Impact of Duty Cycle on End-to-End Performance in a Wireless Sensor Network

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***Abstract*—Duty cycling has been widely used in Wireless Sensor Networks (WSNs). While its general impact on the network performance is intuitive and clear, this understanding is far from sufficient for WSN planning, design and optimization, among others. In this paper, a quantitative study on the relationship between duty cycle and end-to-end network performance, in terms of delay and throughput, is presented, where the network diameter is also taken into consideration. Particularly, closed-form expressions are derived, revealing the impact of duty cycle on the end-to-end performance in a wireless sensor network. Numerical results illustrating the impact are presented and discussed.**

***Index Terms*—Wireless sensor network, duty cycle, network diameter, delay, throughput, stochastic network calculus.**

I. Introduction

In wireless sensor networks (WSNs), duty cycling has been widely used to reduce energy consumption and hence prolong sensor node's (battery) lifetime. It is intuitive that the duty cycle of scheduling in a WSN impacts the network performance, e.g., delay and throughput. However, this intuition is far from sufficient for network planning and design as well as for optimization of WSNs. In order to address this limitation, it is crucial to relate duty cycle to network performance in a quantitative manner.

The objective of this paper is to conduct a quantitative study to find the relationship between duty cycle and network performance in terms of delay and throughput. Particularly, we are interested in end-to-end network performance, of which single node performance is a special case, and establish the relationship on the network diameter [1].

In the literature, there are several studies that take duty cycle / energy consumption and performance together [2], [3] or consider network diameter in the analysis of WSNs [4]–[7]. Specifically, in [2], the duty cycle is considered as a property of the energy consumption and incorporated into the latency-rate service curve for performance analysis. In addition it is also depicted in [2] that the calculations could also start with a given delay/buffer requirement and work out the length of the duty cycle and thus the power consumption level and therefore the network lifetime. In [3], a framework of the energy consumption and delay requirement is built for single-hop and multi-hop scenarios in the video sensor networks and the tradeoff between these two factors are discussed. The analyses in [2] and [3] are based on deterministic network calculus. In [1], it is illustrated that the network diameter can be involved in the complexity analysis of the network performance if the diameter and the performance are proportionally connected. In [4], different routing schemes are discussed based on the diameter. In [5], for some desired performance, a lower bound and an upper bound on the diameter in a random sensor grid are derived and it is shown that the diameter is of the order $\sqrt{n/\log n}$, where n is the number of nodes placed in the square of unit area. In [6], it is shown that the difficulty of reaching a consensus increases with the network diameter of the underlying graph and the consensus can be reached in four deterministic communications steps for a wireless network of diameter two. [7] presents a distributed algorithm, taking advantage of the diameter, for constructing an energy efficient connected dominating sets of wireless networks. The algorithm is further investigated in [8] to take the impact of energy or duty cycle on the diameter and performance together.

However, none of the literature works provides explicit results for the relationship between duty cycle and end-to-end network performance in terms of delay and throughput, particularly under the stochastic network setting and taking network diameter into consideration.

In this paper, we consider a randomly distributed WSN, where the diameter of the network is determined by the maximum end-to-end node number N. The energy consumption is characterized by the sleep scheduling duty cycle and duty period tuple $(\lambda, \mathring{T})$. The network performance is measured by the delay tuple $(\mathcal{D}, \varepsilon)$, where $\mathcal{D}$ is the delay constraint and ε is the delay violation probability, and the throughput tuple $(\mathcal{T}, \xi)$, where $\mathcal{T}$ is the expected throughput and ξ is the violation probability. Fundamentally, we derive relationships between $(\mathcal{D}, \varepsilon)$ and N and $(\lambda, \mathring{T})$, and relationships between $(\mathcal{T}, \xi)$ and N and $(\lambda, \mathring{T})$, based on a newly developed theory, the stochastic network calculus theory [9]. With these obtained relationships, we further investigate the impacts of duty cycle on the allowed network diameter under delay and throughput constraints. These relationships and impacts are all in closed-form expression. Finally, numerical results are presented to illustrate these relationships and impacts.

The remainder is structured as follows. In Sec. II, stochastic network calculus basics are introduced. In Sec. III, the system model is presented. In Sec. IV, the relationships between delay / throughput and duty cycle and network diameter are analyzed, based on which, the impacts of duty cycle on the allowed network diameter are further derived. Numerical results are in Sec. V. Finally, conclusion is made in Sec. VI.

