, Douglas Lee, borrowed Reps's expression in a sharp attack on the large-scale urban modeling movement of the 1960s (Lee 1973). The time seems ripe to introduce yet another requiem—this one for the nonexistent net migrant. This paper is directed at that task. It begins with a discussion of how net migration rates are introduced into the analysis of spatial population dynamics and then goes on to identify the bias that such a formulation produces. A further problem arises when age patterns of migration are taken into account; the paper considers how the strong relationship between migration and the life course is camouflaged by net migration rates.

1. THE NET MIGRATION RATE AND SPATIAL POPULATION DYNAMICS

Net changes in regional population stocks are often dwarfed by the gross migration flows that help to produce them, hiding the spatial dynamics that are at work; a modest net contribution to regional population growth by migration may be generated by large gross flows in both directions:

> Generally, net migratory gains or losses are only the surface ripples of powerful crosscurrents... Between 1965 and 1970, for example, metropolitan San Jose, California, gained 75,000 new residents through migration, but 395,000 people moved either in or out. (Morrison 1975, p. 243)

The net migration rate, m_j, for a particular region j is defined as the difference between the region's in-migration rate, i_j, and its out-migration rate, o_j. The out-migration rate is defined as a true rate (or probability) because it divides the number of times that an event, out-migration, occurred during a year, say, by the number of persons exposed to the risk of experiencing that event. The in-migration rate, on the other hand, is a measure of prevalence rather than of propensity. It too has a numerator that is an occurrence count of a particular event, in-migration in this case, but its denominator is not a count of the number of persons that could have experienced the event. Rather, its denominator is the population in the region of destination that was at risk of experiencing the *out-migration* event. Since the

This is because I take for granted that demographic analyses based on rates are superior to those based on counts. Demographic rates exhibit strong age-specific regularities and temporal stabilities that a projection based on rates can exploit to generate events through demographic accounting identities; a projection based on a count of events ignores this information. Moreover, it is much easier to assess and interpret the reasonableness of results produced by forecasted rates. For instance, a set of one hundred numbers representing deaths by single year of age of decedent is not very informative; nor is a collection of thirty numbers representing numbers of births by single year of age of mother. Yet the meanings of the expectation of life at birth and the net reproduction rate implied by these two sets of numbers (both calculated using age-specific rates) are readily grasped, and unrealistic values for these two variables suggest possible sources of error in the data or in the forecasting procedure.

net migration rate is the difference between a measure of prevalence and a true rate or propensity, its interpretation is necessarily ambiguous.

The net migration rate for a given region is a function of its out-migration rate and the out-migration rates from all other regions to the given region, weighted by the proportion of persons initially residing in each origin region. It therefore follows that changes in the net migration rate are influenced by shifts in the relative numbers living in each region, as well as by their propensities to migrate. Recognizing this, Tucker (1976) shows that the concentration of the U.S. national population in metropolitan areas was an important contributing factor to the metropolitan "turnaround" of the early 1970s, and Lieberson (1979) similarly demonstrates that shifts in the North-South black population distribution significantly affected the observed net migration rates of blacks over the years 1935–1940 to 1965–1970. Tucker points out the interesting fact that although the net in-migration of 352,000 persons into metropolitan areas between 1965 and 1970 turned into a net out-migration of 1,594,000 over the next five years, the out-migration rate of nonmetropolitan residents was higher than that of metropolitan residents during both periods. Lieberson shows how "a significant change in 1970 from the earlier period in the propensity to leave the South is hidden by the changes in population distribution over time" (Lieberson 1979, p. 182).

For each set of fixed out-migration rates, different spatial distributions of a population will give rise to different values of the net migration rate to a region. Also, for each fixed initial spatial distribution of a population, a given value of the net migration rate can be generated by a wide range of in-migration and out-migration rates; but the long-term implications for the geography of the population may be quite different. Thus one must be wary of cross-sectional comparisons of net migration rates of different regions as well as of comparisons of such rates for the same region over time. In both instances the net migration rate will embody the influences of spatial population distribution along with those of movement propensities.

Consider, for example, how projections of urbanization might be carried out with uniregional (net migration) and multiregional (gross directional migration) models. In a uniregional model, the urban population is the central focus of interest and all rural-to-urban migration flows are assessed only with respect to the population in the region of destination, that is, the urban population. Changes in the population at the region of origin are totally ignored, with potentially serious consequences. For example, the rural population ultimately may be reduced to near zero levels, but a fixed and positive net migration into urban areas will nevertheless continue to be generated by the uniregional model.

To see the source of the problem more clearly, consider how the rural-urban migration specification is altered when a biregional model of urban and rural population growth is transformed into a uniregional model. Let urban population growth be described by the equation

$$P_u(t+1) = (1 + b_u - d_u - o_u)P_u(t) + o_v P_v(t) . \tag{1}$$

Equation (1) states that next year's urban population total, $P_u(t+1)$, may be calculated by adding to this year's urban population $[P_u(t)]$ the increment due to the excess of births over deaths, that is, urban natural increase $[(b_u - d_u)P_u(t)]$, the decrement due to urban out-migration to rural (v = village) areas $[o_u P_u(t)]$, and the increment due to rural-to-urban migration $[o_v P_v(t)]$.

