Lecture 04
Structures and Patterns in Functional Programming

T. METIN SEZGIN

Announcements

1. Reading SICP 1.2 (pages 31-50)
2. Etutor 1, 2, 3, 4 all due on Sunday midnight
3. Attend your Pses
4. Quiz on Friday

Lecture 3 – Review
Functional Programming

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Lecture Nuggets

- Lambda expressions creates procedures
  - Formal parameters
  - Body
  - Procedures allow creating abstractions
- We can solve problems by creating functions
- The substitution model is a good mental model of an interpreter

Controlling the process

```
(define sqrt
    (lambda (x)
        (sqrt-loop 1.0 x)))
```

```
(define sqrt-loop (lambda G X)
   (if (close-enuf? G X)
       G
       (sqrt-loop (improve G X) X ) ) )
```

Nugget

The substitution model is a good mental model of an interpreter

Iterative and Recursive versions of fact

```scheme
;; RECURSIVE

(define (fact-r x)
  (if (= x 0) 1 (* x (fact-r (- x 1)))))


;; ITERATIVE
(define (fact-i x)
  (fact-i-helper 1 1 x))

(define fact-i-helper
  (lambda (product counter n)
    (if(> counter n)
       product
       (fact-i-helper (* product counter) (+ counter 1) n))))
```

```
(define fact(lambda (n)
 (if (= n 1)1(* n (fact (- n 1))))))

(fact 3)
(if (= 3 1) 1 (* 3 (fact (- 3 1))))
(if #f 1 (* 3 (fact (- 3 1))))
(* 3 (fact (- 3 1)))
(* 3 (fact 2))
(* 3 (if (= 2 1) 1 (* 2 (fact (- 2 1)))))
(* 3 (if #f 1 (* 2 (fact (- 2 1)))))
(* 3 (* 2 (fact (- 2 1))))
(* 3 (* 2 (fact 1)))
(* 3 (* 2 (if (= 1 1) 1 (* 1 (fact (- 1 1))))))
(* 3 (* 2 (if #t 1 (* 1 (fact (- 1 1))))))
(* 3 (* 2 1))
(* 3 2)
6
```

Iterative algorithm to compute 4! as a table

- In this table:
  - One column for each piece of information used
  - One row for each step

first row handles 0! cleanly

product	counter	N
1	1	4
1	2	4
2	3	4
6	4	4
24	5	4

product * counter

counter +1

answer

- The last row is the one where counter > n
- The answer is in the product column of the last row

Iterative factorial in scheme

- (define ifact (lambda (n) (ifact-helper 1 1 n)))

initial
row of table

```
(define ifact-helper (lambda (product counter n)
    (if     (> counter n)
        product
        (ifact-helper (* product counter) (+ counter 1) n))))
```

compute next row of table

answer is in product column of last row

at last row when counter > n

Partial trace for `(ifact 4)`

```
(define ifact-helper (lambda (product count n)
    (if (> count n) product
        (ifact-helper (* product count)
                      (+ count 1) n))))
```

```
(ifact 4)
(ifact-helper 1 1 4)
(if (> 1 4) 1 (ifact-helper (* 1 1) (+ 1 1) 4))
(ifact-helper 1 2 4)
(if (> 2 4) 1 (ifact-helper (* 1 2) (+ 2 1) 4))
(ifact-helper 2 3 4)
(if (> 3 4) 2 (ifact-helper (* 2 3) (+ 3 1) 4))
(ifact-helper 6 4 4)
(if (> 4 4) 6 (ifact-helper (* 6 4) (+ 4 1) 4))
(ifact-helper 24 5 4)
(if (> 5 4) 24 (ifact-helper (* 24 5) (+ 5 1) 4))
24
```

Iterative = no pending operations when procedure calls itself

- Recursive factorial:

```
(define fact (lambda (n)
      (if (= n 1) 1
        (* n (fact (- n 1)) )
   )))
```

pending operation

- `(fact 4)`
  ```
  (* 4 (fact 3))
  (* 4 (* 3 (fact 2)))
  (* 4 (* 3 (* 2 (fact 1))))
  ```

- Pending ops make the expression grow continuosly

Iterative = no pending operations

- Iterative factorial:

```
(define ifact-helper (lambda (product count n)
  (if (> count n) product
      (ifact-helper (* product count)
                    (+ count 1) n))))
```

- ```
  (ifact-helper 1 1 4)
  (ifact-helper 1 2 4)
  (ifact-helper 2 3 4)
  (ifact-helper 6 4 4)
  (ifact-helper 24 5 4)
  ```

no pending operations

- Fixed size because no pending operations
  ```

Lecture notes

- λ expressions create procedures
  - formal parameters
  - body
  - procedures allow creating abstractions
- problems get solved → func.
- Substitution model → model of an interpeter.

