Lecture 3
Functional Programming

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Announcements

1. Etutor assignment due on Sunday midnight
2. Reading SICP 1.2 (pages 31-50)
3. Labs (PSes) started

Lecture 2
Functional Programming & Scheme

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Main programming paradigms

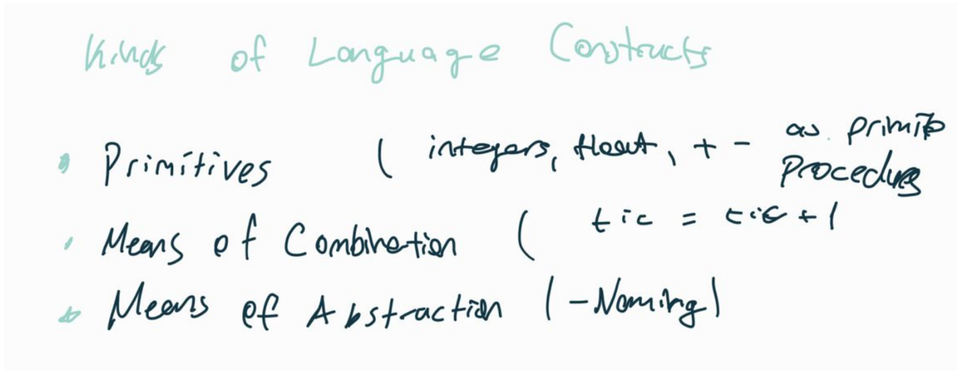
Paradigm	Description	Main traits	Related paradigm(s)	Examples
Imperative	Programs as statements that *directly* change computed state (datafields)	Direct assignments, common data structures, global variables		C, C++, Java, Kotlin, PHP, Python, Ruby
Procedural	Derived from structured programming, based on the concept of modular programming or the *procedure call*	Local variables, sequence, selection, iteration, and modularization	Structured, imperative	C, C++, Lisp, PHP, Python
Functional	Treats computation as the evaluation of mathematical functions avoiding state and mutable data	Lambda calculus, compositionality, formula, recursion, referential transparency, no side effects	Declarative	C++,[1] C#,[2][*circular reference*] Clojure, CoffeeScript,[3] Elixir, Erlang, F#, Haskell, Java (since version 8), Kotlin, Lisp, Python, R,[4] Ruby, Scala, SequenceL, Standard ML, JavaScript, Elm
Object-oriented	Treats datafields as *objects* manipulated through predefined methods only	Objects, methods, message passing, information hiding, data abstraction, encapsulation, polymorphism, inheritance, serialization-marshalling	Procedural	Common Lisp, C++, C#, Eiffel, Java, Kotlin, PHP, Python, Ruby, Scala, JavaScript[8][9]
Declarative	Defines program logic, but not detailed control flow	Fourth-generation languages, spreadsheets, report program generators		SQL, regular expressions, Prolog, OWL, SPARQL, Datalog, XSLT

