Characterization of Heat Exchangers

April 1, 2025

Group TR9

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Heat Exchangers Are Crucial For Industry

Industrial Applications:[1]

- Boilers & Condensers
- Refrigerators
- Heat Pumps
- Car radiators
- Server rooms

Objectives:

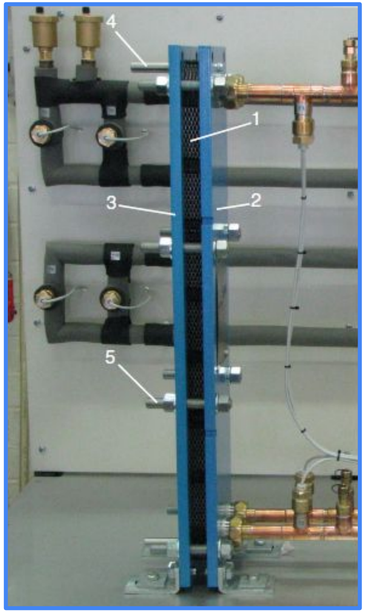
- Characterize overall heat exchanger coefficients (U) and effectiveness (ε) for each heat exchanger
- Compare experimental tubular heat transfer coefficients (h) with theoretical
- Obtain experimental and theoretical tubular temperature profiles

Fig 1. Home Radiator[2]

(1) Behesti-Pour, A. Katz. *Heat Exchange (HX) Revised 2025.docx*; University of California, Berkeley: Berkeley, California, 2025.
(2) Brick Underground. *Why Are NYC Apartments So Hot in the Winter? Blame a Pandemic (from 100 Years Ago)*. https://www.brickunderground.com/blog/2015/12/radiators (accessed Mar. 31, 2025).

Different Heat Exchangers Have Varying Performances

(1)



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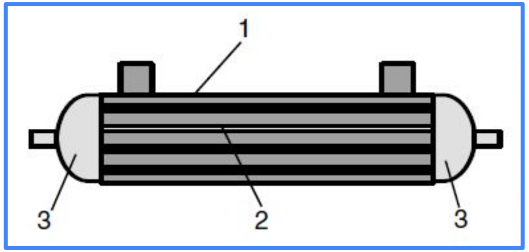

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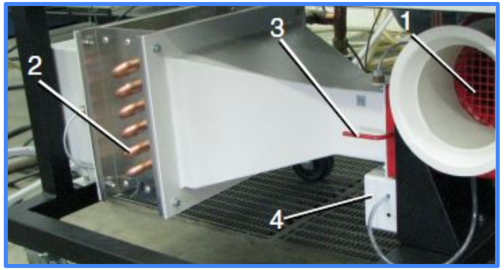

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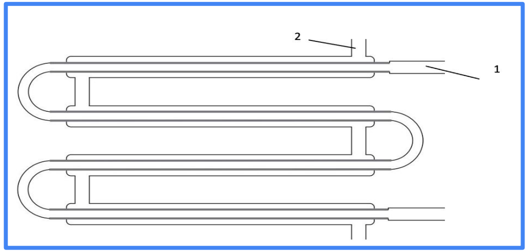


Types Of Heat Exchangers (HX):

1. Plate HX
2. Shell & Tube HX
3. Finned Cross-Flow HX
4. Tubular HX

Identical Conditions:

- Co-current Flow
- Cold Stream: 350 L/h
- Hot Stream: 300 L/h

Varied Conditions for Tubular:

- Counter-current Flow
- Cold Stream (Outer): 350 L/h
- Hot Stream (Inner): 200, 300, and 400 L/h

Images: Behesti-Pour, A. Katz. *Heat Exchanger (HX) Revised 2025.docx*; University of California, Berkeley: Berkeley, California, 2025.

Overall Heat Transfer Coefficient (U) and Effectiveness (ε)

(1)
$$U = \frac{\dot{Q}_M}{A_M \, \Delta T_{\mathrm{LM}}}$$

$$\varepsilon = \frac{\dot{Q}_{\mathrm{actual}}}{\dot{Q}_{\mathrm{max}}} = \frac{\dot{Q}_M}{C_{\mathrm{min}}(T_{h_i} - T_{c_i})}.$$

Q_M = Mean Exchanged Heat Flow

A_M = Mean Area

ΔT_{LM} = Log Mean Temperature Difference

C = Heat Capacity Rate

- U quantifies how efficient each square meter of the heat exchanger is at heat transfer
- ε quantifies the proportion of heat leaving the hot stream that is actually transferred to the cold steam, as opposed to being lost to the surroundings

(1) Geankoplis, C. J.; Hersel, A. A.; Lepek, D. H. Transport Processes and Separation Process Principles. In *Transport Processes and Separation Process Principles*, 5th ed.; Prentice Hall, 2018; Chapter 16.