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II. Stochastic Network Calculus Basics

The analysis in this paper makes extensive use of concepts and results of the newly developed stochastic network calculus theory [9]. To facilitate the later analysis as well as the understanding of the analysis, this section is devoted to an introduction of the stochastic network calculus basics that can all be found from the literature, e.g. [9].

A. Fundamental Concepts

We assume discrete-time systems with time indexed by $t = 0, 1, 2, \cdots$, through which the data amount is not altered. Denote $A(t)$ and $A^*(t)$ as the cumulative traffic that arrives to and departs from a system in time interval $(0, t]$ respectively, and $S(t)$ as the cumulative amount of service provided by the system in time interval $(0, t]$. For any $0 \leq s \leq t$, let $A(s,t) \equiv A(t) - A(s)$, $A^*(s,t) \equiv A^*(t) - A^*(s)$, and $S(s,t) \equiv S(t) - S(s)$. By default, $A(0) = A^*(0) = S(0) = 0$.

In stochastic network calculus, stochastic arrival curve and stochastic service curve are the most fundamental models. Stochastic arrival curve is for traffic modeling and stochastic service curve is for server modeling. While there are several definition variations of stochastic arrival curve and stochastic service curve in the literature [9], we adopt the following in this paper.

Definition 1: A flow is said to have a v.b.c (virtual backlog centric) stochastic arrival curve $\alpha(t)$ with bounding function $\bar{F}$, if its arrival process $A(t)$ satisfies, for any $t \geq 0$,

$$P\{A(t) - A \otimes \alpha(t) > x\} \leq \bar{F}(x), \tag{1}$$

where $\alpha(t)$ is non-negative wide sense increasing on t, $\bar{F}(x)$ is non-negative non-increasing on x, and $\otimes$ denotes the min-plus convolution.

Here and throughout the rest of the paper, the min-plus convolution $\otimes$ of function $f(\cdot)$ and $g(\cdot)$ is defined as

$$f \otimes g(t) \equiv \inf_{0 \leq s \leq t} \{f(s) + g(t-s)\}, \tag{2}$$

where it is easily verified $f \otimes g(t) = g \otimes f(t)$.

Definition 2: A system is said to provide a stochastic service curve $\beta(t)$ with bounding function $\bar{G}$, if there holds, for all $t \geq 0$,

$$P\{A \otimes \beta(t) - A^*(t) > x\} \leq \bar{G}(x), \tag{3}$$

where $\beta(t)$ is non-negative wide sense increasing on t, and $\bar{G}(x)$ non-negative non-increasing on x.

B. Basic Results

For delay analysis, the following result has been proved.

Proposition 1: If a system provides a stochastic service curve $\beta(t)$ with bounding function $\bar{G}$ to a flow, which has v.b.c stochastic arrival curve $\alpha(t)$ with bounding function $\bar{F}$, then the flow has a delay bound as

$$P\{D(t) > h(\alpha + x, \beta)\} \leq \bar{F} \otimes \bar{G}(x), \tag{4}$$

where $h(\alpha + x, \beta)$ denotes the maximum horizontal distance between $\alpha(t) + x$ and $\beta(t)$.

Proposition 2: Under the same condition as Preposition 1, the output has a traffic-amount-centric (t.a.c.) stochastic arrival curve $\alpha \oslash \beta$ with bounding function $\bar{F} \otimes \bar{G}(x)$, i.e., for all $0 \leq s \leq t$,

$$P\{A^*(s,t) - \alpha \oslash \beta(t-s) > x\} \leq \bar{F} \otimes \bar{G}(x), \tag{5}$$

where $\alpha \oslash \beta(t) \equiv \sup_{s \geq 0} \{\alpha(t+s) - \beta(s)\}$.

In stochastic network calculus, the following result, called the *leftover service* property, has been widely used for finding the stochastic service curve characterization of the service when there is crossing traffic that shares the system.