Source: Wikipedia

Write a function for factorial

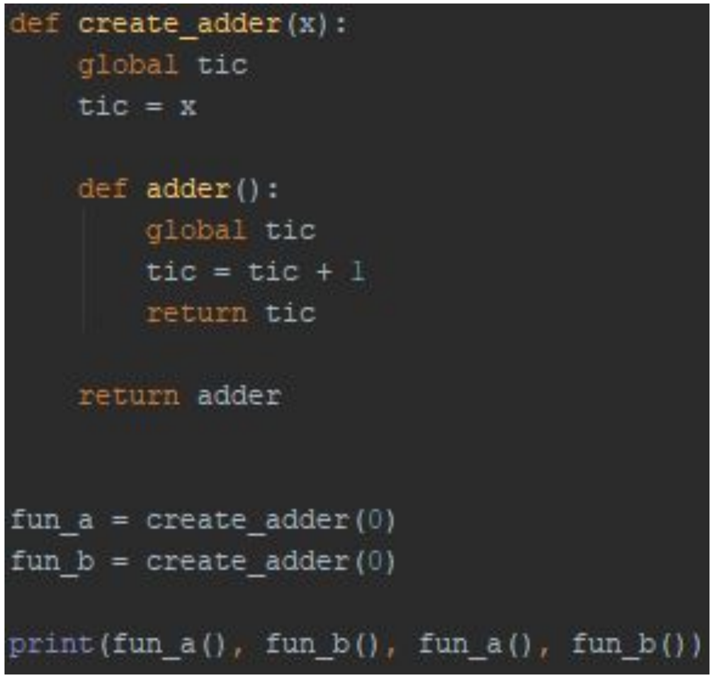
Kinds of Language Constructs

- Primitives
- Means of combination
- Means of abstraction



Kinds of Language Constructs

- Primitives (integers, float, + - as primitive procedures
- Means of Combination (tic = tic + 1
- Means of Abstraction (- Naming)



```python
def create_adder(x):
    global tic
    tic = x

    def adder():
        global tic
        tic = tic + 1
        return tic

    return adder


fun_a = create_adder(0)
fun_b = create_adder(0)

print(fun_a(), fun_b(), fun_a(), fun_b())
```

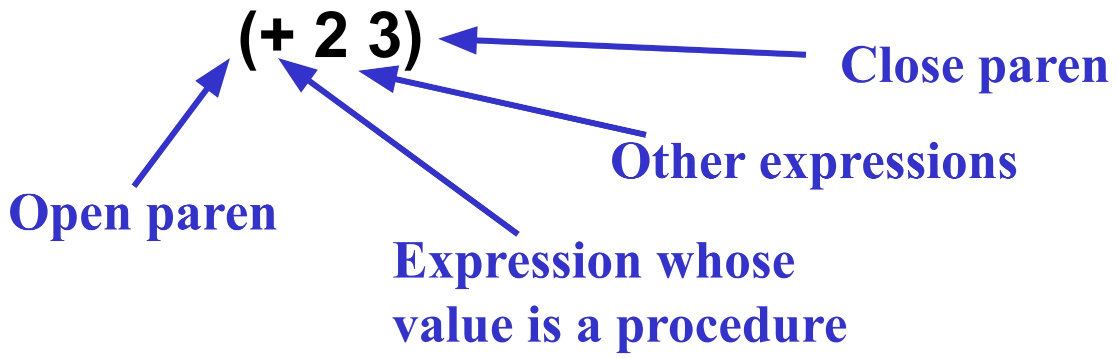
Omer Veysel Cagatan

Language elements – primitives

- Names for built-in procedures
  - +, *, -, /, =, …
  - What is the value of such an expression?
  - + □ [#procedure …]
  - Evaluate by looking up value associated with name in a special table

Language elements – combinations

- How do we create expressions using these procedures?



```
(+ 2 3)
```

- Evaluate by getting values of sub-expressions, then applying operator to values of arguments

Language elements -- abstractions

- In order to abstract an expression, need way to give it a name

(define score 23)

- This is a special form
  - Does not evaluate second expression
  - Rather, it pairs name with value of the third expression
- Return value is unspecified

Nugget

Functions are first class citizens

Scheme Basics

- Rules for evaluation

1. If **self-evaluating,** return value.
2. If a **name,** return value associated with name in environment.
3. If a **special form,** do something special.
4. If a **combination,** then

