European Doctoral School of Demography 2019-20

Population Projections

Lecture 3

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Online course

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Brief course summary

Lecture 1: introduction & first simple model of population projections

Lecture 2: cohort component method

Lecture 3:

- matrix projections
- dynamic visualizations and `shiny` apps

Lecture 4: more matrix projections

Small recap

In Lecture 2, we have seen the most employed model for population projections, the cohort component method

- takes into account age composition of populations
- projection of age groups rather straightforward, except for the youngest and the oldest
- projected population from one interval becomes the baseline for next projection
- several projections can become cumbersome if done one at a time

$\Rightarrow$ we can speed things up with matrix algebra

Introduction

The cohort component method:

- can be compactly written in matrix notation (Bernardelli 1941, Lewis 1942, Leslie 1945, as reported by Smith and Keyfitz 1977)
- rewriting facilitates computer programming and shows the use of matrix algebra for population projections
- simplifies projections over multiple time intervals

Cohort component formulas I

We can write down the female-specific equations for each age group:

$$
\begin{aligned}
N_0(t+5) &= \sum_x N_x(t) b_x \\
N_5(t+5) &= N_0(t)\, s_0 \\
N_{10}(t+5) &= N_5(t)\, s_5 \\
&\vdots \\
N_{45}(t+5) &= N_{40}(t)\, s_{40} \\
N_{50}(t+5) &= N_{45}(t)\, s_{45} \\
&\vdots \\
N_{80}(t+5) &= N_{75}(t)\, s_{75} \\
N_{85}(t+5) &= \big(N_{80}(t) + N_{85}(t)\big)\, s_{80}
\end{aligned}
$$

with $s_x = \frac{L_{x+5}}{L_x}$, $s_{80} = \frac{T_{85}}{T_{80}}$ and $b_x = \frac{1}{1+SRB} \frac{L_0}{2\ell_0} \left(F_x + s_x\, F_{x+5}\right)$

Cohort component formulas II

Note the structure of the equations:

$$
\begin{array}{rcl}
N_0(t+5) & = & N_{10}(t)b_{10} + \cdots + N_{45}(t)b_{45} \\
N_5(t+5) & = & N_0(t)\,s_0 \\
N_{10}(t+5) & = & N_5(t)\,s_5 \\
N_{15}(t+5) & = & N_{10}(t)\,s_{10} \\
 & \vdots & \ddots \\
N_{45}(t+5) & = & N_{40}(t)\,s_{40} \\
N_{50}(t+5) & = & N_{45}(t)\,s_{45} \\
 & \vdots & \ddots \\
N_{80}(t+5) & = & N_{75}(t)\,s_{75} \\
N_{85}(t+5) & = & \big(N_{80}(t) + N_{85}(t)\big)\,s_{80}
\end{array}
$$

$\Rightarrow$ this suggests the use of matrix notation

Matrix notation I

Let's rewrite this in matrix form:

$$
\begin{bmatrix} N_0(t+5) \\ N_5(t+5) \\ N_{10}(t+5) \\ N_{15}(t+5) \\ \vdots \\ N_{45}(t+5) \\ N_{50}(t+5) \\ \vdots \\ N_{80}(t+5) \\ N_{85}(t+5) \end{bmatrix} = \begin{bmatrix} 0 & 0 & b_{10} & \dots & b_{45} & 0 & \dots & 0 & 0 & 0 \\ s_0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & s_5 & 0 & \dots & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & s_{10} & \dots & 0 & 0 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & s_{45} & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & s_{50} & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & \dots & s_{75} & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 & s_{80} & s_{80} \end{bmatrix} \begin{bmatrix} N_0(t) \\ N_5(t) \\ N_{10}(t) \\ N_{15}(t) \\ \vdots \\ N_{45}(t) \\ N_{50}(t) \\ \vdots \\ N_{80}(t) \\ N_{85}(t) \end{bmatrix}
$$

Matrix notation II

More compactly:

$$\boldsymbol{N}(t+5) = \boldsymbol{L}[t, t+5]\boldsymbol{N}(t) \tag{1}$$

If we can assume that the projection (Leslie) matrix $\boldsymbol{L}$ does not change over future projections, then:

$$\boldsymbol{N}(t+5\times n) = \boldsymbol{L}^{\mathbf{n}}\,\boldsymbol{N}(t) \tag{2}$$

Using matrices - exercise

Exercise

Load the data that we saved yesterday, `EDSD.lect2.Rdata`. Project the female population by one period using the matrix notation in Equation (1). Check your results with the previously derived projection.

