Chapter 4 : LISTS

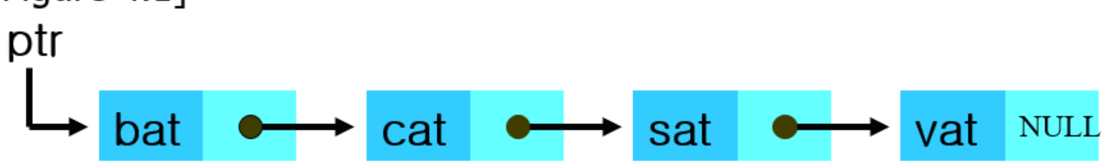
POINTERS

- **Sequential representation**
  - storing successive elements of the data object a fixed distance apart.
  - adequate for many operations.

  But difficulties occurs when
  - insertion and deletion of an arbitrary element (time-consuming)
  - storing several lists of varying sizes in different arrays of maximum size (waste of storage)
  - maintaining the lists in a single array (frequent data movements)

- **Linked representation**
  - A node, associated with an element in the list, contains a *data component* and a *pointer* to the next item in the list. The pointers are often called *links.*

[Figure 4.1]



- C provides extensive support for pointers
  - actual value of a pointer type is an address of memory.
  - operators
    - & : the address operator
    - * : the dereferencing (or indirection) operator.

- Pointer example:

```
int  i, *pi;             // declaration
pi = &i;                 // to assign the address of i as the value of pi
i=10; or *pi = 10;       // to assign a value to i
```

- C allows us to perform arithmetic operations and relational operations on pointers. Also we can convert pointers explicitly to integers.

- The null pointer points to no object or function.
- Typically the null pointer is represented by the integer 0.
- There is a macro called NULL which is defined to be this constant.

- The macro is defined either

  in *stddef.h* for ANSI C or in *stdio.h* for K&R C.

- To test for the null pointer on C

  if (pi == NULL) or if (!pi)

Pointers Can Be Dangerous

- By using pointers we can attain a high degree of flexibility and efficiency.

- But pointer can be dangerous: accessing unexpected memory locations

  - Set all pointers to NULL when they are not actually pointing to an object.
  - Explicit *type casts* when converting between pointer types.

```
pi = malloc(sizeof(int));    /* assign to pi a pointer to int */
pf = (float *) pi;     /* casts an int pointer to a float pointer */
```

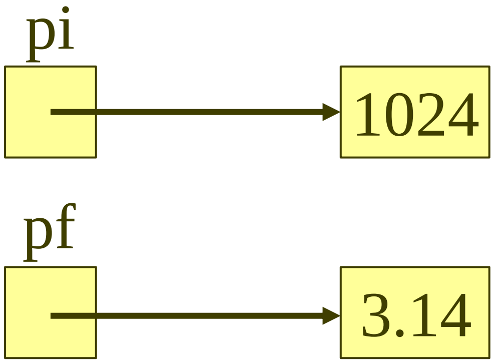
  - Define explicit return types for functions.

Using Dynamically Allocated Storage

- **malloc** to request a new area of memory
- **free** to return an area of memory to the system

[Program 1.1]

```
int *pi;
float *pf;
pi = (int *) malloc(sizeof(int));
pf = (float *) malloc(sizeof(float));
*pi = 1024;
*pf = 3.14;
printf("an integer = %d, a float = %f\n", *pi, *pf);
free(pi);
free(pf);
```



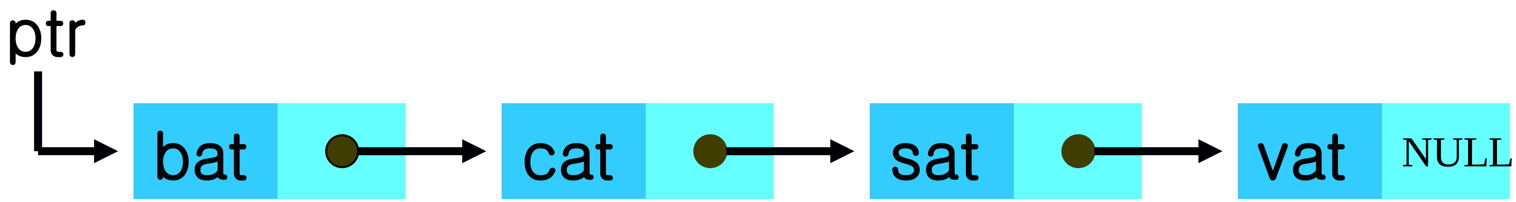
Inserting

pf = (float *) malloc(sizeof(float));

immediately after the printf statement creates *Garbage, Dangling reference*

4.1 SINGLY LINKED LISTS

[Figure 4.2]



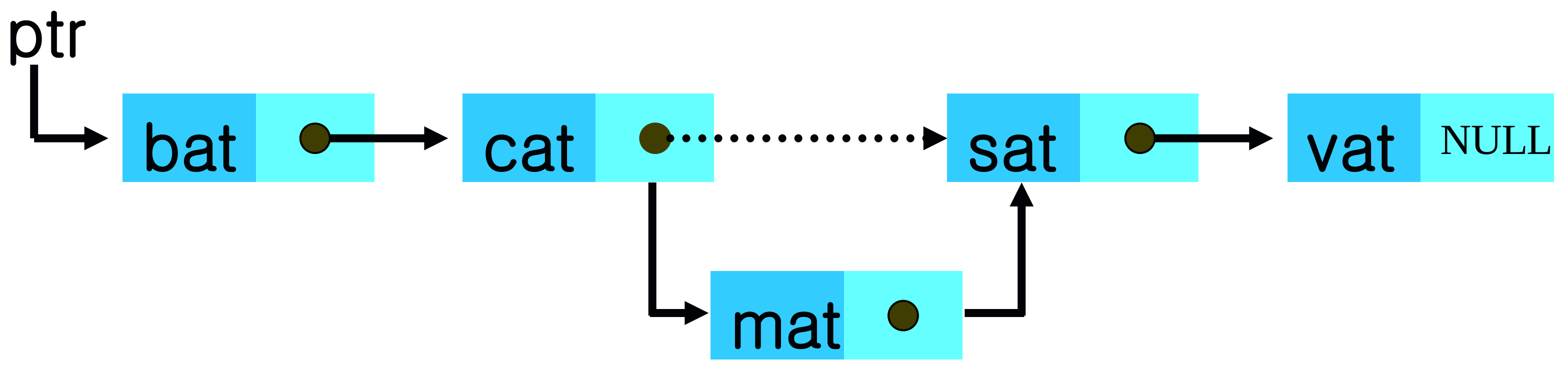
- The name of the pointer to the first node in the list is the name of the list.
- **Note that**

  (1) the nodes do not reside in sequential locations

  (2) the locations of the nodes may change on the different runs.
- When we write a program that works with lists, we almost never look for a specific address except when we test for the end of the list.

- **To insert the word *mat* between *cat* and *sat*, we must :**
  - (1) Get a node that is currently unused; let its address be *paddr*.
  - (2) Set the data field of this node to *mat*.
  - (3) Set *paddr*'s link field to point to the address found in the link field of the node containing *cat*.
  - (4) Set the link field of the node containing *cat* to pointer to *paddr*.

[Figure 4.3]

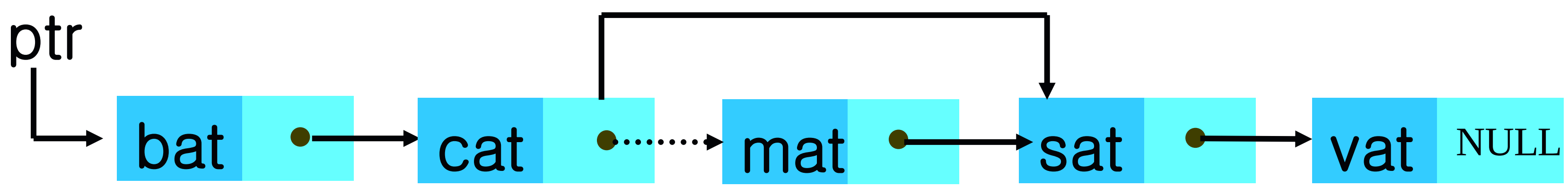


- **To delete *mat* from the list.**

  (1) Find the element (node) that immediately precedes *mat*, which is *cat*.

  (2) Set *cat*'s link field to point to *mat*'s link field.

[Figure 4.3]



4.2 REPRESENTING CHAINS IN C

- **Necessary capabilities to make linked representations possible:**

  (1) A mechanism for defining a node's structure,
  self-referential structures.

  (2) A way to create new nodes when we need them, *malloc*.

  (3) A way to remove nodes that we no longer need, *free.*

- **<u>Example 4.1</u> [List of words]**

  Necessary declarations are :

```
typedef struct  list_node  *list_pointer;
typedef struct  list_node {
      char  data[4];
      list_pointer  link;
};
list_pointer  ptr = NULL;     /* creating a new empty list */
```

- **A macro to test for an empty list :**

  #define IS_EMPTY(ptr) (!(ptr))

- **Creating new nodes :**

  use the *malloc* function provided in <*stdlib.h*>.

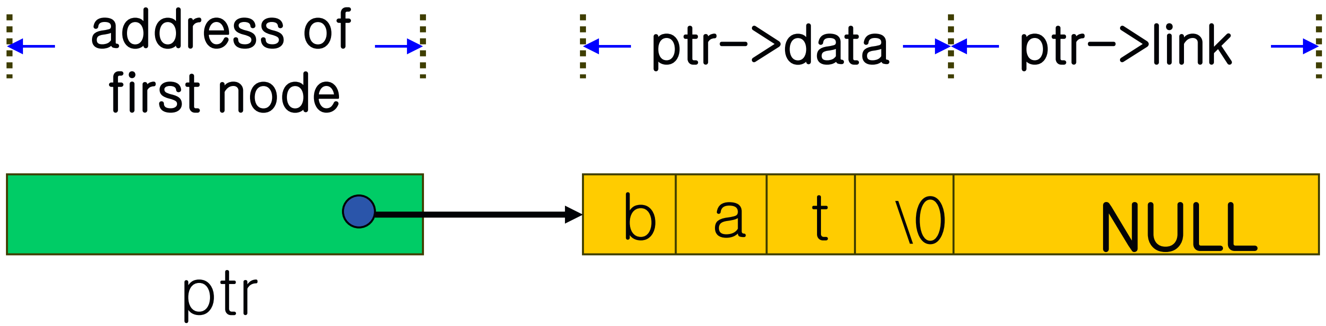
  ptr = (list_pointer) malloc (sizeof(list_node));

- **Assigning values to the fields of the node:**
  - If *e* is a pointer to a structure that contains the field *name*, then *e->name* is a shorthand way of writing the expression (*e).*name*.

- **To place the word bat into the list :**

```
strcpy (ptr->data, "bat");
ptr->link = NULL;
```

[Figure 4.5]



■ Example 4.2 [Two-node linked list]

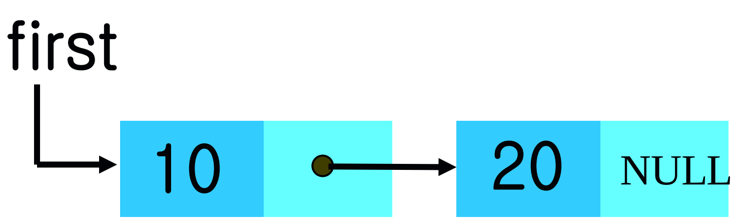
The node structure is defined as:

```
typedef struct  list_node  *list_pointer;
typedef struct  list_node {
      int  data;
      list_pointer  link;
};
list_pointer  ptr = NULL;
```

[Program 4.1] Create a two-node list

```
list_pointer create2()
{
    /*  create a linked list with two nodes  */
    list_pointer first, second;
    first = (list_pointer) malloc(sizeof(list_node));
    second = (list_pointer) malloc(sizeof(list_node));
    second->link = NULL;
    second->data = 20;
    first->data = 10;
    first->link = second;
    return first;
}
```

[Figure 4.6]



- **Example 4.3 [List insertion] :**
  - To insert a node with data field of 50 after some arbitrary node.

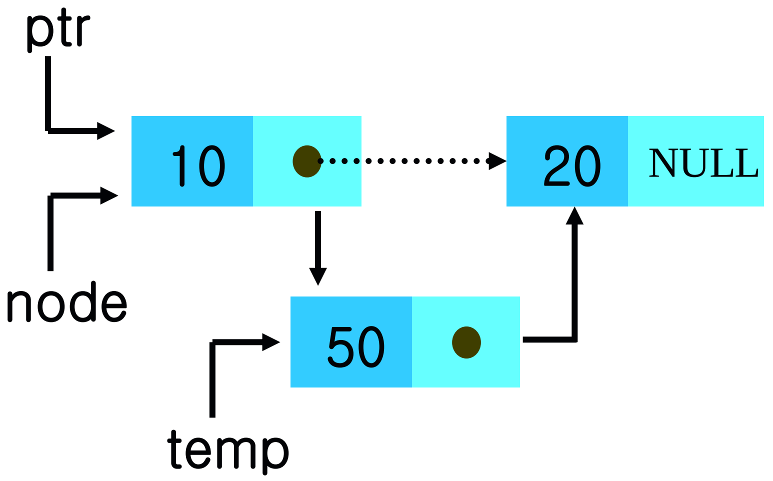
    Note that we pass in the address of the first node in the list, so that we can change it if there are no nodes in the list.
  - We use a new macro, IS_FULL,

    that allows us to determine if we have used all available memory.

```
#define IS_FULL (ptr)  (!(ptr))
```

- **[Program 4.2]**

```
void insert(list_pointer *ptr, list_pointer node)
{
    /* insert a new node with data=50 into the list ptr after node */
    list_pointer temp;
    temp = (list_pointer) malloc(sizeof(list_node));
    if (IS_FULL(temp))  {
        fprintf(stderr, "The memory is full\n");
        exit(1);
    }
    temp->data = 50;
    if (*ptr)  {
        temp->link = node->link;
        node->link = temp;
    }
    else  {
        temp->link = NULL;
        *ptr = temp;
    }
}
```



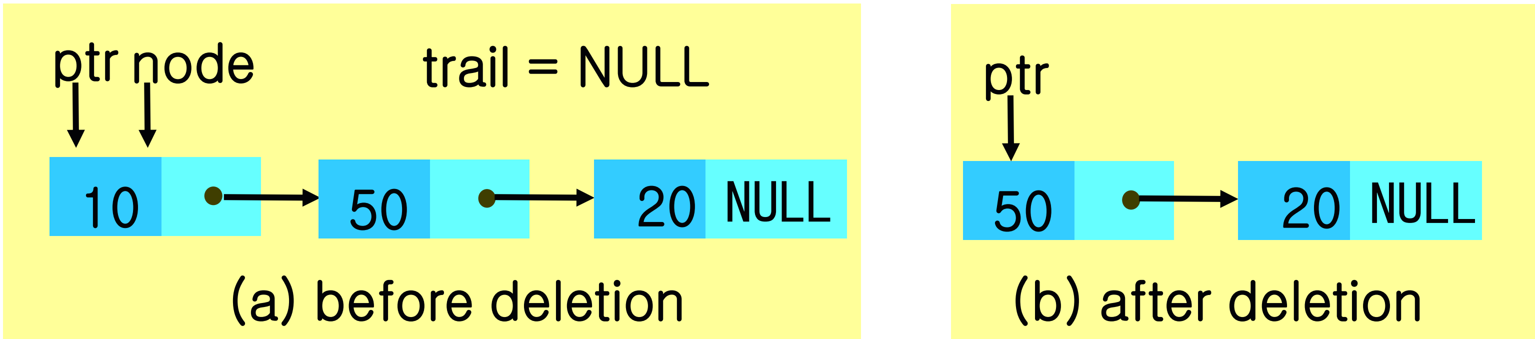
- **Example 4.4 [List deletion] :**
  - Deletion depends on the location of the node to be deleted.
  - Assume three pointers :

    ptr points to the start of the list.

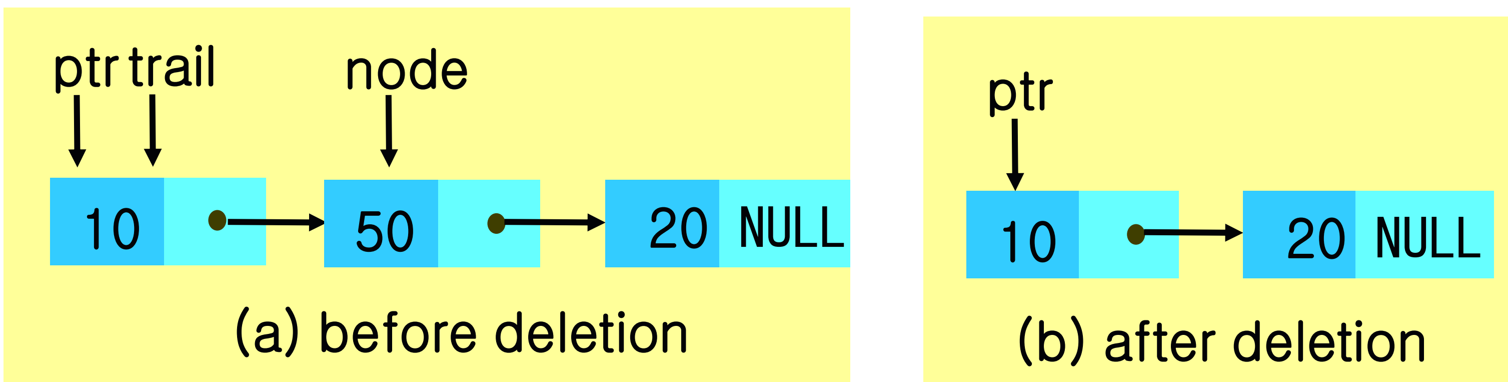
    node points to the node that we wish to delete.

    trail points to the node that precedes the node to be deleted.
- **[Program 4.4]**

```
void delete(list_pointer *ptr, list_pointer trail,  list_pointer node)
{
      /* delete node from the list, trail is the preceding node
ptr is the head of the list  */
      if (trail)
            trail->link = node->link;
      else
            *ptr = (*ptr)->link;
      free(node);
}
```



[Figure 4.8] ***delete(&ptr, NULL, ptr);***



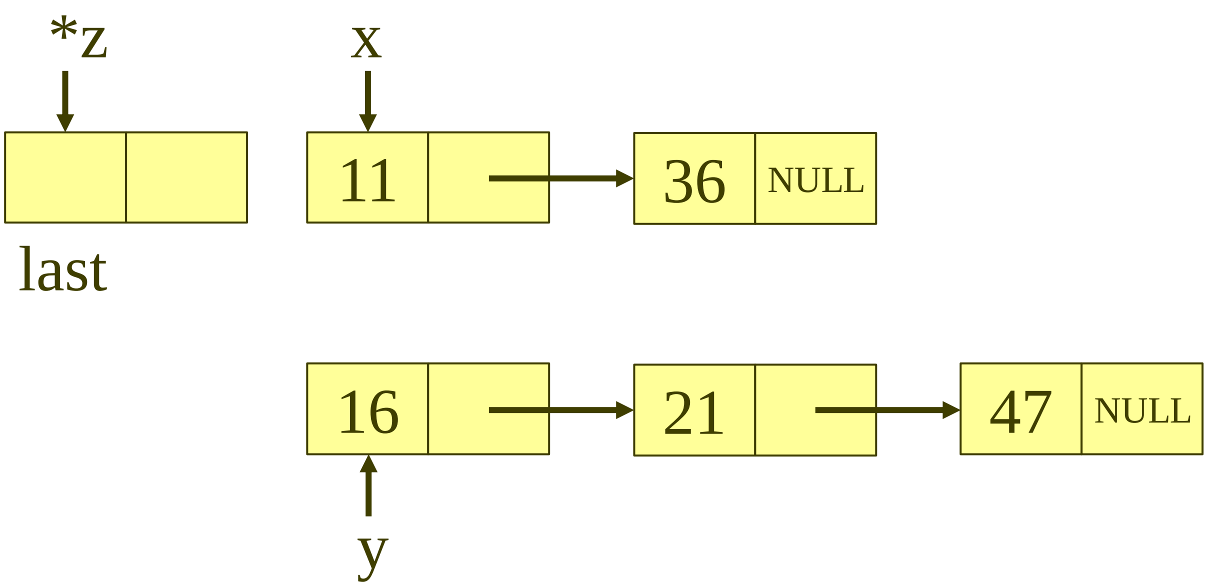
[Figure 4.9] ***delete(&ptr, ptr, ptr->link);***

- **Example 4.5 [Printing out a list] :**
- **[Program 4.4]**

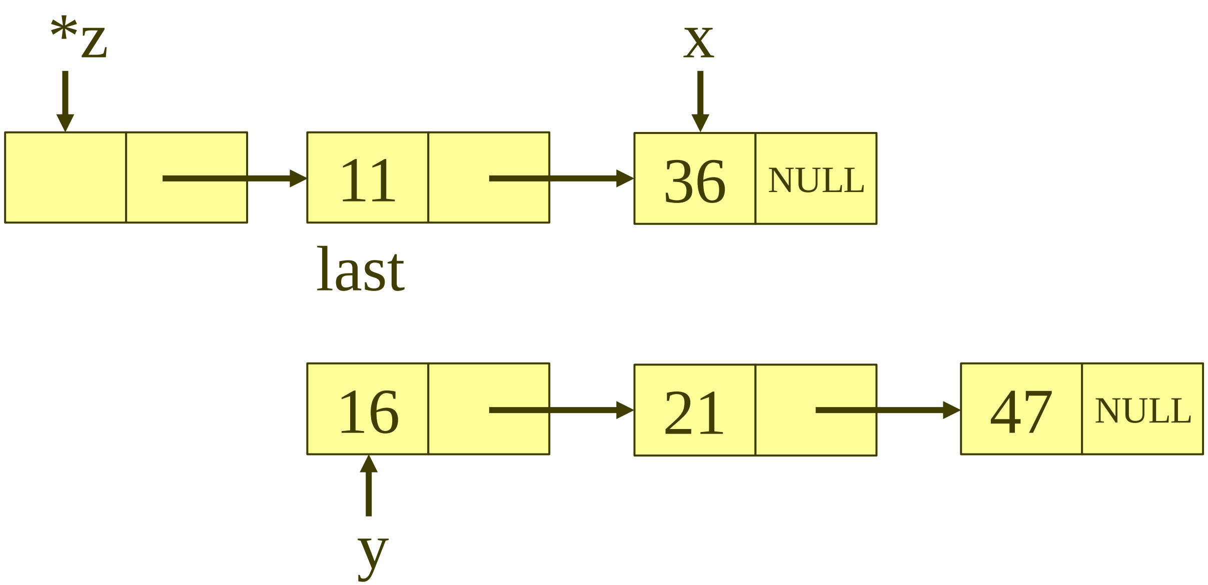
```
void print_list(list_pointer ptr)
{
    printf("The list contains: ");
    for ( ; ptr; ptr = ptr->link)
        printf("%4d", ptr->data);
    printf("\n");
}

list_pointer  search (list_pointer ptr, int num)
{
    for ( ; ptr; ptr = ptr->link)
        if (ptr->data == num)   return  ptr;
    return  ptr;
}
```

```
void  merge (list_pointer x,  list_pointer y,  list_pointer *z)
{
        list_pointer  last;
        last = (list_pointer) malloc(sizeof(list_node));
        *z = last;
        while (x && y)  {
                if (x->data <= y->data)  {
                        last->link = x;
                        last = x;
                        x = x->link;
                }
                else  {
                        last->link = y;
                        last = y;
                        y = y->link;
                }
        }
        if (x)   last->link = x;
        if (y)   last->link = y;
        last = *z;  *z = last->link;  free(last);
}
```



```
void  merge (list_pointer x,  list_pointer y,  list_pointer *z)
{
        list_pointer  last;
        last = (list_pointer) malloc(sizeof(list_node));
        *z = last;
        while (x && y)  {
                if (x->data <= y->data)  {
                        last->link = x;
                        last = x;
                        x = x->link;
                }
                else  {
                        last->link = y;
                        last = y;
                        y = y->link;
                }
        }
        if (x)   last->link = x;
        if (y)   last->link = y;
        last = *z;  *z = last->link;  free(last);
 }
```



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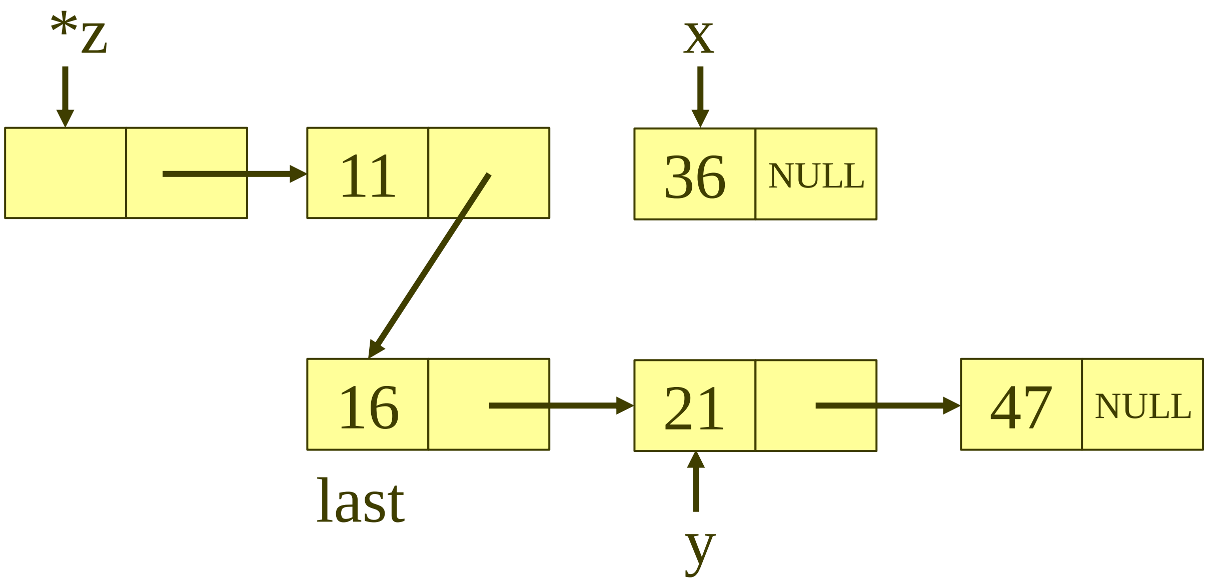
                if (x->data <= y->data)  {
                        last->link = x;
                        last = x;
                        x = x->link;
                }
                else  {
                        last->link = y;
                        last = y;
                        y = y->link;
                }
        }
        if (x)   last->link = x;
        if (y)   last->link = y;
        last = *z;  *z = last->link;  free(last);
}
```



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{
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        *z = last;
        while (x && y)  {
                if (x->data <= y->data)  {
                        last->link = x;
                        last = x;
                        x = x->link;
                }
                else  {
                        last->link = y;
                        last = y;
                        y = y->link;
                }
        }
        if (x)   last->link = x;
        if (y)   last->link = y;
        last = *z;  *z = last->link;  free(last);
}
```