Tubular Heat Exchanger had the Greatest U and Plate Heat Exchanger had the Greatest ε

Heat Exchanger	ΔT_{LM} (°K)	Q_M (W)	A_M (m^2)	U (W/m^2 K)	ε (%)
Tubular	11.1 ± 1.7	3740 ± 180	0.070	4800 ± 800	45 ± 2
Plate	7.4 ± 0.3	3740 ± 40	1.254	403 ± 16	50.2 ± 0.6
Shell and Tube	18.5 ± 0.4	2910 ±40	0.15	1050 ± 30	30.6 ± 0.4
Finned Cross-Flow	14.5 ± 0.3	2050 ± 30	2.77	51.0 ± 1.4	32.2 ± 0.5

Tubular Heat Exchanger Methods

Theoretical h and U:[1,2]

$$\mathrm{Nu} = 0.023\mathrm{Re}^{0.8}\mathrm{Pr}^{0.4}$$

Expected Scaling: $h_i, h_o \;\; \alpha \;\; v^{0.8}$

Dittus-Boelter Equation[1]

$$U_{\mathrm{exp}} = \frac{Q_M}{A_M \, \Delta T_{\mathrm{LM}}} \qquad U_{\mathrm{theo}} = \frac{1}{\frac{1}{h_i} + \frac{s}{k_{\mathrm{copper}}} + \frac{1}{h_o}}$$

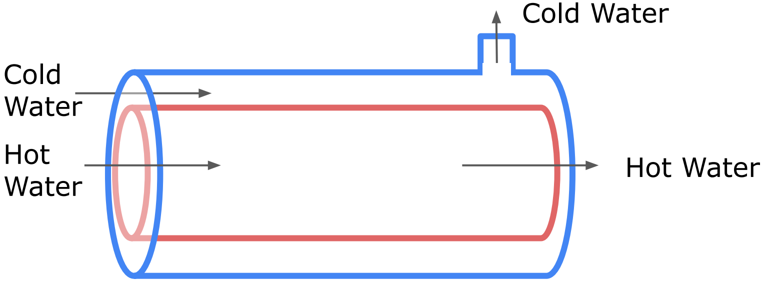
Theoretical temperature profiles:

$$\frac{dT_c}{dV_c} = -\frac{h_c a_c}{\dot{m}_c c_p}(T_c - T_h)$$

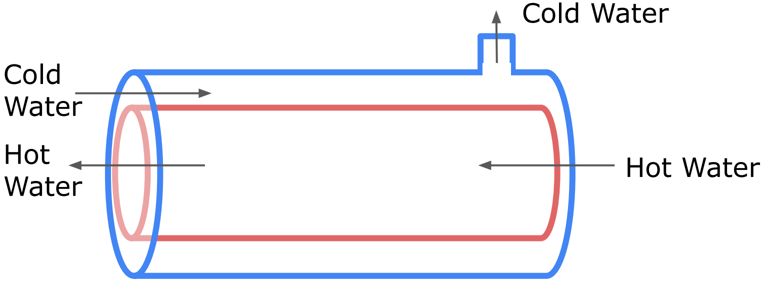
$$\frac{dT_h}{dV_h} = \pm\frac{h_h a_h}{\dot{m}_h c_p}(T_c - T_h)$$

T = temperature (K)
m = mass flow rate (kg/s)
h = heat transfer coefficient (W/m^2/K)
C_p heat capacity of water (J/kg/K)
a = heat exchange area/volume (1/m)
V = volume (m^3)

Co-Current:



Counter-Current:



(1) Geankoplis, C. J.; Hersel, A. A.; Lepek, D. H. Transport Processes and Separation Process Principles. In *Transport Processes and Separation Process Principles*, 5th ed.; Prentice Hall, 2018; Chapter 16
(2) Behesti-Pour, A. Katz. *Heat Exchange (HX) Revised 2025.docx*; University of California, Berkeley: Berkeley, California, 2025.