Hint: insert the vectors `sFx` and `bFx` in the matrix $\boldsymbol{L}$ and compute

$$\boldsymbol{N}(t+5) = \boldsymbol{L}[t, t+5]\boldsymbol{N}(t)$$

Using matrices - one possible solution

Example

```
rm(list = ls())
library(tidyverse); library(viridis)
load("EDSD.lect2.Rdata")
## extract sx, bx and NFx
m <- nrow(dta.swe)
sFx <- dta.swe$sFx; bFx <- dta.swe$bFx; NFx <- dta.swe$NFx
## change NA to 0 in bFx and remove NA from sFx
bFx[is.na(bFx)] <- 0
sFx <- sFx[!is.na(sFx)]
## create empty Leslie matrix
L <- matrix(0,m,m)
## assign bFx and sFx
L[1,] <- bFx
diag(L[-1,]) <- sFx
L[m,m] <- sFx[m-1]
## matrix projection and comparison
NFx5.matrix <- c(L%*%NFx)
NFx5.manual <- dta.swe$NFx5
all.equal(NFx5.matrix,NFx5.manual)
```

```
[1] TRUE
```

Longer projections - exercise

Exercise

Now that we are sure that the matrix formulation works, let us project $n = 20$ periods ahead. Plot your results in a pyramid showing the starting population, the first and the last projected periods

Reminder:

$$\boldsymbol{N}(t + 5 \times n) = \boldsymbol{L}^{\mathbf{n}}\, \boldsymbol{N}(t)$$

Hint: write a function containing a population matrix $\boldsymbol{N}$ with $n + 1$ columns and loop through its columns with matrix multiplication. You can do this recursively (without the need of matrix exponentiation) by updating your baseline population and using the simpler formula

$$\boldsymbol{N}(t + 5) = \boldsymbol{L}[t, t + 5]\boldsymbol{N}(t)$$

Longer projections - one possible solution I

Example

```
## function to project several periods
pop.proj <- function(x,AgeGroup,Nx,sx,bx,n){
    ## number of age groups
    m <- length(x)
    ## create Leslie matrix
    L <- matrix(0,m,m)
    L[1,] <- bx
    diag(L[-1,]) <- sx
    L[m,m] <- sx[m-1]
    ## create population matrix
    N <- matrix(0,m,n+1)
    N[,1] <- Nx
    for (i in 1:n){
    N[,i+1] <- L%*%N[,i]
    }
    out <- cbind(data.frame(x=x,AgeGroup=AgeGroup),N)
    return(out)
}
## actual projection
n <- 20
my.proj <- pop.proj(x=dta.swe$Age,AgeGroup=dta.swe$AgeGroup,
Nx=NFx,sx=sFx,bx=bFx,n=n)
all.equal(my.proj$`1`,NFx)
all.equal(my.proj$`2`,NFx5.manual)
```