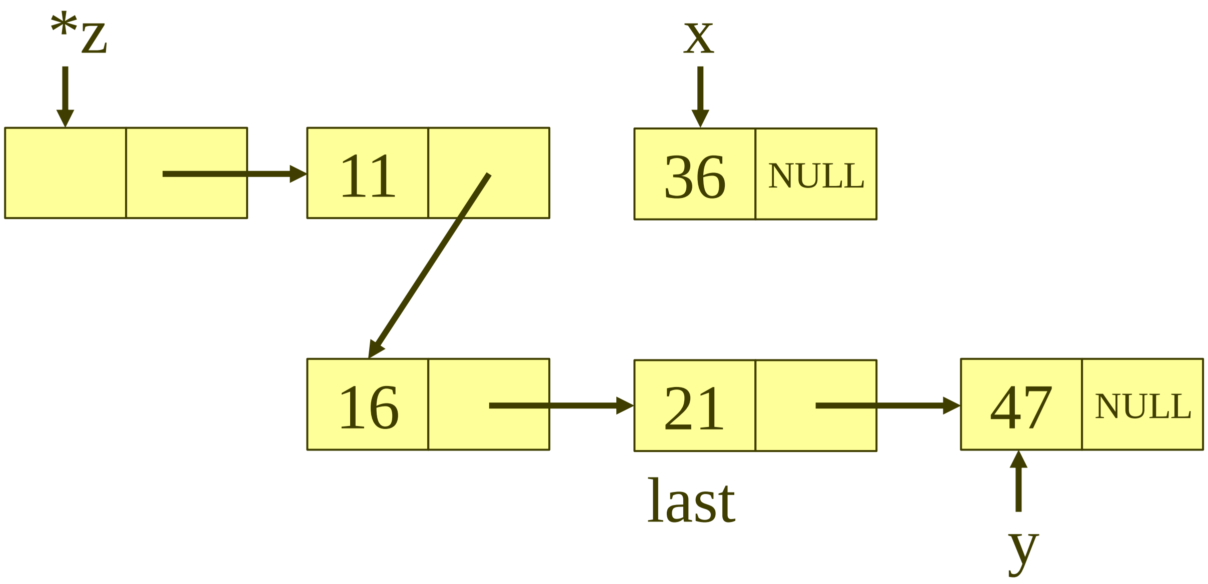

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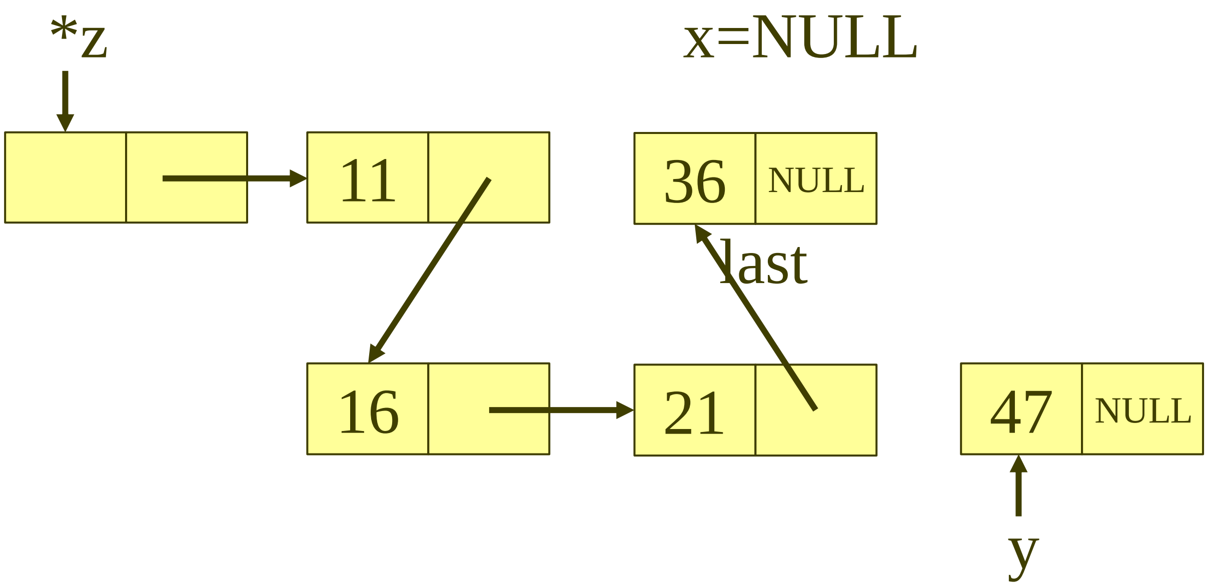
                        last->link = x;
                        last = x;
                        x = x->link;
                }
                else  {
                        last->link = y;
                        last = y;
                        y = y->link;
                }
        }
        if (x)   last->link = x;
        if (y)   last->link = y;
        last = *z;  *z = last->link;  free(last);
}
```



```
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{
        list_pointer  last;
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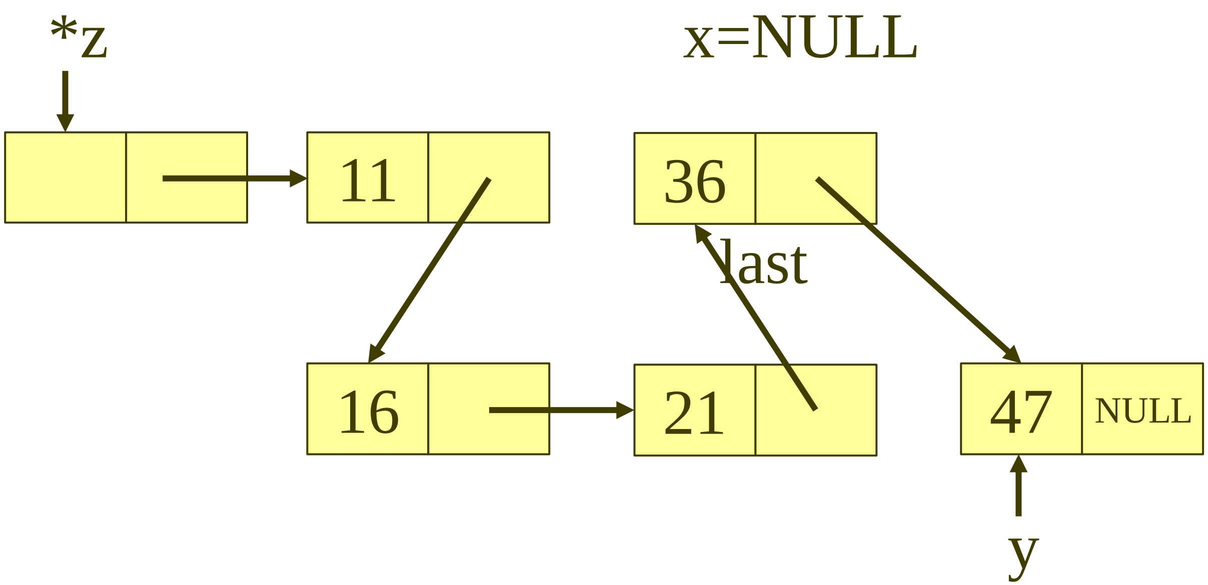
        *z = last;
        while (x && y)  {
                if (x->data <= y->data)  {
                        last->link = x;
                        last = x;
                        x = x->link;
                }
                else  {
                        last->link = y;
                        last = y;
                        y = y->link;
                }
        }
        if (x)   last->link = x;
        if (y)   last->link = y;
        last = *z;  *z = last->link;  free(last);
 }
```



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        *z = last;
        while (x && y)  {
                if (x->data <= y->data)  {
                        last->link = x;
                        last = x;
                        x = x->link;
                }
                else  {
                        last->link = y;
                        last = y;
                        y = y->link;
                }
        }
        if (x)   last->link = x;
        if (y)   last->link = y;
        last = *z;  *z = last->link;  free(last);
}
```

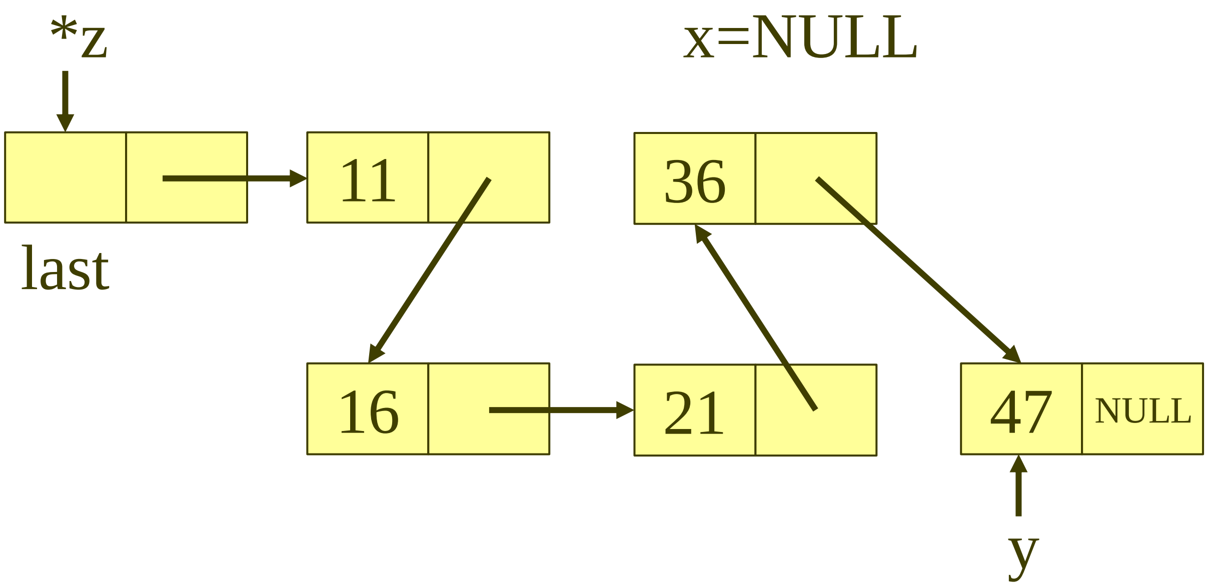

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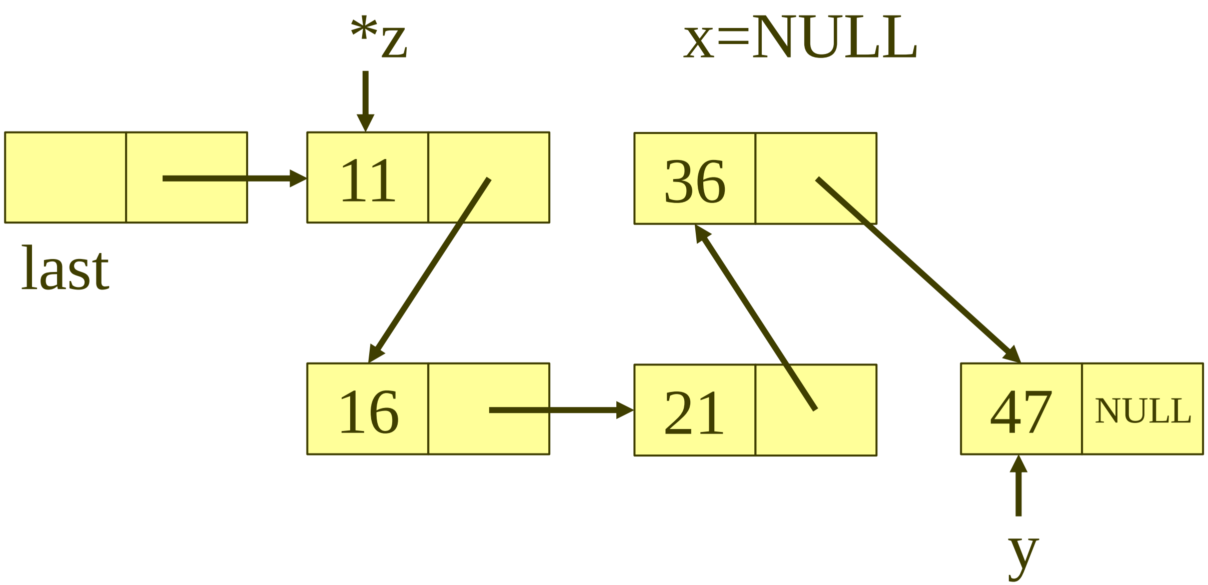
        *z = last;
        while (x && y)  {
                if (x->data <= y->data)  {
                        last->link = x;
                        last = x;
                        x = x->link;
                }
                else  {
                        last->link = y;
                        last = y;
                        y = y->link;
                }
        }
        if (x)   last->link = x;
        if (y)   last->link = y;
        last = *z;  *z = last->link;  free(last);
}
```



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        *z = last;
        while (x && y)  {
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                        last->link = x;
                        last = x;
                        x = x->link;
                }
                else  {
                        last->link = y;
                        last = y;
                        y = y->link;
                }
        }
        if (x)   last->link = x;
        if (y)   last->link = y;
        last = *z;  *z = last->link;  free(last);
}
```
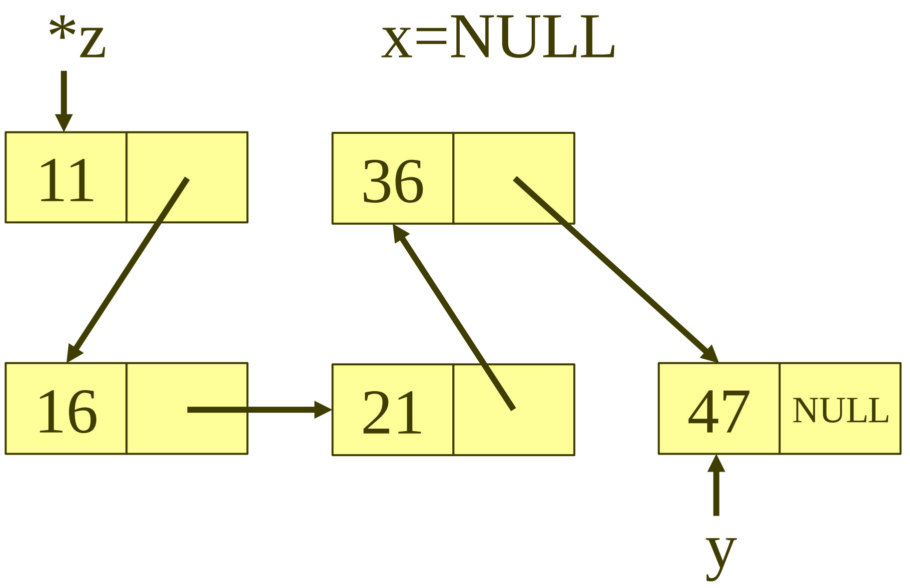
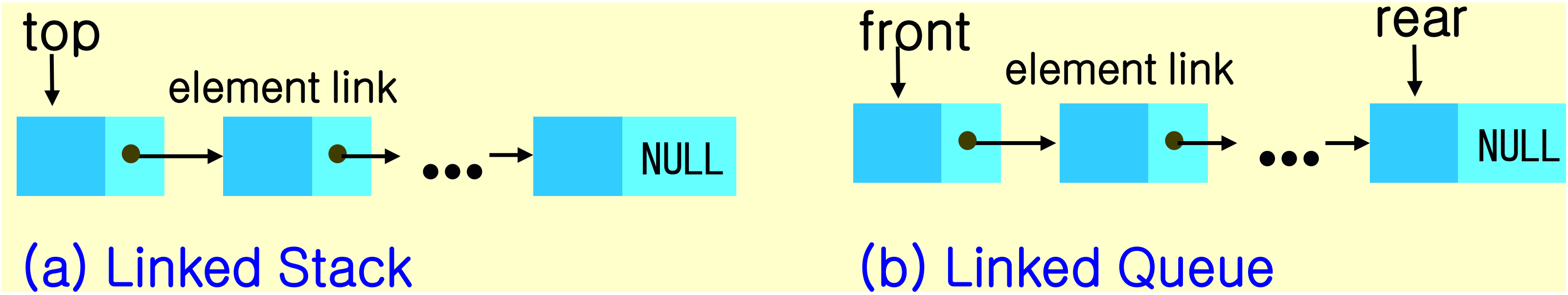



4.3 LINKED STACKS AND QUEUES

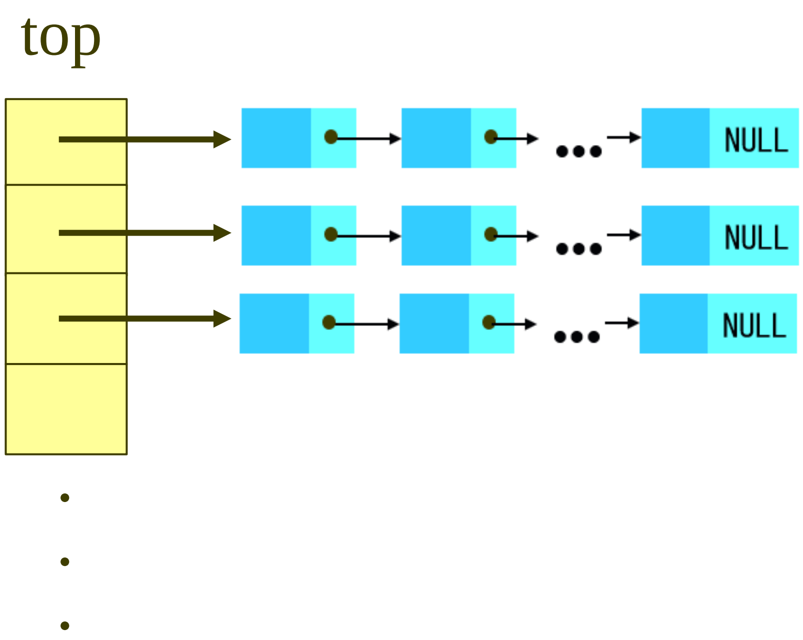
- Sequential representation is proved efficient if we had only one stack or one queue.
- When several stacks and queues coexisted, there was no efficient way to represent them sequentially.
- Linked stacks and linked queues.



Notice that the direction of links for both the stack and the queue facilitate easy insertion and deletion of nodes.

- To represent n(<=MAX_STACKS) stacks simultaneously:

```
#define MAX_STACKS 10  /* maximum number of stacks */
typedef struct  {
      int key;
      /* other fields */
} element;
typedef struct stack *stack_pointer;
typedef struct stack  {
      element  data;
      stack_pointer  link;
};
stack_pointer  top[MAX_STACKS];
```



- **initialize empty stacks :**

  top[*i*] = NULL, 0 <= I < MAX_STACKS

- **the boundary conditions :**

  top[*i*] == NULL *iff* the *i* th stack is empty

and

  IS_FULL(temp) *iff* the memory is full

[Program 4.5] Add to a linked stack

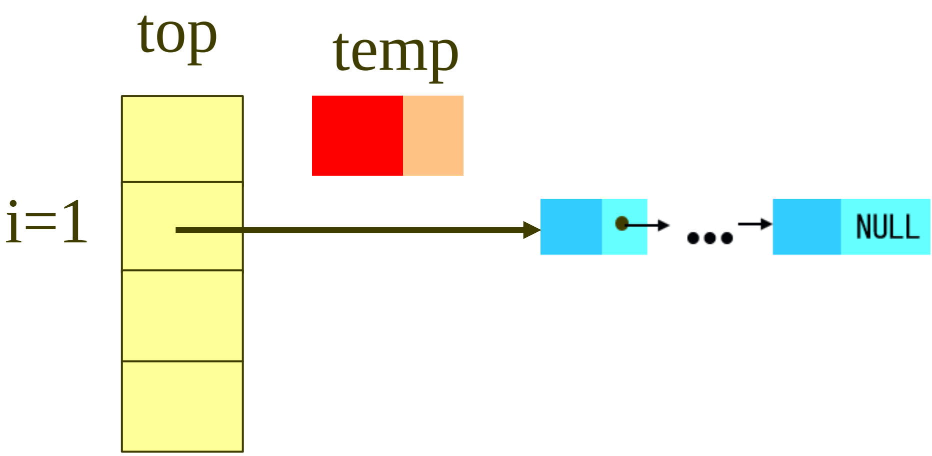
```
void push(int i, element item)
{
    /*  add item to the ith stack  */
    stack_pointer  temp = (stack_pointer) malloc(sizeof(stack));
    if (IS_FULL(temp))  {
        fprintf(stderr, "The memory is full\n");
        exit(1);
    }
    temp->data = item;
    temp->link = top[i];
    top[i] = temp;
}
```

call : ***push(i, item)*****;**

[Program 4.5] Add to a linked stack

```
void push(int i, element item)
{
    /*  add item to the ith stack  */
    stack_pointer  temp = (stack_pointer) malloc(sizeof(stack));
    if (IS_FULL(temp))  {
        fprintf(stderr, "The memory is full\n");
        exit(1);
    }
    temp->data = item;
    temp->link = top[i];
    top[i] = temp;
}
```

call : ***push(i, item)*****;**



■ [Program 4.5] Add to a linked stack

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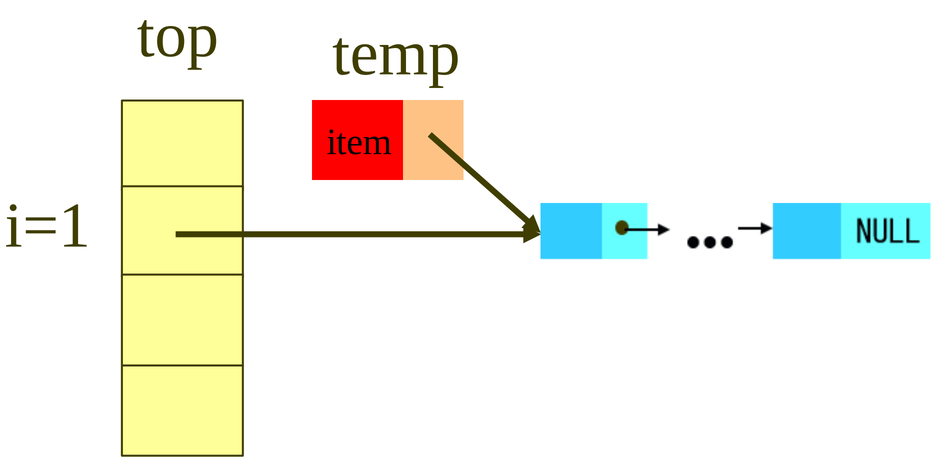
{
    /*  add item to the ith stack  */
    stack_pointer  temp = (stack_pointer) malloc(sizeof(stack));
    if (IS_FULL(temp))  {
        fprintf(stderr, "The memory is full\n");
        exit(1);
    }
    temp->data = item;
    temp->link = top[i];
    top[i] = temp;
}
```

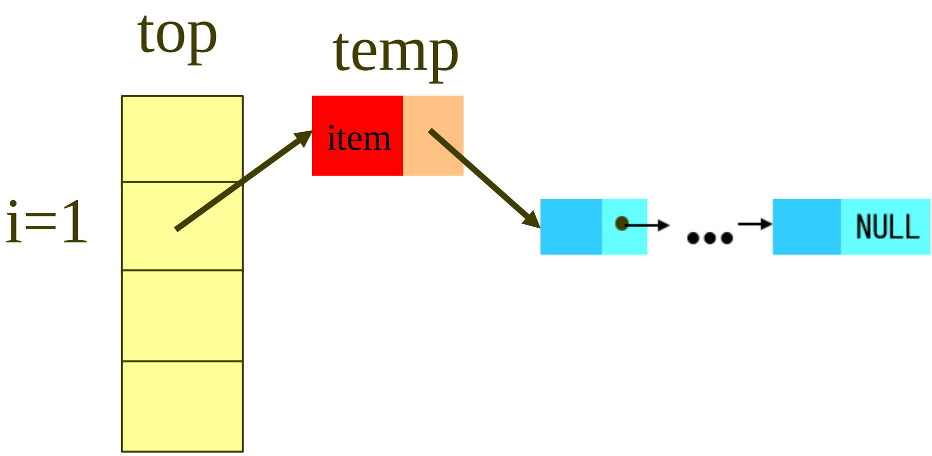
call : ***push(i, item)*****;**



[Program 4.5] Add to a linked stack

```
void push(int i, element item)
{
    /*  add item to the ith stack  */
    stack_pointer  temp = (stack_pointer) malloc(sizeof(stack));
    if (IS_FULL(temp))  {
        fprintf(stderr, "The memory is full\n");
        exit(1);
    }
    temp->data = item;
    temp->link = top[i];
    top[i] = temp;
}
```

call : ***push(i, item)*;**



[Program 4.6] Delete from a linked stack

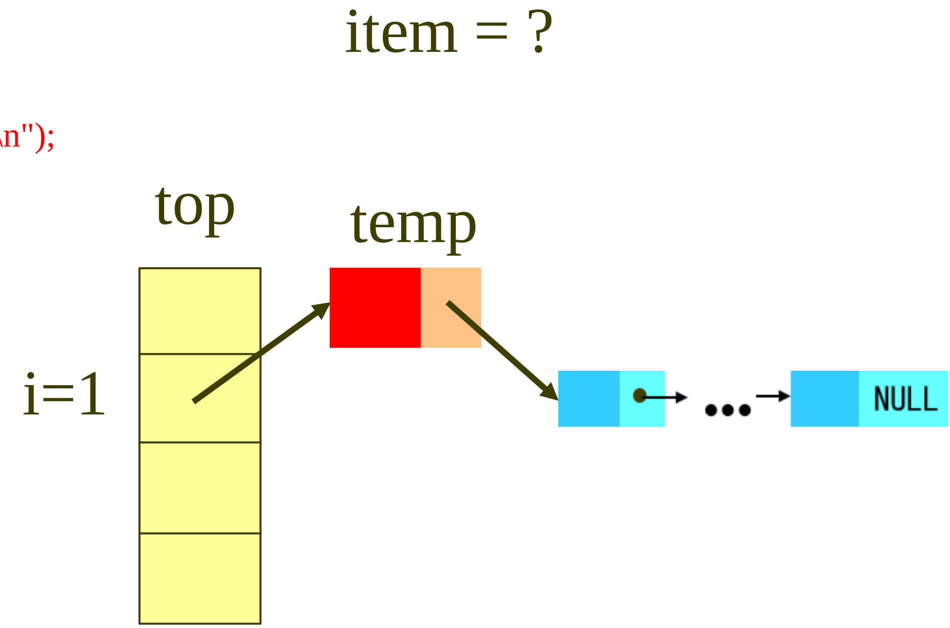
```
element pop(int i)
{
    /*  remove top element from the ith stack */
    stack_pointer  temp = top[i];
    element  item;
    if (IS_EMPTY(temp))  {
        fprintf(stderr, "The stack is empty\n");
        exit(1);
    }
    item = temp->data;
    top[i] = temp->link;
    free(temp);
    return  item;
}
```

call : ***item = pop(i)*;**

[Program 4.6] Delete from a linked stack

```
element pop(int i)
{
    /*  remove top element from the ith stack */
    stack_pointer  temp = top[i];
    element  item;
    if (IS_EMPTY(temp))  {
        fprintf(stderr, "The stack is empty\n");
        exit(1);
    }
    item = temp->data;
    top[i] = temp->link;
    free(temp);
    return  item;
}
```

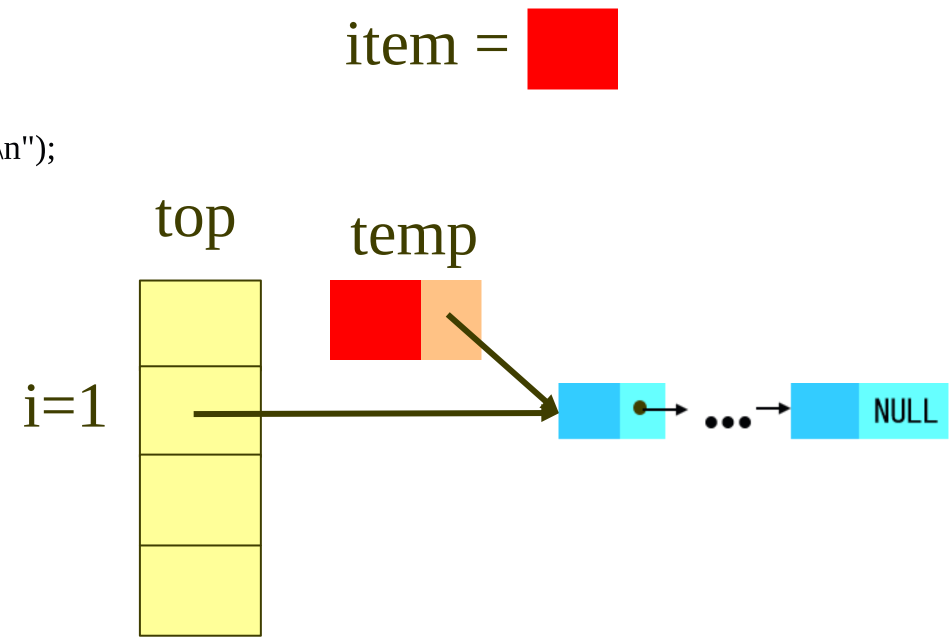
call : ***item = pop(i)*;**



[Program 4.6] Delete from a linked stack