```
(define fact (lambda(x) ---- etc. ))
(define (fact x) ( etc. ------ ))
```
OR

- Zeynep Aydin

Lecture notes

Substitution Model : is a good mental model of an interpreter

$((\text{lambda}\ (x)\ (\cdot\ \overset{5}{x}\ \overset{5}{x}))\ 5)$

self - evaluate 5

substitute into x , now $(\cdot\ 5\ 5) \rightarrow 25$

- Hasan Buhurcu

Lecture notes

Scheme language

Language elements:

primitives: Numbers, Strings, booleans, operators...

combinations: (+ 2 3)

does not evaluated

abstractions: (define x 23)

(define y x) → y = 23

(define myplus +)

(myplus 3 4)

x	23

environment

- Berat Karayilan

Lecture notes

Factorial Example

- Tail Recursion :

```
(define fact (lambda (x)
    (if (= n 1) 1  → base case
        (* n (fact (- n 1))))))
```

pending operation is this multiplication

- Iterative

```
(define (fact-i x)
    (fact-i-helper 1 1 x))

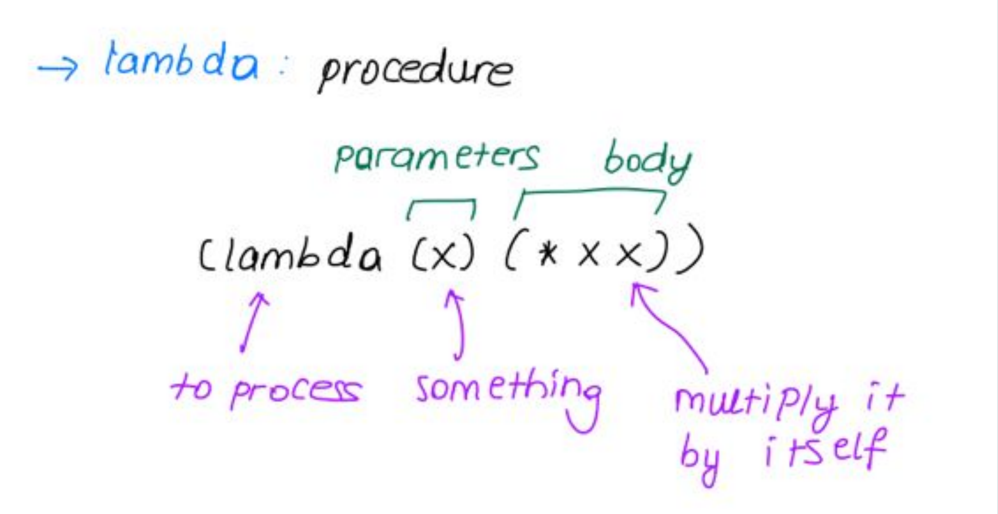
(define fact-i-helper
    (lambda (product counter n)
        (if (> counter n) product
            (fact-i-helper (* product counter) (+ counter 1) n))))
```

next product   next counter

→ no pending operation

○ Mete Erdogan

Lecture notes



- Ceren Tarim

Lecture notes

Scheme Basics

- Rules for evaluation

1. If self evaluating, return value
2. If a name, return value associated with name in environment.
3. If a special form, do something special.
4. If a combination, then
   a. Evaluate all of the subexpressions of combination (in any order)
   b. apply the operator to the values of the operands (arguments) and return result

○ Digdem Yildiz

Lecture notes

General form of recursive algorithms