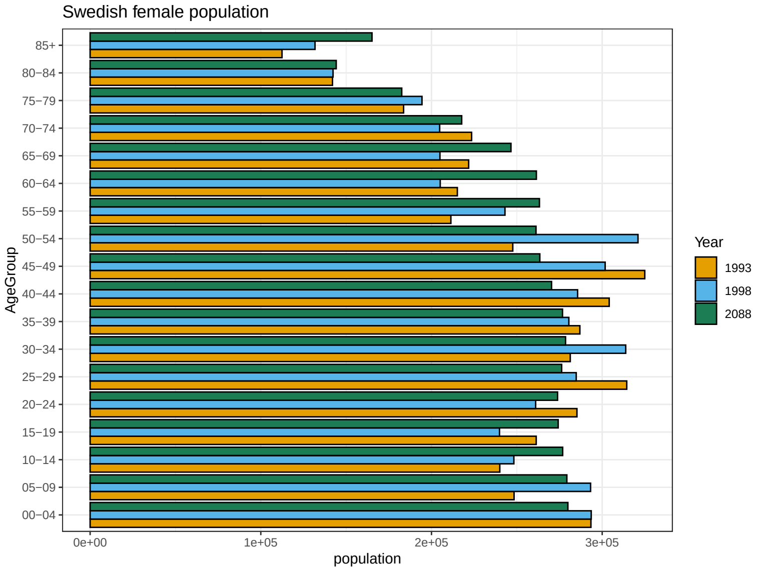
```
[1] TRUE
[1] TRUE
```

Longer projections - one possible solution II

Example

```
## long data
dta.swe.l <- my.proj %>%
    pivot_longer(-c(x,AgeGroup),names_to = "period",values_to = "population") %>%
    mutate(period=as.numeric(period),
           Year=1993 + (period-1)*5,
           YearF=as.factor(Year))
## plotting
ggplot(dta.swe.l,aes(x=AgeGroup,y=population,fill=YearF)) +
    geom_bar(data = subset(dta.swe.l, period %in% c(1,2,20)),
        stat = "identity",position = "dodge",color = "black") +
    coord_flip() +
    theme_bw() +
    ggtitle("Swedish female population") +
    scale_fill_manual(name = 'Year', values=c("#E69F00", "#56B4E9","#1C7C54"))
```

Longer projections - one possible solution III



Intermezzo: animated visualizations & shiny-app

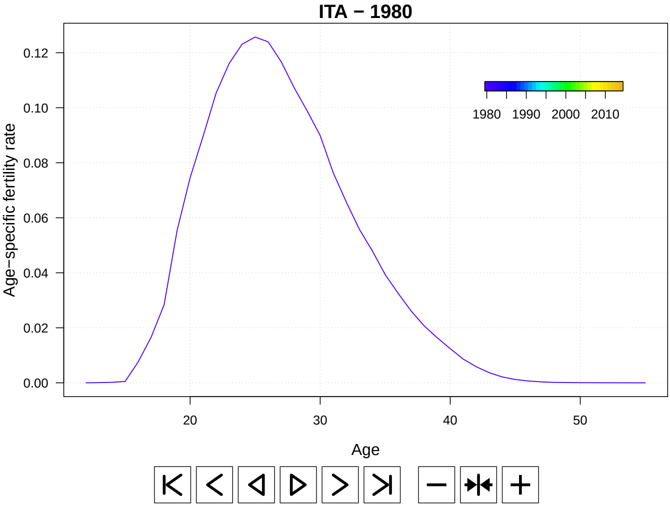
In some instances, it can be useful to introduce dynamic plots, animations and flexible outputs in your presentations or documents:

- can grab the audience's attention
- show time-series evolutions
- to describe your model see my own example
- sensitivity analysis of your results

$\Rightarrow$ We will now look at how to make these

Images: animation with \animategraphics

BEAMER supports animation images: \animategraphics



Images: animation with \animategraphics

\animategraphics[<options>]{frame rate}{file}{first}{last}

- Preamble: \usepackage{animate}
- need to create a multipage PDF file (next frame)
- Options:
  - autoplay: start animation after the page has opened
  - loop: animation restarts immediately after the end
  - palindrome: animation plays forwards and backwards
  - step: step through animation by mouse-click
  - controls: shows control buttons below the animation widget

- make sure to open file with Adobe Acrobat Reader (or the animation will fail)!!

Animated pyramids: an example

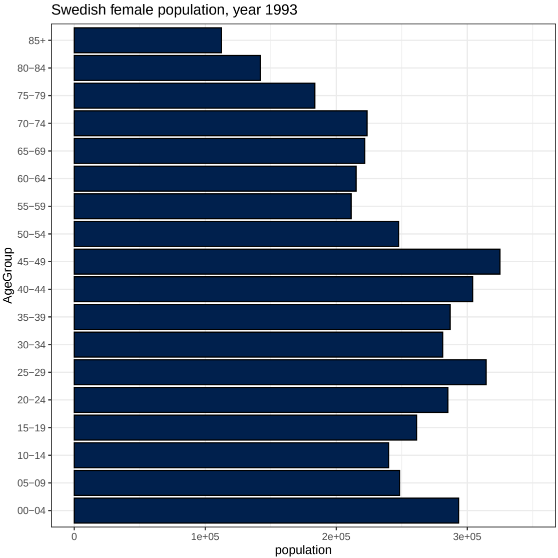
Example

```
plots <- list()
my.cols <- cividis(n+1)
my.years <- unique(dta.swe.l$Year)
for (i in 1:(n+1)){
gg <- ggplot(dta.swe.l,aes(x=AgeGroup,y=population,fill=YearF)) +
        geom_bar(data = subset(dta.swe.l, period == i),
            stat = "identity",color = "black") +
        coord_flip() +
        theme_bw() +
        theme(legend.position = "none") +
        ggtitle(paste("Swedish female population, year",my.years[i])) +
        scale_fill_manual(values=my.cols[i])
plots[[i]] <- gg
}
## saving plots in a single file
pdf("myAnimFig.pdf")
invisible(lapply(plots, print))
dev.off()
```