```
element pop(int i)
{
    /*  remove top element from the ith stack */
    stack_pointer  temp = top[i];
    element  item;
    if (IS_EMPTY(temp))  {
        fprintf(stderr, "The stack is empty\n");
        exit(1);
    }
    item = temp->data;
    top[i] = temp->link;
    free(temp);
    return  item;
}
```

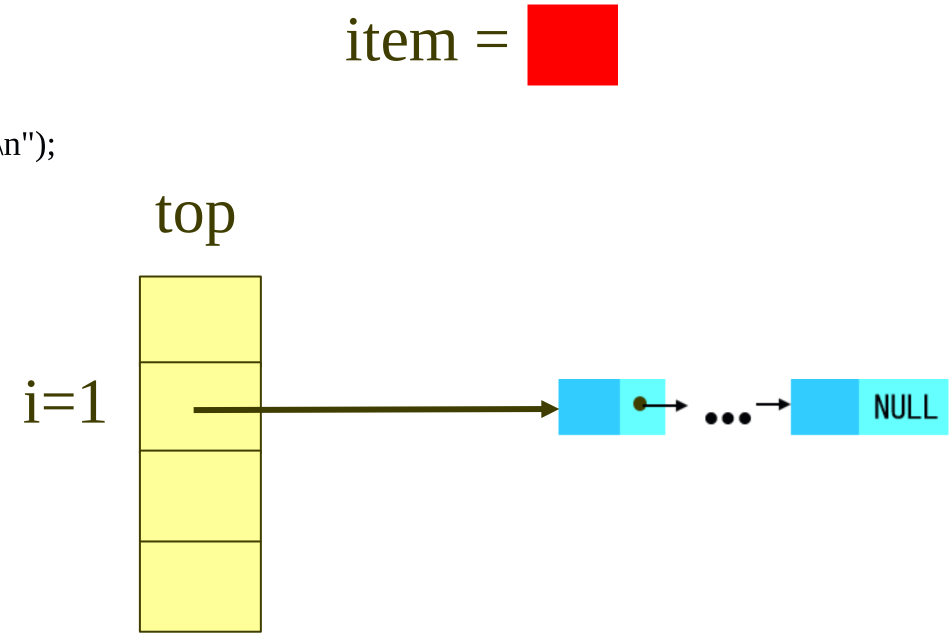
call : ***item = pop(i)*;**



[Program 4.6] Delete from a linked stack

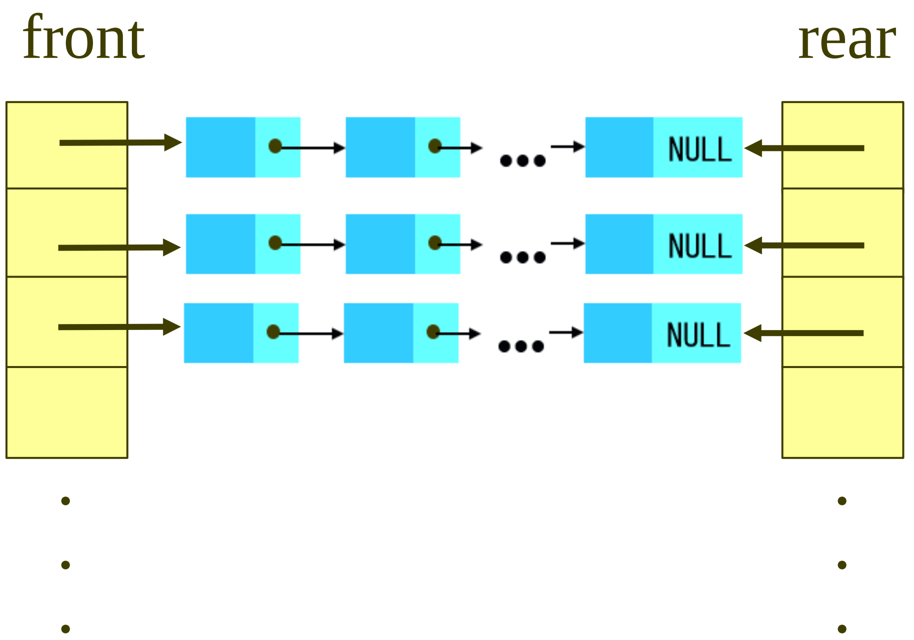
```
element pop(int i)
{
    /*  remove top element from the ith stack */
    stack_pointer  temp = top[i];
    element  item;
    if (IS_EMPTY(temp))  {
        fprintf(stderr, "The stack is empty\n");
        exit(1);
    }
    item = temp->data;
    top[i] = temp->link;
    free(temp);
    return  item;
}
```

call : ***item = pop(i)*;**



- To represent m(<=MAX_QUEUES) queues simultaneously:

```
#define MAX_QUEUES 10  /* maximum number of queues */
typedef struct  {
      int key;
      /* other fields */
} element;
typedef struct queue *queue_pointer;
typedef struct queue  {
      element  data;
      queue_pointer  link;
};
queue_pointer  front[MAX_QUEUES], rear[MAX_QUEUES];
```



- **initialize empty queues :**

  front[*i*] = NULL, 0<=i<MAX_QUEUES

- **the boundary conditions :**

  front[*i*] == NULL *iff* the *i* th queue is empty

and

  IS_FULL(temp) *iff* the memory is full

[Program 4.7] Add to the rear of a linked queue

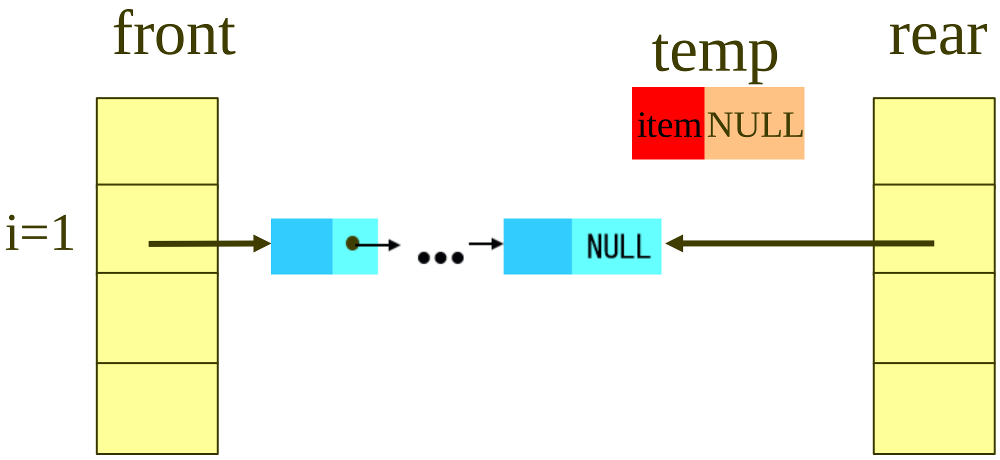
```
void addq(int i, element item)
{
    /*  add item to the rear of queue i  */
    queue_pointer  temp =  (queue_pointer) malloc(sizeof(queue));
    if (IS_FULL(temp))  {
        fprintf(stderr, "The memory is full\n");
        exit(1);
    }
    temp->data = item;
    temp->link = NULL;
    if (front[i])  rear[i]->link = temp;
    else  front[i] = temp;
    rear[i] = temp;
}
```

call : ***addq*****(i, item);**

- **[Program 4.7] Add to the rear of a linked queue**

```
void addq(int i, element item)
{
    /* add item to the rear of queue i */
    queue_pointer temp = (queue_pointer) malloc(sizeof(queue));
    if (IS_FULL(temp)) {
        fprintf(stderr, "The memory is full\n");
        exit(1);
    }
    temp->data = item;
    temp->link = NULL;
    if (front[i]) rear[i]->link = temp;
    else front[i] = temp;
    rear[i] = temp;
}
```

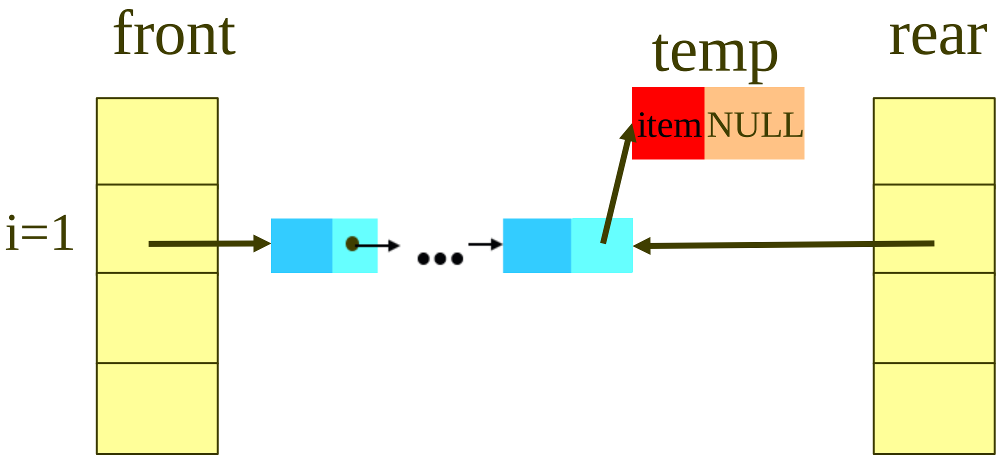
call : ***addq*****(i, item);**



[Program 4.7] Add to the rear of a linked queue

```
void addq(int i, element item)
{
    /*  add item to the rear of queue i  */
    queue_pointer  temp =  (queue_pointer) malloc(sizeof(queue));
    if (IS_FULL(temp))  {
        fprintf(stderr, "The memory is full\n");
        exit(1);
    }
    temp->data = item;
    temp->link  = NULL;
    if (front[i])  rear[i]->link  = temp;
    else  front[i] = temp;
    rear[i] = temp;
}
```

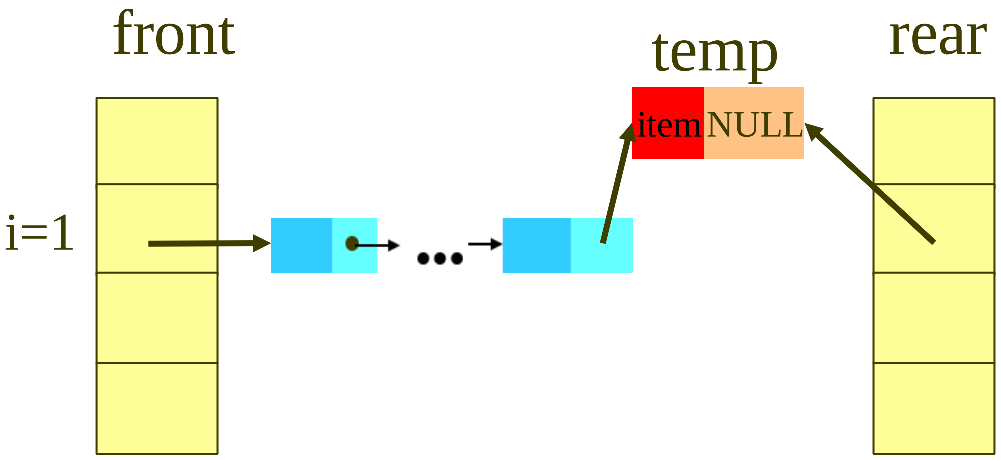
call : *addq*(i, item);



[Program 4.7] Add to the rear of a linked queue

```
void addq(int i, element item)
{
    /*  add item to the rear of queue i  */
    queue_pointer  temp =  (queue_pointer) malloc(sizeof(queue));
    if (IS_FULL(temp))  {
        fprintf(stderr, "The memory is full\n");
        exit(1);
    }
    temp->data = item;
    temp->link  = NULL;
    if (front[i])  rear[i]->link  = temp;
    else  front[i]  = temp;
    rear[i] = temp;
}
```

call : *addq*(i, item);



[Program 4.8] Delete from the front of a linked queue

```
element deleteq(int i)
{
    /*  delete an element from queue i */
    queue_pointer  temp = front[i];
    element  item;
    if (IS_EMPTY(front[i]))  {
        fprintf(stderr, "The queue is empty\n");
        exit(1);
    }
    item = temp->data;
    front[i] = temp->link;
    free(temp);
    return  item;
}
```

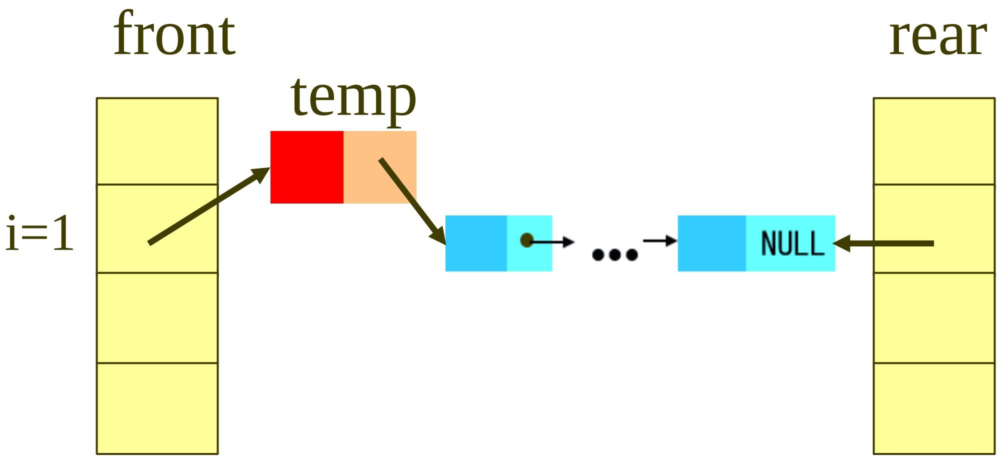
call : ***item*** **=** ***deleteq*****(i);**

- **[Program 4.8] Delete from the front of a linked queue**

```
element deleteq(int i)
{
    /* delete an element from queue i */
    queue_pointer temp = front[i];
    element item;
    if (IS_EMPTY(front[i])) {
        fprintf(stderr, "The queue is empty\n");
        exit(1);
    }
    item = temp->data;
    front[i] = temp->link;
    free(temp);
    return item;
}
```

call : *item* = *deleteq*(i);

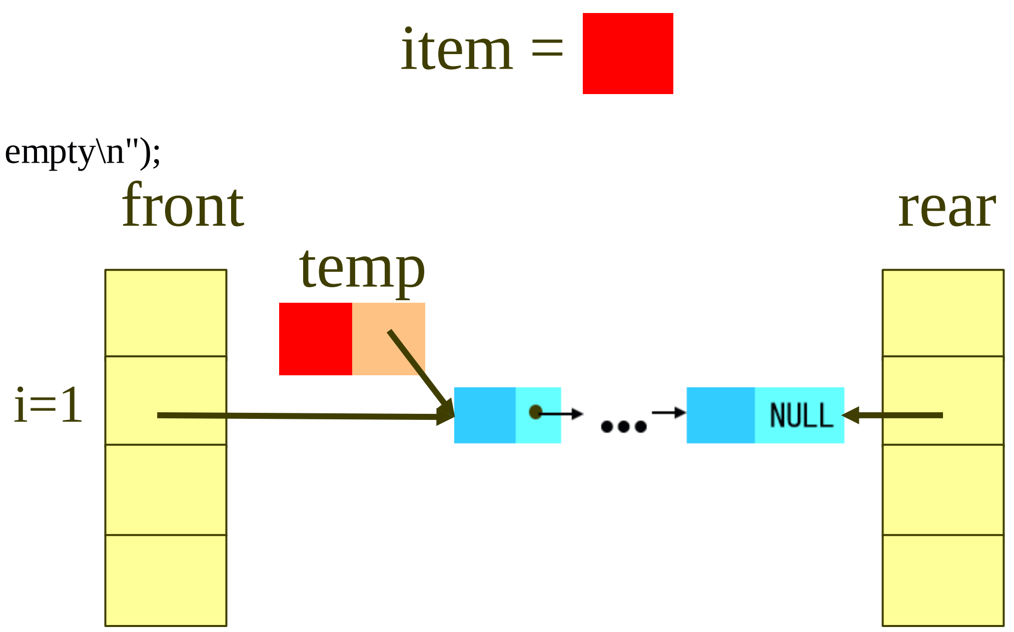
item = ?



- **[Program 4.8] Delete from the front of a linked queue**

```
element deleteq(int i)
{
    /* delete an element from queue i */
    queue_pointer  temp = front[i];
    element  item;
    if (IS_EMPTY(front[i]))  {
        fprintf(stderr, "The queue is empty\n");
        exit(1);
    }
    item = temp->data;
    front[i] = temp->link;
    free(temp);
    return  item;
}
```

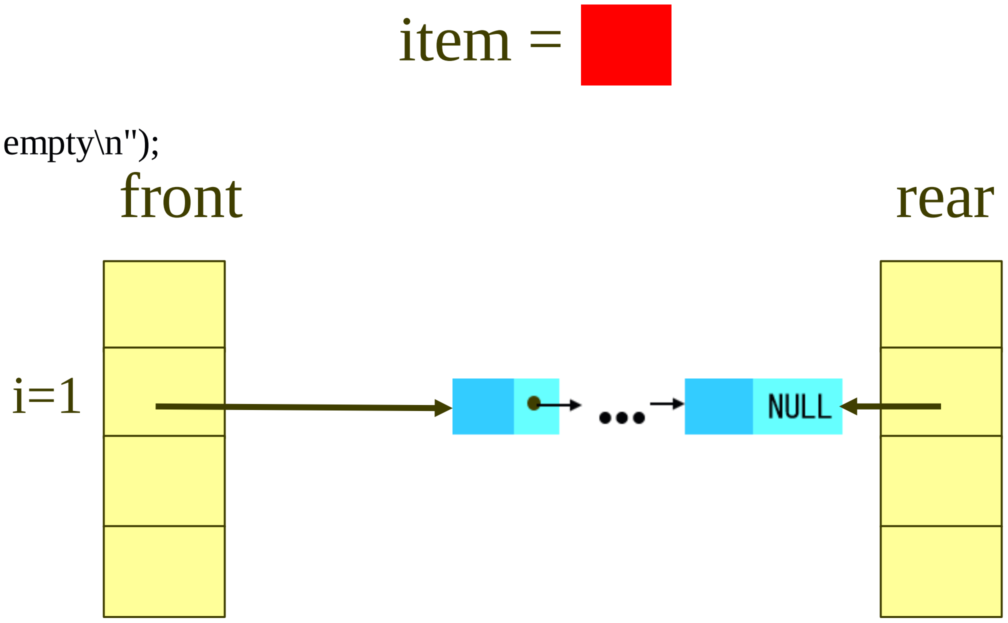
call : *item* = *deleteq*(i);



- **[Program 4.8] Delete from the front of a linked queue**

```
element deleteq(int i)
{
    /* delete an element from queue i */
    queue_pointer temp = front[i];
    element item;
    if (IS_EMPTY(front[i])) {
        fprintf(stderr, "The queue is empty\n");
        exit(1);
    }
    item = temp->data;
    front[i] = temp->link;
    free(temp);
    return item;
}
```

call : *item* = *deleteq*(i);



4.4 POLYNOMIALS

4.4.1 Polynomial Representation

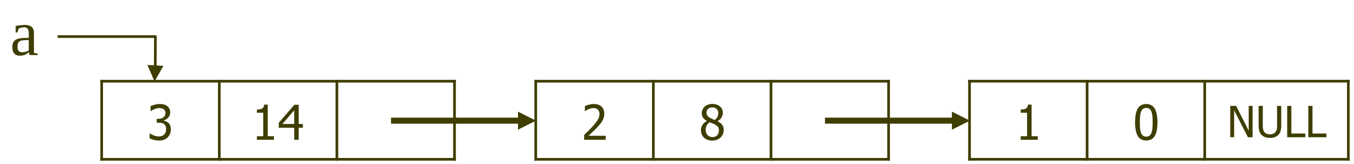
- We want to represent the polynomial $A(x) = a_{m-1}x^{e_{m-1}} + \cdots + a_0x^{e_0}$
  - where the a_i are nonzero coefficients and the e_i are nonnegative integer exponents such that $e_{m-1} > e_{m-2} > \cdots > e_1 > e_0 \geq 0$.

```
typedef struct poly_node *poly_pointer;
typedef struct poly_node {
    int coef;
    int expon;
    poly_pointer link;
};
poly_pointer a, b;
```

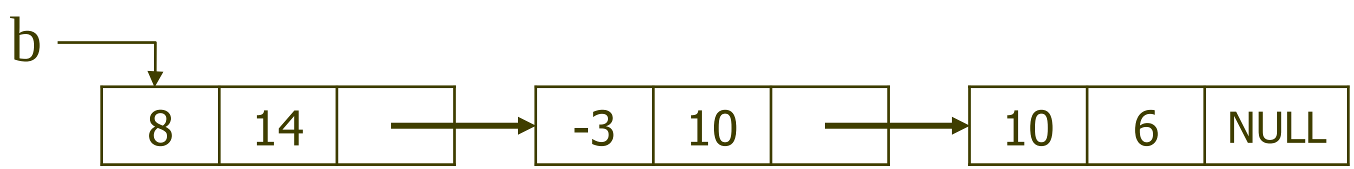
coef	expon	link

- **[Figure 4.11]**

$$a = 3x^{14} + 2x^{8} + 1$$



$$b = 8x^{14} - 3x^{10} + 10x^{6}$$



4.4.2 Adding Polynomials

- Compare Program 4.9 and Program 4.10 with Program 2.6 and Program 2.7.

- **[Program 4.9] Add two polynomials**

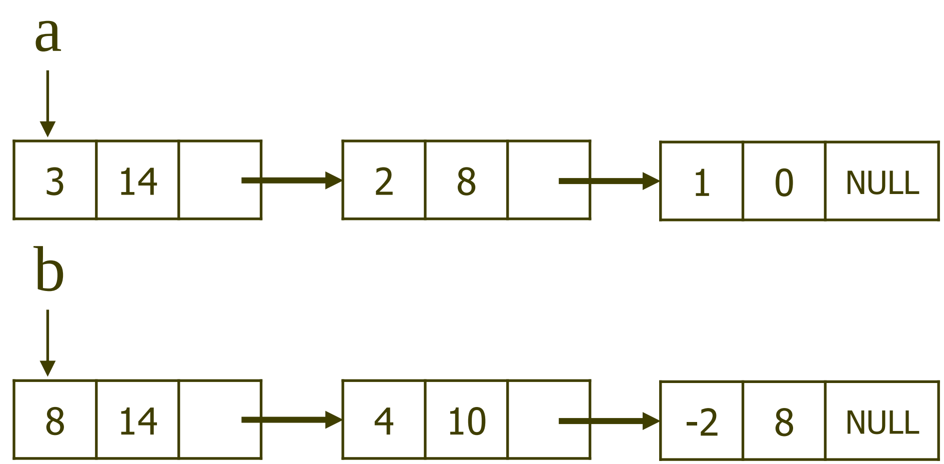
```
poly_pointer padd(poly_pointer a, poly_pointer b)
{
    /*  return a polynomial which is the sum of a and b  */
    poly_pointer  c, rear, temp;
    int sum;
    rear = (poly_pointer) malloc(sizeof(poly_node));
    if  (IS_FULL(rear))  {
        fprintf(stderr, "The memory is full\n");
        exit(1);
    }
    c = rear;
```

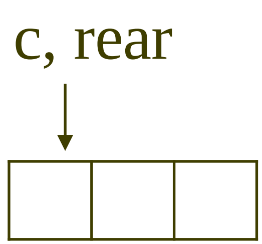
```
    while (a && b)
        switch (COMPARE(a->expon, b->expon)){
        case –1 :  /* a->expon < b->expon */
            attach (b->coef, b->expon, &rear);
            b = b->link;      break;
        case  0 : /* a->expon = b->expon */
            sum =  a->coef + b->coef;
            if (sum)  attach(sum, a->expon, &rear);
            a = a->link;   b = b->link;   break;
        case  1 :  /* a->expon > b->expon */
            attach (a->coef, a->expon, &rear);
            a = a->link;
    }
    /*  copy rest of list a and then list b  */
    for ( ; a; a = a->link) attach (a->coef, a->expon, &rear);
    for ( ; b; b = b->link) attach (b->coef, b->expon, &rear);
    rear->link = NULL;
    /*  delete extra initial node  */
    temp = c;  c = c->link;  free(temp);
    return  c;
}
```

- **[Program 4.10] Attach a node to the end of a list**

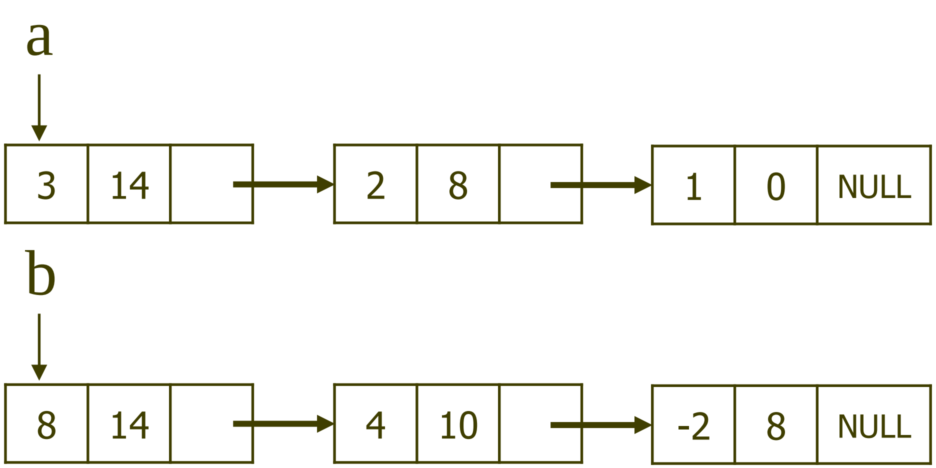
```
void attach(int coefficient, int exponent, poly_pointer *ptr)
{
    /* create a new node with coef = coefficient and expon = exponent,
    attach it to the node pointed to by ptr.  ptr is updated to point to this new node  */
    poly_pointer  temp;
    temp = (poly_pointer)malloc(sizeof(poly_node));
    if  (IS_FULL(temp))  {
        fprintf(stderr, "The memory is full\n");
        exit(1);
    }
    temp->coef = coefficient;
    temp->expon = exponent;
    (*ptr)->link = temp;
    *ptr = temp;
}
```

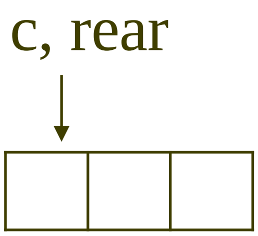
```
poly_pointer  c, rear, temp;
int sum;
rear = (poly_pointer) malloc(sizeof(poly_node));
if  (IS_FULL(rear))  {
      fprintf(stderr, "The memory is full\n");
      exit(1);
}
c = rear;
```



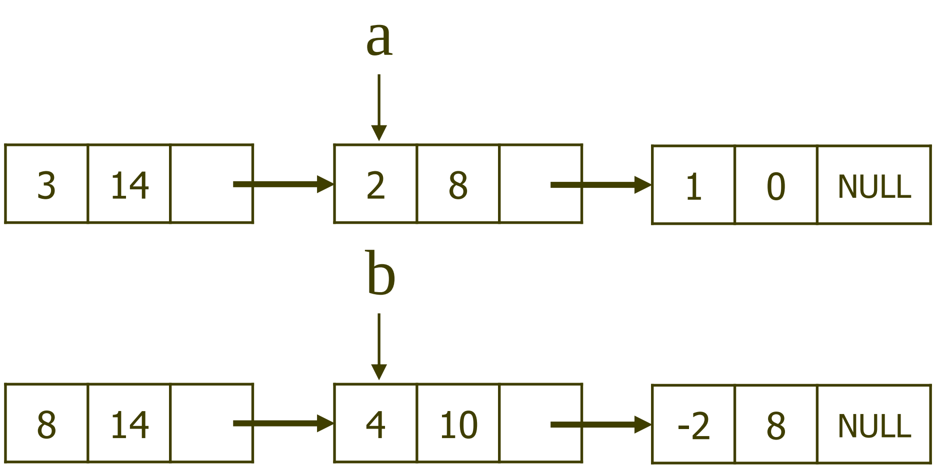


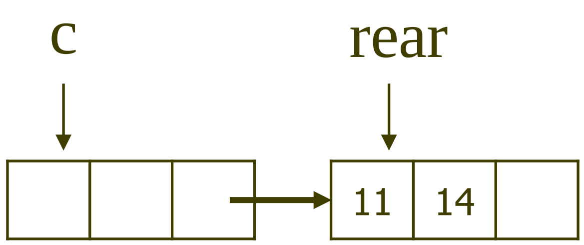
```
while (a && b)
        switch (COMPARE(a->expon, b->expon)){
        case –1 :  /* a->expon < b->expon */
                attach (b->coef, b->expon, &rear);
                b = b->link;      break;
        case  0 : /* a->expon = b->expon */
                sum =  a->coef + b->coef;
                if (sum)  attach(sum, a->expon, &rear);
                a = a->link;   b = b->link;   break;
        case  1 :  /* a->expon > b->expon */
                attach (a->coef, a->expon, &rear);
                a = a->link;
}
```



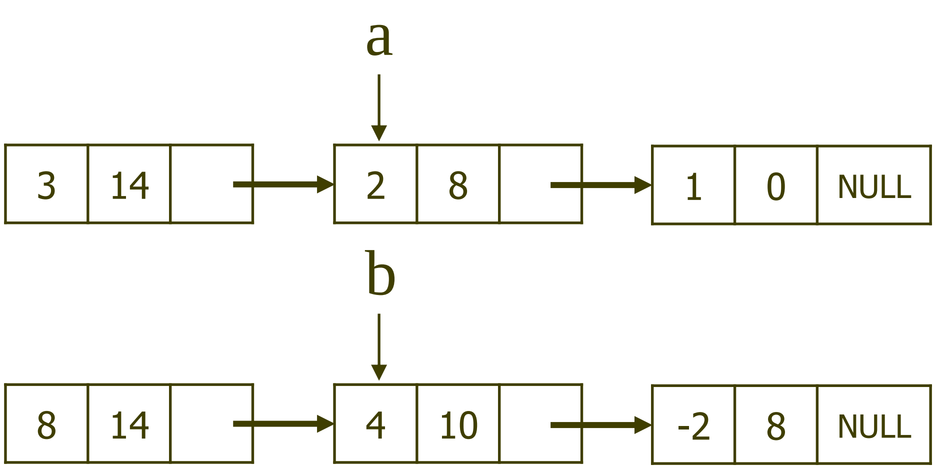


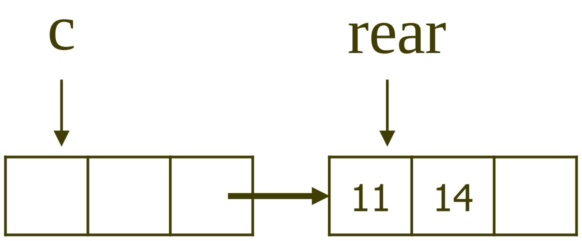
```
while (a && b)
        switch (COMPARE(a->expon, b->expon)){
        case –1 :  /* a->expon < b->expon */
                attach (b->coef, b->expon, &rear);
                b = b->link;        break;
        case  0 : /* a->expon = b->expon */
                sum =  a->coef + b->coef;
                if (sum)  attach(sum, a->expon, &rear);
                a = a->link;    b = b->link;   break;
        case  1 :  /* a->expon > b->expon */
                attach (a->coef, a->expon, &rear);
                a = a->link;
}
```



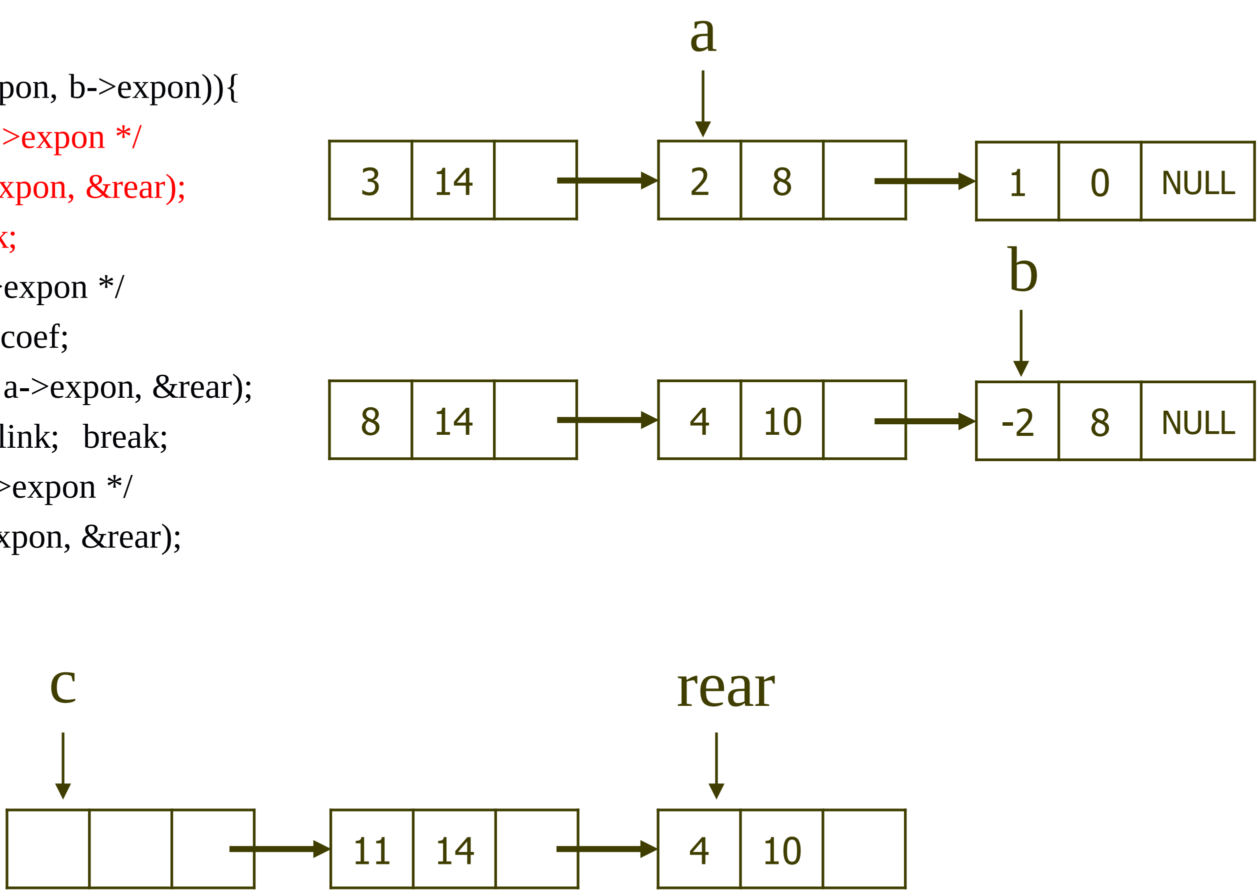