Now, multiplying the last term in equation (1) by unity expressed as $P_u(t)/P_u(t)$ transforms that equation into its uniregional counterpart (Rogers 1985, p. 30):

$$\begin{aligned} P_u(t+1) &= (1 + b_u - d_u - o_u)P_u(t) + o_v\left[\frac{P_v(t)}{P_u(t)}\right]P_u(t) \\ &= (1 + b_u - d_u - o_u + i_u)P_u(t) \\ &= (1 + b_u - d_u + m_u)P_u(t) = (1 + r_u)P_u(t) \end{aligned} \tag{2}$$

where

$$i_u = o_v\left[\frac{P_v(t)}{P_u(t)}\right] = o_v\left[\frac{1 - U(t)}{U(t)}\right],$$

$$m_u = i_u - o_u,$$

and $U(t)$ is the fraction of the total national population that is urban at time t. If all annual rates are assumed to be fixed in the biregional projection, then in the uniregional model i_u, and therefore also m_u, depend on $U(t)$, which varies in the course of the projection, thereby introducing a bias. The dependence of the urban net migration rate m_u on the level of urbanization at time t means that m_u *must decrease as the level of urbanization increases*. Consequently, it seems inappropriate to use such a model to answer, for example, the question whether it is natural increase or net migration that is the principal source of urban population growth over time, as does Keyfitz (1980).

2. NET MIGRATION, THE UNIREGIONAL FALLACY, AND BIAS

The notion that the spatial dynamics of a system of multiple interacting regional populations can be analyzed profitably by a set of *independent* uniregional models, which apply net migration rates to each regional population, dies hard. The biases and inconsistencies that are created by such decompositions of multiregional population projection models are generally ignored. Thus despite more than two decades of published work on multiregional demography exposing the "uniregional fallacy," it is unfortunately still not uncommon to find articles in prominent journals that ignore this literature, for example, the recent essay on heterogeneity and regional growth rates by Kephart (1988), who to illustrate "aggregate bias" borrows Keyfitz's (1977, pp. 14–18) argument on the impact of mixing populations having different rates of increase and the conclusion that "separate projection of each of the various elements of a heterogeneous population gives a total greater than is obtained by projection of the whole population at its average rate" (Keyfitz 1977, p. 17). In section 2.4 we set out a simple numerical example that illustrates the opposite to be the case.

In applying Keyfitz's argument to analyze county population growth rates, Kephart commits the uniregional fallacy, thereby possibly invalidating some of his substantive conclusions. (Indeed Keyfitz [1977, p. 17] himself commits the same error in his numerical example involving the populations of the United States and Mexico.) Kephart's methodological argument, however, provides a useful illustration of the bias that is introduced by adopting the uniregional perspective.

2.1. Aggregation Bias

Imagine, once again, a closed two-region population system consisting of an urban population, $P_u(t)$, and a rural population, $P_v(t)$. In its discrete-time formulation, Kephart's equation (1) becomes

$$\begin{aligned} P(t) &= P_u(t) + P_v(t) \\ &= (1 + r_u)^t P_u(0) + (1 + r_v)^t P_v(0) \\ &= (1 + r_u) P_u(t-1) + (1 + r_v) P_v(t-1) \end{aligned} \tag{3}$$

and his equation (2) becomes

$$r(t) = U(t-1) r_u + [1 - U(t-1)] r_v \tag{4}$$

where, as before, $U(t) = P_u(t)/P(t)$ is the fraction urban at time t.

By definition, the urban growth rate, r_u, is equal to the birth rate, b_u, minus the death rate, d_u, minus the out-migration rate, o_u, plus the in-migration rate, i_u:

$$r_u = b_u - d_u - o_u + i_u . \tag{5}$$

If r_u is to remain constant, as Kephart's model assumes, then the component rates on the right-hand side of equation (5) must sum to a constant, and

$$P_u(t) = (1 + r_u)^t P_u(0) . \tag{6}$$

But, we have earlier shown that

$$i_u = o_v \left[\frac{P_v(t)}{P_u(t)} \right] = o_v \left[\frac{1 - U(t)}{U(t)} \right] \tag{7}$$

which means that i_u (and therefore r_u) *changes over time as urbanization proceeds*. Bias and inconsistency are therefore the probable result of viewing this biregional population system through a uniregional perspective.

Although Kephart (1988, p. 101) seems to recognize this problem by asserting that "...changes in the magnitude of migration streams may occur apart from changes in the propensity to move...," he doesn't follow up on this observation in developing his model. He recognizes the need for a method of analysis that has the "ability to distinguish between changes in rates that reflect actual social changes from changes in rates that are merely the result of existing processes" (Kephart 1988, p. 101). But his "existing processes" only consider changes in the urbanization *weights* in equation (4) and not changes in the rates themselves (that is, the r_u and r_v values). The uniregional perspective simply does not have this ability. To see this, assume a behaviorally fixed and totally *homogeneous* population in equation (5). Then

$$\begin{aligned} r_u(t) &= b - d - o + o \left[\frac{1 - U(t)}{U(t)} \right] \\ &= b - d + \left[\frac{1 - 2U(t)}{U(t)} \right] o = n + a(t) o \end{aligned} \tag{8}$$

and, similarly,

$$r_v(t) = b - d - o + o\left[\frac{U(t)}{1 - U(t)}\right]$$
$$= b - d + \left[\frac{2U(t) - 1}{1 - U(t)}\right]o = n + b(t)o \tag{9}$$

where $n = b - d$, $a(t) = [1 - 2U(t)]/U(t)$, and $b(t) = [2U(t) - 1]/[1 - U(t)]$.

Since all members of the population exhibit identical and constant behavior, one might expect both regional growth rates, $r_u(t)$ and $r_v(t)$, to be identical and to remain fixed at the value of the natural increase rate $n = b - d$; but this will only occur either if (1) the two regional populations do not interact with each other via migration (that is, $o = 0$), or (2) the entire population is currently experiencing stable growth, a condition that in this illustration can only arise if the two regional populations happen to be identical in size (that is, $U(t) = \frac{1}{2}$, whence $a(t) = b(t) = 0$).

Unlike the case of "perfect aggregation," total homogeneity is not a sufficient condition for "perfect deconsolidation," that is, for avoiding a bias in transformations of multiregional models to uniregional ones; indeed homogeneity is irrelevant and distributional stability is essential. We now turn to this issue.