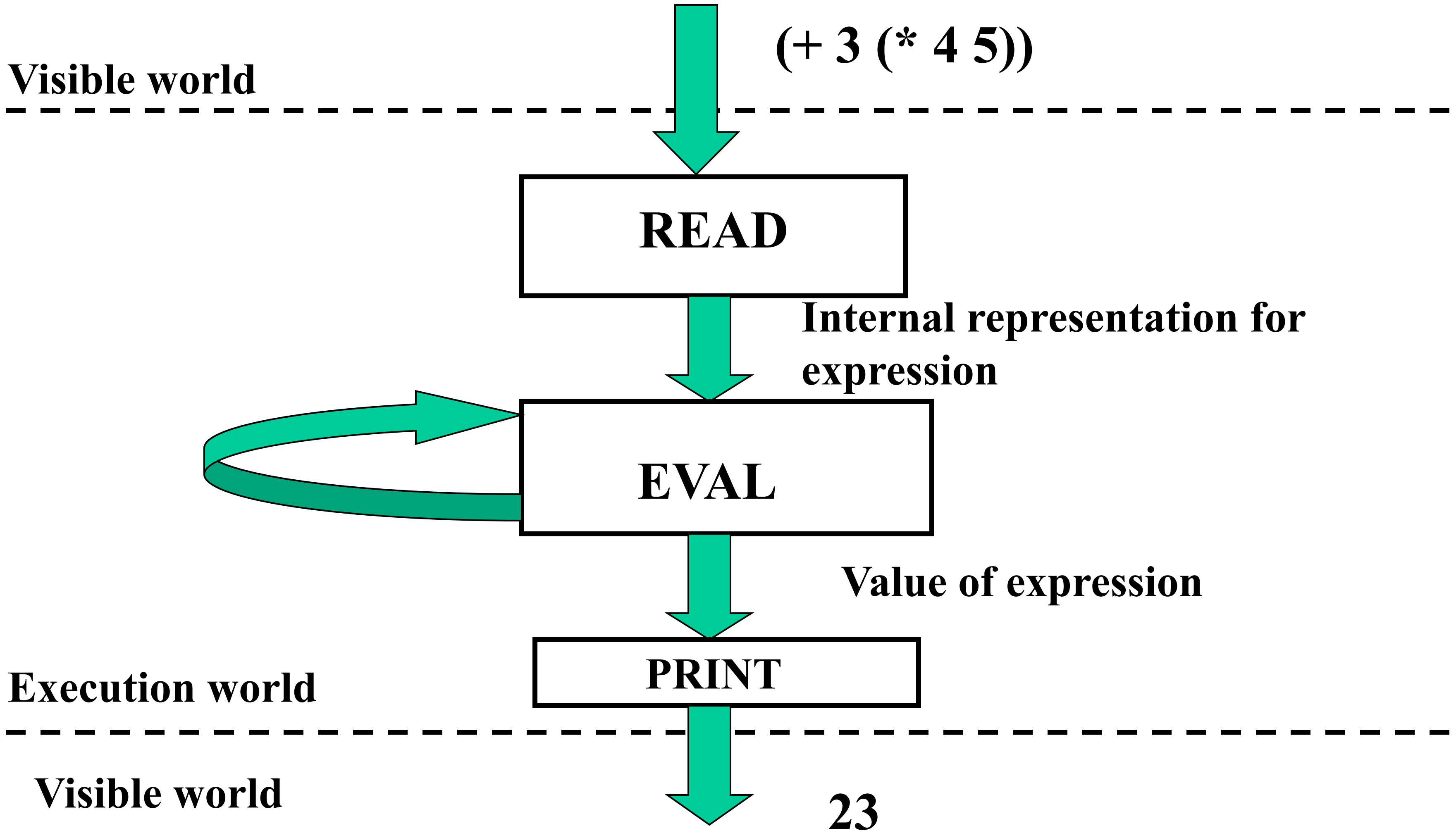
   a. *Evaluate* all of the subexpressions of combination (in any order)

   b. *apply* the operator to the values of the operands (arguments) and return result

- Rules for application

1. If procedure is **primitive procedure,** just do it.
2. If procedure is a **compound procedure,** then:
   evaluate the body of the procedure with each formal parameter replaced by the corresponding actual argument value.

Read-Eval-Print



Lecture 3
Functional Programming

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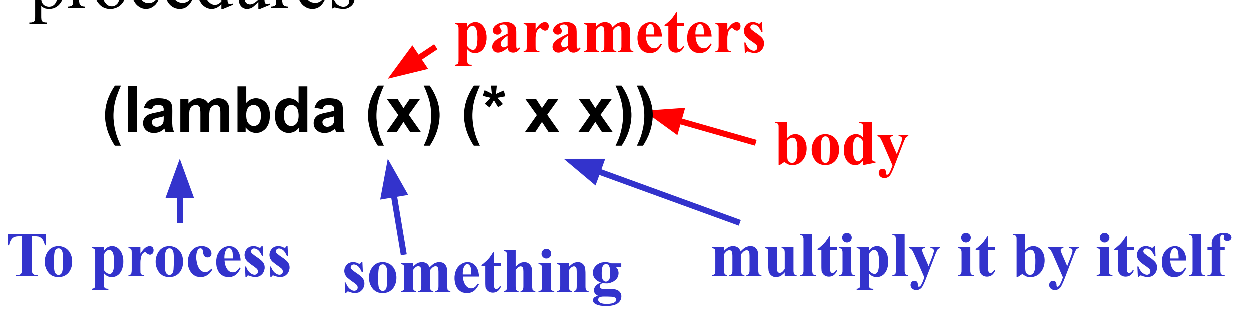
Lecture Nuggets