```
while (a && b)
    switch (COMPARE(a->expon, b->expon)){
    case –1 :  /* a->expon < b->expon */
        attach (b->coef, b->expon, &rear);
        b = b->link;      break;
    case  0 : /* a->expon = b->expon */
        sum =  a->coef + b->coef;
        if (sum)  attach(sum, a->expon, &rear);
        a = a->link;    b = b->link;   break;
    case  1 :  /* a->expon > b->expon */
        attach (a->coef, a->expon, &rear);
        a = a->link;
}
```

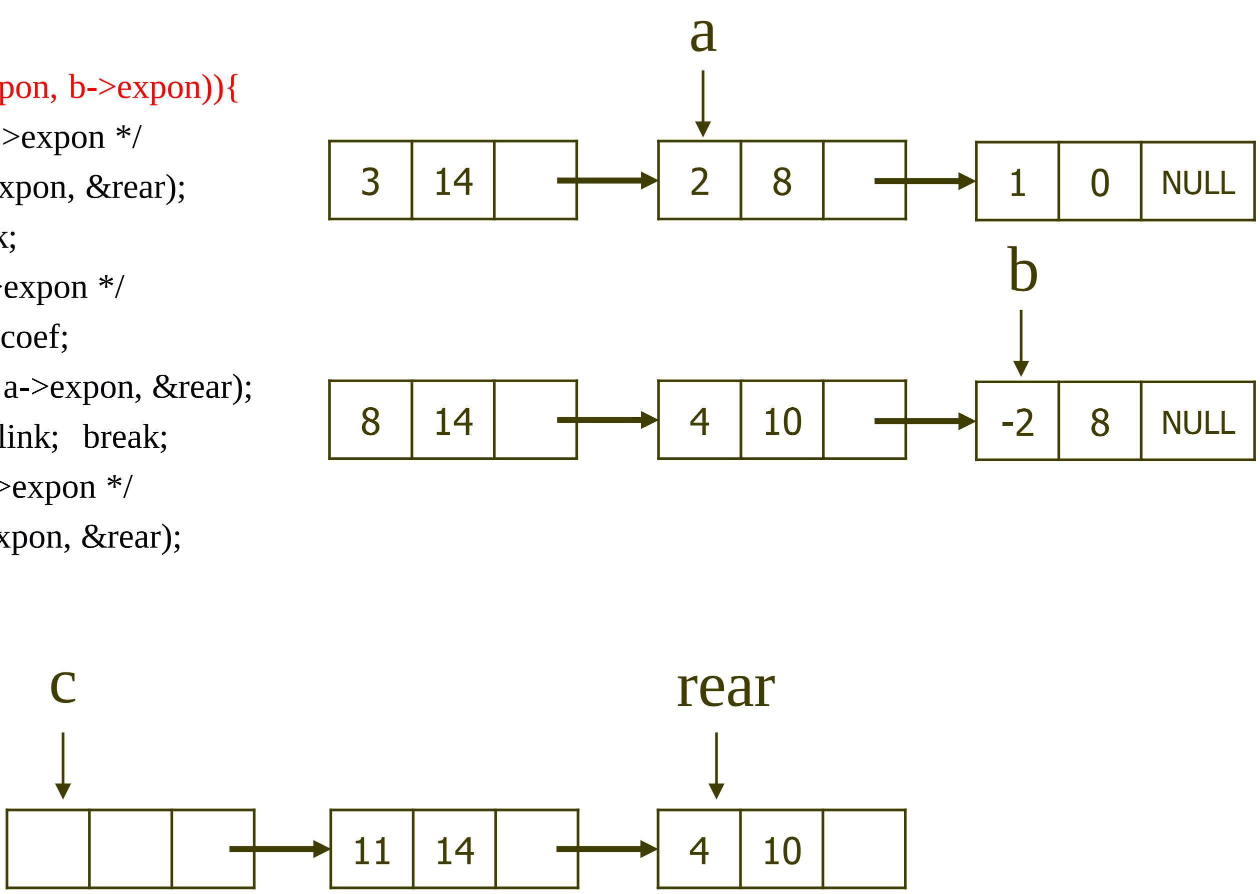




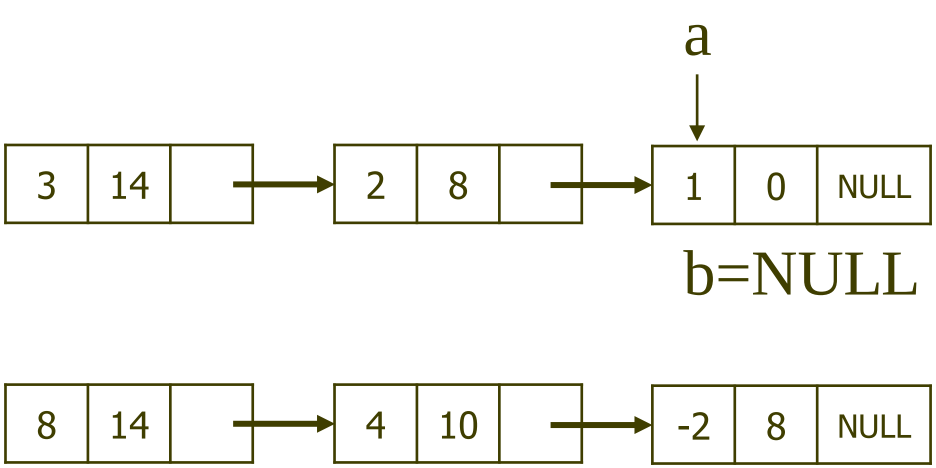
```
while (a && b)
        switch (COMPARE(a->expon, b->expon)){
        case –1 :  /* a->expon < b->expon */
                attach (b->coef, b->expon, &rear);
                b = b->link;        break;
        case  0 : /* a->expon = b->expon */
                sum =  a->coef + b->coef;
                if (sum)  attach(sum, a->expon, &rear);
                a = a->link;    b = b->link;   break;
        case  1 :  /* a->expon > b->expon */
                attach (a->coef, a->expon, &rear);
                a = a->link;
}
```

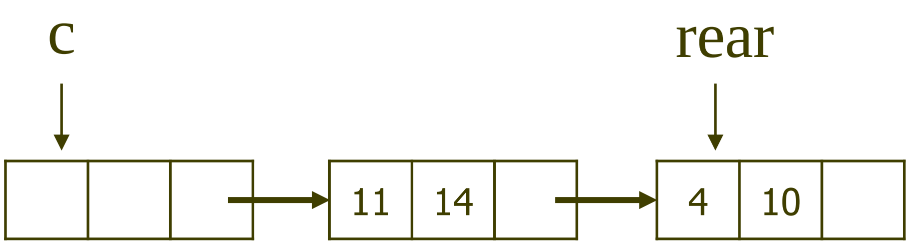


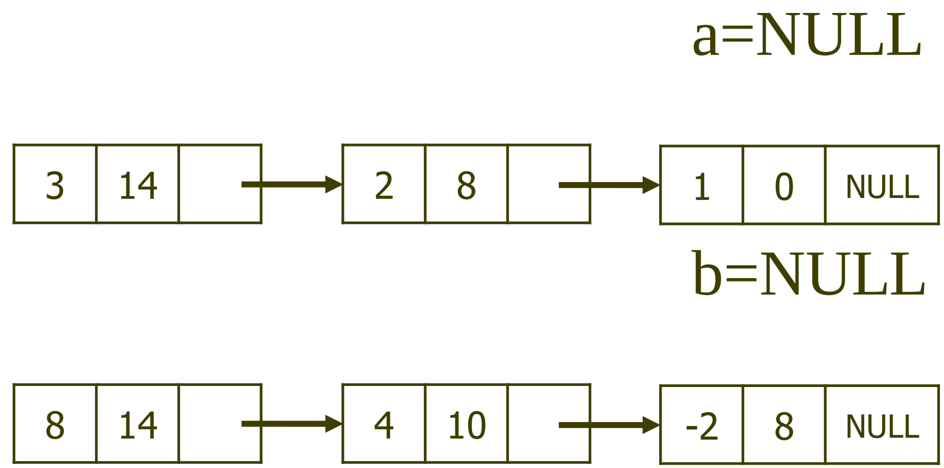
```
while (a && b)
        switch (COMPARE(a->expon, b->expon)){
        case –1 :  /* a->expon < b->expon */
                attach (b->coef, b->expon, &rear);
                b = b->link;      break;
        case  0 : /* a->expon = b->expon */
                sum =  a->coef + b->coef;
                if (sum)  attach(sum, a->expon, &rear);
                a = a->link;    b = b->link;   break;
        case  1 :  /* a->expon > b->expon */
                attach (a->coef, a->expon, &rear);
                a = a->link;
}
```



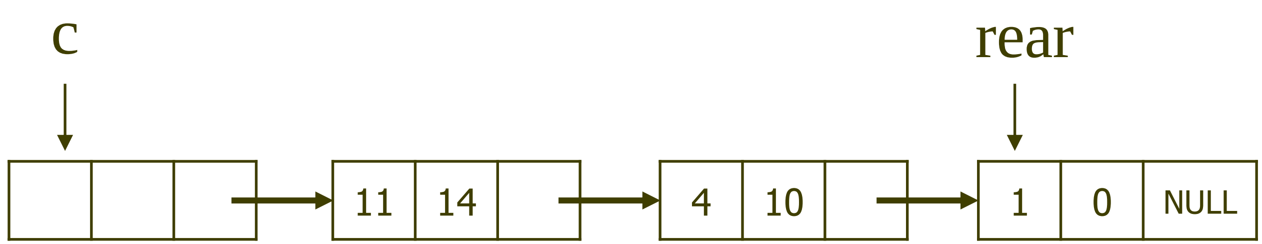
```
while (a && b)
        switch (COMPARE(a->expon, b->expon)){
        case –1 :  /* a->expon < b->expon */
                attach (b->coef, b->expon, &rear);
                b = b->link;       break;
        case  0 : /* a->expon = b->expon */
                sum =  a->coef + b->coef;
                if (sum)  attach(sum, a->expon, &rear);
                a = a->link;    b = b->link;   break;
        case  1 :  /* a->expon > b->expon */
                attach (a->coef, a->expon, &rear);
                a = a->link;
}
```

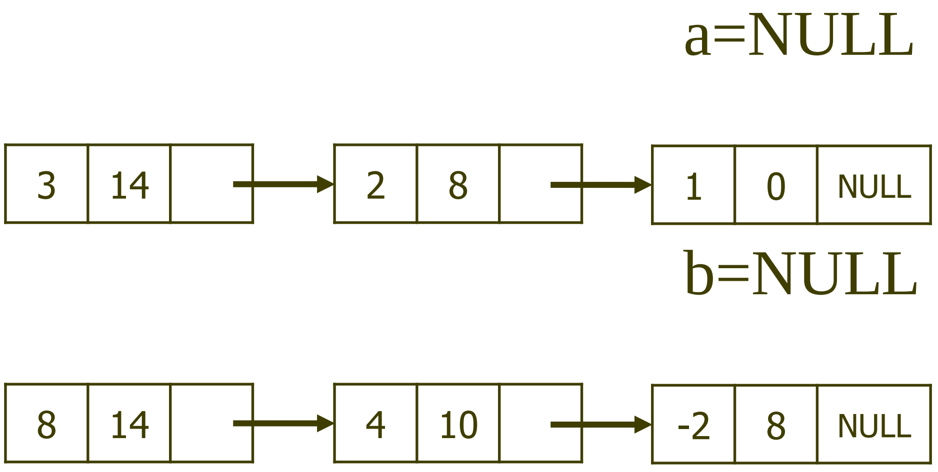




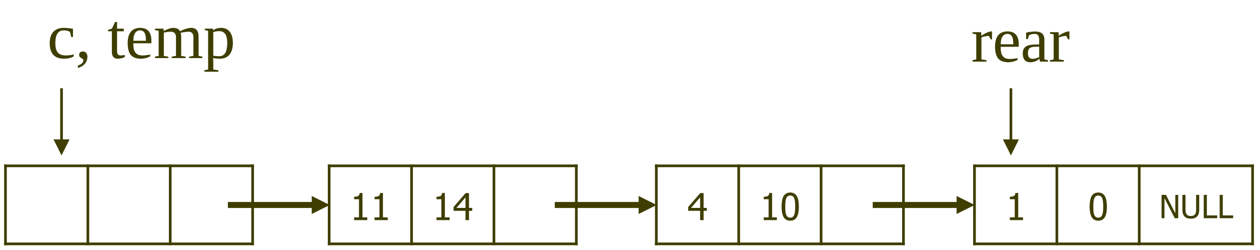


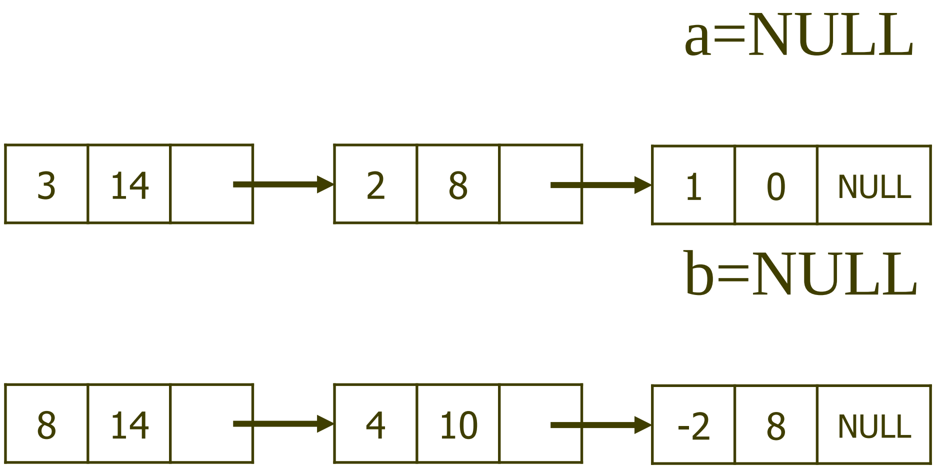

```
for ( ; a; a = a->link) attach (a->coef, a->expon, &rear);
for ( ; b; b = b->link) attach (b->coef, b->expon, &rear);
rear->link = NULL;
/*  delete extra initial node  */
temp = c;  c = c->link;  free(temp);
```



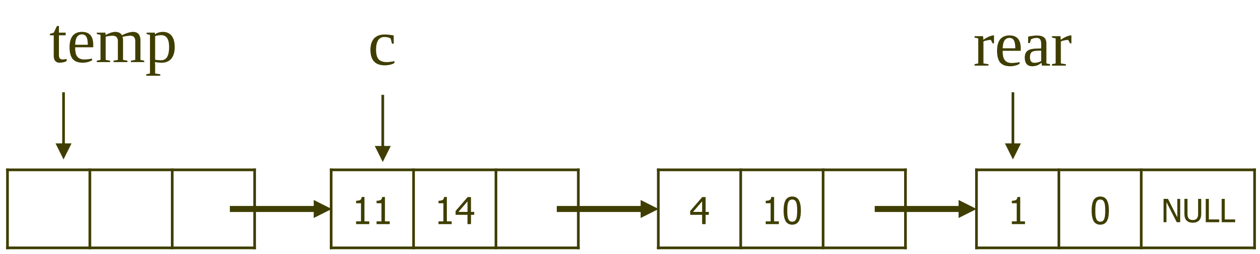



```
for ( ; a; a = a->link) attach (a->coef, a->expon, &rear);
for ( ; b; b = b->link) attach (b->coef, b->expon, &rear);
rear->link = NULL;
/*  delete extra initial node  */
temp = c;  c = c->link;  free(temp);
```

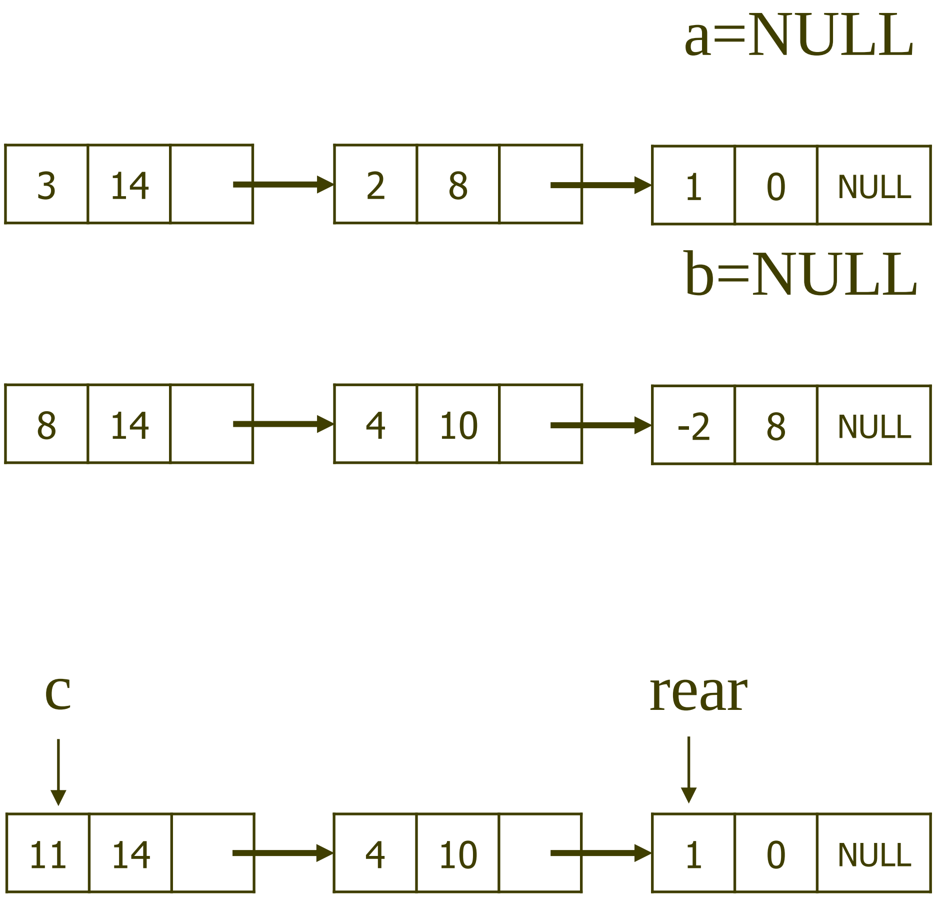





```
for ( ; a; a = a->link) attach (a->coef, a->expon, &rear);
for ( ; b; b = b->link) attach (b->coef, b->expon, &rear);
rear->link = NULL;
/*  delete extra initial node  */
temp = c;  c = c->link;  free(temp);
```



```
for ( ; a; a = a->link) attach (a->coef, a->expon, &rear);
for ( ; b; b = b->link) attach (b->coef, b->expon, &rear);
rear->link = NULL;
/*  delete extra initial node  */
temp = c;  c = c->link;  free(temp);
```



- **Analysis of *padd* :**
  - Similar to the analysis of Program 2.6.

    Three cost measures :

    (1) coefficient additions

    (2) exponent comparisons

    (3) creation of new nodes for c

- Assume that a and b have m and n terms, respectively:

$$A(x) = a_{m-1}x^{e_{m-1}} + \cdots + a_0 x^{e_0}$$
$$B(x) = b_{n-1}x^{f_{n-1}} + \cdots + b_0 x^{f_0}$$

  Clearly, $0 \le \text{number of coefficient additions} \le \min\{m, n\}$, number of exponent comparisons and creation of new nodes is at most *m+n*.

- Therefore, its time complexity is $\mathrm{O}(m + n)$.

4.4.3 Erasing Polynomials

- Let's assume that we are writing a collection of functions for input, output, addition, subtraction, and multiplication of polynomials using linked lists as the means of representation.

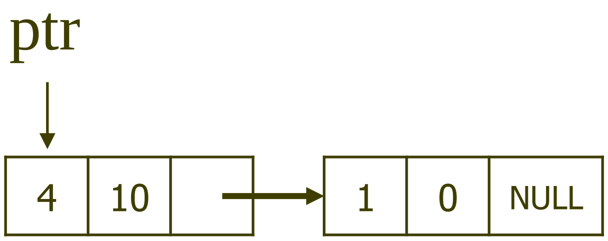
- Suppose we wish to compute e(x) = a(x) * b(x) + d(x) :

```
poly_pointer  a, b, d, e;
    ⋮
a = read_poly();
b = read_poly();
d = read_poly();
temp = pmult(a, b);
e = padd(temp, d);
print_poly(e);
```

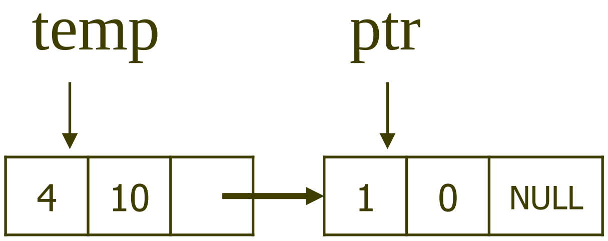
- Note that we created polynomial *temp(x)* only to hold a partial result for d(x).
- By returning the nodes of *temp(x)*, we may use them to hold other polynomials.
- **[Program 4.11] Erasing a polynomial**