In LaTeX:

```
\animategraphics[autoplay,scale=0.42]{3}{myAnimFig.pdf}{}{}
```

Animated pyramids: output



Images: animation with .GIFs

- if you use Powerpoint* for your presentations, you can include animations by creating .GIFs in R using:
  - the gganimate package (for ggplots)
  - a series of .png or .pdf files and converting them to a .gif
- .GIFs are also useful for other media outlets, for example Twitter, ...

*please consider switching to LaTeX for your presentations. You can find here some materials on how to get started with BEAMER: https://github.com/ubasellini/LaTeXpresentations

Animations with .GIFs: gganimate

Example

```
## gganimate
library(gganimate)
gg <- ggplot(dta.swe.l,aes(x=AgeGroup,y=population,fill=YearF)) +
    geom_bar(stat = "identity",position = "dodge",color = "black") +
    coord_flip() +
    theme_bw() +
    theme(legend.position = "none") +
    scale_fill_viridis_d(option="cividis")

gg + transition_states(YearF) +
    ggtitle('Swedish female population, year {closest_state}')

anim_save("F2.gif")
```

Animations with .GIFs: converting .pdf or .png

Example

```
## convert the previously created .pdf into a .gif using ImageMagick.
## this is done internally by the terminal
## the "-delay" sets the time between the frames, i.e. the speed of the animation.
system("convert -delay 40 myAnimFig.pdf example.gif")

## save your data
save(dta.swe.l,file="EDSD.lect3.Rdata")
```

Shiny app

- `shiny` is an `R` package that makes it easy to build interactive web apps straight from `R`
- keep them on webpages or embed them in `R Markdown` documents
- a user-friendly interface to interact with your `R` analysis and show your results
- it is composed by
    - an `UI` (user interface), where you can create the inputs for your `shiny` and decide the outputs to display
    - a `server`, where you assemble the outputs from your given inputs
    - the `shinyApp`, putting the two together
- to learn more, visit `https://shiny.rstudio.com` to get started, plenty of video and written tutorials

Shiny app - a simple example

Example

```
rm(list = ls())
library("shiny"); library("ggplot2"); library("viridis")
load("EDSD.lect3.Rdata")
## general parameters
n1 <- max(dta.swe.l$period)
my.cols <- cividis(n1)
my.years <- unique(dta.swe.l$Year)
## build your user interfact
ui <- fluidPage(
    ## title of your shiny
    titlePanel('My first shiny app'),
    ## display a slider that returns input$year to pass to the server function
    sliderInput(inputId = "year", label = "Year", step = 5,
        value = min(my.years), min = min(my.years), max = max(my.years)),
    ## display a plot returned from the server
    plotOutput("plot_pyr1")
)
```