- Lambda expressions create procedures
  - Formal parameters
  - Body
  - Procedures allow creating abstractions
- We can solve problems by creating functions
- The substitution model is a good mental model of an interpreter

Hold your breath

Language elements -- abstractions

- Need to capture ways of doing things – use procedures



```
(lambda (x) (* x x))
```

•Special form – creates a procedure and returns it as value

Nugget

Lambda expressions creates procedures

Language elements -- abstractions

- Use this anywhere you would use a procedure

```
((lambda (x) (* x x)) 5)

 (* 5 5)

  25
```

- Can give it a name

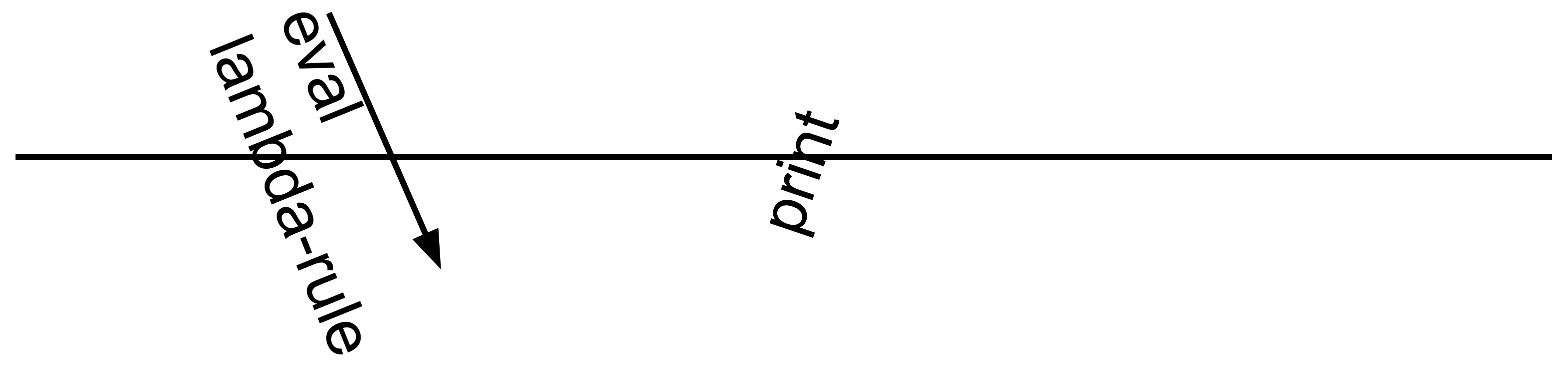
```
(define square (lambda (x) (* x x)))
(square 5)  25
```

Lambda: making new procedures

expression

printed representation of value

```
(lambda (x) (* x x))        #[compound-procedure 9]
```



A compound
proc
that squares its
argument

value

Interaction of define and lambda

```
1.   (lambda (x) (* x x))
           ==> #[compound-procedure 9]
2.   (define square (lambda (x) (* x x)))
           ==> undef
3.   (square 4)                ==> 16
4.   ((lambda (x) (* x x)) 4) ==> 16
5.   (define (square x) (* x x)) ==> undef
```

This is a convenient shorthand (called "syntactic sugar") for 2 above – this is a use of lambda!