```
void erase(poly_pointer  *ptr)
{
      /* erase the polynomial pointed by ptr */
      poly_pointer  temp;
      while (*ptr) {
            temp = *ptr;
            *ptr = (*ptr) -> link;
            free(temp);
      }
}
```

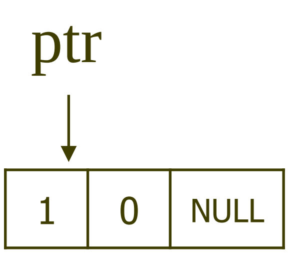
```
void erase(poly_pointer  *ptr)
{
      /* erase the polynomial pointed by ptr */
      poly_pointer  temp;
      while (*ptr) {
            temp = *ptr;
            *ptr = (*ptr) -> link;
            free(temp);
      }
}
```



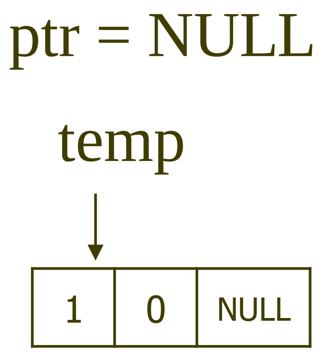
```
void erase(poly_pointer  *ptr)
{
    /* erase the polynomial pointed by ptr */
    poly_pointer  temp;
    while (*ptr) {
        temp = *ptr;
        *ptr = (*ptr) -> link;
        free(temp);
    }
}
```



```
void erase(poly_pointer  *ptr)
{
      /* erase the polynomial pointed by ptr */
      poly_pointer  temp;
      while (*ptr) {
            temp = *ptr;
            *ptr = (*ptr) -> link;
            free(temp);
      }
}
```



```
void erase(poly_pointer  *ptr)
{
      /* erase the polynomial pointed by ptr */
      poly_pointer  temp;
      while (*ptr) {
            temp = *ptr;
            *ptr = (*ptr) -> link;
            free(temp);
      }
}
```

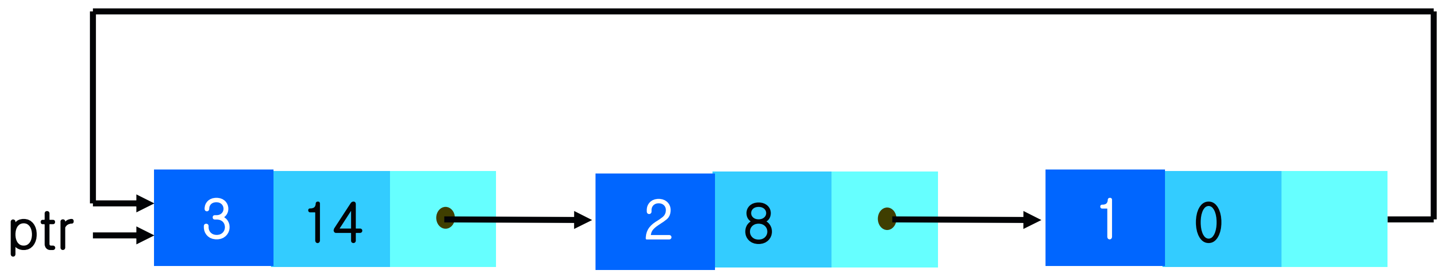


```
void erase(poly_pointer  *ptr)
{
    /* erase the polynomial pointed by ptr */
    poly_pointer  temp;
    while (*ptr) {
        temp = *ptr;
        *ptr = (*ptr) -> link;
        free(temp);
    }
}
```

ptr = NULL

4.4.4 Circular List Representation of Polynomials

- To free all the nodes of a polynomial more efficiently, we modify our list structure so that the link field of the last node points to the first node in the list.



- We call this a *circular list*.
- A *chain* : a singly linked list in which the last node has a null link.

- We want to free nodes that are no longer in use so that we may reuse these nodes later.

- We can obtain an efficient erase algorithm for circular lists, by maintaining our own list (as a chain) of nodes that have been "freed".

- When we need a new node, we examine this list.

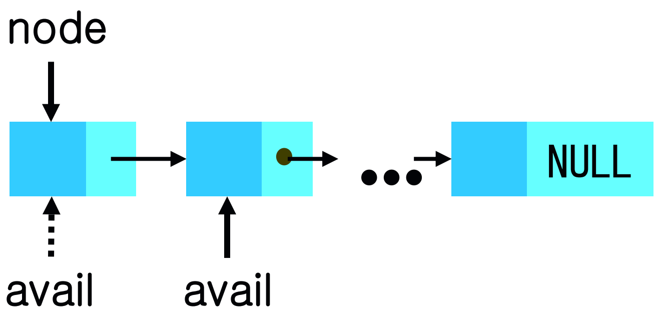
  If the list is not empty, then we may use one of its nodes.

  Only when the list is empty, use *malloc* to create a new node.

- Let *avail* be a variable of type *poly_pointer* that points to the first node in the list of freed nodes.

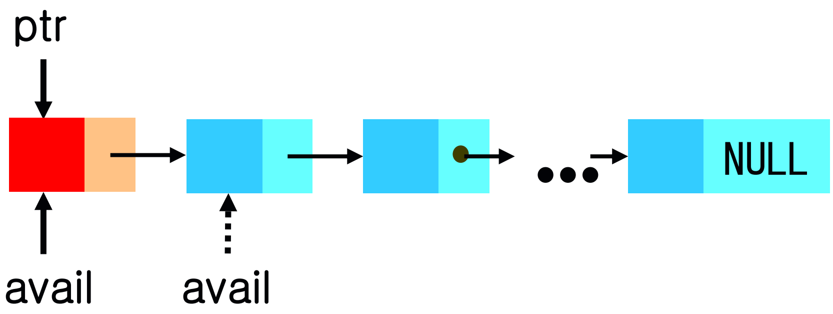
- **[Program 4.12]**

```
poly_pointer get_node(void) {
    /* provide a node for use */
    poly_pointer node;
    if (avail)  {
        node = avail;
        avail = avail->link;
    }
    else  {
        node = (poly_pointer) malloc(sizeof(poly_node));
        if (IS_FULL(node))  {
            fprintf(stderr, "The memory is full\n");
            exit(1);
        }
    }
return node;
}
```



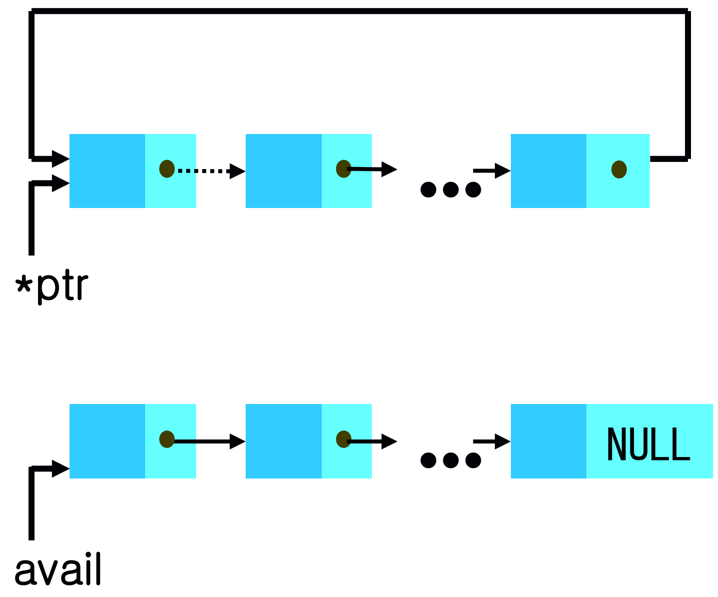
- **[Program 4.13]**

```
void ret_node(poly_pointer ptr)  {
      /*  return a node to the available list */
      ptr->link = avail;
      avail = ptr;
}
```



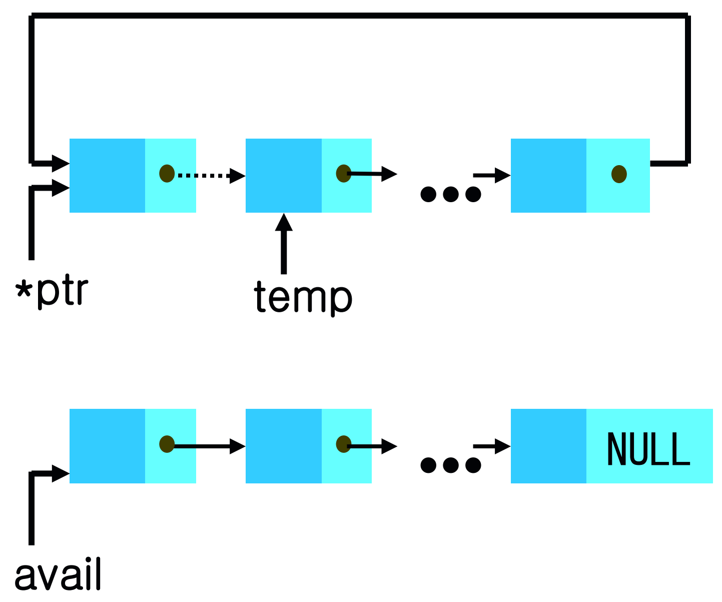
■ **[Program 4.14]**

```
void cerase(poly_pointer *ptr)  {
      /* erase the circular list ptr */
      poly_pointer  temp;
      if (*ptr)  {
            temp = (*ptr)->link;
            (*ptr)->link = avail;
            avail = temp;
            *ptr = NULL;
      }
}
```



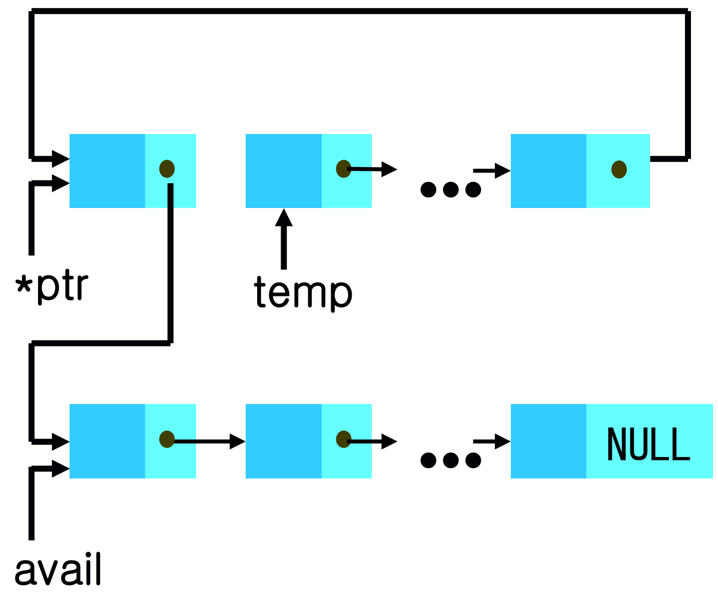
- **[Program 4.14]**

```
void cerase(poly_pointer *ptr)  {
    /* erase the circular list ptr */
    poly_pointer  temp;
    if (*ptr)  {
        temp = (*ptr)->link;
        (*ptr)->link = avail;
        avail = temp;
        *ptr = NULL;
    }
}
```



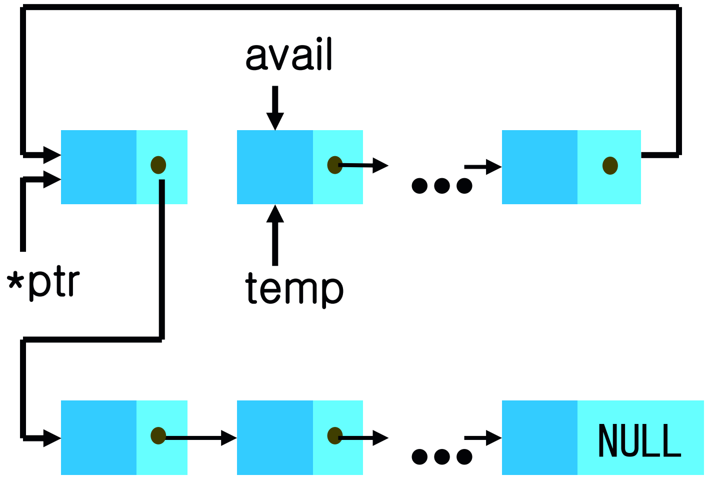
- **[Program 4.14]**

```
void cerase(poly_pointer *ptr)  {
    /* erase the circular list ptr */
    poly_pointer  temp;
    if (*ptr)  {
        temp = (*ptr)->link;
        (*ptr)->link = avail;
        avail = temp;
        *ptr = NULL;
    }
}
```



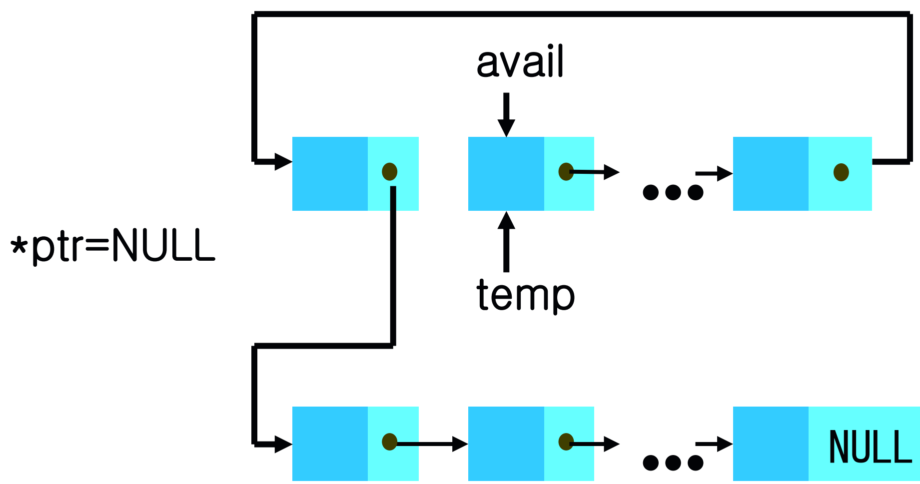
- **[Program 4.14]**

```
void cerase(poly_pointer *ptr)  {
      /* erase the circular list ptr */
      poly_pointer  temp;
      if (*ptr)  {
            temp = (*ptr)->link;
            (*ptr)->link = avail;
            avail = temp;
            *ptr = NULL;
      }
}
```



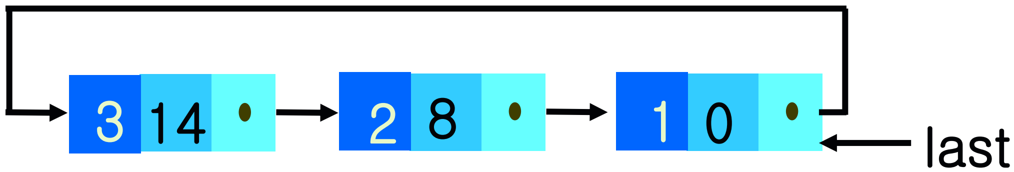
[Program 4.14]

```
void cerase(poly_pointer *ptr)  {
      /* erase the circular list ptr */
      poly_pointer  temp;
      if (*ptr)  {
            temp = (*ptr)->link;
            (*ptr)->link = avail;
            avail = temp;
            *ptr = NULL;
      }
}
```



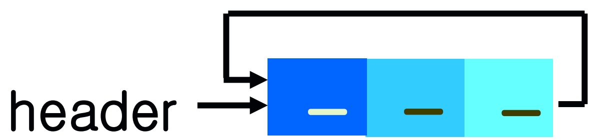
- A direct changeover to the structure of Figure 4.14 creates problems when we implement the other polynomial operations since we must handle the zero polynomial as a special case.

[Figure 4.14] Circular representation of $3x^{14} + 2x^{8} + 1$

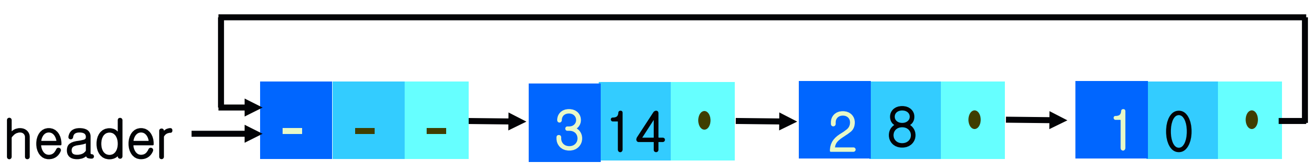


- We introduce a *header node* into each polynomial.

[Figure 4.15] Example polynomials with header nodes



(a) zero polynomial



(b) $3x^{14} + 2x^8 + 1$

- For the circular list with header node representation, we may remove the test for *(*ptr)* from *cerase*.
- The only changes that we need to make to *padd* are :

(1) Add two variables, *starta = a* and *startb =* b.

(2) Prior to the *while* loop, assign *a = a->link* and *b = b->link*.

(3) Change the *while* loop to *while* (*a* != *starta* && *b* != *startb*).

(4) Change the first *for* loop to *for* (; *a* != *starta*; *a* = *a*->*link*).

(5) Change the second *for* loop to *for* (; *b* != *startb*; *b* = *b*->*link*).

(6) Delete the line :

```
rear -> link = NULL;
```

(7) Change the lines :

```
temp = c;
c = c -> link;
free(temp);
```

to

```
lastc -> link = c;
```

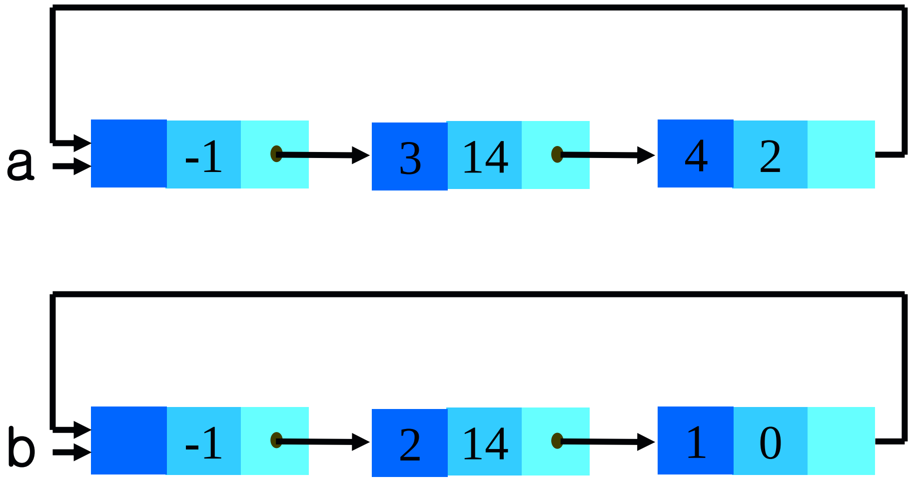
- We may further simplify the addition algorithm if we set the *expon* field of the header node to -1.

- **[Program 4.15]**

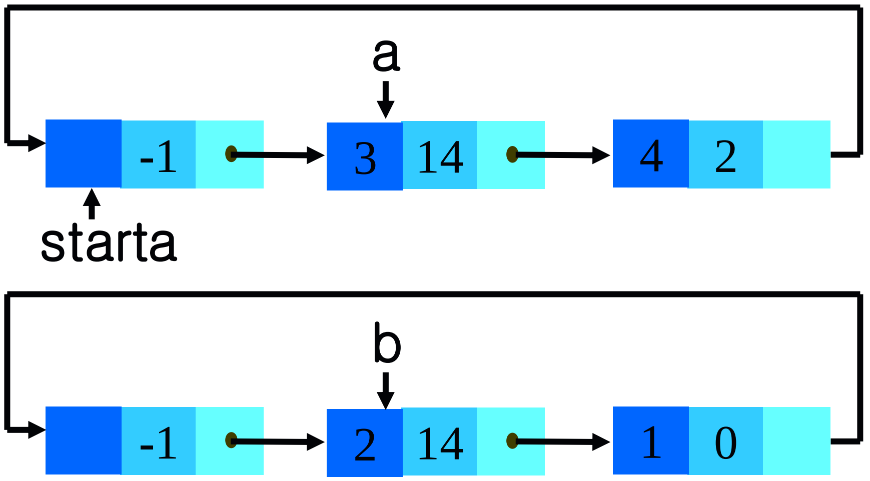
```
poly_pointer cpadd(poly_pointer a, poly_pointer b)
{
/* polynomials a and b are singly linked circular lists with a header
  node.  Return a polynomial which is the sum of a and b */
    poly_pointer starta, c, lastc;
    int sum,  done = FALSE;
    starta = a;                    /*  record start of a  */
    a = a->link;                   /*  skip header node for a and b  */
    b = b->link;
    c = get_node();          /*  get a header node for sum  */
    c->expon = -1;                 lastc = c;
```

```
do  {
    switch (COMPARE(a->expon, b->expon)){
    case –1 :  /* a->expon < b->expon */
          attach (b->coef, b->expon, &lastc);
          b = b->link;          break;
    case  0 : /* a->expon = b->expon */
          if (starta == a)  done = TRUE;
          else  {
                    sum =  a->coef + b->coef;
                    if (sum)   attach(sum, a->expon, &lastc);
                    a = a->link;   b = b->link;
          }
          break;
     case  1 :  /* a->expon > b->expon */
          attach (a->coef, a->expon, &lastc);
          a = a->link;
     }
} while (!done)
lastc->link = c;
return c;
}
```

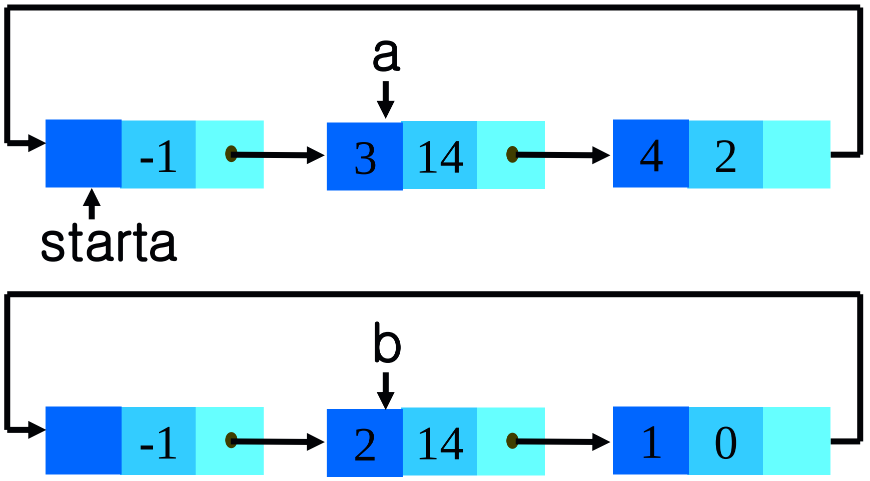
```
poly_pointer starta, c, lastc;
int sum,  done = FALSE;
starta = a;
a = a->link;
b = b->link;
c = get_node();
c->expon = -1;
lastc = c;
```

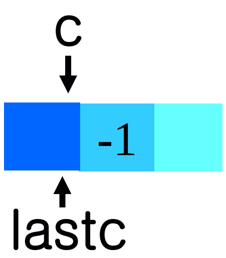


```
poly_pointer starta, c, lastc;
int sum,  done = FALSE;
starta = a;
a = a->link;
b = b->link;
c = get_node();
c->expon = -1;
lastc = c;
```

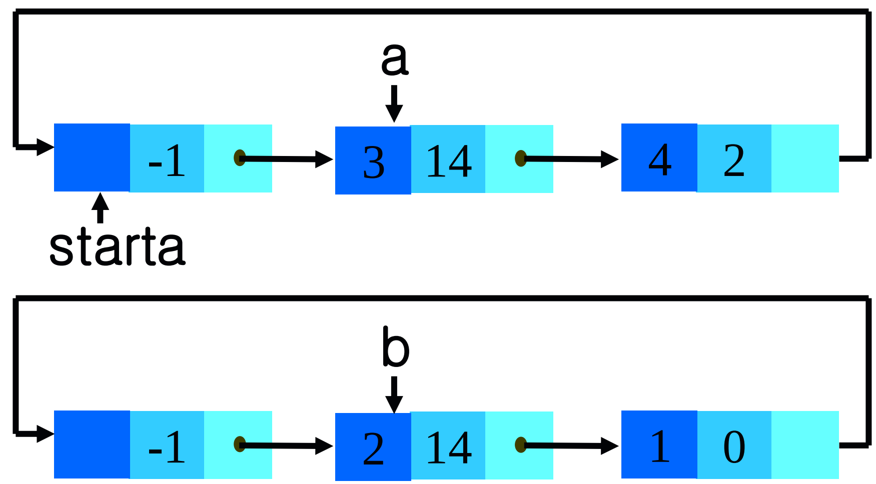


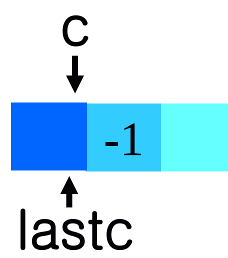
```
poly_pointer starta, c, lastc;
int sum,  done = FALSE;
starta = a;
a = a->link;
b = b->link;
c = get_node();
c->expon = -1;
lastc = c;
```



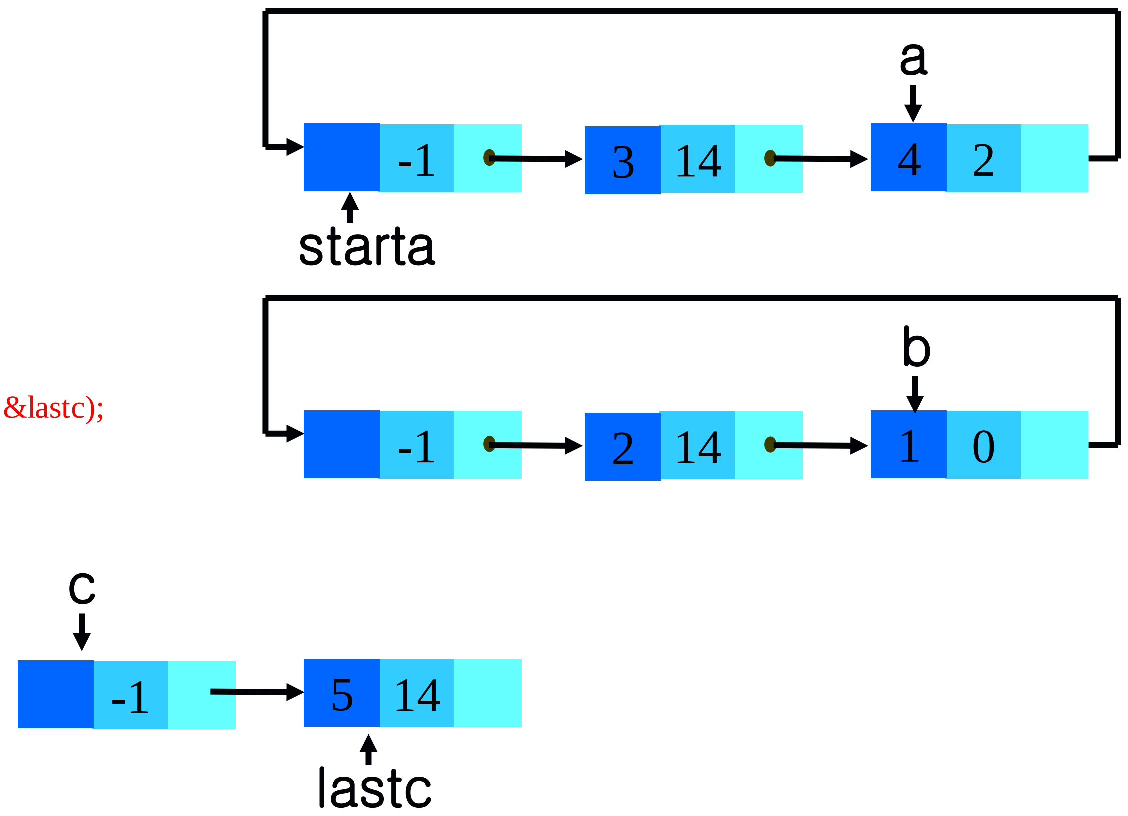