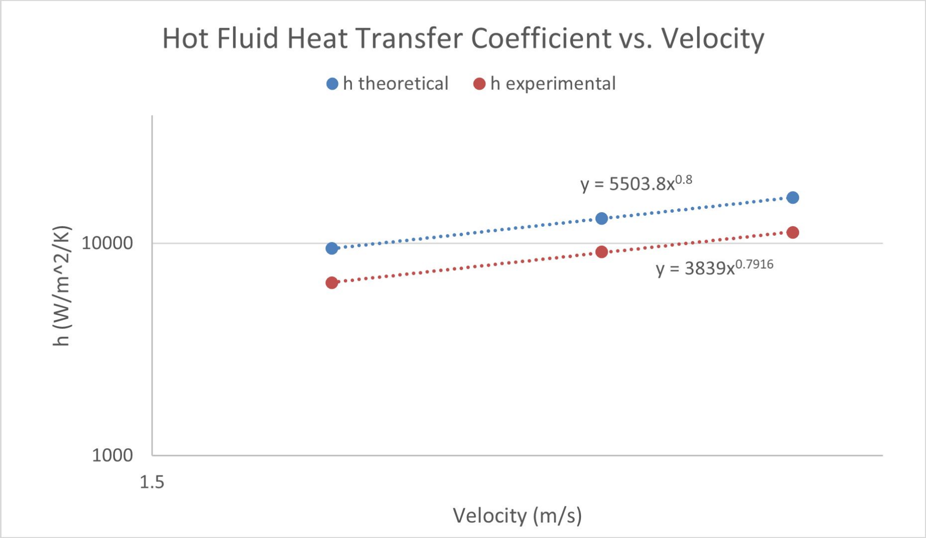
Inner Tube Heat Transfer Coefficient Scales with h ∝ $v^{0.79 \pm 0.02}$

U_{exp} and U_{theo} both **increase** with inner tube flow rate.

$$h_{\text{theo}} = \left(\frac{0.023\, k_{\text{water}}}{D_{t,i}}\right)\left(\frac{\rho V D_{t,i}}{\mu}\right)^{0.8} \text{Pr}^{0.4}$$

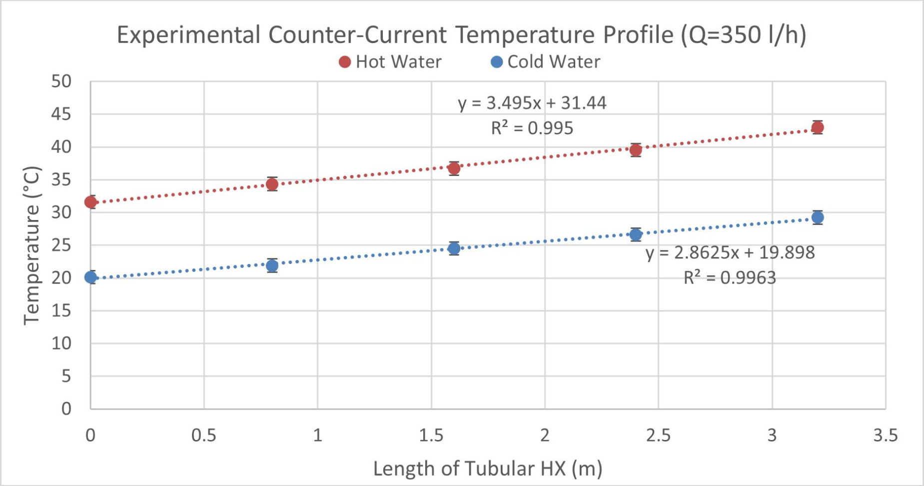
$$h_{\text{exp}} = \frac{1}{\frac{1}{U_{\text{exp}}} - \frac{s}{k_{\text{copper}}} - \frac{1}{h_o}}$$