Proposition 3: Consider a system shared by a traversing flow with a (possibly aggregate) crossing flow. If the system provides to the whole input a stochastic service curve $\beta(t)$ with bounding function $\bar{G}(x)$ and the crossing flow has a v.b.c. stochastic arrival curve $\alpha^c(t)$ with bounding function $\bar{F}^c(x)$, then the *leftover service* provided to the traversing flow has a stochastic service curve $\beta^f(t) = (\beta(t) - \alpha^c(t))^+$ with bounding function $\bar{G}^f(x) = \bar{F}^c \otimes \bar{G}(x)$.

When the system is shared by the traversing flow and the crossing flow, the following delay bound follows immediately from Proposition 1 and Proposition 3.

Proposition 4: Under the same condition as Preposition 3, if the traversing flow has a stochastic arrival curve $\alpha^f(t)$ with bounding function $\bar{F}^f(x)$, then the delay of the flow is bounded as

$$P\{D^f(t) > h(\alpha^f + x, \beta^f)\} \leq \bar{F}^f \otimes \bar{F}^c \otimes \bar{G}(x), \tag{6}$$

where $\beta^f(t) = (\beta(t) - \alpha^c(t))^+$.

With some approximation for simplifying the representation, the following result allows to easily extend single node analysis to the network case.

Proposition 5: Consider a flow passing through a network of N nodes in tandem. If each node $n (= 1, 2, \cdots, N)$ provides stochastic service curve $S^n \sim \langle \bar{G}^n(x), \beta^n \rangle$ to its input, then the network guarantees to the flow a stochastic service curve $S \sim \langle \bar{G}(x), \beta \rangle$ with

$$\begin{aligned} \beta(t) &= \beta^1 \otimes \beta^2 \otimes \cdots \otimes \beta^N(t), \\ \bar{G}(x) &= \bar{G}^1 \otimes \bar{G}^2 \otimes \cdots \otimes \bar{G}^N(x). \end{aligned}$$

III. The System Model

We consider a WSN with a sink, where nodes may be randomly distributed in an area. Each node employs a sleep scheduling mechanism and is in charge of both data sensing and data routing. The sleep scheduling is characterized by the duty cycle and duty period tuple $(\lambda, \mathring{T})$. Specifically, we assume that the scheduling in this network is synchronized for each node with duty period $\mathring{T}$, active time interval $\tilde{T}$, and duty cycle $\lambda = \frac{\tilde{T}}{\mathring{T}}$.

Sensed data are transmitted from the source to the sink with a multi-hop style. Specifically, for any path from the edge to the sink of any sensed data, it is composed of a series of nodes in tandem. Let the edge node where the sensed data are generated be the first hop, the diameter N of the network equals the maximum end-to-end node number from the network edge to the sink.

We refer to the node at the edge as S_1 and the other nodes as S_i, $i = 2, \cdots, N$. For flow A_1 of the sensed data originating from S_1 and passing through the path till the sink S_N, we call it the traversing flow and call the aggregation of other flows the crossing flow, i.e., A_i transmitted by S_i, $i = 2, \cdots, N$. We assume that there are no crossing flows for the nodes at the edge of the network.

The network performance is measured by the delay tuple $(\mathcal{D}, \varepsilon)$, where $\mathcal{D}$ is the delay constraint and ε is the delay violation probability, and the throughput tuple $(\mathcal{T}, \xi)$, where $\mathcal{T}$ is the expected throughput and ξ is the violation probability. In the next section, we derive relationships between $(\mathcal{D}, \varepsilon)$ and N and $(\lambda, \mathring{T})$, and relationships between $(\mathcal{T}, \xi)$ and N and $(\lambda, \mathring{T})$, under the following assumptions on the traffic of sensed data flows and on the service provided by each node.

For traffic, we assume every flow of sensed data can be characterized by an affine function

$$\alpha(t) = \rho \cdot t + \sigma,$$

where ρ is the average data rate and σ is the short-term data burst [2]. Specifically, we assume that the flow has a stochastic arrival curve as

$$P\{A(t) - A \otimes (\rho \cdot t + \sigma) > x\} \leq a_f e^{-b_f x}, \tag{7}$$

where the bounding function is a negative exponential bounding function, $f(x) = a_f e^{-b_f x}$, with some $a_f > 0$, $b_f > 0$. In [9], it is discussed that a wide range of traffic types including Poisson, periodic and long-range dependent traffic can be characterized using the above form.