```
do  {
   switch (COMPARE(a->expon, b->expon)){
          case –1 :  /* a->expon < b->expon */
              attach (b->coef, b->expon, &lastc);
              b = b->link;         break;
          case  0 : /* a->expon = b->expon */
              if (starta == a)  done = TRUE;
              else  {
                     sum =  a->coef + b->coef;
                     if (sum)   attach(sum, a->expon, &lastc);
                     a = a->link;    b = b->link;
              }
              break;
           case  1 :  /* a->expon > b->expon */
              attach (a->coef, a->expon, &lastc);
              a = a->link;
           }
   } while (!done)
   lastc->link  = c;
```

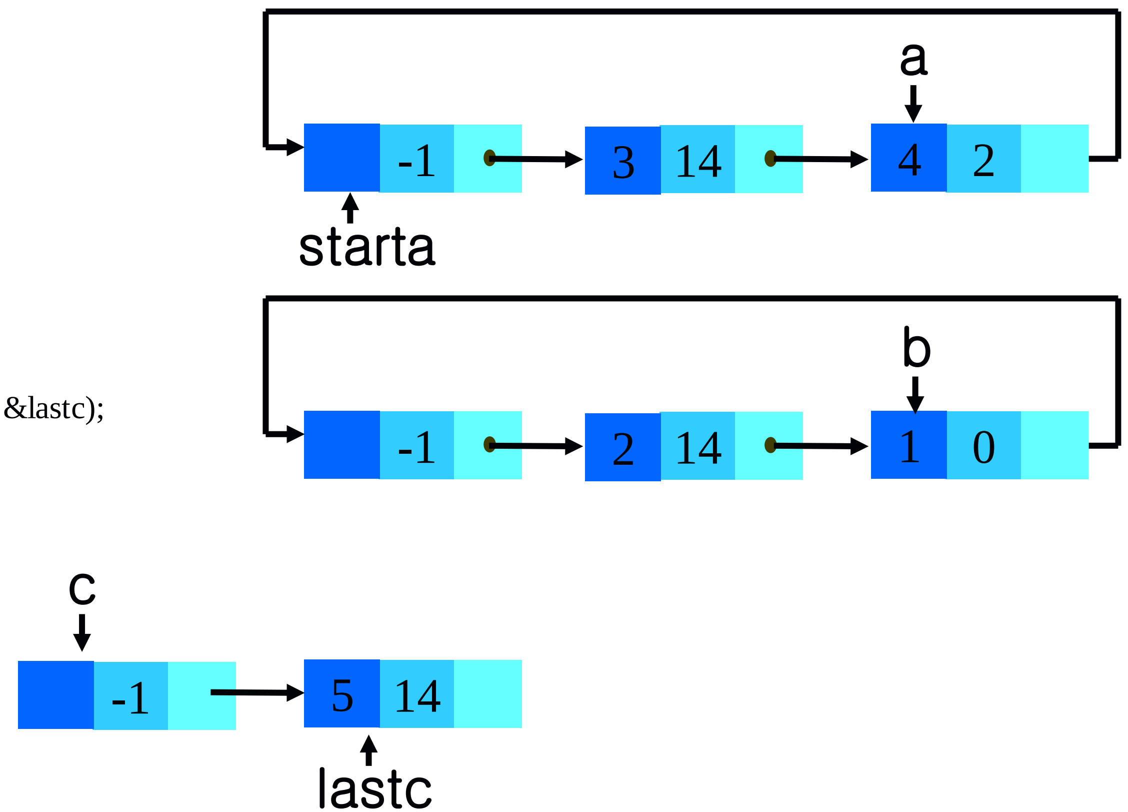




```
do  {
   switch (COMPARE(a->expon, b->expon)){
          case –1 :  /* a->expon < b->expon */
              attach (b->coef, b->expon, &lastc);
              b = b->link;          break;
          case  0 : /* a->expon = b->expon */
              if (starta == a)  done = TRUE;
              else  {
                     sum =  a->coef + b->coef;
                     if (sum)    attach(sum, a->expon, &lastc);
                     a = a->link;    b = b->link;
              }
              break;
           case  1 :  /* a->expon > b->expon */
              attach (a->coef, a->expon, &lastc);
              a = a->link;
           }
   } while (!done)
   lastc->link  = c;
```



```
do  {
   switch (COMPARE(a->expon, b->expon)){
          case –1 :  /* a->expon < b->expon */
              attach (b->coef, b->expon, &lastc);
              b = b->link;         break;
          case  0 : /* a->expon = b->expon */
              if (starta == a)  done = TRUE;
              else  {
                     sum =  a->coef + b->coef;
                     if (sum)   attach(sum, a->expon, &lastc);
                     a = a->link;    b = b->link;
              }
              break;
          case  1 :  /* a->expon > b->expon */
              attach (a->coef, a->expon, &lastc);
              a = a->link;
          }
   } while (!done)
   lastc->link  = c;
```



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do  {
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                attach (b->coef, b->expon, &lastc);
                b = b->link;         break;
            case  0 : /* a->expon = b->expon */
                if (starta == a)  done = TRUE;
                else  {
                        sum =  a->coef + b->coef;
                        if (sum)   attach(sum, a->expon, &lastc);
                        a = a->link;    b = b->link;
                }
                break;
            case  1 :  /* a->expon > b->expon */
                attach (a->coef, a->expon, &lastc);
                a = a->link;
            }
    } while (!done)
    lastc->link  = c;
```
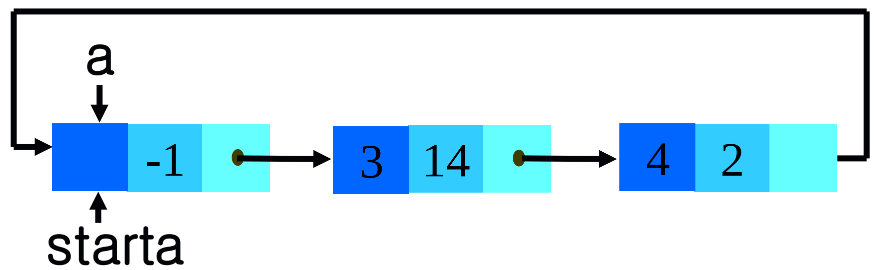
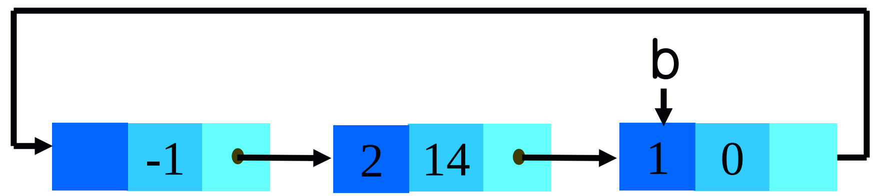
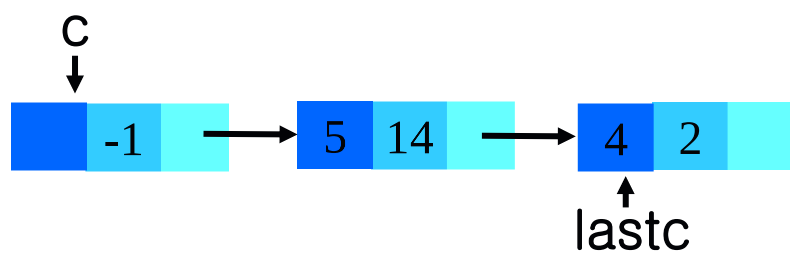
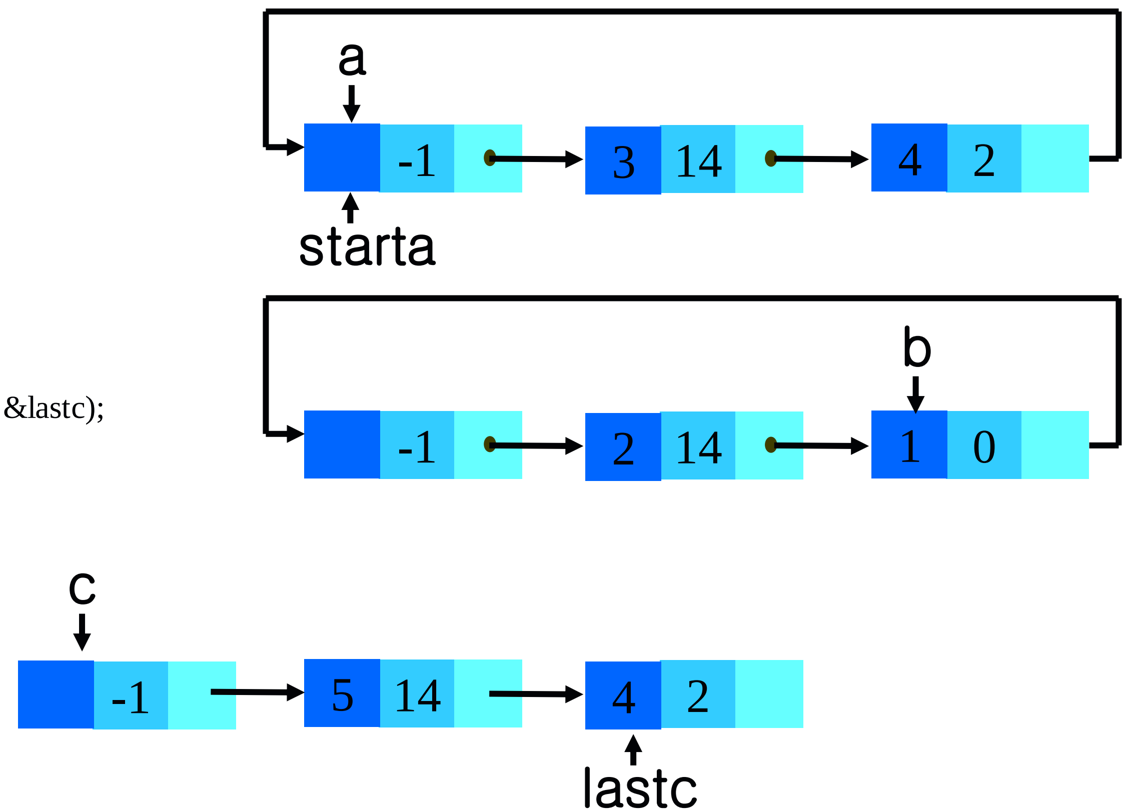



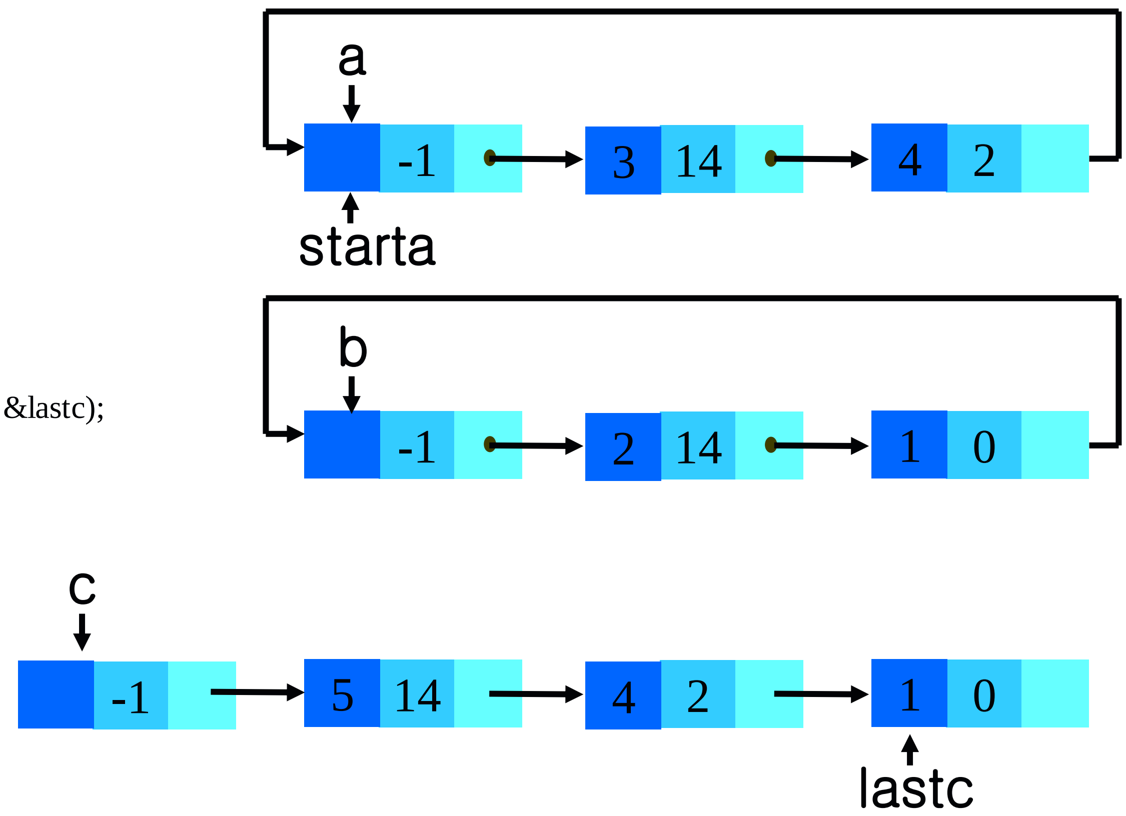




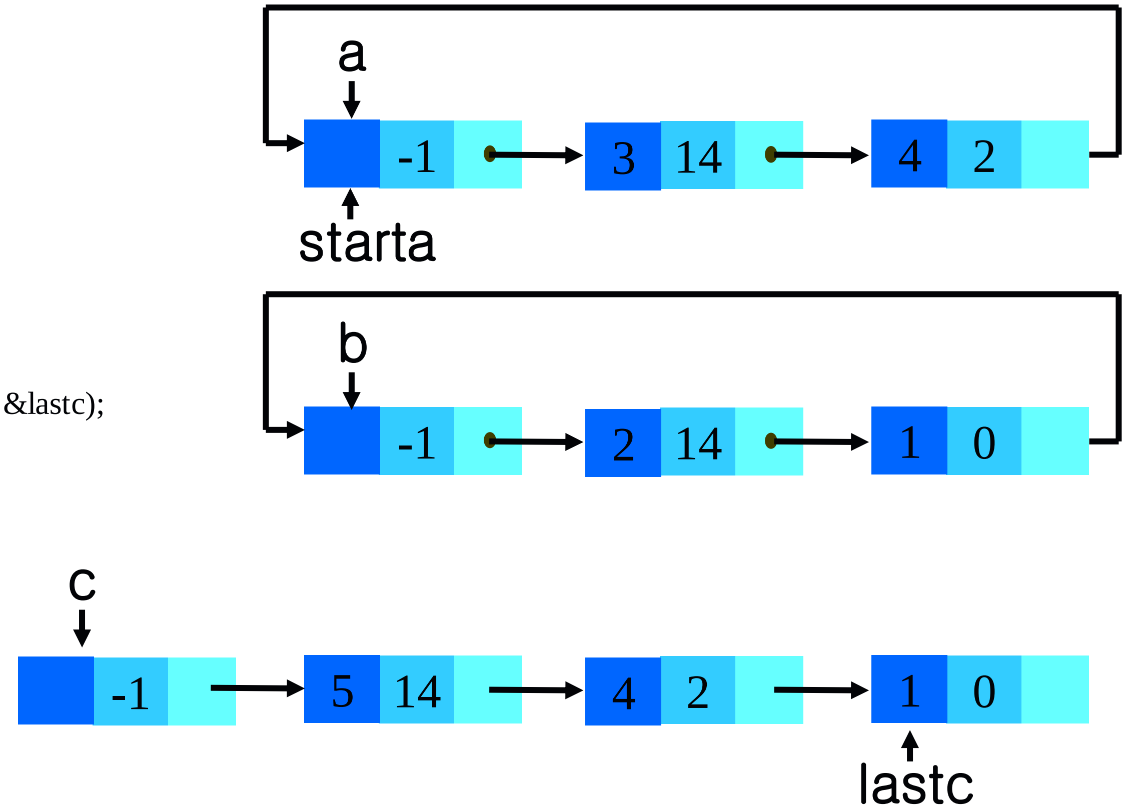
```
do  {
   switch (COMPARE(a->expon, b->expon)){
          case –1 :  /* a->expon < b->expon */
              attach (b->coef, b->expon, &lastc);
              b = b->link;         break;
          case  0 : /* a->expon = b->expon */
              if (starta == a)  done = TRUE;
              else  {
                     sum =  a->coef + b->coef;
                     if (sum)   attach(sum, a->expon, &lastc);
                     a = a->link;    b = b->link;
              }
              break;
           case  1 :  /* a->expon > b->expon */
              attach (a->coef, a->expon, &lastc);
              a = a->link;
           }
   } while (!done)
   lastc->link  = c;
```



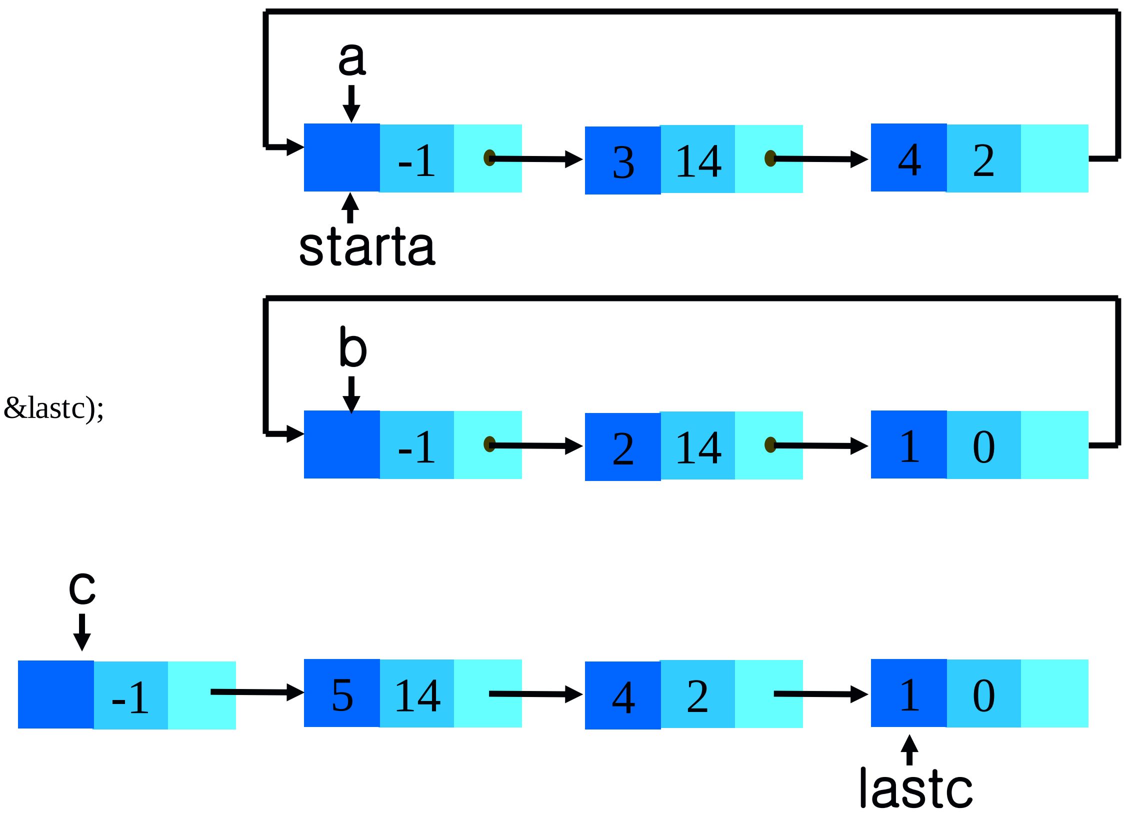
```
do  {
   switch (COMPARE(a->expon, b->expon)){
         case –1 :  /* a->expon < b->expon */
             attach (b->coef, b->expon, &lastc);
             b = b->link;         break;
         case  0 : /* a->expon = b->expon */
             if (starta == a)  done = TRUE;
             else  {
                    sum =  a->coef + b->coef;
                    if (sum)   attach(sum, a->expon, &lastc);
                    a = a->link;    b = b->link;
             }
             break;
          case  1 :  /* a->expon > b->expon */
             attach (a->coef, a->expon, &lastc);
             a = a->link;
          }
   } while (!done)
   lastc->link  = c;
```



```
do  {
   switch (COMPARE(a->expon, b->expon)){
          case –1 :  /* a->expon < b->expon */
              attach (b->coef, b->expon, &lastc);
              b = b->link;        break;
          case  0 : /* a->expon = b->expon */
              if (starta == a)  done = TRUE;
              else  {
                     sum =  a->coef + b->coef;
                     if (sum)   attach(sum, a->expon, &lastc);
                     a = a->link;    b = b->link;
              }
              break;
          case  1 :  /* a->expon > b->expon */
              attach (a->coef, a->expon, &lastc);
              a = a->link;
          }
   } while (!done)
   lastc->link = c;
```



```
do  {
   switch (COMPARE(a->expon, b->expon)){
          case –1 :  /* a->expon < b->expon */
              attach (b->coef, b->expon, &lastc);
              b = b->link;          break;
          case  0 : /* a->expon = b->expon */
              if (starta == a)  done = TRUE;
              else  {
                     sum =  a->coef + b->coef;
                     if (sum)   attach(sum, a->expon, &lastc);
                     a = a->link;    b = b->link;
              }
              break;
           case  1 :  /* a->expon > b->expon */
              attach (a->coef, a->expon, &lastc);
              a = a->link;
           }
   } while (!done)
   lastc->link  = c;
```



```
do  {
   switch (COMPARE(a->expon, b->expon)){
          case –1 :  /* a->expon < b->expon */
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              b = b->link;          break;
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              if (starta == a)  done = TRUE;
              else  {
                     sum =  a->coef + b->coef;
                     if (sum)   attach(sum, a->expon, &lastc);
                     a = a->link;    b = b->link;
              }
              break;
           case  1 :  /* a->expon > b->expon */
              attach (a->coef, a->expon, &lastc);
              a = a->link;
           }
   } while (!done)
   lastc->link  = c;
```
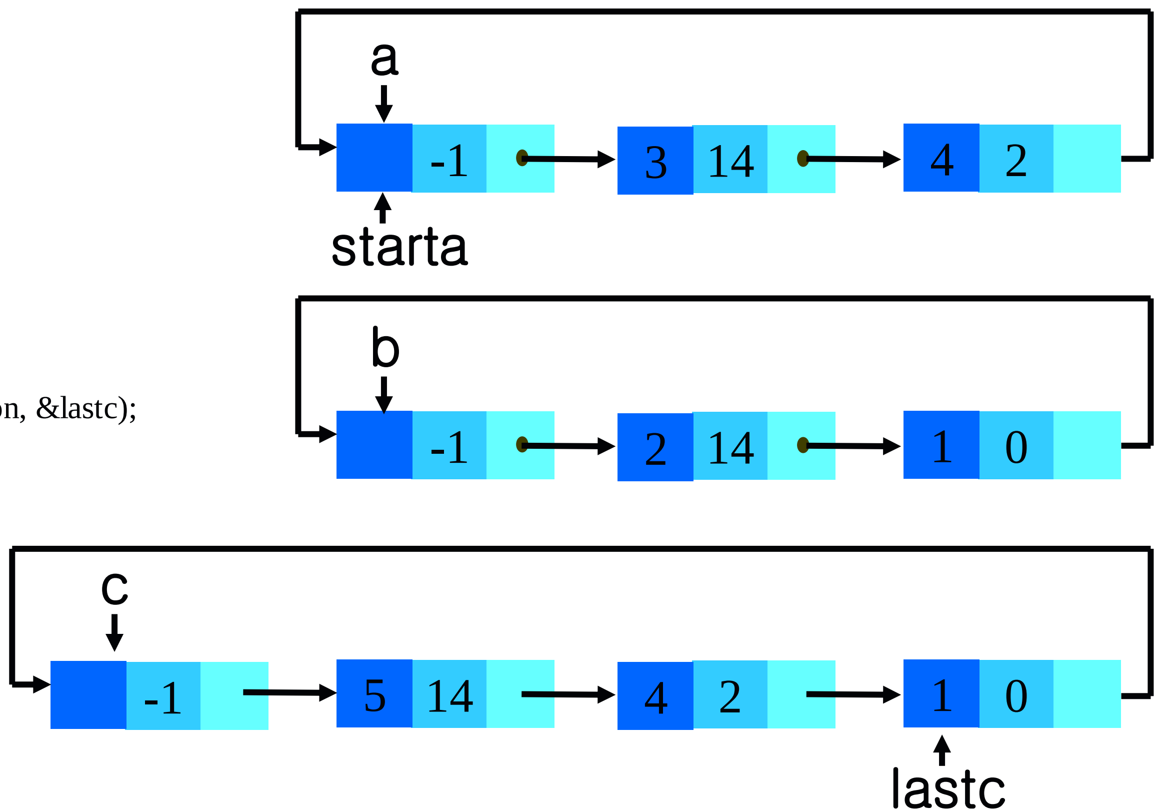



4.5 ADDITIONAL LIST OPERATIONS

4.5.1 Operations For Chains

- It is often necessary, and desirable to build a variety of functions for manipulating singly linked lists. We have seen *get_node* and *ret_node*.