2.2. *Decomposition Bias*

The transformation of a multiregional model that describes the matrix of interregional migrations between the constituent regions of the population system into the corresponding separate uniregional models can be viewed as a process of compensated decomposition in which net migration rates carry out the "compensation" (Rogers 1976). Before such a transformation, the population of the jth region, $P_j(t + 1)$ say, can be defined as

$$P_j(t + 1) = (1 + b_j - d_j - o_j)P_j(t) + \sum_{i \neq j} o_{ij}P_i(t) . \tag{10}$$

Denoting $1 + b_j - d_j - o_j$ by o_{jj}, and multiplying the last term in the equation by unity, in the form of $P_j(t)/P_j(t)$, gives

$$P_j(t + 1) = o_{jj}P_j(t) + \left[\sum_{i \neq j} o_{ij}\frac{P_i(t)}{P_j(t)}\right]P_j(t)$$
$$= \left[o_{jj}(t) + \sum_{i \neq j} o_{ij}\frac{P_i(t)}{P_j(t)}\right]P_j(t)$$

or

$$P_j(t + 1) = o_j(t)P_j(t) = [1 + r_j(t)]P_j(t) \tag{11}$$

where $o_j(t)$ represents the quantity in the square brackets, and $r_j(t)$ is the annual rate of growth.

In the constant coefficient multiregional specification all o_{ji} elements are assumed to be fixed, but the transformation into a uniregional model introduces a time dependence into the $o_j(t)$ terms and thereby generates bias and inconsistency in the analysis, except when either one of the two conditions for perfect decomposition is satisfied, namely,

(i.) if the regional populations do not interact, that is, if all o_{ij} are equal to zero:

$$\begin{aligned} P_j(t+1) &= o_{jj}P_j(t) = (1 + b_j - d_j)P_j(t) \\ &= (1 + r_j)P_j(t) \end{aligned} \tag{12}$$

or

(ii.) if the multiregional population is stable, with each pairwise ratio $P_i(t)/P_j(t)$ equal to a constant, say k_{ij}:

$$\begin{aligned} P_j(t+1) &= \left[o_{jj} + \sum_{i \neq j} o_{ij}k_{ij}\right]P_j(t) = (1 + b_j - d_j + m_j)P_j(t) \\ &= (1 + r_j)P_j(t). \end{aligned} \tag{13}$$

In either extreme case, no bias or inconsistency is introduced by the transformation of the multiregional model into a collection of corresponding independent uniregional models, and perfect decomposition is achieved. In all other instances a bias will be introduced into the projection dynamics.

2.3. *The Projection Bias of Uniregional Models*

Comparing two alternative projections of Florida's population, published in the early 1980s; Carl Haub of the Population Reference Bureau observed:

> The Census Bureau projects Florida to rise from 9.7 million in 1980 to 17.4 in 2000, while BEA [Bureau of Economic Analysis] projects Florida at only 14.6 million in 2000. Based on Florida's continued rapid growth from 1980 to 1985, the Census Bureau's prognosis looks more feasible. (Haub 1986, p. 10)

Writing in 1986, Haub could not have foreseen that two years later, the same Census Bureau would issue a revised set of projections that lowered Florida's expected total in 2000 by two million people, bringing the earlier projected total down to roughly the level projected by the BEA. Was the principal cause of the drop a downward revision of the future attraction of Florida as a destination? Not at all. The principal cause was the Bureau's shift from the use of net migration rates in the earlier projection to gross rates in the more recent effort, that is, the adoption of a multiregional perspective in place of the former uniregional one.

> Net migration is defined with respect to the particular population being projected. If that population is currently experiencing an excess of in-migrants over out-migrants, this feature will be built in as part of the projection process, and its effects will multiply and increase cumulatively over time. The converse applies, of course, to regions experiencing net out-migration. In short, regional populations with a positive net migration rate are likely to be overprojected and those with a negative net migration rate are likely to be underprojected. (Rogers 1976, p. 527)

The above conclusion anticipates the results of Smith's (1986) subsequent experimental simulations. Comparing the projection performance of a biregional model (Model I) against a uniregional model based on net migration rates (Model II), he found that:

> For rapidly growing states... Model I produced the lowest population projections... and Model II produced the highest. The differences were huge... Even for Florida, a large state with a considerably lower growth rate... the Model II projection was more than 50 percent larger than the Model I projection by 2030... for the slowly growing states... Model I produced the highest projections. (Smith 1986, pp. 129–31)

The guaranteed over-projection by a uniregional model with positive net migration can be explained (without loss of generality) with a simple fixed-coefficient biregional example of convergence to stable growth. In the uniregional representation, the population gaining from the migration exchange will be projected as an exponentially increasing total. So will the corresponding population in the biregional model, but in the latter case the exponential trajectory of the gaining region will be continually "dampened" by the negative contribution of the second characteristic vector associated with the subdominant (second) characteristic root of the projection matrix. This process is best illustrated with a numerical example.