For service, under the duty cycle scheduling assumption, if the service rate R of a node is constant, it can be easily verified that the average transmission rate of the node is $\frac{R\tilde{T}}{\mathring{T}}$, and the maximum latency at the node is $\mathring{T} - \tilde{T}$ [3]. Take the stochastic nature of wireless communication into consideration, we generalize this service analysis and assume that each node provides a stochastic service curve as

$$\beta(t) = \lambda R(t - \tau),$$

where $\lambda = \frac{\tilde{T}}{\mathring{T}}$ and $\tau = \mathring{T} - \tilde{T}$, with bounding function $g(x) = a_g e^{-b_g x}$, $a_g > 0$, $b_g > 0$, i.e.,

$$P\{A(t) \otimes (\lambda R(t - \tau)) - A^*(t) > x\} \leq a_g e^{-b_g x}. \tag{8}$$

IV. Analysis

We devote this section to analyzing the relationships between delay / throughput and duty cycle and network diameter, based on which, the impacts of duty cycle on the allowed network diameter are derived. We start with some preliminary results that are the building blocks of the later analysis.

A. Preliminary Results

Consider an end-to-end path. The stochastic service curve for the sensing data at the network edge S_1 is $\beta_1(t)$ with bounding function $g_1(x)$. For other nodes on the path, the leftover service for the traversing flow at nodes S_i, $i = 2, \cdots, N$, is $\beta^i(t) = \beta_i(t) - \alpha_i(t)$ with bounding function $g^i(x) = f_i \otimes g_i(x)$. Consequently, the network stochastic service curve for the sensing data is

$$\beta(t) = \beta_1 \otimes (\beta_2 - \alpha_2) \otimes \cdots \otimes (\beta_N - \alpha_N)(t), \tag{9}$$
$$g(x) = g_1(x) \otimes \bigotimes_{2 \leq i \leq N} f_i \otimes g_i(x). \tag{10}$$

In the following, explicit expressions for $\beta(t)$ and $g(x)$ are derived.

1) Derivation of $\beta(t)$: First, we derive the formula for the initial two nodes,

$$\begin{aligned}
&\beta_1 \otimes (\beta_2 - \alpha_2)(x) \\
&= \lambda_1 R_1(t - \tau_1) \otimes [\lambda_2 R_2(t - \tau_2) - (\sigma_2 + \rho_2 t)] \\
&= \inf_{0 \leq y \leq x} [(\lambda_1 R_1 - \lambda_2 R_2 + \rho_2) y + (\lambda_2 R_2 - \rho_2) x \\
&\quad - (\lambda_1 R_1 \tau_1 + \lambda_2 R_2 \tau_2 + \sigma_2)] \\
&= (\lambda_2 R_2 - \rho_2) x - (\lambda_1 R_1 \tau_1 + \lambda_2 R_2 \tau_2 + \sigma_2).
\end{aligned}$$

Then, we increase the node count to three,

$$\begin{aligned}
&\beta_1 \otimes (\beta_2 - \alpha_2) \otimes (\beta_3 - \alpha_3)(x) \\
&= \inf_{0 \leq y \leq x} [(\lambda_2 R_2 - \rho_2 - \lambda_3 R_3 + \rho_3) y + (\lambda_3 R_3 - \rho_3) x \\
&\quad - (\lambda_1 R_1 \tau_1 + \lambda_2 R_2 \tau_2 + \lambda_3 R_3 \tau_3 + \sigma_2 + \sigma_3)],
\end{aligned}$$

when $\lambda_2 R_2 - \rho_2 \geq \lambda_3 R_3 - \rho_3$,

$$\begin{aligned}
&\beta_1 \otimes (\beta_2 - \alpha_2) \otimes (\beta_3 - \alpha_3)(x) \\
&= (\lambda_3 R_3 - \rho_3) x \\
&\quad - (\lambda_1 R_1 \tau_1 + \lambda_2 R_2 \tau_2 + \lambda_3 R_3 \tau_3 + \sigma_2 + \sigma_3),
\end{aligned}$$

when $\lambda_2 R_2 - \rho_2 < \lambda_3 R_3 - \rho_3$,

$$\begin{aligned}
&\beta_1 \otimes (\beta_2 - \alpha_2) \otimes (\beta_3 - \alpha_3)(x) \\
&= (\lambda_2 R_2 - \rho_2) x \\
&\quad - (\lambda_1 R_1 \tau_1 + \lambda_2 R_2 \tau_2 + \lambda_3 R_3 \tau_3 + \sigma_2 + \sigma_3).
\end{aligned}$$