```
(define fact
   (lambda (n)
      (if (= n 1)   ; test for base case
          1         ; base case
          (* n (fact (- n 1)))   ; recursive
                                   case
)))
```

- Design recursive → decompose the problem
  → identify non-decomposable (smallest) problems

Digdem Yildiz

Lecture 04
Structures and Patterns in Functional Programming

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Lecture Nuggets

- Order of growth matters
- Support for compound data allows data abstraction
  - Pairs
  - Lists
  - Others
- Two main patterns when dealing with lists
  - Consing up – to build
  - Cdring down – to process

Nugget

Order of growth matters

Orders of growth of processes

- Suppose `n` is a parameter that measures the size of a problem
- Let `R(n)` be the amount of resources needed to compute a procedure of size `n`.
- We say `R(n)` has order of growth Θ(`f(n)`) if there are constants k_1 and k_2 such that $k_1 f(n) <= R(n) <= k_2 f(n)$ for large `n`
- Two common resources are space, measured by the number of deferred operations, and time, measured by the number of primitive steps.

Examples of orders of growth

- FACT
  - Space Θ(n) – linear
  - Time Θ(n) – linear

- IFACT
  - Space Θ(1) – constant
  - Time Θ(n) – linear

Nugget

Support for compound data allows data abstraction

Language Elements

- Primitives
  - prim. data: numbers, strings, booleans
  - primitive procedures
- Means of Combination
  - procedure application
  - compound data (today)
- Means of Abstraction
  - naming
  - compound procedures
    - block structure
    - higher order procedures (next time)
  - conventional interfaces – lists (today)
  - data abstraction

Compound data

- Need a way of gluing data elements together into a unit that can be treated as a simple data element
- Need ways of getting the pieces back out

- Need a contract between the “glue” and the “unglue”

- Ideally want the result of this “gluing” to have the property of **closure:**
  - “the result obtained by creating a compound data structure can itself be treated as a primitive object and thus be input to the creation of another compound object”

Pairs (cons cells)

- **`(cons <x-exp> <y-exp>) ==> <P>`**
  - Where **`<x-exp>`** evaluates to a value **`<x-val>`**, and **`<y-exp>`** evaluates to a value **`<y-val>`**
  - **Returns a** pair **`<P>`** whose car-part is **`<x-val>`** and whose cdr-part is **`<y-val>`**
- **`(car <P>) ==> <x-val>`**
  - Returns the car-part of the pair **`<P>`**
- **`(cdr <P>) ==> <y-val>`**
  - Returns the cdr-part of the pair **`<P>`**

Compound Data

- Treat a PAIR as a single unit:
  - Can pass a pair as argument
  - Can return a pair as a value

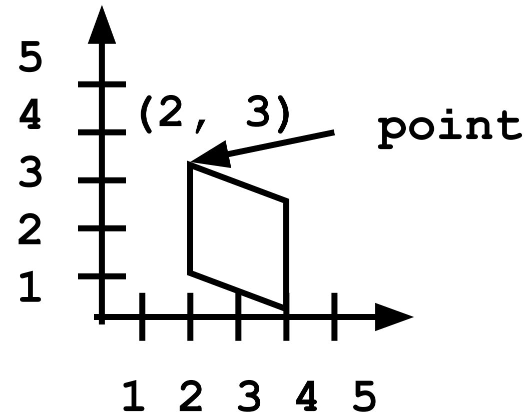
```
(define (make-point x y)
  (cons x y))

(define (point-x point)
  (car point))

(define (point-y point)
  (cdr point))

(define (make-seg pt1 pt2)
  (cons pt1 pt2))

(define (start-point seg)
   (car seg))
```



Pair Abstraction

- Constructor

```
; cons: A,B -> A X B
; cons: A,B -> Pair<A,B>
  (cons <x> <y>) ==> <P>
```

- Accessors

```
; car: Pair<A,B> -> A
  (car <P>) ==> <x>
; cdr: Pair<A,B> -> B
  (cdr <P>) ==> <y>
```

- Predicate

```
; pair? anytype -> boolean
  (pair? <z>)
```

`==>` `#t` if `<z>` evaluates to a pair, else `#f`

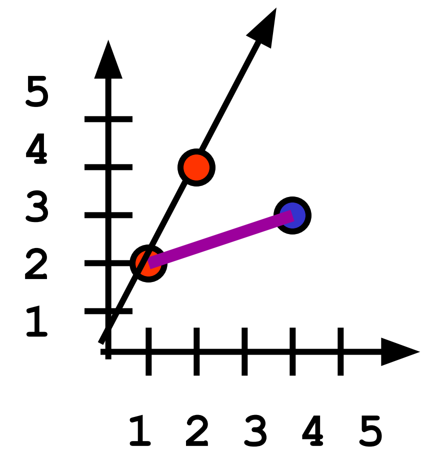
Pair abstraction

- Note how there exists a contract between the constructor and the selectors:
  - *(car (cons <a> <b>)) □ <a>*
  - *(cdr (cons <a> <b>)) □ <b>*
- Note how pairs have the property of closure – we can use the result of a pair as an element of a new pair:
  - *(cons (cons 1 2) 3)*

Using pair abstractions to build procedures

- Here are some data abstractions

```
(define p1 (make-point 1 2))
(define p2 (make-point 4 3))
(define s1 (make-seg p1 p2))
```




```
(define stretch-point
  (lambda (pt scale)
    (make-point (* scale (point-x pt))
                (* scale (point-y pt)))))
(stretch-point p1 2) □ (2 . 4)
p1 □ (1 . 2)
```

Grouping together larger collections

- Suppose we want to group together a set of points. Here is one way