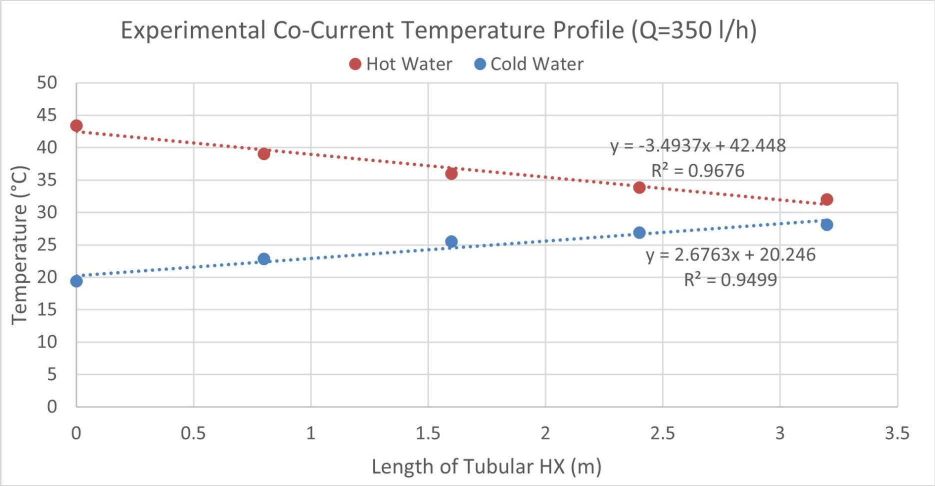
Inner Flow Rate (L/hr)	U_{exp} ($W/m^2/K$)	U_{theo} ($W/m^2/K$)
200	4520 ± 160	5800 ± 400
300	5630 ± 160	6900 ± 400
400	6390 ± 170	7800 ± 300



Experiment strongly validates theoretical model since scaling relative error of **~1%**

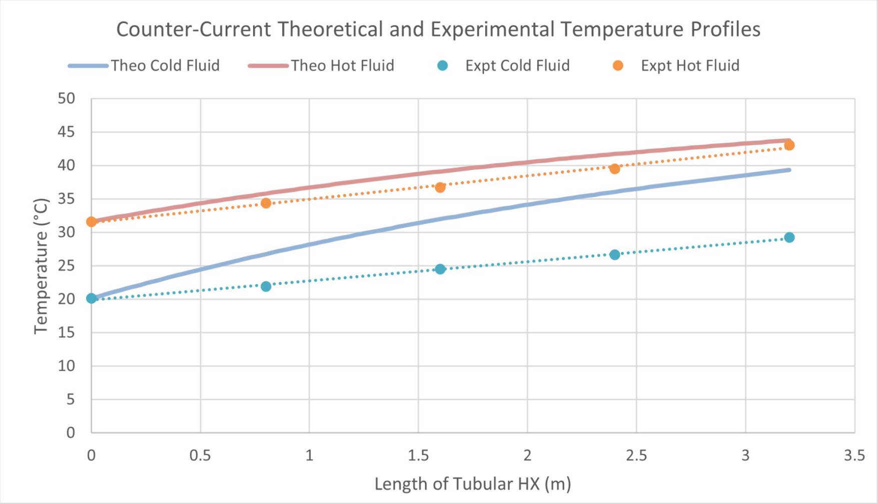
Counter-Current Heat Exchanger has Greater Heat Transfer

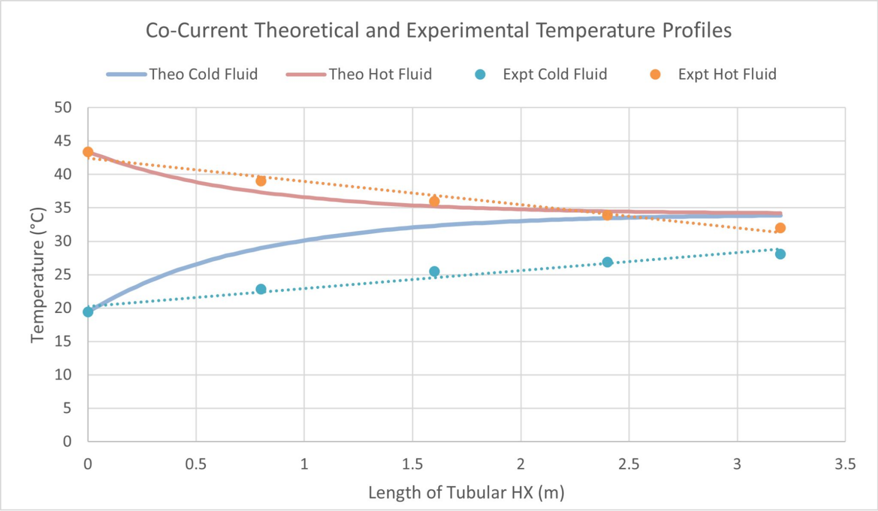




HX	**U (W/m^2 K)**	**ε (%)**
Co-Current	4800 ± 800	45 ± 2
Counter Current	6100 ± 1200	48 ± 2

