

Lecture 13 – Review Proc



T. METIN SEZGIN

Review



PROC

⊗ allows to create new procedures

Ex Val = Int + Bool + Proc

Den Val = Int + Bool + Proc

↳ set of values

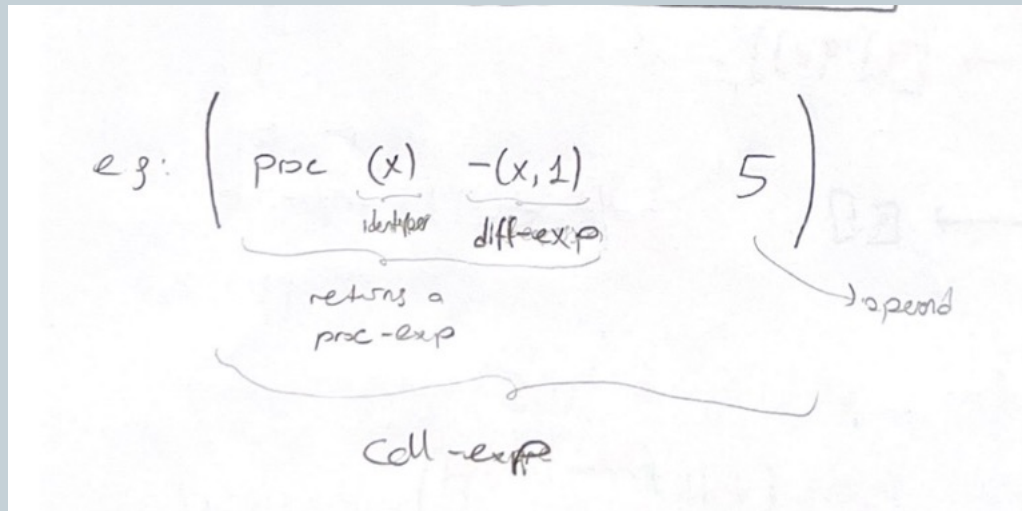
representing procedures

↳ abstract data type

(consider as)

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Review



Numan Batur

Review



ⓧ Procedure creating and calling

Expression ::= proc (Identifier) Expression

proc-exp (var body) $\xrightarrow{\text{bound variable or formal parameter}}$

Expression ::= (Expression Expression)

call-exp (rator rand) $\xrightarrow{\text{value of operand is argument}}$
 $\xrightarrow{\text{operand / actual param.}}$
 $\xrightarrow{\text{operator}}$

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⊗ Why constructor procedure must take 3 args :

Constructor → procedure

↳ (value-of (proc-exp var body) p)

= (proc-val (procedure var body p))

! constructor → like bool-val or num-val

let x = 200

in let f = proc (z) - (z, x)

in let x = 100

in let g = proc (z) - (z, x)

in - ((f 1), (g 1))

let f = proc(x) - (x, 1)
in (f 5)

saved-env

env/p

done twice

once this is evaluated in environment it is saved env.

① x bound to 200
subtracts 200 from its arg. } procedure f

② x bound to 100
subtracts 100 from its arg. } procedure g

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- ⊗ if a value of the operator is `proc-val`,
then apply it to the value of the operand.

$$\begin{aligned} & (\text{value-of } (\text{call-exp } \text{rator } \text{rand}) \text{ } p) \\ = & (\text{let } ((\text{proc } (\text{expval} \rightarrow \text{proc } (\text{value-of } \text{rator } p))) \\ & \quad (\text{arg } (\text{value-of } \text{rand } p)))) \\ & (\text{apply-procedure } \text{proc } \text{arg})) \end{aligned}$$

- ⊗ What happens when ^{observer → apply-procedure} `apply-procedure` is invoked?

- 1 a procedure applied
- 2 body evaluated
- 3 in environment that binds the formal parameter of the procedure to the argument

$$\begin{aligned} & (\text{apply-procedure } (\text{procedure } \text{var } \text{body } p) \text{ } \text{val}) \\ = & (\text{value-of } \text{body } [\text{var} = \text{val}] \text{ } p) \end{aligned}$$

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Lecture 14

PROC



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LET is ex; long live PROC



- LET had its limitations
 - No procedures
- Define a language with procedures
 - Specification
 - ✦ Syntax
 - ✦ Semantics
 - Representation
 - Implementation

Expressed and Denoted values



- Before

ExpVal = Int + Bool
DenVal = Int + Bool

- After

ExpVal = Int + Bool + Proc
DenVal = Int + Bool + Proc

Examples



Expression ::= **proc** (*Identifier*) *Expression*

proc-exp (**var** *body*)

Expression ::= (*Expression* *Expression*)

call-exp (**rator** *rand*)

- Concepts

- In definition

- ✦ **var**

- Bound variable (a.k.a. formal parameter)

- In procedure call

- ✦ **Rand**

- Actual parameter (the value → argument)

- ✦ **Rator**

- Operator

Syntax for constructing and calling procedures



Expression ::= `proc (Identifier) Expression`
`proc-exp (var body)`

Expression ::= `(Expression Expression)`
`call-exp (rator rand)`

```
let f = proc (x) - (x, 11)
in (f (f 77))
```

```
(proc (f) (f (f 77))
  proc (x) - (x, 11))
```

Syntax for constructing and calling procedures



Expression ::= `proc (Identifier) Expression`
`proc-exp (var body)`

Expression ::= `(Expression Expression)`
`call-exp (rator rand)`

```
let x = 200
in let f = proc (z) - (z,x)
    in let x = 100
        in let g = proc (z) - (z,x)
            in -((f 1), (g 1))
```