```
(cons (cons (cons (cons p1 p2)
                  (cons p3 p4))
            (cons (cons p5 p6)
                  (cons p7 p8)))
      p9)
```

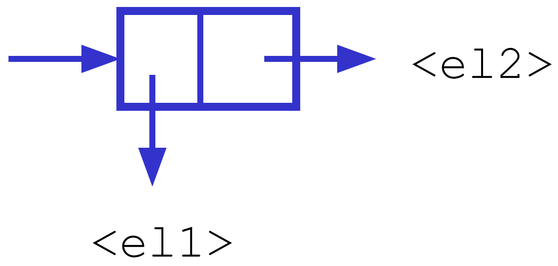
- **UGH!!** How do we get out the parts to manipulate them?

Conventional interfaces -- lists

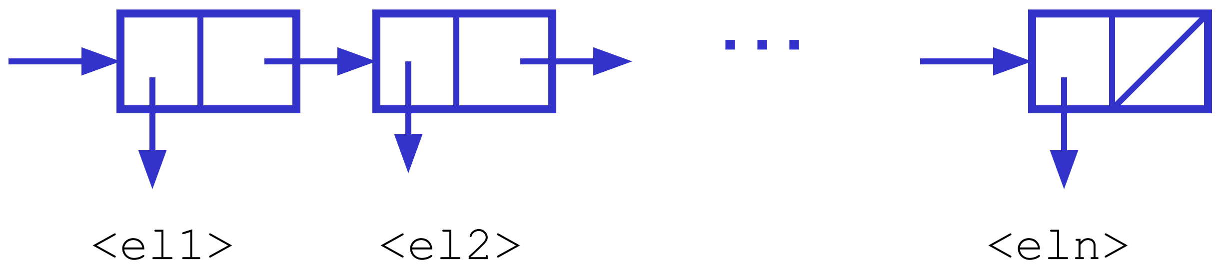
- A list is a data object that can hold an arbitrary number of ordered items.
- More formally, a list is a sequence of pairs with the following properties:
  - Car-part of a pair in sequence – holds an item
  - Cdr-part of a pair in sequence – holds a pointer to rest of list
  - Empty-list **`nil`** – signals no more pairs, or end of list
- Note that lists are closed under operations of **`cons`** and **`cdr`**.

Conventional Interfaces - Lists

`(cons <el1> <el2>)`



`(list <el1> <el2> ... <eln>)`



Predicate

`(null? <z>)`

`==> #t` if **`<z>`** evaluates to empty list

… to be really careful

- For today we are going to create different constructors and selectors for a list
  - `(define first car)`
  - `(define rest cdr)`
  - `(define adjoin cons)`
- Note how these abstractions inherit closure from the underlying abstractions!

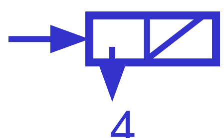
Nugget

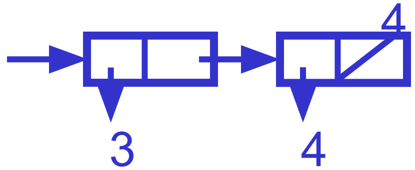
Two patterns for dealing with lists

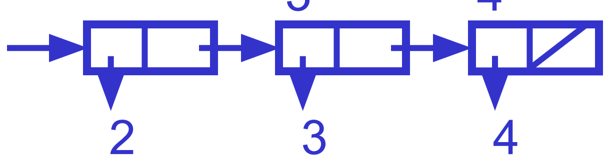
Common Pattern #1: cons'ing up a list

```
(define (enumerate-interval from to)
  (if (> from to)
      nil
      (adjoin from
              (enumerate-interval
                (+ 1 from)
                to))))
```

```
(e-i 2 4)
(if (> 2 4) nil (adjoin 2 (e-i (+ 1 2) 4)))
(if #f nil (adjoin 2 (e-i 3 4)))
(adjoin 2 (e-i 3 4))
(adjoin 2 (adjoin 3 (e-i 4 4)))
(adjoin 2 (adjoin 3 (adjoin 4 (e-i 5 4))))
(adjoin 2 (adjoin 3 (adjoin 4 nil)))
```

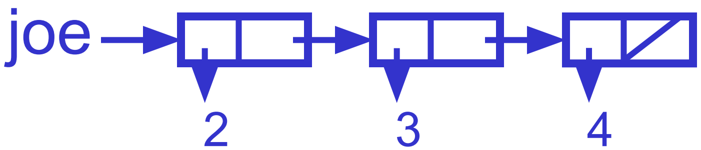
(adjoin 2 (adjoin 3 ))

(adjoin 2 )

 ==> (2 3 4)

Common Pattern #2: cdr'ing down a list

```
(define (list-ref lst n)
  (if (= n 0)
      (first lst)
      (list-ref (rest lst)
                (- n 1))))
```




```
(list-ref joe 1)
```

```
(define (length lst)
  (if (null? lst)
      0
      (+ 1 (length (rest lst)))))
```