Iteratively using the method above, we obtain

$$\beta(t) = \inf_{1 \leq i \leq N} (\lambda_i R_i - \rho_i) t - \left(\sum_{i=1}^{N} \lambda_i R_i \tau_i + \sum_{i=2}^{N} \sigma_i \right).$$

2) Derivation of $\bigotimes_{1 \leq i \leq N} f_i \otimes g_i(x)$: This derivation makes use of the following result (see e.g. [9]). For any positive numbers $a_k, b_k, k = 1, \ldots, K$ and any $x \geq 0$, we have

$$\inf_{x_1 + \ldots + x_K = x} \sum_{k=1}^{K} a_k e^{-b_k x_k} = e^{\frac{-x}{w}} \prod_{k=1}^{K} (a_k b_k w)^{\frac{1}{b_k w}},$$

where $w = \sum_{k=1}^{K} \frac{1}{b_k}$.

With above, we have

$$\begin{aligned}&\bigotimes_{1\le i\le N} f_i \otimes g_i(x)\\ &= \bigotimes_{1\le i\le N} \inf_{0\le y\le x}\left\{a_{f_i}e^{-b_{f_i}y}+a_{g_i}e^{-b_{g_i}(x-y)}\right\}\\ &= \bigotimes_{1\le i\le N} a_i e^{-b_i x}\\ &= e^{-\frac{x}{\mathcal{W}}}\prod_{i=1}^{N}(a_i b_i \mathcal{W})^{\frac{1}{b_i\mathcal{W}}},\end{aligned}$$

where

$$a_i = (a_{f_i}b_{f_i}w_i)^{\frac{1}{b_{f_i}w_i}} \times (a_{g_i}b_{g_i}w_i)^{\frac{1}{b_{g_i}w_i}},$$

$$b_i = \frac{1}{w_i},\ w_i = \frac{1}{b_{f_i}}+\frac{1}{b_{g_i}},$$

$$\mathcal{W} = \sum_{i=1}^{N}\frac{1}{b_i} = \sum_{i=1}^{N}\left(\frac{1}{b_{f_i}}+\frac{1}{b_{g_i}}\right).$$

B. Impact on Delay

In this subsection, we focus on the impact of duty cycle on the delay performance and on the allowed network diameter subject to delay constraint.

1) Impact of duty cycle on delay: With the end-to-end stochastic service curve derived in the previous subsection as well as Proposition 1, a delay bound for flow A_1 transmitting through the network is easily found as

$$P\{D > h(\alpha_1 + x, \beta)\} \le \bigotimes_{1\le i\le N} f_i \otimes g_i(x), \quad (11)$$

where $h(\alpha_1 + x, \beta)$ represents the maximum horizontal distance between the arrival curve and service curve of the network [9].

With β derived in the previous subsection, the horizontal distance is easily verified as:

$$h(\alpha_1 + x, \beta) = \frac{\sum_{i=1}^{N}\lambda_i R_i \tau_i + \sum_{i=1}^{N}\sigma_i + x}{\inf_{1\le i\le N}(\lambda_i R_i - \rho_i)}.$$

Let $D = h(\alpha_1 + x, \beta)$ and set the right side of (11) equal to ε, we have for the delay bound D

$$D = \frac{\sum_{i=1}^{N}\lambda_i R_i \tau_i + \sum_{i=1}^{N}\sigma_i}{R_{inf}} + \frac{\mathcal{W}}{R_{inf}} * ln\frac{\psi}{\varepsilon}, \quad (12)$$

with

$$R_{inf} = \inf_{1\le i\le N}(\lambda_i R_i - \rho_i),\ \psi = \prod_{i=1}^{N}(a_i b_i \mathcal{W})^{\frac{1}{b_i\mathcal{W}}},$$

where a_i, b_i, and $\mathcal{W}$ are shown in the previous subsection.