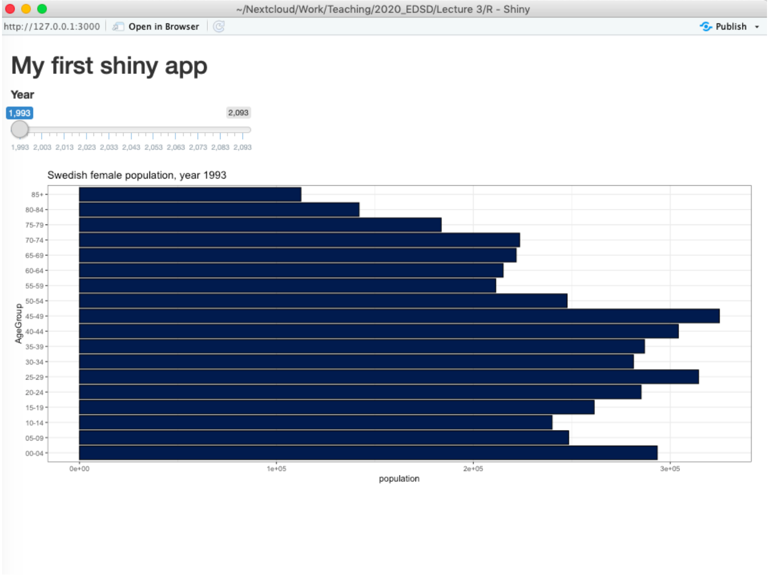
Shiny app - a simple example

Example

```
## build your server
server <- function(input, output){
    ## create an output that renders a plot
    output$plot_pyr1 <- renderPlot({
    ## any ggplot or plot,
    ## here subsetting the year of the given input$year
    ggplot(dta.swe.l,aes(x=AgeGroup,y=population,fill=YearF)) +
        geom_bar(data = subset(dta.swe.l, Year == input$year),
            stat = "identity",color = "black") +
        coord_flip() +
        theme_bw() +
        theme(legend.position = "none") +
        ggtitle(paste("Swedish female population, year",input$year)) +
        scale_fill_manual(values=my.cols[which(input$year==my.years)])
})
}

## run the shiny app, which puts together the ui and server
shinyApp(ui = ui, server = server)
```

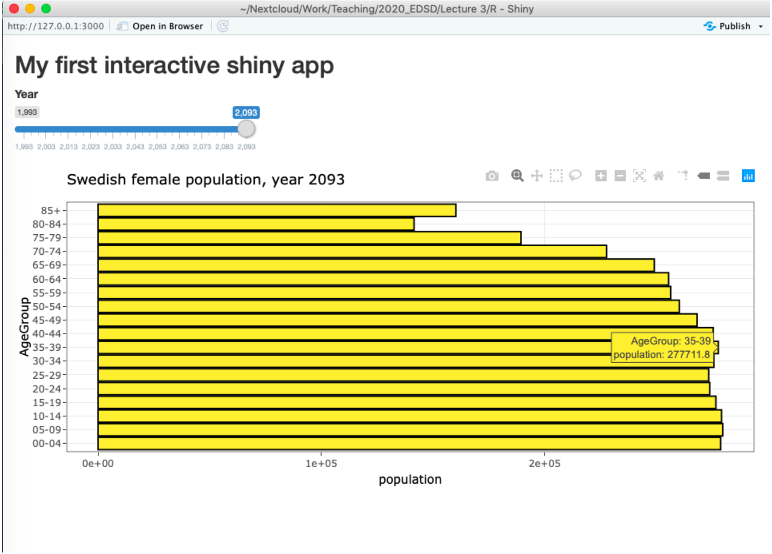
Shiny app - a simple example



Interactive Shiny apps

- interactivity can be easily implemented into `shiny` using `plotly`
- only need small changes to previous code:
  - load `plotly` package
  - from `plotOutput` to `plotlyOutput`
  - from `renderPlot` to `renderPlotly`
  - can select what to interactively show by using `ggplotly(fig,tooltip = c('AgeGroup','population'))`

Interactive Shiny apps - an example



Including the male population

- For males, we have seen that we can use (almost) the same formulas as for females
- we can express the equations for both sexes using matrix notation and a block diagonal Leslie matrix:

$$\begin{bmatrix} N^F(t+5) \\ N^M(t+5) \end{bmatrix} = \begin{bmatrix} L^F & 0 \\ B^M & L^M \end{bmatrix} \begin{bmatrix} N^F(t) \\ N^M(t) \end{bmatrix}$$

where

$$B^M = \begin{bmatrix} 0 & 0 & b_{10} & \dots & b_{45} & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 \end{bmatrix}, \; L^M = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ s_0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & s_5 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & s_{10} & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & s_{75} & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & s_{80} & s_{80} \end{bmatrix}$$

Two-sex projections - exercise

Exercise

Starting again from the data of the previous lecture, write a function to project both sexes contemporaneously. Check if your projection for the first period are are the same as those of the previous class. Plot a pyramid of the projections after 20 periods.