Experimental Results Align with Theoretical Model





Theoretical data overestimates heat exchange by ignoring loss due to insulation and exposed surfaces

Conclusions

1. **Tubular** Heat exchanger transfers the most **heat per area**.

2. **Plate** Heat Exchanger is the most **effective**.

3. Experimental scaling of $\mathbf{h \propto v^{0.79 \pm 0.02}}$ aligns with theoretical scaling of $\mathbf{h \propto v^{0.8}}$.

4. **Counter-current** configuration is preferred for both heat transfer and effectiveness for the **tubular heat exchanger**.

5. Theoretical temperature profile **overestimates** heat transfer due to assumption of **adiabatic conditions**.

Appendix A: Error Analysis

$$\Delta U = U\sqrt{\left(\frac{\Delta Q_M}{Q_M}\right)^2 + \left(\frac{\Delta(\Delta T_{LM})}{(\Delta T_{LM})}\right)^2}$$

Derived from:

$$\delta y = \sqrt{\sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 (\delta x_i)^2}$$

$$\Delta Q_M = \frac{1}{2}\sqrt{(\Delta Q_C)^2 + (\Delta Q_H)^2}$$

$$\Delta Q_H = Q_H\sqrt{\left(\frac{\Delta m_h}{m_h}\right)^2 + \left(\frac{\Delta(\Delta T_h)}{(\Delta T_h)}\right)^2}$$

$$\Delta(\Delta T_h) = \sqrt{(\Delta T_{h,i})^2 + (\Delta T_{h,o})^2}$$

Appendix B: Equations

$$\dot{Q}_M = \frac{|\dot{Q}_C| + |\dot{Q}_H|}{2}$$

$$\dot{Q}_H = \dot{m}_h \cdot c_{p,h} \cdot (\Delta T)_h$$

$$\dot{Q}_C = \dot{m}_c \cdot c_{p,c} \cdot (\Delta T)_c$$

$$\Delta T_{\mathrm{LM}} = \frac{(T_{1i} - T_{2i}) \; - \; (T_{1o} - T_{2o})}{\ln\left(\dfrac{T_{1i} - T_{2i}}{T_{1o} - T_{2o}}\right)}$$

$$A_M = \frac{A_{outer} - A_{inner}}{ln(\frac{A_{outer}}{A_{inner}})} = \frac{\pi L(D_{t,o} - D_{t,i})}{ln(\frac{D_{t,o}}{D_{t.i}})}$$