Assume that all flows have the same stochastic service curve $\alpha(t) = \sigma + \rho t$ with bounding function $f(x) = a_f e^{-b_f x}$ and all nodes have the same stochastic service curve $\beta(t) = \lambda R(t-\tau)$ with bounding function $g(x) = a_g e^{-b_g x}$, then,

$$D = \frac{N(\lambda R\tau + \sigma)}{\lambda R - \rho} - \frac{Nw}{\lambda R - \rho} \times \ln\frac{\varepsilon}{abwN}, \quad (13)$$

where $w = \frac{1}{b_f}+\frac{1}{b_g}$, by the nature, we assume that the required violation probability ε is small, satisfying

$$0 < \frac{\varepsilon}{abwN} \le 1. \quad (14)$$

2) Impact of duty cycle on diameter under delay constraint: The series representation of $\ln(x)$ is [10]

$$\ln(x) = \sum_{k=1}^{\infty}(-1)^{k+1}\frac{(x-1)^k}{k},\ 0 < x \le 2.$$

Ignoring the high order terms,

$$\begin{aligned}\ln\frac{\varepsilon}{abwN} &\approx \left(\frac{\varepsilon}{abwN}-1\right)-\frac{1}{2}\left(\frac{\varepsilon}{abwN}-1\right)^2\\ &= -\frac{1}{2}\left(\frac{\varepsilon}{abwN}\right)^2+\frac{2\varepsilon}{abwN}-\frac{3}{2}. \quad (15)\end{aligned}$$

Insert (15) into (13), we obtain, with a few manipulations,

$$\left(\frac{\lambda R\tau+\sigma}{\lambda R-\rho}+\frac{3w}{2(\lambda R-\rho)}\right)N^2-\left(\mathcal{D}+\frac{2\varepsilon}{ab(\lambda R-\rho)}\right)N + \frac{\varepsilon^2}{2w(\lambda R-\rho)(ab)^2} = 0,$$

namely,

$$N = \frac{\left(\mathcal{D}+\frac{2\varepsilon}{ab(\lambda R-\rho)}\right)\pm\sqrt{\Delta}}{2\left(\frac{\lambda R\tau+\sigma}{\lambda R-\rho}+\frac{3w}{2(\lambda R-\rho)}\right)}, \quad (16)$$

where

$$\Delta = \left(\mathcal{D}+\frac{2\varepsilon}{ab(\lambda R-\rho)}\right)^2 - \frac{2\varepsilon^2}{w(\lambda R-\rho)(ab)^2}\times\left(\frac{\lambda R\tau+\sigma}{\lambda R-\rho}+\frac{3w}{2(\lambda R-\rho)}\right).$$

C. Impact on Throughput

In this subsection, we focus on the impact of duty cycle on the throughput performance and on the allowed network diameter subject to throughput constraint.

1) Impact of duty cycle on throughput: The (end-to-end) throughput up to time t is defined as the time-average output rate, i.e., $T_t = \frac{A^*(t)}{t}$. As we have assumed there is no loss, the average output rate equals the average input rate at the steady state, i.e., $T_{t\to\infty} = \lim_{t\to\infty}\frac{A(t)}{t}$, which however does not mean no throughput fluctuations over time.

With the end-to-end stochastic service curve derived in the previous subsection as well as Proposition 2, a bound on the throughput for flow A_1 transmitting through the network can be found as

$$P\{T_t t > \alpha_1 \oslash \beta(t) + x\} \le \bigotimes_{1\le i\le N} f_i \otimes g_i(x), \quad (17)$$

where

$$\begin{aligned}\alpha_1 \oslash \beta(t) &= \sup_{s\geq 0}\{\alpha_1(t+s) - \beta(s)\}\\ &= \sup_{s\geq 0}\Bigg\{\sigma_1 + \rho_1(t+s) - \inf_{1\leq i\leq n}\{\lambda_i R_i - \rho_i\}s\\ &\quad + \sum_{i=1}^{N}\lambda_i R_i \tau_i + \sum_{i=2}^{N}\sigma_i\Bigg\}\\ &= \rho_1 t + \sum_{i=1}^{N}\lambda_i R_i \tau_i + \sum_{i=1}^{N}\sigma_i.\end{aligned}$$

Let $T = [\alpha_1 \oslash \beta(\Delta t)]|_{\Delta t=1} + x$ and set the right side of (17) equal to ξ, we get a throughput upper bound

$$T_t = \rho_1 + \frac{\sum_{i=1}^{N}(\lambda_i R_i \tau_i + \sigma_i) - \mathcal{W}\times \ln\frac{\xi}{\psi}}{t}. \tag{18}$$

Following the same assumption and analysis procedure in Section IV-B, we obtain the end-to-end throughput as

$$T_t = \rho + \frac{N(\lambda R\tau + \sigma) - Nw\times \ln\frac{\xi}{abwN}}{t}, \tag{19}$$

with $\lim_{t\to\infty} T_t = \rho$, which justifies our above discussion on the throughput at the steady-state and reveals that the impact of duty cycle on throughput fades away with time, i.e., the impact only exists during the transition period.