Lambda special form

- lambda syntax `(lambda (x y) (/ (+ x y) 2))`

- 1st operand position: the parameter list `(x y)`
  - a list of names (perhaps empty)
  - determines the number of operands required

- 2nd operand position: the body `(/ (+ x y) 2)`
  - may be any expression
  - not evaluated when the lambda is evaluated
  - evaluated when the procedure is applied

- semantics of lambda:

THE VALUE OF

A LAMBDA EXPRESSION

IS

A PROCEDURE

Nugget

We can solve problems by creating functions

Procedures allow abstraction

- Breaking computation into modules that capture commonality
  - Enables reuse in other places (e.g. square)
- Isolates details of computation within a procedure from use of the procedure
- May be many ways to divide up

```
(define square (lambda (x) (* x x)))
(define sum-squares
    (lambda (x y) (+ (square x) (square y))))
(define pythagoras
    (lambda (y x) (sqrt (sum-squares y x))))
```

Abstracting the process

- Stages in capturing common patterns of computation
  - Identify modules or stages of process
  - Capture each module within a procedural abstraction
  - Construct a procedure to control the interactions between the modules
  - Repeat the process within each module as necessary

A more complex example

- Remember our method for finding sqrts
  - To find the square root of X
    - Make a guess, called G
    - If G is close enough, stop
    - Else make a new guess by averaging G and X/G

Imperative Knowledge

- "How to" knowledge

To find an approximation of square root of x:

- Make a guess G
- Improve the guess by averaging G and x/G
- Keep improving the guess until it is good enough

Example: $\sqrt{x}$ for $x = 2$.

X = 2	G = 1
X/G = 2	G = ½ (1+ 2) = 1.5
X/G = 4/3	G = ½ (3/2 + 4/3) = 17/12 = 1.416666
X/G = 24/17	G = ½ (17/12 + 24/17) = 577/408 = 1.4142156

The stages of “SQRT”

- When is something “close enough”
- How do we create a new guess
- How to we control the process of using the new guess in place of the old one

Procedural abstractions

For "close enough":

```
(define close-enuf?
  (lambda (guess x)
     (< (abs (- (square guess) x)) 0.001)))
```

$\Uparrow$

Note use of procedural abstraction!

Procedural abstractions

For “improve”:

```
(define average
    (lambda (a b) (/ (+ a b) 2)))
(define improve
    (lambda (guess x)
        (average guess (/ x guess))))
```

Why this modularity?

- "Average" is something we are likely to want in other computations, so only need to create once
- Abstraction lets us separate implementation details from use
  - E.g. could redefine as

```
(define average
  (lambda (x y) (* (+ x y) 0.5)))
```

  - No other changes needed to procedures that use **`average`**
  - Also note that variables (or parameters) are internal to procedure – cannot be referred to by name outside of scope of lambda

Controlling the process

- Basic idea:
  - Given X, G, want `(improve G X)` as new guess
  - Need to make a decision – for this need a new *special form*

```
(if <predicate> <consequence> <alternative>)
```

The `IF` special form

```
(if <predicate> <consequence> <alternative>)
```

- Evaluator first evaluates the `<predicate>` expression.
- If it evaluates to a TRUE value, then the evaluator evaluates and returns the value of the `<consequence>` expression.
- Otherwise, it evaluates and returns the value of the `<alternative>` expression.
- Why must this be a special form?

Controlling the process

- Basic idea:
  - Given X, G, want `(improve G X)` as new guess
  - Need to make a decision – for this need a new *special form*

```
(if <predicate> <consequence> <alternative>)
```

  - So heart of process should be:

```
    (if (close-enuf? G X)
          G
                          (improve G X)          )
```

  - But somehow we want to use the value returned by "improving" things as the new guess, and repeat the process

Controlling the process

- Basic idea:
  - Given X, G, want `(improve G X)` as new guess
  - Need to make a decision – for this need a new *special form*

```
(if <predicate> <consequence> <alternative>)
```

  - So heart of process should be:

```
(define sqrt-loop (lambda G X)
    (if (close-enuf? G X)
        G
        (sqrt-loop (improve G X) X        )
```

  - But somehow we want to use the value returned by “improving” things as the new guess, and repeat the process
  - Call process `sqrt-loop` and reuse it!