2.4. A Simple Numerical Illustration

The urban population of the Pacific island of Mora-Bora increased by three quarters last year ($r_u = \frac{3}{4}$), while the rural population grew by an eighth ($r_v = \frac{1}{8}$). At the start of the year the two populations were enumerated to be 16 and 32 thousand, respectively. During the course of the year a half of the rural population migrated to urban areas ($o_v = \frac{1}{2}$), while a fourth of the urban population moved to the rural areas ($o_u = \frac{1}{4}$). Given these rates and the initial populations, it is a simple matter to define the matrix growth process that will project the island's biregional population forward two consecutive years:

$$\begin{bmatrix} 28 \\ 36 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{1}{2} \\ \frac{1}{4} & 1 \end{bmatrix} \begin{bmatrix} 16 \\ 32 \end{bmatrix}$$

and

$$\begin{bmatrix} 39 \\ 43 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{1}{2} \\ \frac{1}{4} & 1 \end{bmatrix} \begin{bmatrix} 28 \\ 36 \end{bmatrix}.$$

The island's population increased from 48 thousand to 64 thousand after a year, and then it grew to 82 thousand after the following year. The demographic accounting equations for the first year are:

$$\begin{aligned} P_u(1) &= [1 + (b_u - d_u) - o_u]P_u(0) + o_v P_v(0) \\ &= (1 + n_u - o_u)P_u(0) + o_v P_v(0) \\ &= (1 + 0 - \tfrac{1}{4})16 + (\tfrac{1}{2})32 = 28 \end{aligned}$$

and

$$\begin{aligned} P_v(1) &= o_u P_u(0) + (1 + n_v - o_v)P_v(0) \\ &= (\tfrac{1}{4})16 + (1 + \tfrac{1}{2} - \tfrac{1}{2})32 = 36 \end{aligned}$$

where n denotes the natural increase rate. Notice that the urban population is experiencing replacement level fertility, that is, $b_u = d_u$, and the rate of natural increase, n_u, is zero. The natural increase rate of the rural population is a half.

The above disaggregated model produces a projected evolution of the national population that is: $48, 64, 82, \ldots$. Notice that the corresponding consolidated uniregional model for the national total [that is, $P(1) = \frac{4}{3}P(0)$] leads to a *higher*, not lower, projected set of totals: $48, 64, 85\frac{1}{3}, \ldots$. Hence, Keyfitz's (1977, p. 17) proof of guaranteed overprojection by the more disaggregated model does not apply in this case.

The above two fundamental equations define the biregional model. The corresponding uniregional models may be obtained by a compensated decomposition. In that event, the net migration rate for the urban region is

$$\begin{aligned} m_u &= i_u - o_u = o_v\left[\frac{P_v(0)}{P_u(0)}\right] - o_u \\ &= \tfrac{1}{2}[2] - \tfrac{1}{4} = \tfrac{3}{4} \end{aligned}$$

and, correspondingly,

$$m_v = -\tfrac{3}{8}.$$

Thus

$$\begin{bmatrix} 28 \\ 36 \end{bmatrix} = \begin{bmatrix} 1 + 0 + \frac{3}{4} & 0 \\ 0 & 1 + \frac{1}{2} - \frac{3}{8} \end{bmatrix} \begin{bmatrix} 16 \\ 32 \end{bmatrix}$$

and

$$\begin{bmatrix} 49 \\ 40\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{7}{4} & 0 \\ 0 & \frac{9}{8} \end{bmatrix} \begin{bmatrix} 28 \\ 36 \end{bmatrix}.$$

The island's population after the initial "calibration" period is overprojected by seven and a half thousand people relative to the biregional projection, with the rural population's *underprojection* of two and a half thousand being overcompensated by the urban population's *overprojection* of ten thousand. Notice that the former was losing net migrants, whereas the latter was gaining net migrants. This demonstrates the decomposition bias discussed earlier, and it illustrates what happened in the Census Bureau's two projections of Florida's year 2000 population.

During the 1975–80 period, according to Census Bureau data, 399,807 internal migrants entered Florida and 209,610 left the state, giving the state's population a net gain of 190,197 people. In terms of five-year rates, the migration rate out of Florida was

$$\frac{209{,}610}{8{,}504{,}523} = 0.024647$$

while that out of the Rest of the United States was

$$\frac{399{,}807}{211{,}168{,}686} = 0.001893$$

The corresponding net rates for the two regions were 0.022364 and −0.000901, respectively. The redistributional dynamics generated by the two alternative models: biregional (or multiregional) and uniregional produced relative overprojection for Florida and underprojection for the Rest of the United States, in a manner analogous to that illustrated above with the simpler-to-understand numerical example.

Further illumination of the redistributional dynamics generated by the two models is possible by extending the projection process of the hypothetical urban-rural population to stability. In both representations the regional (urban and rural) and the national populations increase without bound. In the uniregional model, they do so exponentially, with the urban population increasingly dominating the national scene, since $(\frac{7}{4})^n$ becomes ever larger relative to $(\frac{9}{8})^n$ as n increases. In the biregional model, however, a dampening of the exponential growth and an augmentation of the rural population's exponential growth are introduced, and it is this feature that ultimately creates a stable spatial distribution of the national population, allocating a half of that population to each of the two regions. To see this process, one needs to introduce a diagonalization transformation of the growth matrix and observe what happens as n increases without bound.

The two characteristic roots of the growth matrix

$$\mathbf{G} = \begin{bmatrix} \frac{3}{4} & \frac{1}{2} \\ \frac{1}{4} & 1 \end{bmatrix}$$

are $\lambda_1 = \frac{5}{4}$ and $\lambda_2 = \frac{1}{2}$. Associated with these two roots are the two characteristic vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \text{ and } \mathbf{v}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