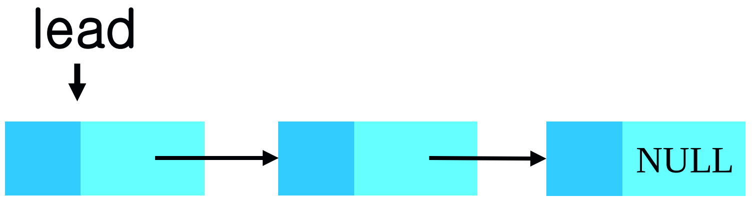
- We use the following declarations :

```
typedef struct list_node *list_pointer;
typedef struct list_node {
        char data;
        list_pointer link;
};
```

- ***Inverting a chain*** **:**
  - we can do it "in place" if we use three pointers.
- **[Program 4.16] Inverting a singly linked list**

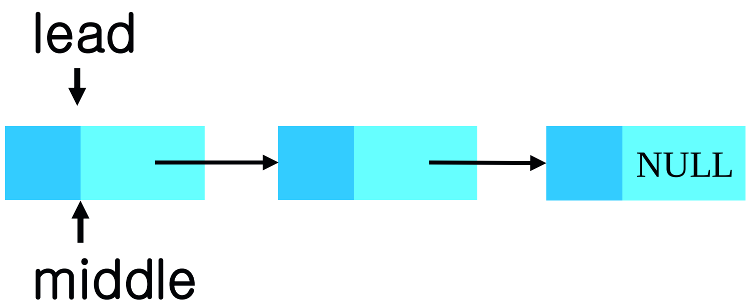
```
list_pointer invert(list_pointer lead)
{
    /* invert the list pointed to by lead  */
    list_pointer  middle, trail;
    middle = NULL;
    while (lead)  {
        trail = middle;
        middle = lead;
        lead = lead->link;
        middle->link = trail;
}
    return  middle;
}
```

```
list_pointer invert(list_pointer lead)
{
    /* invert the list pointed to by lead  */
    list_pointer  middle, trail;
    middle = NULL;
    while (lead)  {
        trail = middle;
        middle = lead;
        lead = lead->link;
        middle->link = trail;
}
    return  middle;
}
```



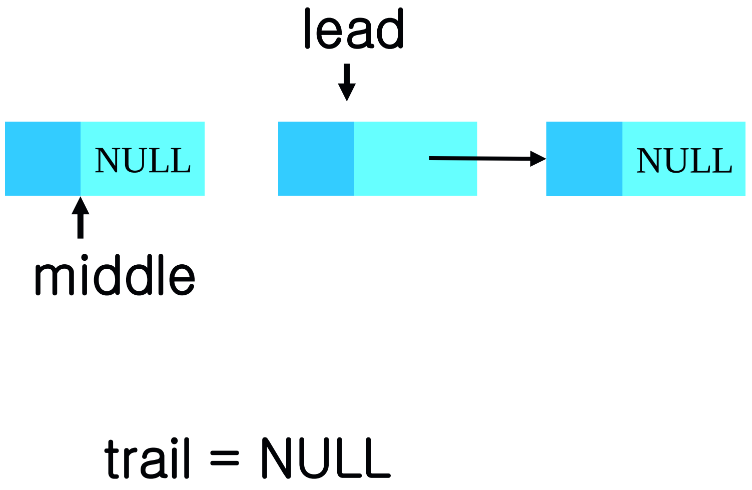
middle = NULL

```
list_pointer invert(list_pointer lead)
{
    /* invert the list pointed to by lead  */
    list_pointer  middle, trail;
    middle = NULL;
    while (lead)  {
        trail = middle;
        middle = lead;
        lead = lead->link;
        middle->link = trail;
}
    return  middle;
}
```

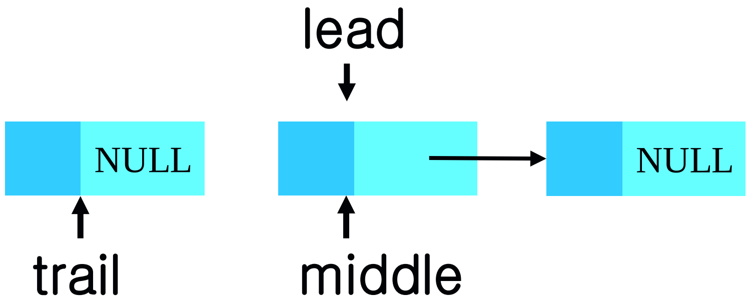


trail = NULL

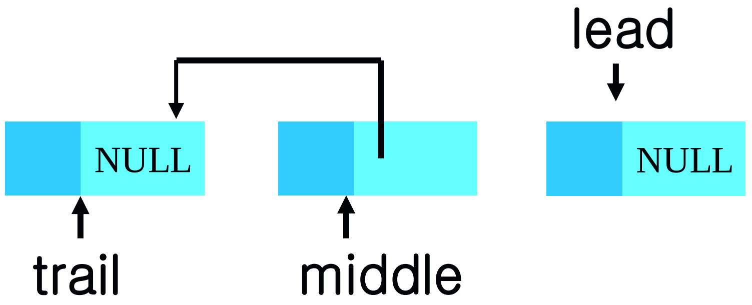
```
list_pointer invert(list_pointer lead)
{
    /* invert the list pointed to by lead  */
    list_pointer  middle, trail;
    middle = NULL;
    while (lead)  {
        trail = middle;
        middle = lead;
        lead = lead->link;
        middle->link = trail;
}
    return  middle;
}
```



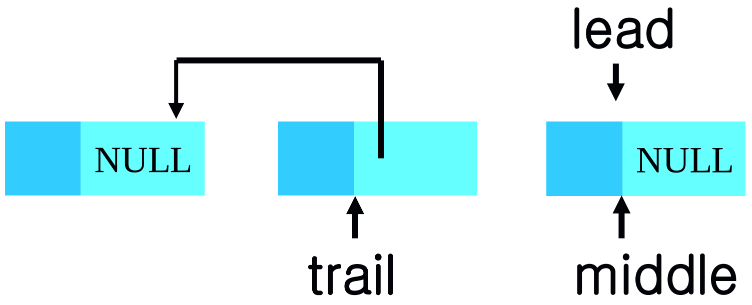
```
list_pointer invert(list_pointer lead)
{
    /* invert the list pointed to by lead  */
    list_pointer  middle, trail;
    middle = NULL;
    while (lead)  {
        trail = middle;
        middle = lead;
        lead = lead->link;
        middle->link = trail;
}
    return  middle;
}
```



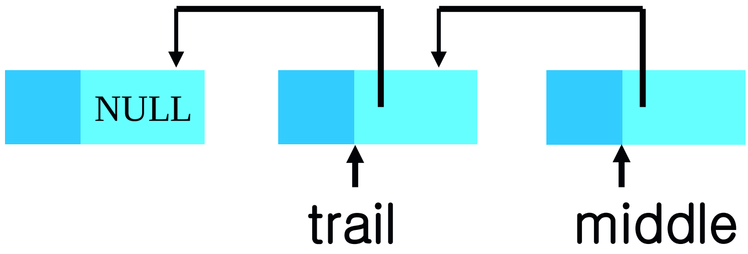
```
list_pointer invert(list_pointer lead)
{
    /* invert the list pointed to by lead  */
    list_pointer  middle, trail;
    middle = NULL;
    while (lead)  {
        trail = middle;
        middle = lead;
        lead = lead->link;
        middle->link = trail;
}
    return  middle;
}
```



```
list_pointer invert(list_pointer lead)
{
        /* invert the list pointed to by lead  */
        list_pointer  middle, trail;
        middle = NULL;
        while (lead)  {
                trail = middle;
                middle = lead;
                lead = lead->link;
                middle->link = trail;
}
        return  middle;
}
```



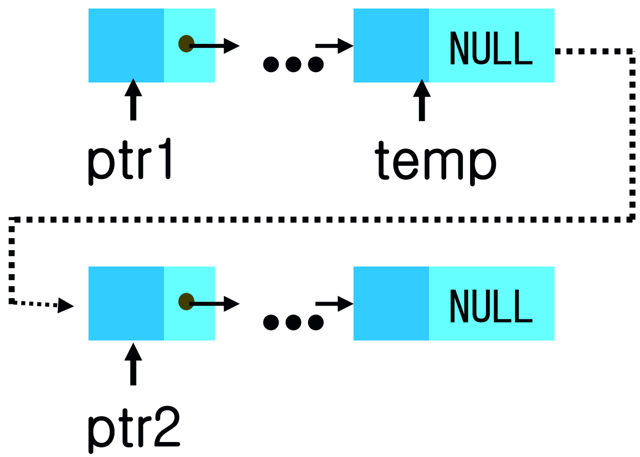
```
list_pointer invert(list_pointer lead)
{
      /* invert the list pointed to by lead  */
      list_pointer  middle, trail;
      middle = NULL;
      while (lead)  {
            trail = middle;
            middle = lead;
            lead = lead->link;
            middle->link = trail;
}
      return  middle;
}
```



lead = NULL

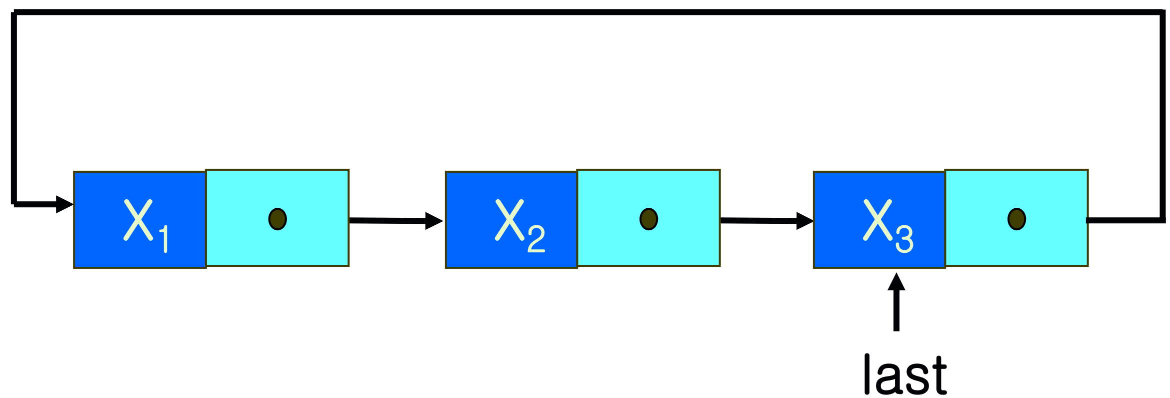
- ***Concatenating two chains*:**
- **[Program 4.17]**

```
list_pointer concatenate(list_pointer ptr1, list_pointer ptr2)
{
    /* produce a new list that contains the list ptr1 followed
    by the list ptr2. The list pointed to by ptr1 is changed permanently */
    list_pointer  temp;
    /* check for empty lists */
    if (IS_EMPTY(ptr1)) return ptr2;
    if (IS_EMPTY(ptr2)) return ptr1;

    /* neither list is empty, find end of first list */
    for(temp=ptr1; temp->link; temp=temp->link) ;

    /* link end of first to start of second */
    temp->link = ptr2;
}
```



4.5.2 Operations For Circularly Linked Lists

- ***Inserting a new node at the front of a circular list*** :
  - Since we have to change the link field of the last node, we must move down the list until we find the last node.
  - It is more convenient if the name of the circular list points to the last node rather than the first.



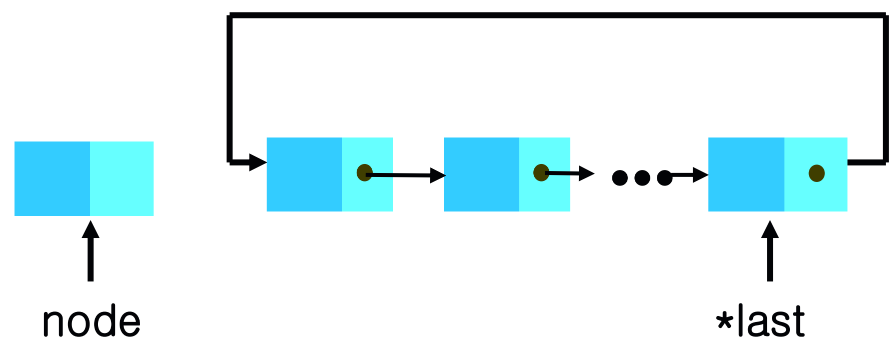
- **[Program 4.18] Inserting at the front of a list**

```
void insert_front(list_pointer *last, list_pointer node)
{
/* insert node at the front of the circular list whose last node is last */
        if (IS_EMPTY(*last))  {
        /* list is empty, change last to point to new entry */
        *last = node;
        node->link = node;
    }
    else  {
        /* list is not empty, add new entry at front */
        node->link = (*last)->link;
        (*last)->link = node;
    }
}
```

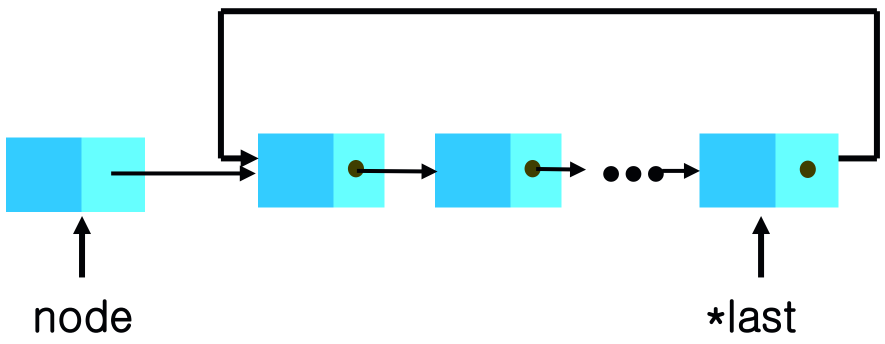
- ***Inserting a new node at the rear of a circular list*** **:**

  We only need to add the additional statement **last = node* to the *else* clause of *insert_front*.

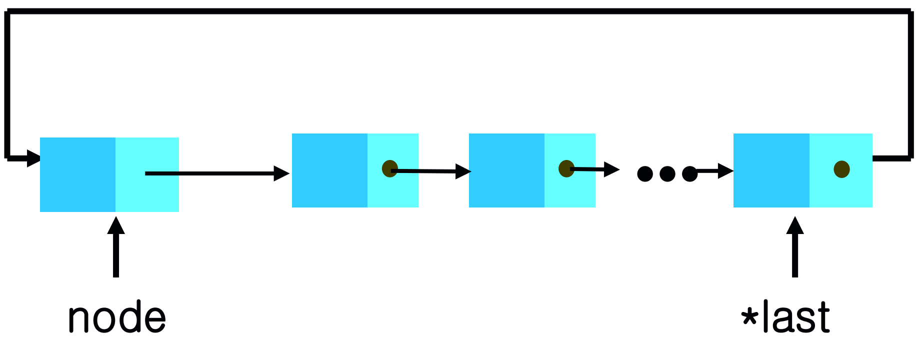
```
void insert_front(list_pointer *last, list_pointer node)
{
/* insert node at the front of the circular list whose last node is last */
        if (IS_EMPTY(*last))  {
        /* list is empty, change last to point to new entry */
        *last = node;
        node->link = node;
    }
    else  {
        /* list is not empty, add new entry at front */
        node->link = (*last)->link;
        (*last)->link = node;
    }
}
```



```
void insert_front(list_pointer *last, list_pointer node)
{
/* insert node at the front of the circular list whose last node is last */
        if (IS_EMPTY(*last))  {
        /* list is empty, change last to point to new entry */
        *last = node;
        node->link = node;
    }
    else  {
        /* list is not empty, add new entry at front */
        node->link = (*last)->link;
        (*last)->link = node;
    }
}
```

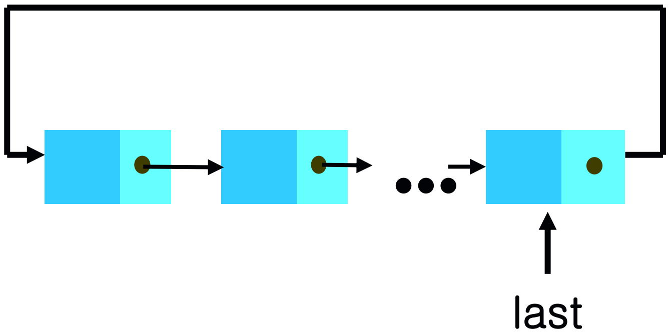


```
void insert_front(list_pointer *last, list_pointer node)
{
/* insert node at the front of the circular list whose last node is last */
        if (IS_EMPTY(*last))  {
        /* list is empty, change last to point to new entry */
        *last = node;
        node->link = node;
    }
    else  {
        /* list is not empty, add new entry at front */
        node->link = (*last)->link;
        (*last)->link = node;
    }
}
```



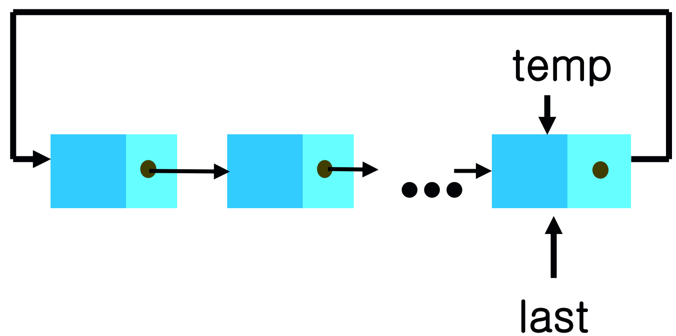
[Program 4.19] Finding the length of a circular list

```
int length(list_pointer last)
{
    /* find the length of the circular list last */
list_pointer temp;
int count = 0;
if (last)  {
        temp = last;
        do  {
            count++;
        temp = temp->link;
        } while (temp != last);
    }
    return count;
}
```



- **[Program 4.19] Finding the length of a circular list**

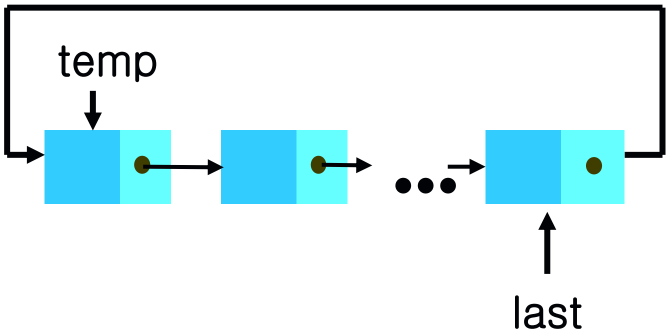
```
int length(list_pointer last)
{
    /* find the length of the circular list last */
list_pointer temp;
int count = 0;
if (last)  {
        temp = last;
        do  {
            count++;
        temp = temp->link;
        } while (temp != last);
    }
    return count;
}
```



count = 0

- **[Program 4.19] Finding the length of a circular list**

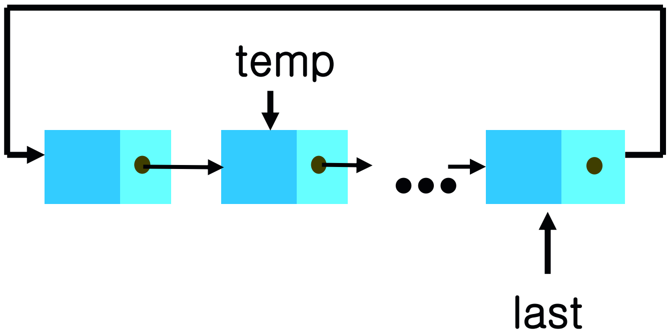
```
int length(list_pointer last)
{
    /* find the length of the circular list last */
list_pointer temp;
int count = 0;
if (last)  {
        temp = last;
        do  {
            count++;
        temp = temp->link;
        } while (temp != last);
    }
    return count;
}
```



count = 1

- **[Program 4.19] Finding the length of a circular list**

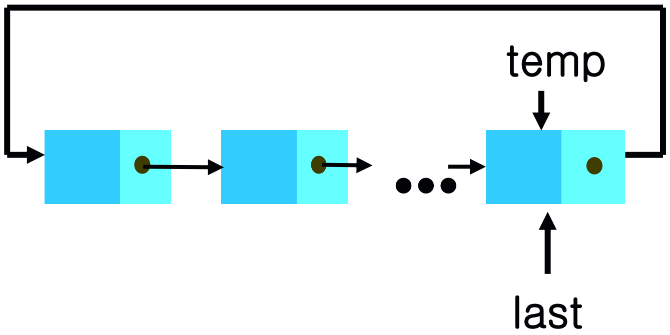
```
int length(list_pointer last)
{
    /* find the length of the circular list last */
list_pointer temp;
int count = 0;
if (last)  {
        temp = last;
        do  {
            count++;
        temp = temp->link;
        } while (temp != last);
    }
    return count;
}
```



count = 2

- **[Program 4.19] Finding the length of a circular list**

```
int length(list_pointer last)
{
      /* find the length of the circular list last */
list_pointer temp;
int count = 0;
if (last)  {
            temp = last;
            do  {
                  count++;
            temp = temp->link;
            } while (temp != last);
      }
      return count;
}
```



count = n

4.6 EQUIVALENCE RELATIONS

- R is a *binary relation* on a set S if $R \subseteq S \times S$.
  If $(a, b) \in R$ then we may write aRb.

  - R is *reflexive* if aRa for all $a \in S$.
  - R is *symmetric* if aRb implies bRa.
  - R is *transitive* if aRb and bRc implies aRc.

  - R is an *equivalence relation* over S
    if R is reflexive, symmetric and transitive over S.

- **[Example]**
  - One of the steps in the manufacture of a VLSI circuit involves exposing a silicon wafer using a series of masks. Each mask consists of several polygons. Polygons that overlap electrically are equivalent and electrical equivalence specifies an equivalence relation ≡ over the set of mask polygons.

(1) For any polygon x, x≡x, that is, x is electrically equivalent to itself. Thus, ≡ is reflexive.

(2) For any two polygons, x and y, if x≡y then y≡x. Thus, the relation ≡ is symmetric.

(3) For any three polygons, x, y, and z, if x≡y and y≡z then x≡z. For example, if x and y are electrically equivalent and y and z are also equivalent, then x and z are also electrically equivalent.

Thus the relation ≡ is transitive.

- Any equivalence relation R over S can partition the set S into disjoint subsets called *equivalence classes*.

- An *equivalence class* E is a subset of S such that if x is in E then E contains every element which is related to x by R. That is, for any $x \in S$, $[x] = \{y \mid y \in S \text{ and } x \equiv y\}$, where [x] denotes the equivalence class of an element x.

- For any x and y in S, either $[x] = [y]$ or $[x] \cap [y] = \emptyset$.

- **Example :**
  - If we have 12 polygons numbered 0 through 11

    and the following pairs overlap :

    0≡4, 3≡1, 6≡10, 8≡9, 7≡4, 6≡8, 3≡5, 2≡11, 11≡0

  - as a result of the reflexivity, symmetry, and transitivity of the relation ≡, we can obtain the following equivalence classes :

    {0, 2, 4, 7, 11}; {1, 3, 5}; {6, 8, 9, 10}

- **The algorithm to determine equivalence works in two phases :**

  - *First phase* : read in and store the equivalence pairs <i, j>.
  - *Second phase* : determining equivalence class as follows

    we begin at 0 and find all pairs of the form <0, j>.

    Then, find all pairs of the form <j, k>.

    - By transitivity,

      <0, j> and <j, k> ⇒ <0, k> i.e., $0 \equiv j$ and $j \equiv k \Rightarrow 0 \equiv k$

  We continue in this way until we have found, marked, and printed the entire equivalence class containing 0.

  Then we continue on.

- **<u>Our first design attempt</u> :**
- **[Program 4.20]**

```
void equivalence()
{
      initialize;
      while (there are more pairs)  {
            read the next pair <i, j>;
            process this pair;
      }
      initialize the output;
      do
            output a new equivalence class;
      while  (not done);
}
```

- Let *m* and *n* represent the number of related pairs and the number of objects, respectively.
- We must first figure out which data structure we should use to hold these pairs.
- The pair <i, j> is essentially two random integers in the range 0 to n-1.
- Use an array, *pairs*[n][m], for easy random access.

  This could waste a lot of space since very few of the array elements would be used.

  It also might require considerable time or use more storage to insert a new pair.

- These considerations lead us to a linked list representation for each row.

- Since we still need random access to the *i* th row, we use a one-dimensional array, *seq*[n], to hold the header nodes of the *n* lists.

- In the second phase of the algorithm, we need to check whether or not the object, *i*, has been printed.

  -> We use the array *out*[n].

- **[Program 4.21] A more detailed version of the equivalence algorithm**

```
void equivalence()
{
    initialize seq to NULL and out to TRUE;
    while (there are more pairs)  {
        read the next pair <i, j>;
        put j on the seq[i] list;
        put i on the seq[j] list;
    }
    for (i=0; i<n; i++)
        if (out[i])  {
            out[i] = FALSE;
            output this equivalence class;
    }
}
```

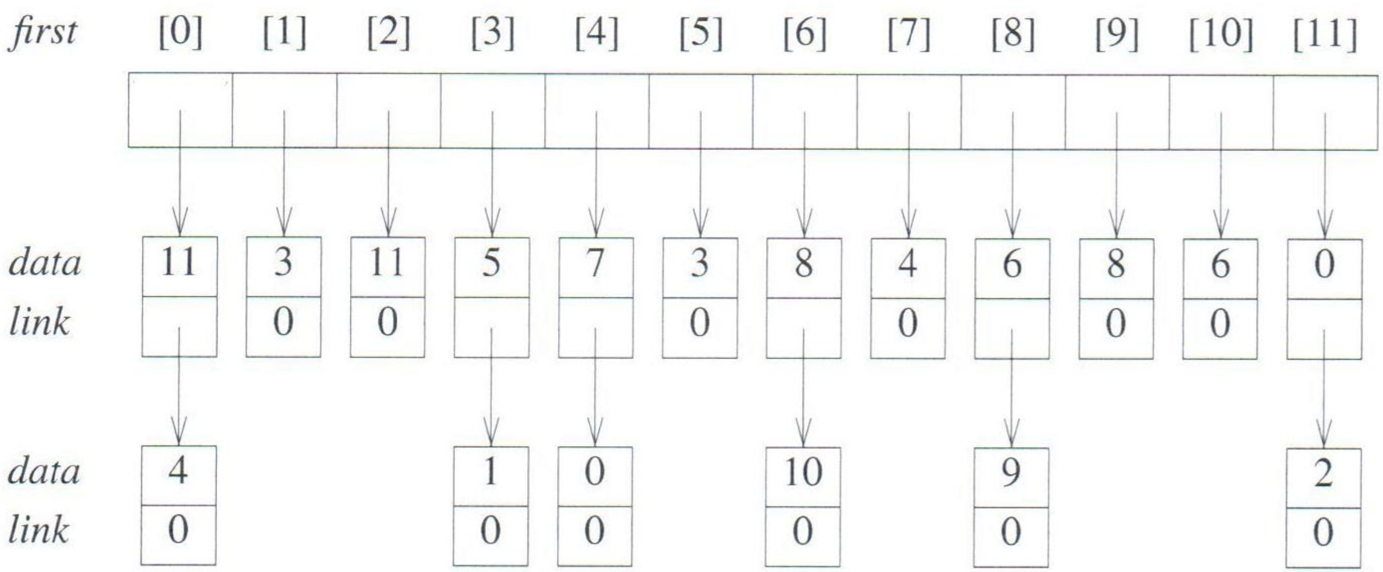


Figure 4.16: Lists after pairs have been input

- In phase two :
  - We scan the *seq* array for the first *i*, $0 \le i < n$, such that *out*[i] = TRUE.
  - Each element in the list *seq*[i] is printed.

- To process the remaining lists which, by transitivity, belong in the same class as *i*, we create a stack of their nodes.

- For the complete equivalence algorithm, see the following declaration and Program 4.22.