The interface for PROC



- Procedures have

- Constructor \rightarrow **procedure**

```
(value-of (proc-exp var body)  $\rho$ )  
= (proc-val (procedure var body  $\rho$ ))
```

- Observer \rightarrow **apply-procedure**

```
(value-of (call-exp rator rand)  $\rho$ )  
= (let ((proc (expval->proc (value-of rator  $\rho$ )))  
      (arg (value-of rand  $\rho$ )))  
  (apply-procedure proc arg))
```

The intuition behind application



- Extend the environment
- Evaluate the body

```
(apply-procedure (procedure var body  $\rho$ ) val)  
= (value-of body [var=val]  $\rho$ )
```

```

(value-of
  <<let x = 200
    in let f = proc (z) -(z,x)
      in let x = 100
        in let g = proc (z) -(z,x)
          in -((f 1), (g 1))>>
  ρ)

```

```

= (value-of
  <<let f = proc (z) -(z,x)
    in let x = 100
      in let g = proc (z) -(z,x)
        in -((f 1), (g 1))>>
  [x=[200]]ρ)

```

```

= (value-of
  <<let x = 100
    in let g = proc (z) -(z,x)
      in -((f 1), (g 1))>>
  [f=(proc-val (procedure z <<-(z,x)>> [x=[200]]ρ))]
  [x=[200]]ρ)

```

```

= (value-of
  <<let g = proc (z) -(z,x)
    in -((f 1), (g 1))>>
  [x=[100]]
  [f=(proc-val (procedure z <<-(z,x)>> [x=[200]]ρ))]
  [x=[200]]ρ)

```

```

= (value-of
  <<-((f 1), (g 1))>>
  [g=(proc-val (procedure z <<-(z,x)>>
    [x=[100]] [f=...] [x=[200]] ρ))])
  [x=[100]]
  [f=(proc-val (procedure z <<-(z,x)>> [x=[200]] ρ))])
  [x=[200]] ρ)

= [(-
  (value-of <<(f 1)>>
    [g=(proc-val (procedure z <<-(z,x)>>
      [x=[100]] [f=...] [x=[200]] ρ))])
    [x=[100]]
    [f=(proc-val (procedure z <<-(z,x)>> [x=[200]] ρ))])
    [x=[200]] ρ)
  (value-of <<(g 1)>>
    [g=(proc-val (procedure z <<-(z,x)>>
      [x=[100]] [f=...] [x=[200]] ρ))])
    [x=[100]]
    [f=(proc-val (procedure z <<-(z,x)>> [x=[200]] ρ))])
    [x=[200]] ρ)

= [(-
  (apply-procedure
    (procedure z <<-(z,x)>> [x=[200]] ρ)
    [1])
  (apply-procedure
    (procedure z <<-(z,x)>> [x=[100]] [f=...] [x=[200]] ρ)
    [1]))]

```


An example



```
= [(-
  (value-of <<(f 1)>>
    (g=(proc-val (procedure z <<-(z,x)>>
      [x=[100]] [f=...] [x=[200]] ρ))
      [x=[100]]
      [f=(proc-val (procedure z <<-(z,x)>> [x=[200]] ρ))
      [x=[200]] ρ)
    (value-of <<(g 1)>>
      (g=(proc-val (procedure z <<-(z,x)>>
        [x=[100]] [f=...] [x=[200]] ρ))
        [x=[100]]
        [f=(proc-val (procedure z <<-(z,x)>> [x=[200]] ρ))
        [x=[200]] ρ))
      [x=[200]] ρ))
  )]

= [(-
  (apply-procedure
    (procedure z <<-(z,x)>> [x=[200]] ρ)
    [1])
  (apply-procedure
    (procedure z <<-(z,x)>> [x=[100]] [f=...] [x=[200]] ρ)
    [1]))]

= [(-
  (value-of <<-(z,x)>> [z=[1]] [x=[200]] ρ)
  (value-of <<-(z,x)>> [z=[1]] [x=[100]] [f=...] [x=[200]] ρ))]

= [(- -199 -99)]

= [-100]
```

Implementation



```
proc? : SchemeVal  $\rightarrow$  Bool  
(define proc?  
  (lambda (val)  
    (procedure? val)))
```

```
procedure : Var  $\times$  Exp  $\times$  Env  $\rightarrow$  Proc  
(define procedure  
  (lambda (var body env)  
    (lambda (val)  
      (value-of body (extend-env var val env))))))
```

```
apply-procedure : Proc  $\times$  ExpVal  $\rightarrow$  ExpVal  
(define apply-procedure  
  (lambda (proc1 val)  
    (proc1 val)))
```

Alternative implementation



```
proc? : SchemeVal → Bool
procedure : Var × Exp × Env → Proc
(define-datatype proc proc?
  (procedure
    (var identifier?)
    (body expression?)
    (saved-env environment?)))

apply-procedure : Proc × ExpVal → ExpVal
(define apply-procedure
  (lambda (proc1 val)
    (cases proc proc1
      (procedure (var body saved-env)
        (value-of body (extend-env var val saved-env)))))))
```

Other changes to the interpreter



```
(define-datatype expval expval?
  (num-val
    (num number?))
  (bool-val
    (bool boolean?))
  (proc-val
    (proc proc?)))

(proc-exp (var body)
  (proc-val (procedure var body env)))

(call-exp (rator rand)
  (let ((proc (expval->proc (value-of rator env)))
        (arg (value-of rand env)))
    (apply-procedure proc arg)))
```