Putting it together

- Then we can create our procedure, by simply starting with some initial guess:

```
(define sqrt
    (lambda (x)
        (sqrt-loop 1.0 x)))
```

Checking that it does the "right thing"

- Next lecture, we will see a formal way of tracing evolution of evaluation process
- For now, just walk through basic steps
  - `(sqrt 2)`
    - `(sqrt-loop 1.0 2)`
    - `(if (close-enuf? 1.0 2) … …)`
    - `(sqrt-loop (improve 1.0 2) 2)`

    This is just like a normal combination
    - `(sqrt-loop 1.5 2)`
    - `(if (close-enuf? 1.5 2) … …)`
    - `(sqrt-loop 1.4166666 2)`
- And so on…

Nugget

The substitution model is a good mental model of an interpreter

Remainder of this lecture

- Substitution model
- An example using the substitution model
- Designing recursive procedures
- Designing iterative procedures

Substitution model

- a way to figure out what happens during evaluation
  - not really what happens in the computer

- to apply a compound procedure:
  - evaluate the body of the procedure, with each parameter replaced by the corresponding operand

- to apply a primitive procedure: just do it

```
(define square (lambda (x) (* x x)))

1.       (square 4)
2.       (* 4 4)
3.       16
```

Substitution model details

```
(define square (lambda (x) (* x x)))
(define average (lambda (x y) (/ (+ x y) 2)))
```

```
(average 5 (square 3))
(average 5 (* 3 3))
(average 5 9)
```

first evaluate operands, then substitute (applicative order)

```
(/ (+ 5 9) 2)
(/ 14 2)
7
```

if operator is a primitive procedure, replace by result of operation

End of part 1

- how to use substitution model to trace evaluation

A less trivial procedure: factorial

- Compute n factorial, defined as n! = n(n-1)(n-2)(n-3)...1

•Notice that n! = n * [(n-1)(n-2)...] = n * (n-1)!   if n > 1

```
(define fact
      (lambda (n)
          (if (= n 1)
            1
            (* n (fact (- n 1))))))
```

•predicate =tests numerical equality

```
(= 4 4) ==> #t
(= 4 5) ==> #f
```
(true)
(false)

•if special form

```
(if (= 4 4) 2 3) ==> 2
(if (= 4 5) 2 3) ==> 3
```

predicate   consequent   alternative

```
 (define fact(lambda (n)
  (if (= n 1)1(* n (fact (- n 1))))))

(fact 3)
(if (= 3 1) 1 (* 3 (fact (- 3 1))))
(if #f 1 (* 3 (fact (- 3 1))))
(* 3 (fact (- 3 1)))
(* 3 (fact 2))
(* 3 (if (= 2 1) 1 (* 2 (fact (- 2 1)))))
(* 3 (if #f 1 (* 2 (fact (- 2 1)))))
(* 3 (* 2 (fact (- 2 1))))
(* 3 (* 2 (fact 1)))
(* 3 (* 2 (if (= 1 1) 1 (* 1 (fact (- 1 1))))))
(* 3 (* 2 (if #t 1 (* 1 (fact (- 1 1))))))
(* 3 (* 2 1))
(* 3 2)
6
```

The fact procedure is a recursive algorithm

- A recursive algorithm:
  - In the substitution model, the expression keeps growing

    ```
    (fact 3)
    (* 3 (fact 2))
    (* 3 (* 2 (fact 1)))
    ```

  - Other ways to identify will be described next time

End of part 2

- how to use substitution model to trace evaluation
- how to recognize a recursive procedure in the trace

How to design recursive algorithms

- follow the general pattern:
    1. wishful thinking
    2. decompose the problem
    3. identify non-decomposable (smallest) problems

1. Wishful thinking

- Assume the desired procedure exists.

- want to implement fact? OK, assume it exists.
- BUT, only solves a smaller version of the problem.