2) Impact of duty cycle on network diameter under throughput constraint: Following the same analysis procedure as in Sec. IV-B2, we have

$$\left((\lambda R\tau + \sigma) + \frac{3w}{2}\right)N^2 - \left((\mathcal{T} - \rho)t + \frac{2\xi}{ab}\right)N + \frac{\xi^2}{2w(ab)^2} = 0,$$

namely,

$$N = \frac{\left((\mathcal{T} - \rho)t + \frac{2\xi}{ab}\right) \pm \sqrt{\Delta}}{2\left((\lambda R\tau + \sigma) + \frac{3w}{2}\right)}, \tag{20}$$

where

$$\Delta = \left((\mathcal{T} - \rho)t + \frac{2\xi}{ab}\right)^2 - \frac{2\xi^2}{w(ab)^2}\times\left((\lambda R\tau + \sigma) + \frac{3w}{2}\right).$$

Note the time in (20) must be limited to the longest transition period of all the nodes in the network. It is obvious that the diameter is dependent on time in the transition period.

V. Numerical Results

To illustrate the analytical results obtained in the previous section, numerical results are presented in this section. In obtaining the numerical results, the following parameter values are used. The average arrival rate is normalized to $\rho = 1$ with data burst $\sigma = 2\rho$, and the service capacity is normalized to the average arrival rate $R = 1000$. The duty period is set to $\mathring{T} = 1$. When delay constraint is fixed, it is normalized to $\mathring{T}$ with $\mathcal{D} = 30$ and violation probability $\varepsilon = 0.001$. When throughput requirement is considered, it is set in the transition period with $\mathcal{T} = (1 + 0.05)\rho$ and $\xi = 0.001$. Whenever necessary, the network diameter is fixed to $N = 9$. For bounding functions, $a_f = a_g = 10$ and $b_f = b_g = 100$ are adopted.

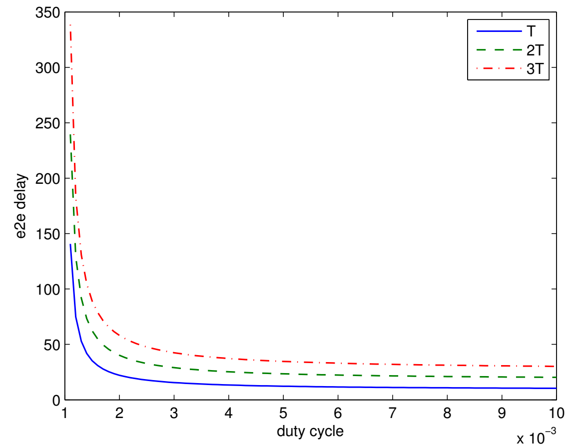

Fig. 1. E2e delay subject to duty cycle, when diameter is fixed.

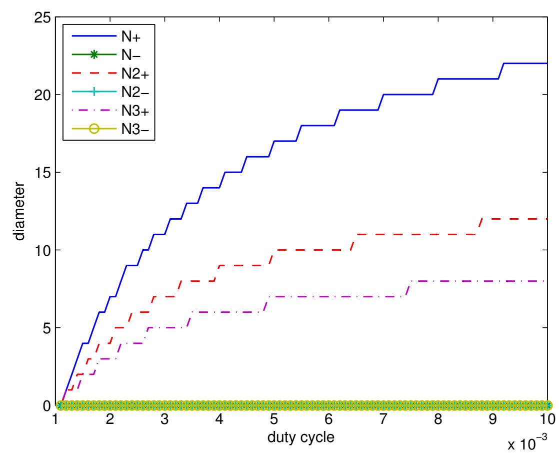

Fig. 2. Diameter subject to duty cycle, when e2e delay constraint is fixed.