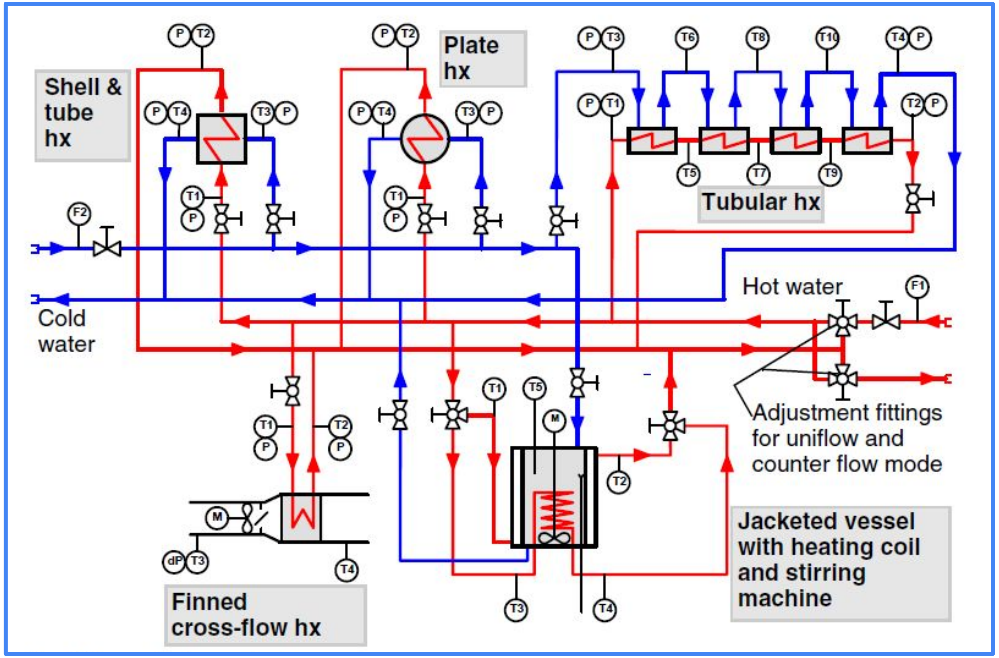
$$C_{\min} = \min(C_h, \, C_c)$$

$$C_c = \dot{m}_c \, c_{p,c}$$

$$C_h = \dot{m}_h \, c_{p,h}$$

Geankoplis, C. J.; Hersel, A. A.; Lepek, D. H. Transport Processes and Separation Process Principles. In *Transport Processes and Separation Process Principles*, 5th ed.; Prentice Hall, 2018; Chapter 16.

Appendix C: Apparatus - Process Flow Diagram



Measur-ing point	Tubular hx	Finned cross-flow hx	Plate hx	Shell & tube hx	Jacketed vessel heating coil
T1	HW in	HW in	HW in	HW in	HW in jacket
T2	HW out	HW out	HW out	HW out	HW out jacket
T3	CW in	Air in	CW in	CW in	HW in coil
T4	CW out	Air out	CW out	CW out	HW out coil
T5, T6	HW 800mm CW 800mm				
T7, T8	HW 1600mm CW 1600mm				
T9, T10	HW 2400mm CW 2400mm				

Key

off / on:	Outlet / inlet	F:	Flow meter
CW / HW:	Cold water / hot water	P:	Pressure measuring connection
hx:	Heat exchanger		
T:	Temperature		
dP:	Differential pressure sensor		

HOT WATER Path

- T1(0mm -> T5(800mm) -> T7(1600mm) -> T9(2400mm -> T2(3200mm)

COLD WATER Path

- T3(0mm) -> T6(800mm) -> T8(1600mm) -> T10(2400mm) -> T4(3200mm)

1) Behesti-Pour, A. Katz. *Heat Exchange (HX) Revised 2025.docx*; University of California, Berkeley: Berkeley, California, 2025.

Appendix D: Heat Exchanger Instrument Data

Tubular

Material	Cu
Inner tube diameter	6 mm
Annulus diameter	13 mm
Total length	3200 mm
Segment length	800 mm
Area, total	0.0698 m^2

Shell and Tube

Heat exchanger Surf. Area	0.15 m^2
Thermal output power	14071 W

Jacket side

Medium	Water
Throughput	0.35 m^3/h
Inlet temperature	80.00 °C
Outlet temperature	46.66 °C
Flow speed	0.29 m/s
Pressure loss	0.03 bar

Tube side

Medium	Water
Throughput	0.77 m^3/h
Inlet temperature	15.00 °C
Outlet temperature	29.93 °C
Flow speed	0.42 m/s
Pressure loss	0.00 bar

1) Behesti-Pour, A. Katz. *Heat Exchange (HX) Revised 2025.docx*; University of California, Berkeley: Berkeley, California, 2025.

Appendix D: Heat Exchanger Instrument Data (cont.d)

Plate

Type	GC-008 PI
Width	180 mm
Height	774 mm
Plate material	1.4401
Number of plates	10
Volume, side 1	0.5 L
Volume, side 2	0.7 L
Max. operating temperature	100 °C
Max. operating pressure	10 bar

Finned Cross-Flow

Power output	0.25 kW
Max. differential pressure	430 Pa
Max. volumetric flow	13 m^3/min
Differential pressure sensor	
Measuring range	0 - 1000 mbar

1) Behesti-Pour, A. Katz. *Heat Exchange (HX) Revised 2025.docx*; University of California, Berkeley: Berkeley, California, 2025.