Reminder:

$$\begin{bmatrix} N^F(t+5) \\ N^M(t+5) \end{bmatrix} = \begin{bmatrix} L^F & 0 \\ B^M & L^M \end{bmatrix} \begin{bmatrix} N^F(t) \\ N^M(t) \end{bmatrix}$$

Two-sex projections - one possible solution I

Example

```
sMx <- dta.swe$sMx; bMx <- dta.swe$bMx; NMx <- dta.swe$NMx
## similar adjustments to bMx and sMx
bMx[is.na(bMx)] <- 0
sMx <- sMx[!is.na(sMx)]
## female Leslie matrix (derived before)
LF <- L
## male Leslie matrix
BM <- LM <- matrix(0,m,m)
BM[1,] <- bMx
diag(LM[-1,]) <- sMx
LM[m,m] <- sMx[m-1]
## putting male and females together
ZEROS <- diag(0,m)
Lup <- cbind(LF,ZEROS)
Ldown <- cbind(BM,LM)
L <- rbind(Lup,Ldown)
## matrix projection and comparison
Nx <- c(NFx,NMx)
Nx5.matrix <- c(L%*%Nx)
NFx5.manual <- dta.swe$NFx5
NMx5.manual <- dta.swe$NMx5
all.equal(NFx5.manual,Nx5.matrix[1:m])  ## [1] TRUE
all.equal(NMx5.manual,Nx5.matrix[1:m+m])  ## [1] TRUE
```

Two-sex projections - one possible solution II

Example

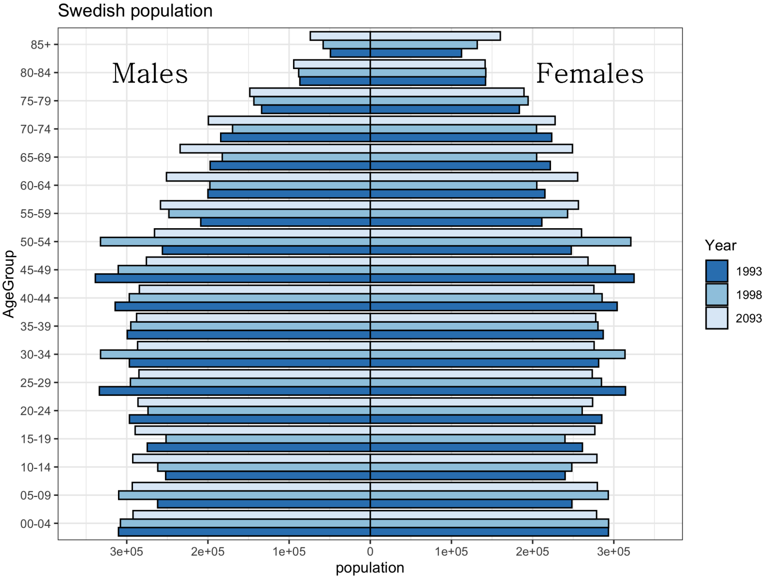
```
## function to project several periods
pop.proj.v2 <- function(x,AgeGroup,NFx,sFx,bFx,NMx,sMx,bMx,n){
    ## number of age groups
    m <- length(x); m2 <- m*2
    ## female Leslie matrix
    LF <- matrix(0,m,m)
    LF[1,] <- bFx
    diag(LF[-1,]) <- sFx
    LF[m,m] <- sFx[m-1]
    ## male Leslie matrix
    BM <- LM <- matrix(0,m,m)
    BM[1,] <- bMx
    diag(LM[-1,]) <- sMx
    LM[m,m] <- sMx[m-1]
    ## putting them together
    ZEROS <- diag(0,m)
    Lup <- cbind(LF,ZEROS)
    Ldown <- cbind(BM,LM)
    L <- rbind(Lup,Ldown)
    ## create population matrix
    N <- matrix(0,m2,n+1)
    N[,1] <- c(NFx,NMx)
    for (i in 1:n){
        N[,i+1] <- L%*%N[,i]
    }
    out <- cbind(data.frame(x=rep(x,2),AgeGroup=rep(AgeGroup,2),
        sex=rep(c("Females","Males"),each=m)),N)
    return(out)
}
```