A. Impact on Delay

The impact of duty cycle and duty period on end-to-end (e2e) delay is illustrated in Fig. 1. The figure shows that when the duty cycle is small, its impact on the e2e delay is prominent, while when the duty cycle increases the impact fades away. In addition, a biger duty cycle reduces the e2e delay. When the duty period is doubled and tripled with the duty cycle fixed, the e2e delay is reduced. This is because a bigger duty period comprises of a longer sleep time and hence triggers a longer delay.

Fig. 2 shows that under delay constraint, the allowed network diameter without violating the delay constraint increases

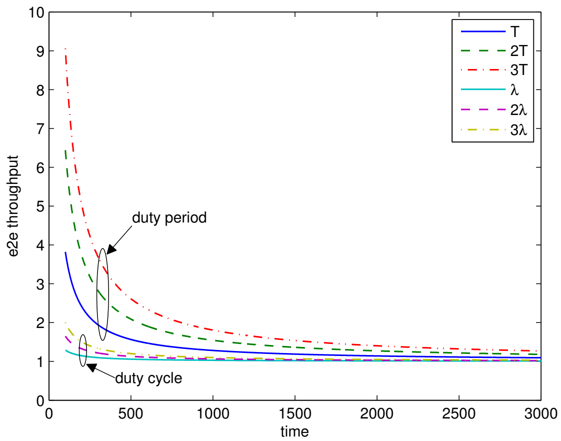

Fig. 3. Throughput subject to duty cycle, when diameter is fixed.

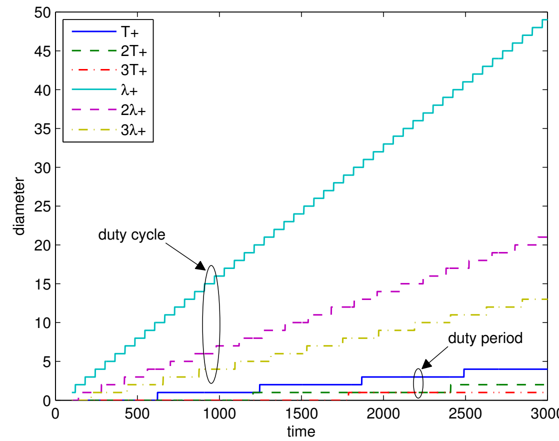

Fig. 4. Diameter subject to duty cycle, when throughput constraint is fixed.

with the duty cycle and decrease with the duty period. This is consistent with the common sense that when the duty cycle increases there will be less delay hence allowing more hops while satisfying the delay requirement, and for the duty period when the duty period increases maintaining the same duty cycle, the delay increases hence reducing hops under delay constraint. It is also shown that the smaller root in (16), e.g., $N-$, $N2-$, and $N3-$, is close to zero and makes no sense, which is the same for the smaller root in (20).

B. Impact on Throughput

In Fig. 3, the impact of duty cycle and duty period on throughput is explored in two cases, i.e., when the duty cycle is fixed the duty period is doubled and tribled, and when the duty period is fixed the duty cycle is doubled and tripled. Fig. 3 shows that the throughput of the network equals the average input of the network under the assumption of no data amount alteration. It is also shown that the throughput during the transition period increases with the duty cycle and duty period and the impact of the duty cycle and duty period on the throughput fades away with time.

Under throughput constraint, the impact of duty cycle and duty period on the allowed network diameter is also explored in two cases in Fig. 4. The diameter decreases with the duty cycle and duty period during the transition period. A heuristic explanation of the increase of diameter with time is that the transition periods of the network triggers a maximum e2e node number, i.e., diameter, and a bigger diameter requires a longer time to transmit to the steady state.

VI. Conclusion

In this paper, we considered a wireless sensor network, which is sleep-scheduled and synchronized, and explored the impact of duty cycle / period on the network performance, taking the network diameter into consideration. Specifically, we investigated the relationship between duty cycle, network diameter and network performance, and derived closed-form expressions of end-to-end delay and throughput subject to the duty cycle. Taking advantage of Taylor series, we also obtained closed-form expressions of network diameter subject to the duty cycle under performance constraints. The above procedure forms a reference analysis loop, i.e., the performance metrics are derived based on a generic network specification, then the network parameters are calculated satisfying a given performance requirement in a reverse style. We believe the specific results obtained in this paper can serve as a reference for network planning, design and optimization of WSNs.

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