2. Decompose the problem

- Solve a problem by
  1. solve a smaller instance   (using wishful thinking)
  2. convert that solution to the desired solution

- Step 2 requires creativity!
  - Must design the strategy before coding.
  - n! = n(n-1)(n-2)... = n[(n-1)(n-2)...] = n * (n-1)!
  - solve the smaller instance, multiply it by n to get solution

```
(define fact
    (lambda (n) (* n (fact (- n 1)))))
```

3. Identify non-decomposable problems

- Decomposing not enough by itself
- Must identify the "smallest" problems and solve directly

- Define 1! = 1

```
(define fact
    (lambda (n)
     (if (= n 1) 1
        (* n (fact (- n 1))))))
```

General form of recursive algorithms

- test, base case, recursive case

```
(define fact
  (lambda (n)
    (if (= n 1)          ; test for base case
      1                  ; base case
      (* n (fact (- n 1)))   ; recursive case
)))
```

- base case: smallest (non-decomposable) problem
- recursive case: larger (decomposable) problem

End of part 3

- Design a recursive algorithm by
  1. wishful thinking
  2. decompose the problem
  3. identify non-decomposable (smallest) problems

- Recursive algorithms have
  1. test
  2. recursive case
  3. base case

Iterative algorithms

- In a recursive algorithm, bigger operands => more space

```
                (define fact (lambda (n)
                    (if (= n 1) 1
                       (* n (fact (- n 1))))))
     (fact 4)
     (* 4 (fact 3))
     (* 4 (* 3 (fact 2)))
     (* 4 (* 3 (* 2 (fact 1))))
     (* 4 (* 3 (* 2 1)))
     ...
     24
```

•An iterative algorithm uses constant space

Intuition for iterative factorial

- same as you would do if calculating 4! by hand:

1. multiply 4 by 3	gives 12
2. multiply 12 by 2	gives 24
3. multiply 24 by 1	gives 24

- At each step, only need to remember:

  previous product, next multiplier

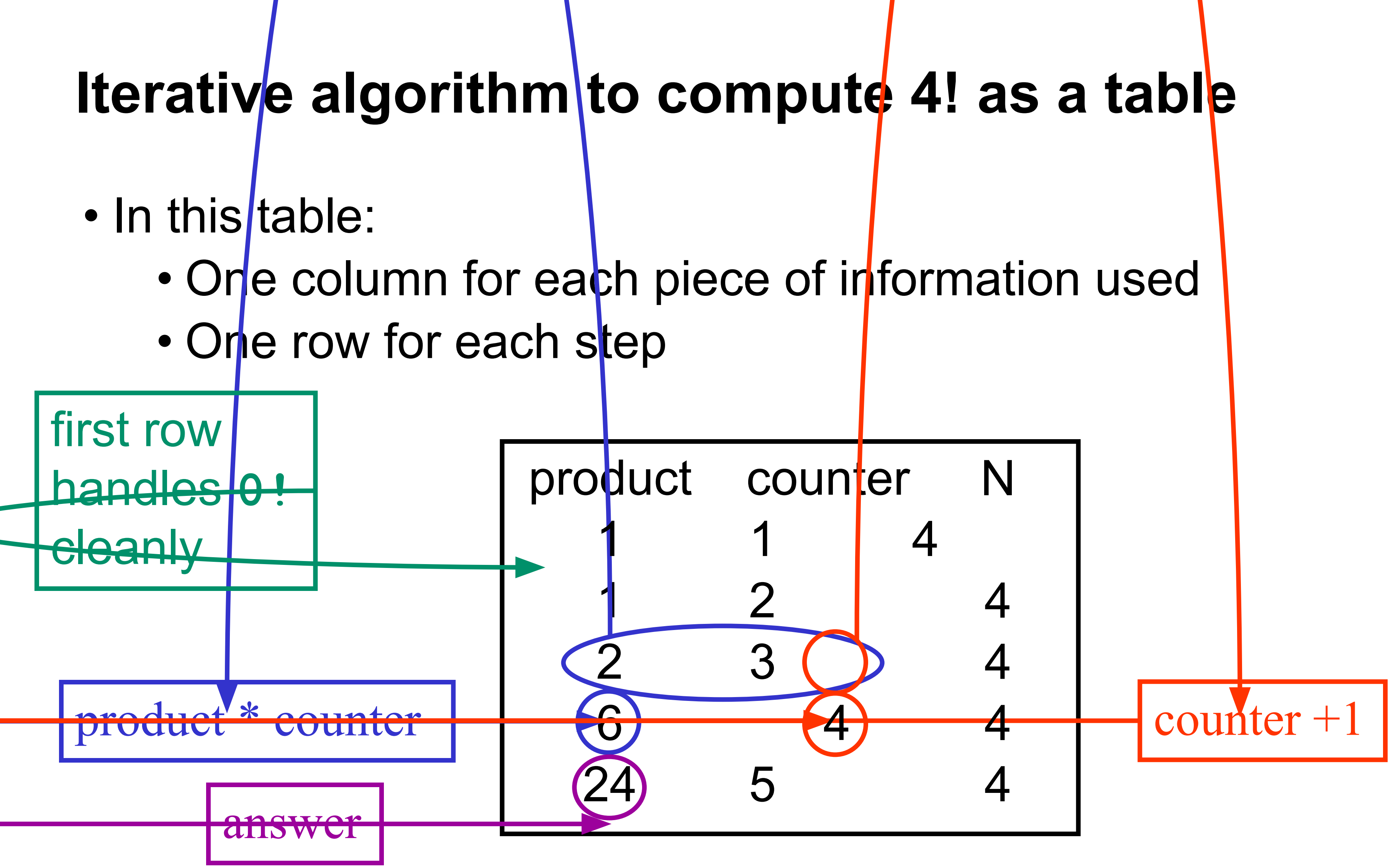
- Therefore, constant space

- Because multiplication is associative and commutative:

1. multiply 1 by 2	gives 2
2. multiply 2 by 3	gives 6
3. multiply 6 by 4	gives 24

Iterative algorithm to compute 4! as a table

- In this table:
  - One column for each piece of information used
  - One row for each step



product	counter	N
1	1	4
1	2	4
2	3	4
6	4	4
24	5	4

- The last row is the one where counter > n
- The answer is in the product column of the last row

Iterative factorial in scheme

- (define ifact (lambda (n) (ifact-helper 1 1 n)))

initial row of table

```
(define ifact-helper (lambda (product counter n)
   (if    (> counter n)
      product
      (ifact-helper (* product counter) (+ counter 1) n))))
```

compute next row of table

answer is in product column of last row

at last row when counter > n

Partial trace for `(ifact 4)`

```
(define ifact-helper (lambda (product count n)
    (if (> count n) product
        (ifact-helper (* product count)
                      (+ count 1) n))))
```

```
(ifact 4)
(ifact-helper 1 1 4)
(if (> 1 4) 1 (ifact-helper (* 1 1) (+ 1 1) 4))
(ifact-helper 1 2 4)
(if (> 2 4) 1 (ifact-helper (* 1 2) (+ 2 1) 4))
(ifact-helper 2 3 4)
(if (> 3 4) 2 (ifact-helper (* 2 3) (+ 3 1) 4))
(ifact-helper 6 4 4)
(if (> 4 4) 6 (ifact-helper (* 6 4) (+ 4 1) 4))
(ifact-helper 24 5 4)
(if (> 5 4) 24 (ifact-helper (* 24 5) (+ 5 1) 4))
24
```

Iterative = no pending operations when procedure calls itself

- Recursive factorial:

```
(define fact (lambda (n)
      (if (= n 1) 1
        (* n (fact (- n 1)) )
  )))
```

pending operation

- `(fact 4)`

```
(* 4 (fact 3))
(* 4 (* 3 (fact 2)))
(* 4 (* 3 (* 2 (fact 1))))
```

- Pending ops make the expression grow continuosly

Iterative = no pending operations

- Iterative factorial:

```
(define ifact-helper (lambda (product count n)
   (if (> count n) product
       (ifact-helper (* product count)
                     (+ count 1) n))))
```

-
  ```
  (ifact-helper 1 1 4)
  (ifact-helper 1 2 4)
  (ifact-helper 2 3 4)
  (ifact-helper 6 4 4)
  (ifact-helper 24 5 4)
  ```

no pending operations

- Fixed size because no pending operations

End of part 4

- Iterative algorithms have constant space
- How to develop an iterative algorithm
    - figure out a way to accumulate partial answers
    - write out a table to analyze precisely:
        - initialization of first row
        - update rules for other rows
        - how to know when to stop
    - translate rules into scheme code

- Iterative algorithms have no pending operations when the procedure calls itself

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