Two-sex projections - one possible solution III

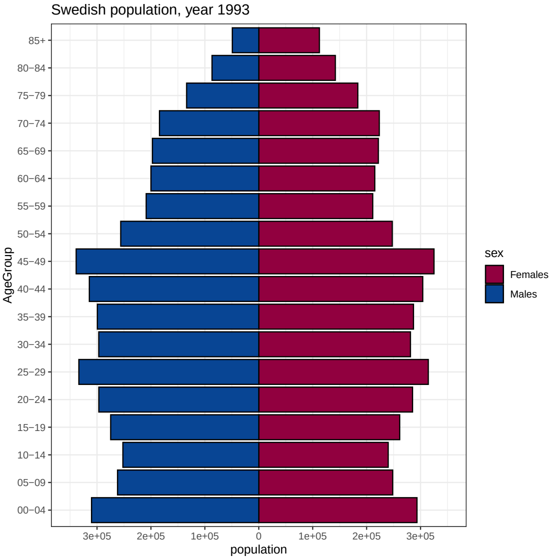
Example

```
my.proj <- pop.proj.v2(x=dta.swe$Age,AgeGroup=dta.swe$AgeGroup,NFx=NFx,sFx=sFx,bFx=bFx,
    NMx=NMx,sMx=sMx,bMx=bMx,n=20)
## long data
dta.swe.l <- my.proj %>%
    pivot_longer(-c(x,AgeGroup,sex),names_to = "period",values_to = "population") %>%
    mutate(period=as.numeric(period),
    Year=1993 + (period-1)*5,
    YearF=as.factor(Year))
## plotting
ggplot(dta.swe.l,aes(x=AgeGroup,y=population,fill=YearF)) +
    geom_bar(data = subset(dta.swe.l, period %in% c(1,2,21) & sex == "Males"),
        stat = "identity",position = "dodge",color = "black",mapping = aes(y = -population)) +
    geom_bar(data = subset(dta.swe.l, period %in% c(1,2,21) & sex == "Females"),
        stat = "identity",position = "dodge",color = "black") +
    coord_flip() +
    theme_bw() +
    ggtitle("Swedish population") +
    scale_y_continuous(limits=c(-3.5e5,3.5e5),
        breaks = seq(-4e5,4e5,1e5),
        labels = abs(seq(-4e5,4e5,1e5))) +
    scale_fill_brewer(name="Year",palette = 'Blues', direction = -1) +
    geom_text(data = subset(dta.swe.l, period %in% c(1)),
        aes(y = max(population)/1.25, x = 17, label='Females'),size=7) +
    geom_text(data = subset(dta.swe.l, period %in% c(1)),
        aes(y = -max(population)/1.25, x = 17, label='Males'),size=7)
```

Two-sex projections - one possible solution IV



Two-sex projections - one possible solution V



Some final remarks

- matrix algebra can significantly speed up your population projections
- can incorporate female and male populations within the same setting
- great care is needed to construct the Leslie matrix
- we still have not considered migration, nor time-specific assumptions on future demographic components

$\Rightarrow$ we'll look at these in tomorrow's lecture

Assignment

Exercise #4

Take again the population that you used for Exercise #2. Project the population for 5 years ahead, but this time use matrix formulas. Compare your results with the projections that you obtained in Exercise #2. Are they the same?

Exercise #5

Project your chosen population by sex for $n = 20$ periods ahead and show your results.

Bonus: present your results with the aid of a shiny app or an animation.

References

- Basellini, U. and Camarda, C.G. (2019). Modelling and forecasting adult age-at-death distributions. *Population Studies*, **73**(1), 119-138.
- Bernardelli, H. (1941). Population Waves. *Journal of the Bourma Research Society*, **31**(1), 1–18
- Lewis, E.G. (1942). On the generation and growth of a population. *Sankhya*, **6**, 93–96
- Leslie, P.H. (1945). On the use of matrices in certain population dynamics. *Biometrika*, **33**, 183–212
- Preston, S. H., Heuveline, P., and Guillot, M. (2001). *Demography. Measuring and Modeling Population Processes*. Blackwell.
- Smith, D.P. and Keyfitz, N. (*eds.*) (1977). *Mathematical Demography: Selected Papers*. Berlin: Springer Verlag

Animation example: the STAD model (Basellini & Camarda 2019)

go back