This gives rise to the transformation

$$\mathbf{G} = \mathbf{V\Lambda V}^{-1}$$

$$\begin{bmatrix} \frac{3}{4} & \frac{1}{2} \\ \frac{1}{4} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} \frac{5}{4} & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix} \tag{3}$$

and the results that

$$\begin{aligned} \mathbf{G}^n &= \mathbf{V\Lambda}^n\mathbf{V}^{-1} \\ &= \begin{bmatrix} 1 & 2 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} (\frac{5}{4})^n & 0 \\ 0 & (\frac{1}{2})^n \end{bmatrix} \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix} \\ &= \begin{bmatrix} \frac{1}{3}(\frac{5}{4})^n + \frac{2}{3}(\frac{1}{2})^n & \frac{2}{3}(\frac{5}{4})^n - \frac{2}{3}(\frac{1}{2})^n \\ \frac{1}{3}(\frac{5}{4})^n - \frac{1}{3}(\frac{1}{2})^n & \frac{2}{3}(\frac{5}{4})^n + \frac{1}{3}(\frac{1}{2})^n \end{bmatrix} \end{aligned}$$

and that the corresponding projected population vector is

$$\mathbf{P}(n) = \begin{bmatrix} \frac{80}{3}\left(\frac{5}{4}\right)^n - \frac{32}{3}\left(\frac{1}{2}\right)^n \\ \frac{80}{3}\left(\frac{5}{4}\right)^n + \frac{16}{3}\left(\frac{1}{2}\right)^n \end{bmatrix}.$$

As n increases without bound, the terms with $(\frac{1}{2})^n$ drop out, and the national population becomes distributed equally across the two regions. During the projection process to stability, however, the contribution brought about by the second characteristic root dampens the exponential trajectory of the urban population, at the same time that it augments that of the rural population.

Finally, consider the same growth matrix, but now imagine that the initial national population of 48 thousand is distributed equally among the two regions. Then the biregional projection gives

$$\begin{bmatrix} 30 \\ 30 \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{1}{2} \\ \frac{1}{4} & 1 \end{bmatrix} \begin{bmatrix} 24 \\ 24 \end{bmatrix}$$

and

$$\begin{bmatrix} 37\frac{1}{2} \\ 37\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{1}{2} \\ \frac{1}{4} & 1 \end{bmatrix} \begin{bmatrix} 30 \\ 30 \end{bmatrix}.$$

The relevant net migration rates now are $m_u = \frac{1}{4}$ and $m_v = -\frac{1}{4}$, and the corresponding uniregional projection becomes

$$\begin{bmatrix} 30 \\ 30 \end{bmatrix} = \begin{bmatrix} \frac{5}{4} & 0 \\ 0 & \frac{5}{4} \end{bmatrix} \begin{bmatrix} 24 \\ 24 \end{bmatrix}$$

and

$$\begin{bmatrix} 37\frac{1}{2} \\ 37\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{5}{4} & 0 \\ 0 & \frac{5}{4} \end{bmatrix} \begin{bmatrix} 30 \\ 30 \end{bmatrix}.$$

Because the initial population has a stable initial distribution, perfect decomposition results. No bias is introduced by shifting to a uniregional model by means of compensated decomposition.

The above illustrations ignored age groups and dealt with two regional populations. Exactly analogous results would arise if instead we were to imagine the two populations to represent two adjacent age groups in a single region (not the first or last age groups, however). Decomposition bias would arise if the age distribution did not follow the stable age composition.

2.5. *Conclusions*

Aggregating the separate projections of several *noninteracting* heterogeneous populations will give rise to a total greater than the one that would be obtained by projecting the aggregate population at its average rate of growth at the outset (Keyfitz 1977). Aggregation prior to projection introduces an aggregation bias that is guaranteed to be *negative*, giving rise to an underprojection relative to the consolidation of the corresponding disaggregated projection. The aggregation of noninteracting heterogenous populations prior to projection, then, always produces an underprojection: the aggregate population never stabilizes, the aggregate rate of growth forever increases, and the population's composition varies continuously.

Does the same guaranteed *negative* bias also arise in the aggregation introduced by the consolidation of *interacting* heterogeneous populations? The answer is no. Rogers (1986), for example, offers an illustration of a positive aggregation bias which is introduced by the consolidation of the Swedish female population across four "regions" that are marital status categories. The consolidated projection, in this instance, produces an overprojection relative to the aggregation of the deconsolidated projection. So clearly the answer in this situation is an ambiguous one; the aggregation bias can be positive or negative. This can be readily demonstrated by carrying out a projection across two time intervals with both the deconsolidated and consolidated models and then comparing the two projections, as we did with the simple numerical example in section 2.5. The aggregation of interacting heterogeneous multiregional populations prior to projection, it can be shown, produces either under- or overprojection: the aggregate population ultimately stabilizes and both its aggregate rate of growth and its composition become fixed.

What about decomposition bias? The separation of each region from the others in a multiregional system by means of a net migration rate form of compensated decomposition will always create a bias in the projected regional totals, except in the two relatively uninteresting cases of an interregionally immobile population or one that is experiencing stable growth. Because net migration rates confound movement propensities with population stocks, the conditions for "perfect decomposition" turn out to be even more stringent that those for "perfect aggregation" (Rogers 1969). A particularly simple, yet pervasive, form of decomposition bias is the relative overprojection of the population experiencing net gains from migration and the underprojection of the corresponding population that is the net loser of migrants.

3. NETTING OUT THE AGE PATTERNS OF GROSS MIGRATION RATES

The death rates of Costa Rica in 1960 were higher at every age than those of Sweden during 1958–1962, yet the aggregate crude death rate of the latter was higher than that of the former (Cohen 1986). The cause of this apparent contradiction is age composition. To see this more clearly we shall borrow Nathan Keyfitz's (1983) numerical illustration of what is commonly referred to as "Simpson's Paradox."

Imagine a population of a million people in Country A and another of the same number in Country B. During the course of a year, ten thousand individuals die in the former and nine thousand die in the latter. A comparison of the mortality regimes prevailing in the two countries suggests that mortality is higher in Country A (1.0 percent against 0.9 percent).

Suppose that the population of Country A is equally divided among Young and Old people, half-a-million being in each age group. Country B, on the other hand, has a younger age composition, with 70 percent of its population being in the Young age group. Suppose, further, that of the ten thousand deaths in Country A a quarter occurred among the Young, whereas in Country B the corresponding total was 4.2 thousand. Then the age-specific death rates in Country A were 0.5 percent among the Young population and 1.5 percent among the Old, both lower than the corresponding percentages for Country B: 0.6 percent and 1.6 percent, respectively.