Appendix E: Material Properties

Variables	Value	Description
ρ	997 kg/m^3	Density of water
μ	8.9 x 10^{-4} Pa*s	Dynamic viscosity of water
c_p	4180 J/kg/K	Heat capacity of water
Pr	6.20	Prandtl number of water
k_{water}	0.6 W/m/K	Thermal conductivity of water
k_{copper}	401 W/m/K	Thermal conductivity of copper

Appendix F: Theoretical Temperature Profile Derivation

Hot Fluid:

$$\frac{dU_{\text{sys}}}{dt} = \dot{Q} - \dot{W}_s + \sum \dot{m}_{j0} H_j \Big|_V - \sum \dot{m}_j H_j \Big|_{V+dV}$$

$$h_h a_h (T_h - T_c) = \dot{m}_h H_h \Big|_V - \dot{m}_h H_h \Big|_{V+dV}$$

$$\frac{dT_h}{dV_h} = \frac{h_h a_h (T_h - T_c)}{\dot{m}_h c_p}$$

$$\frac{dT_h}{dV_h} = \pm \frac{h_h a_h}{\dot{m}_h c_p}(T_c - T_h)$$

$$\frac{dT_h}{dz} = -\frac{h_h a_h (T_c - T_h) A_h}{\dot{m}_h c_p}$$ (Co-current)

$$\frac{dT_h}{dz} = \frac{h_h a_h (T_c - T_h) A_h}{\dot{m}_h c_p}$$ (Counter-current)

$$a_h = \frac{2\pi r_1 L}{\pi r_1^2 L} = \frac{2}{r_1} \qquad a_c = \frac{2\pi r_2 L}{\pi (r_3^2 - r_2^2) L} = \frac{2 r_2}{(r_3^2 - r_2^2)}$$

Cold Fluid:

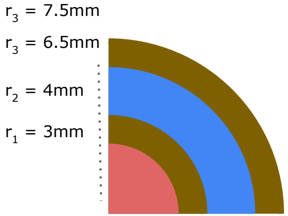
$$\frac{dU_{\text{sys}}}{dt} = \dot{Q} - \dot{W}_s + \sum \dot{m}_{j0} H_j \Big|_V - \sum \dot{m}_j H_j \Big|_{V+dV}$$

$$h_c a_c (T_h - T_c) = \dot{m}_c H_c \Big|_V - \dot{m}_c H_c \Big|_{V+dV}$$

$$\frac{dT_c}{dV_c} = \frac{h_c a_c (T_h - T_c)}{\dot{m}_c c_p}$$

$$\frac{dT_c}{dV_c} = -\frac{h_c a_c}{\dot{m}_c c_p}(T_c - T_h)$$

$$\frac{dT_c}{dz} = -\frac{h_c a_c (T_c - T_h) A_c}{\dot{m}_c c_p}$$ (Co/Counter Current)



Appendix G: Derivation of U_{exp} and U_{theo}

Theoretical Correlation

Dittus-Boelter Equation: $\mathrm{Nu} = 0.023\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{0.4}.$

Assumptions:
- Turbulent flow regime (Re>10000)
- Constant fluid properties at given temperature range (c_p, μ, k_{water}, ρ)

Characteristic Lengths and Flow Rates:
- For inner tube, h_i we use $D_{t,i}$ = 0.006m and V = 200, 300, 400L/hr
- For annular region, h_o we use $D_{annular} - D_{t,o}$ = 0.005m and V = 350L/hr

Theoretical Scaling: $h_i \propto v^{0.8}, \quad h_o \propto v^{0.8}.$

Overall Heat Transfer Coefficient Formulas

$$U_{\mathrm{exp}} = \frac{Q_M}{A_M\,\Delta T_{\mathrm{LM}}}$$

- $$Q_M = \frac{\dot{m}\,c_p\left((T_2 - T_1) + (T_4 - T_3)\right)}{2}$$

(2)
- $$\Delta T_{\mathrm{LM}} = \frac{(T_1 - T_4) - (T_2 - T_3)}{\ln\left(\frac{T_1 - T_4}{T_2 - T_3}\right)}$$

- $$A_M = \frac{\pi L\,(D_{t,o} - D_{t,i})}{\ln\left(\frac{D_{t,o}}{D_{t,i}}\right)}$$

$$U_{\mathrm{theo}} = \frac{1}{\frac{1}{h_i} + \frac{s}{k_{\mathrm{copper}}} + \frac{1}{h_o}}$$

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