A comparison of these percentages indicates that mortality is *lower* in Country A at each age. The cause of this apparent contradiction with our earlier finding is the relatively younger age composition of Country B. Since crude rates are weighted sums of the constituent disaggregated rates, the relatively heavier weight accorded to the death rate of the Young population in Country B lowered its aggregate crude rate with respect to Country A:

$$\textit{Country A}: \ 0.5(0.5 \text{ percent}) + 0.5(1.5 \text{ percent}) = 1.0 \text{ percent}.$$

$$\textit{Country B}: \ 0.7(0.6 \text{ percent}) + 0.3(1.6 \text{ percent}) = 0.9 \text{ percent}.$$

What is true of crude mortality rates is, of course, also true of crude out-migration rates and, therefore, of crude net migration rates. Assume that the above figures now refer to emigration from one country to the other. The aggregate flows then reveal that Country B gains a thousand net migrants from the exchange. This total results from the combination of a net loss of Young people (−1.7 thousand) and a net gain of Old people (+2.7 thousand). *Thus Country B gains net migrants, even though its rates of emigration are higher at each age than those of Country* A. This compositional artifact could possibly be a contributing factor to the counter-intuitive directional behavior of *net* interstate migrants that puzzles David Plane (1988):

> A somewhat embarrassing empirical observation for many mainline economic papers on migration has been that in recent years something like two-thirds of all the net interstate streams of population movement in the United States point in the direction of the lower average wage state. (p. 10)

But, of course, another contributing factor also could have been the decomposition bias introduced by a net migration perspective. Consider, for example, *identical* Young-Old age compositions of a half and a half, say, and the same directional age-specific emigration rates. But now assume that Region A has twice the population of Region B, say two million to Country B's one million. Then,

$$\textit{Country A}: \ 0.5(0.5 \text{ percent}) + 0.5(1.5 \text{ percent}) = 1.0 \text{ percent}$$

$$\textit{Country B}: \ 0.5(0.6 \text{ percent}) + 0.5(1.6 \text{ percent}) = 1.1 \text{ percent}$$

During the course of a year, then, twenty thousand individuals emigrate from Country A and only eleven thousand leave Country B. The result is that Country B shows a positive *net* migration rate of 0.9 percent, while Country A exhibits a corresponding negative rate of 0.45 percent. *And Country B gains net migrants once again, even though its rates of emigration are higher at each age than those of Country A.*

Our numerical illustration also clearly reveals how similar age profiles of gross migration rates may be hidden in the corresponding age profiles of net migration rates. For example, the age-specific immigration rates for Country A in the first illustration are

$$i_A(Y) = 0.6(\tfrac{7}{5}) = 0.84 \text{ percent},$$

$$i_A(O) = 1.6(\tfrac{3}{5}) = 0.96 \text{ percent},$$

and for Country B they are

$$i_B(Y) = 0.5(\tfrac{5}{7}) = 0.36 \text{ percent},$$

$$i_B(O) = 1.5(\tfrac{5}{3}) = 2.50 \text{ percent}.$$

Hence the corresponding net migration rates are

$$m_A(Y) = 0.84 - 0.5 = +0.34 \text{ percent},$$

$$m_A(O) = 0.96 - 1.5 = -0.54 \text{ percent},$$

$$m_B(Y) = 0.36 - 0.6 = -0.24 \text{ percent},$$

$$m_B(O) = 2.5 - 1.6 = +0.90 \text{ percent}.$$

Figure 1 sets out these age-specific patterns of migration and illustrates how the netting out of similar age patterns of gross migration rates gives rise to totally different corresponding age patterns of net migration rates. Note that merely reversing the Young-Old proportional relationship between the two nations, totally reverses the corresponding age pattern of net migration rates (compare Figure 1.1 with Figure 1.2).

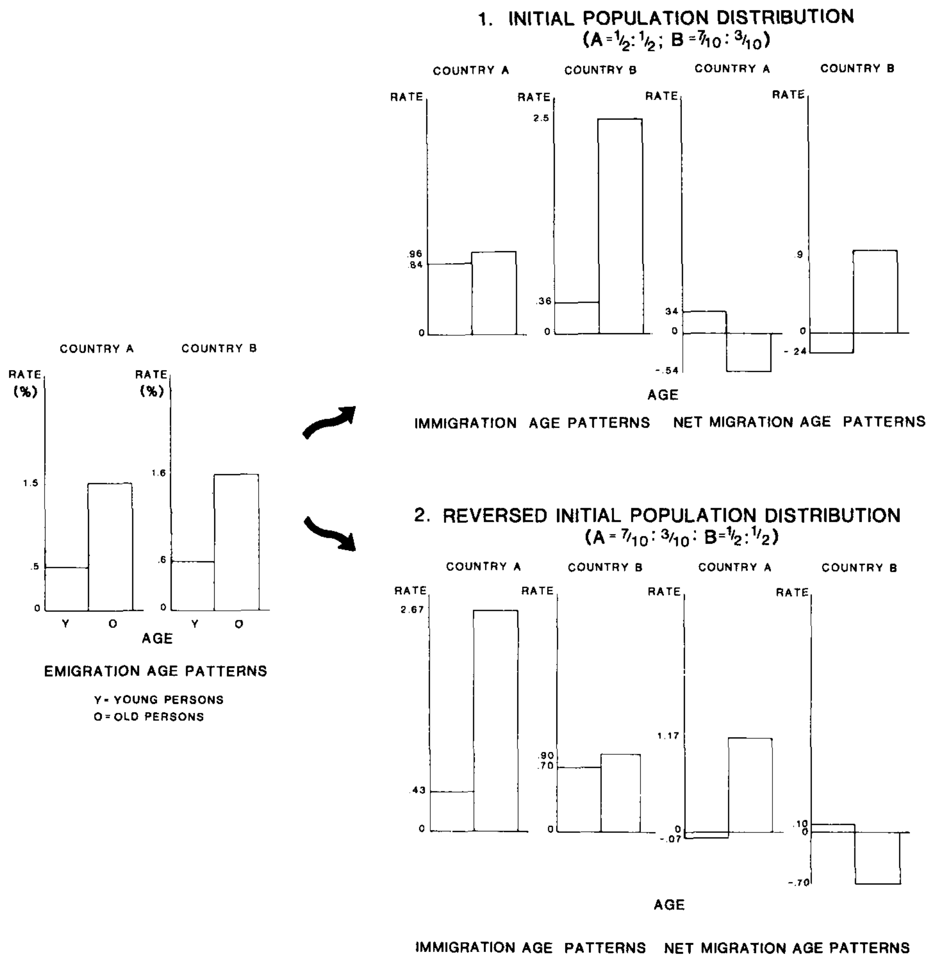

FIG. 1. Various Migration Rates for Numerical Illustration under Two Alternative Population Distributions

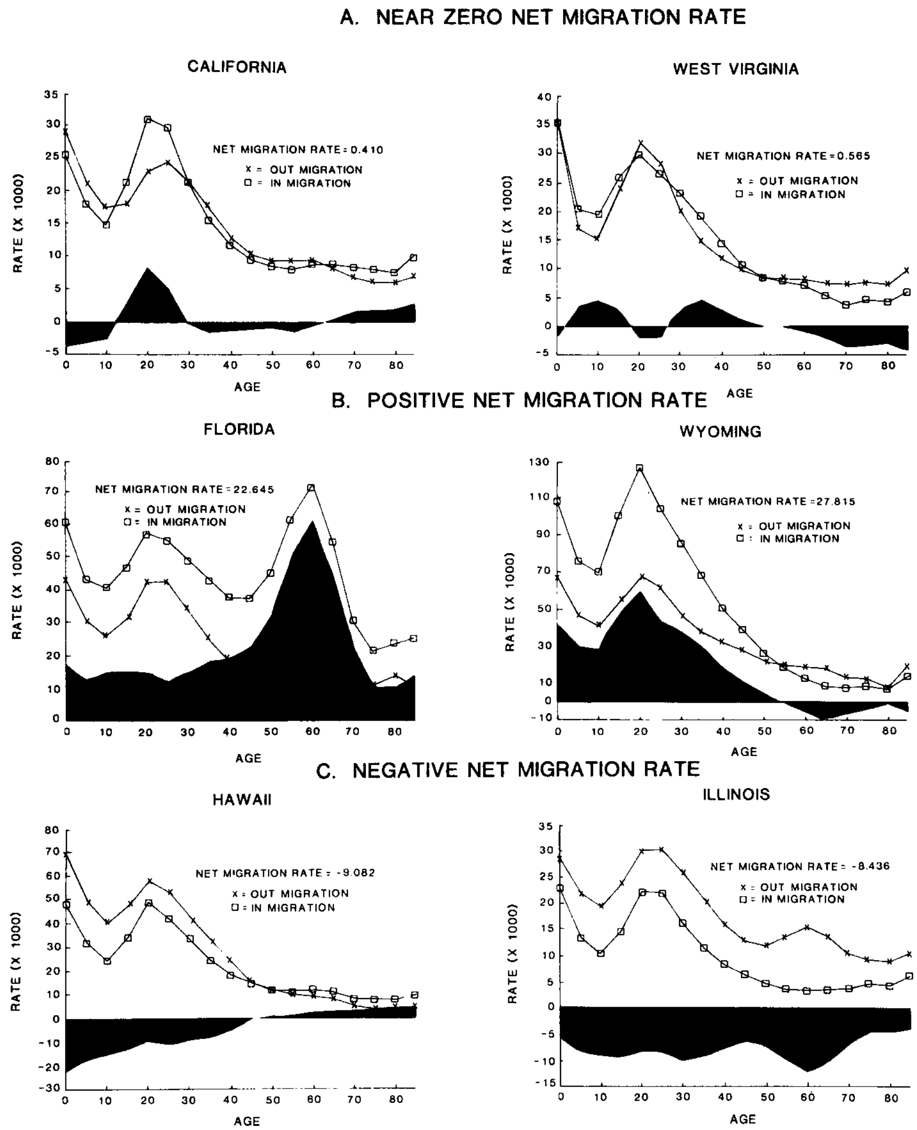

FIG. 2. Out-, In-, and Net Migration Schedules for Six U.S. States, 1975–1980. Source: Data tape provided by the U.S. Bureau of the Census. Rates computed by John Watkins.

Age-specific patterns of in-, out-, and net migration rates are also set out in Figure 2; however, in this case the data come from observed migration schedules and not from a contrived numerical example. Nonetheless, the same observation may be made regarding the effects of the netting out process. Similar age patterns of directional migration give rise to totally different age patterns of net migration. To use the latter in an econometric causal model, therefore, would create a misspecification.

Age patterns of gross migration rates are similar in profile because migration is related to the life course. The top age profile in Figure 3 illustrates the typical migration schedule, with its three sequences of rising rates, two of which occur at the elderly ages. The first rise starts around age sixteen and is triggered by a move

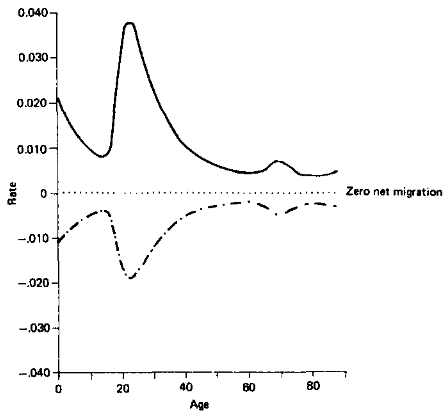

FIG. 3. Net Migration Schedules

away from the *parental* home, a move that reflects the transition from adolescence to adulthood. Entry into the job or marriage markets, military enlistment, or university enrollment all are life course events that often generate migration. The first rise generally peaks at some age in the early twenties and then begins a monotonic decline until the start of the second rise, around age sixty for males and earlier for females. This second upswing in the age pattern of migration reflects movement away from the *family* home, a move that often is motivated by amenity-oriented retirement migration. Finally, the third rise in the schedule occurs at around age seventy-five and is migration away from the *retirement* home, a move that often is a consequence of entry into dependent status and the onset of illness, disability, or the death of a spouse.

Rates of migration also are high among children, starting with a peak during the first years of life, dropping to a low point around age sixteen. The age pattern of these rates for children usually mirrors that of their parents, that is, the pattern of rates occurring some twenty-five to thirty years later.

Net migration rates are often viewed as crude indices that reflect differences in propensities of movement. But as we have seen, net migration rates also reflect the relative sizes of population stocks. The consequence for age patterns of migration rates is the disintegration of a well-established regularity in age profile. To see this, imagine a migration exchange between two neighboring regions of a multiregion system, regions i and j, say, that initially contain populations of equal size, $P_i = P_j$, say. Assume that the gross migraproduction rates (the areas under the migration schedules) are equal to unity in both directions, and that the age profile of both flows is that of the top age profile in Figure 3. Under these conditions the net migration rate into region i is zero at all ages, as shown by the dotted line in Figure 3. At each age, the number of migrants from region j to region i exactly equals the number in the reverse direction, and the equality also holds for the corresponding rates.

Now imagine that because of higher fertility and immigration levels, say, the population in region j grows more rapidly than that of region i, such that it becomes twice as large as its neighbor, that is, $P_j = 2P_i$. Assume that the propensities to migrate in both directions and the associated age profiles remain the same as before [that is, $M_{ij}(x) = M_{ji}(x)$ for all ages x]. Then the resulting net migration rate schedule of region i becomes that of the solid line in Figure 3, that is, the "standard" profile with a gross migraproduction rate of unity. We also include the corresponding net migration rate schedule when $P_j = P_i/2$ (the broken line in Figure 3).

The three net migration schedules in Figure 3 all reflect the same pair of gross migration schedules. In each instance the propensity to migrate in the two directions is the same, and so is the age profile. Yet the net migration rate for region i, say, varies directly with the relative sizes of the two populations, that is, with the ratio P_j/P_i. The net rate is zero at all ages when the ratio is unity, positive at all ages when the ratio exceeds unity, and negative at all ages when the ratio falls short of unity, in the latter two instances following the age profile of the migration schedule standard. Thus in this illustration, net migration once again depends on relative population sizes; the effects of flows are confounded with the effects of changes in stocks.

Because net rates confound flows with changes in stocks, they hide regularities that seem to prevail among gross flows. Although the latter tend always to follow the conventional age profile, the former exhibit a surprisingly wide variety of shapes, a few of which appeared earlier in Figure 2.

4. CONCLUSION

Much of the literature on aggregate, cross-sectional behavioral models of internal migration continues to exhibit a curiously ambivalent position with regard to the measurement of geographical mobility. This ambivalence is particularly surprising because it stands in striking contrast to the corresponding studies of mortality and fertility (natality) which often devote considerable attention to measurement problems. Haenszel (1967) attributes this paradox to the strong influence of Ravenstein's early contributions to migration analysis:

> Work on migration and population redistribution appears to have been strongly influenced by the early successes of Ravenstein in formulating "laws of migration." Subsequent papers have placed a premium on the development and testing of new hypotheses rather than on descriptions of facts and their collation... This is in contrast to the history of vital statistics. While Graunt more than two centuries before Ravenstein, had made several important generalizations from the study of "bills of mortality" in London, his successors continued to concentrate on descriptions of the forces of mortality and natality by means of rates based on populations at risk. (Haenszel 1967, p. 260)

The ambivalence regarding proper measurement of migration is reflected in the concept of the net migration rate, which continues to occupy an important position in the migration literature despite repeated calls by prominent demographers for a fuller view of migration phenomena:

> There are no "net migrants"; there are, rather, people who are arriving at places or leaving them. Why they are doing so is central to understanding the dynamics of... growth and decline. (Morrison 1977, p. 61)

Causal models of migration that seek to explain patterns of net migration are founded on inadequate perspectives. Net migration rates confound movement propensities with relative population stock levels. They hide well-established regularities in the age pattern of geographical mobility. They can lead to misspecified explanatory (causal) models, and they make it virtually impossible to consider properly the impacts of important violations of the basic assumptions underlying many spatial demographic studies: homogeneity, stationarity, and temporal independence (Pickles, Davies, and Crouchley 1982).

Gross migration stream (multiregional) models, on the other hand, more realistically depict the phenomenon being modeled (since there are no net migrants). The rates that they use to represent directional movements are linked to the populations at risk of moving and therefore measure true propensities of migrating (a feature that net migration rates lack). Gross migration models can generate changes in migration streams that arise out of changes in the sizes of the various populations at risk of moving (something that net migration rate models cannot do since they only consider the size of the destination population). And, finally, gross migration models permit their users to keep track of important population attributes such as places of birth and places of former residence, a feature that allows one to differentiate the migration rates of return migrants from those of nonreturn migrants, for example.

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