[Program 4.22] Program to find equivalence classes

```
#include <stdio.h>
#include <stdlib.h>
#define MAX_SIZE  24
#define IS_FULL (ptr)  (!(ptr))
#define FALSE 0
#define TRUE 1

typedef  struct node *node_pointer;
typedef  struct node {
      int data;
      node_pointer link;
};
```

```
void main(void)
{
    short  int out[MAX_SIZE];
    node_pointer  seq[MAX_SIZE];
    node_pointer x, y, top;
    int  i, j, n;

    printf("Enter the size (<= %d) ", MAX_SIZE);
    scanf("%d", &n);
    for (i = 0; i < n; i++)  {
        /*  initialize seq and out  */
        out[i] = TRUE;          seq[i] = NULL;
    }
```

```
/*  Phase 1: Input the equivalence pairs :  */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf("Enter a pair of numbers (-1 -1 to quit): ");
        scanf("%d %d", &i, &j);
}
```

```
    /* Phase 2 : output the equivalence classes  */
    for (i = 0; i < n; i++)  {
          if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;   /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
               for ( ; ; )   {    /*  find rest of class  */
                      while (x)   {      /*  process list  */
                            j = x->data;
                            if  (out[j])   {
                                  printf("%5d", j);   out[j] = FALSE;
                                  y = x->link;  x->link = top;  top = x;  x = y;
                            }
                            else  x = x->link;
                      }
                      if  (!top)   break;
                      x = seq[top->data];  top = top->link;   /* unstack */
                }
          }
    }
}
```

```
/*  initialize seq and out  */
    out[i] = TRUE;        seq[i] = NULL;
```

n = 7

seq

[0]	[1]	[2]	[3]	[4]	[5]	[6]
NULL	NULL	NULL	NULL	NULL	NULL	NULL

out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

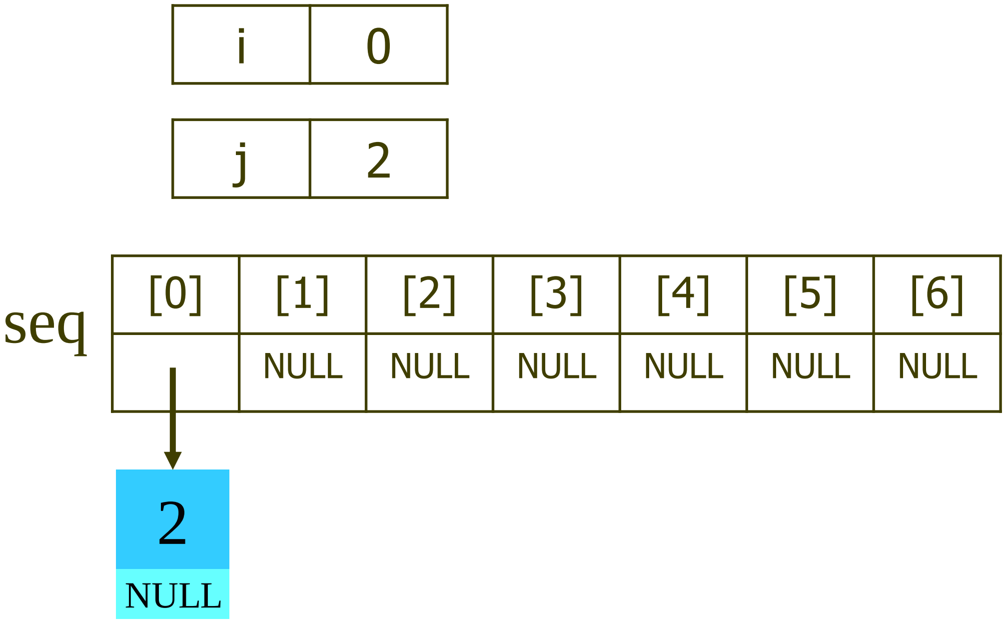
```
/*  Phase 1: Input the equivalence pairs :  */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf("Enter a pair of numbers (-1 -1 to quit): ");
        scanf("%d %d", &i, &j);
}
```

i	0
j	2

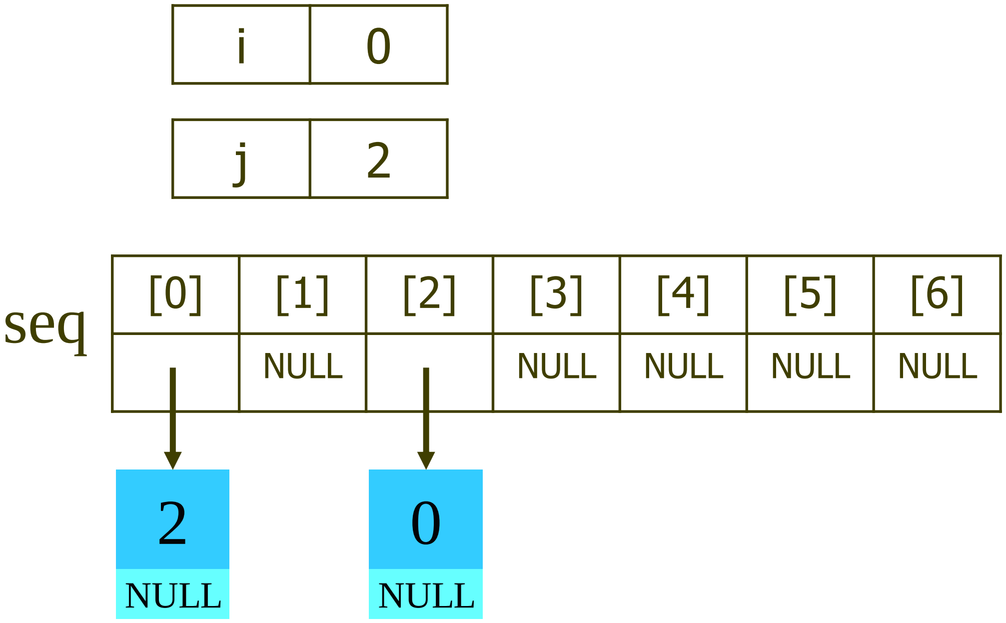
seq

[0]	[1]	[2]	[3]	[4]	[5]	[6]
NULL	NULL	NULL	NULL	NULL	NULL	NULL

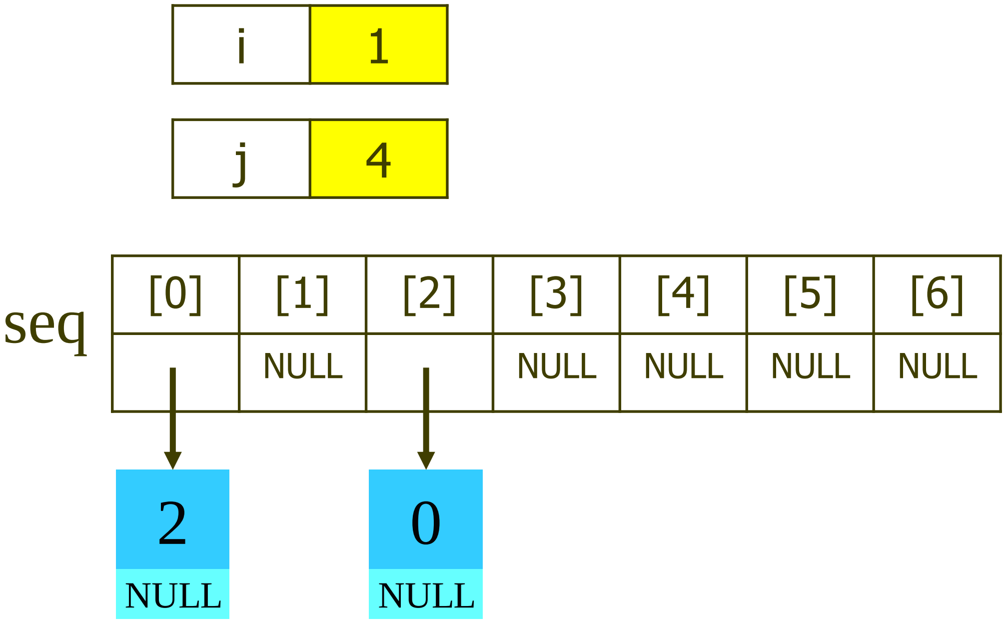
```
/*  Phase 1: Input the equivalence pairs :  */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf("Enter a pair of numbers (-1 -1 to quit): ");
        scanf("%d %d", &i, &j);
}
```



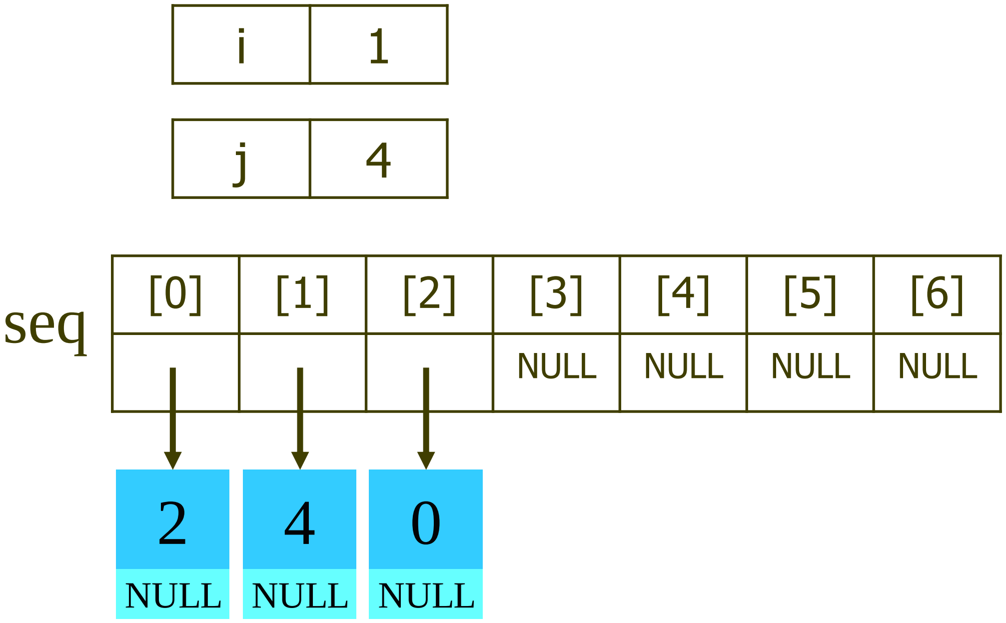
```
/*  Phase 1: Input the equivalence pairs :  */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf("Enter a pair of numbers (-1 -1 to quit): ");
        scanf("%d %d", &i, &j);
}
```



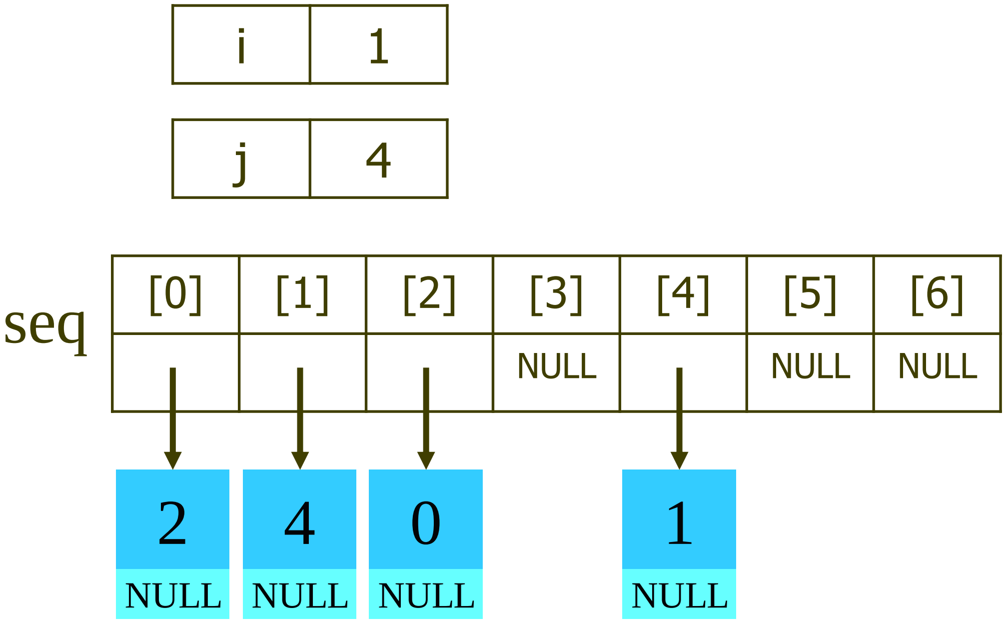
```
/*  Phase 1: Input the equivalence pairs :  */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf("Enter a pair of numbers (-1 -1 to quit): ");
        scanf("%d %d", &i, &j);
}
```



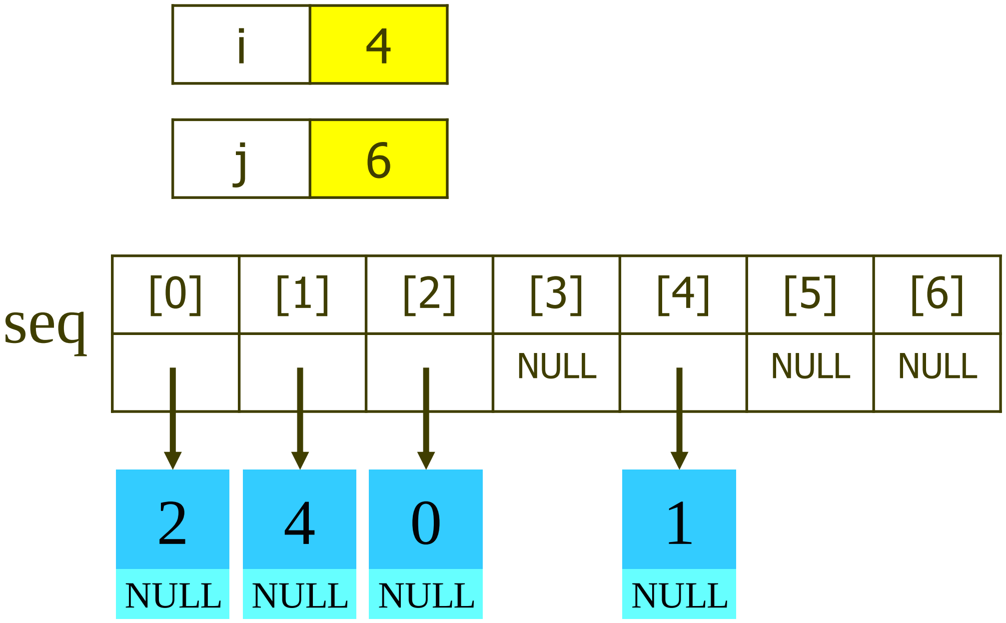
```
/*  Phase 1: Input the equivalence pairs :  */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf("Enter a pair of numbers (-1 -1 to quit): ");
        scanf("%d %d", &i, &j);
}
```



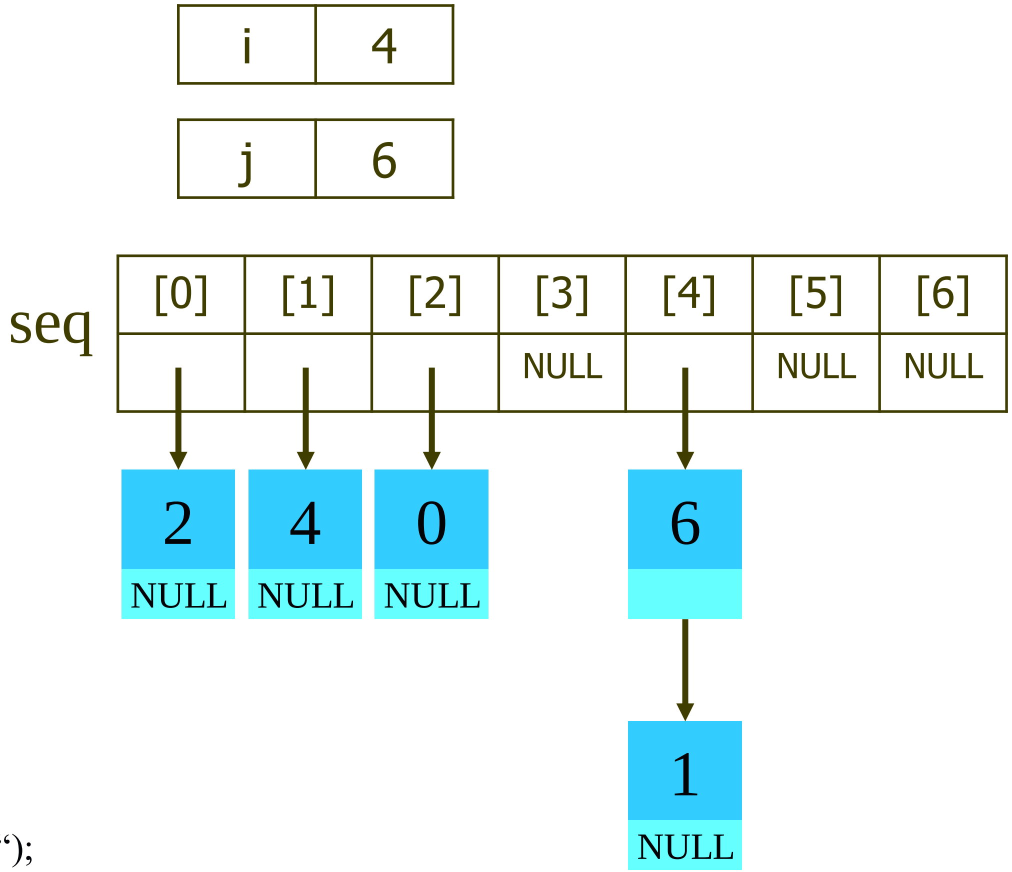
```
/*  Phase 1: Input the equivalence pairs :  */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf("Enter a pair of numbers (-1 -1 to quit): ");
        scanf("%d %d", &i, &j);
}
```



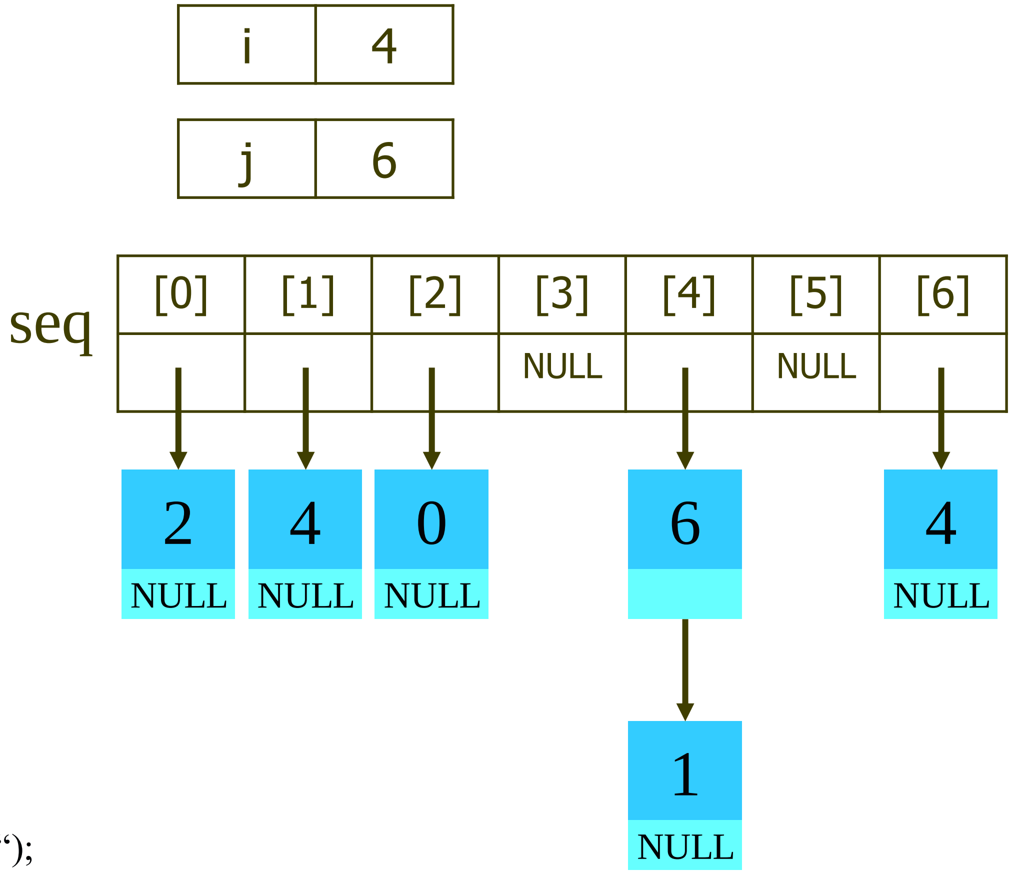
```
/*  Phase 1: Input the equivalence pairs :  */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf("Enter a pair of numbers (-1 -1 to quit): ");
        scanf("%d %d", &i, &j);
}
```



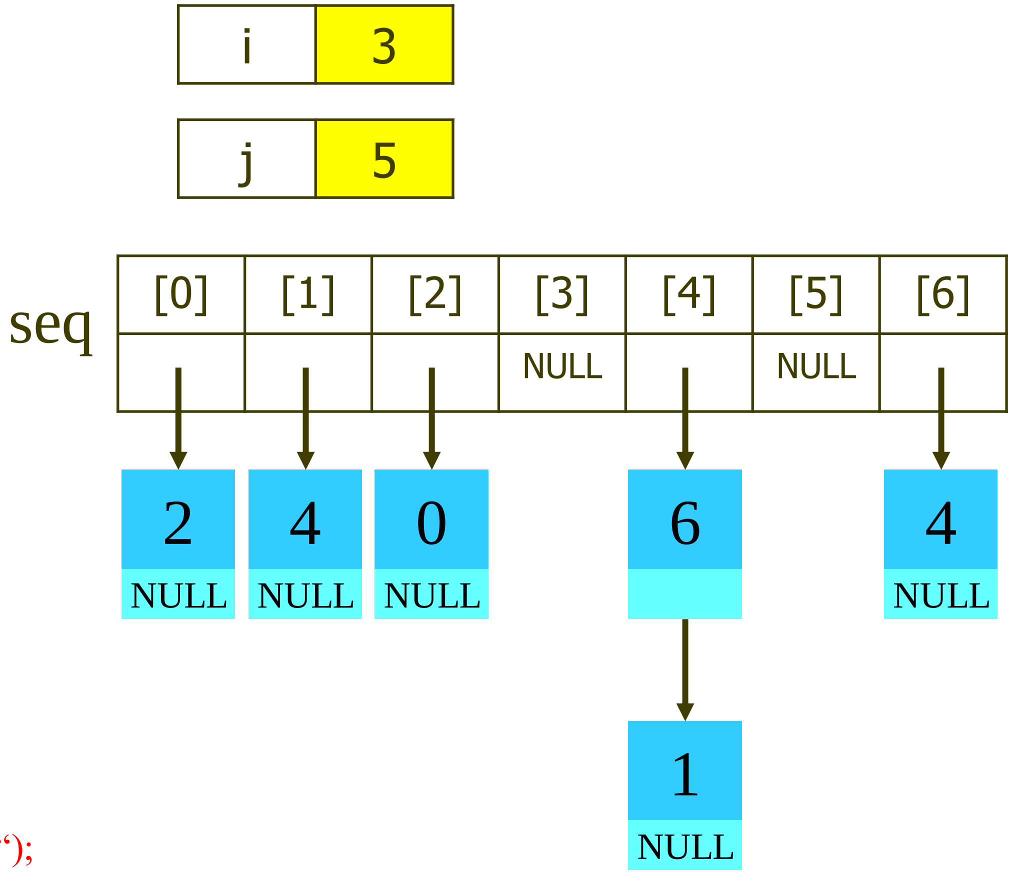
```
/*  Phase 1: Input the equivalence pairs :  */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)  {
      x = (node_pointer)malloc(sizeof(struct node));
      if (IS_FULL(x))  {
            fprintf(stderr, "The memory is full\n");
            exit(1);
      }
      x->data = j;  x->link = seq[i];  seq[i] = x;
      x = (node_pointer)malloc(sizeof(struct node));
      if (IS_FULL(x))  {
            fprintf(stderr, "The memory is full\n");
            exit(1);
      }
      x->data = i;  x->link = seq[j];  seq[j] = x;
      printf("Enter a pair of numbers (-1 -1 to quit): ");
      scanf("%d %d", &i, &j);
}
```



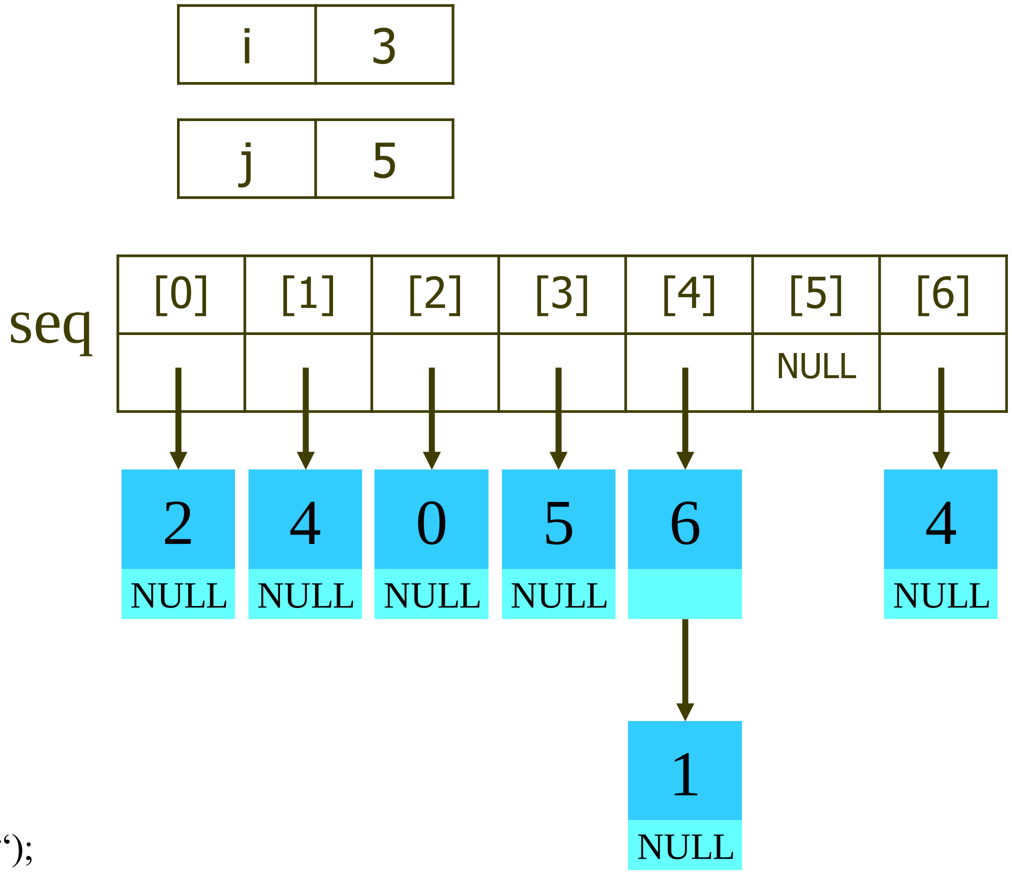
```c
/*  Phase 1: Input the equivalence pairs :  */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf("Enter a pair of numbers (-1 -1 to quit): ");
        scanf("%d %d", &i, &j);
}
```



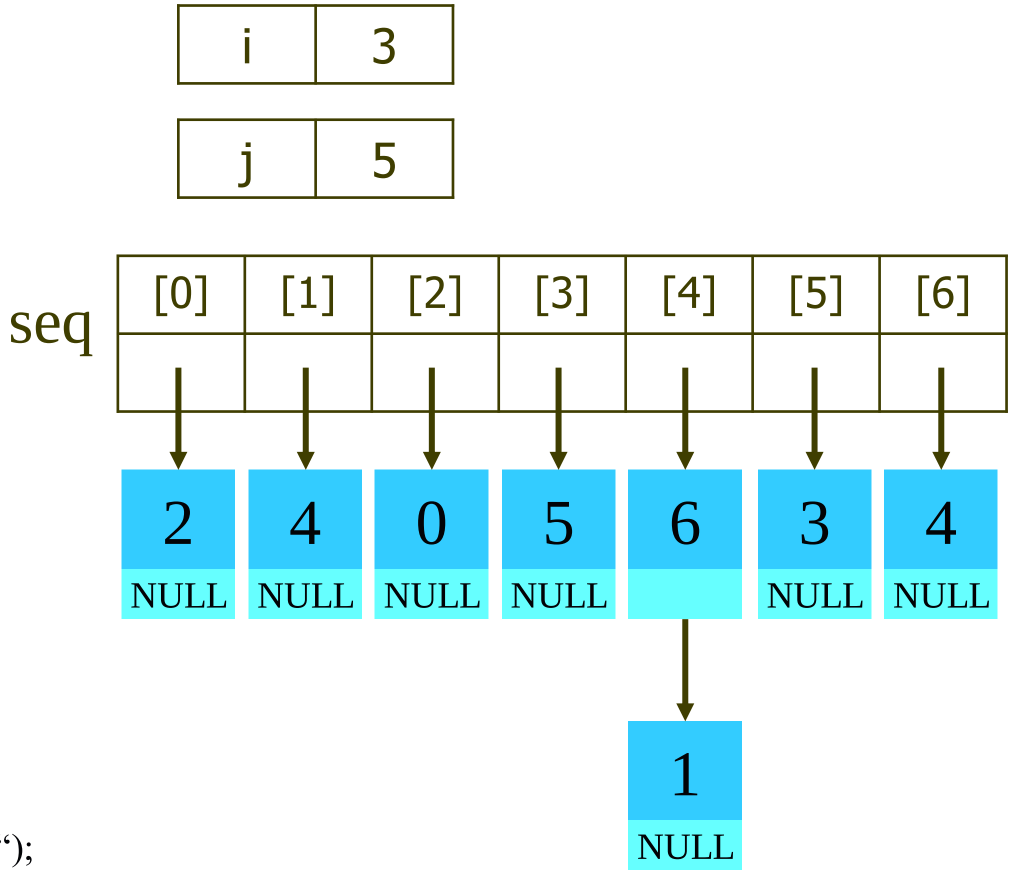
```
/*  Phase 1: Input the equivalence pairs :  */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf("Enter a pair of numbers (-1 -1 to quit): ");
        scanf("%d %d", &i, &j);
}
```



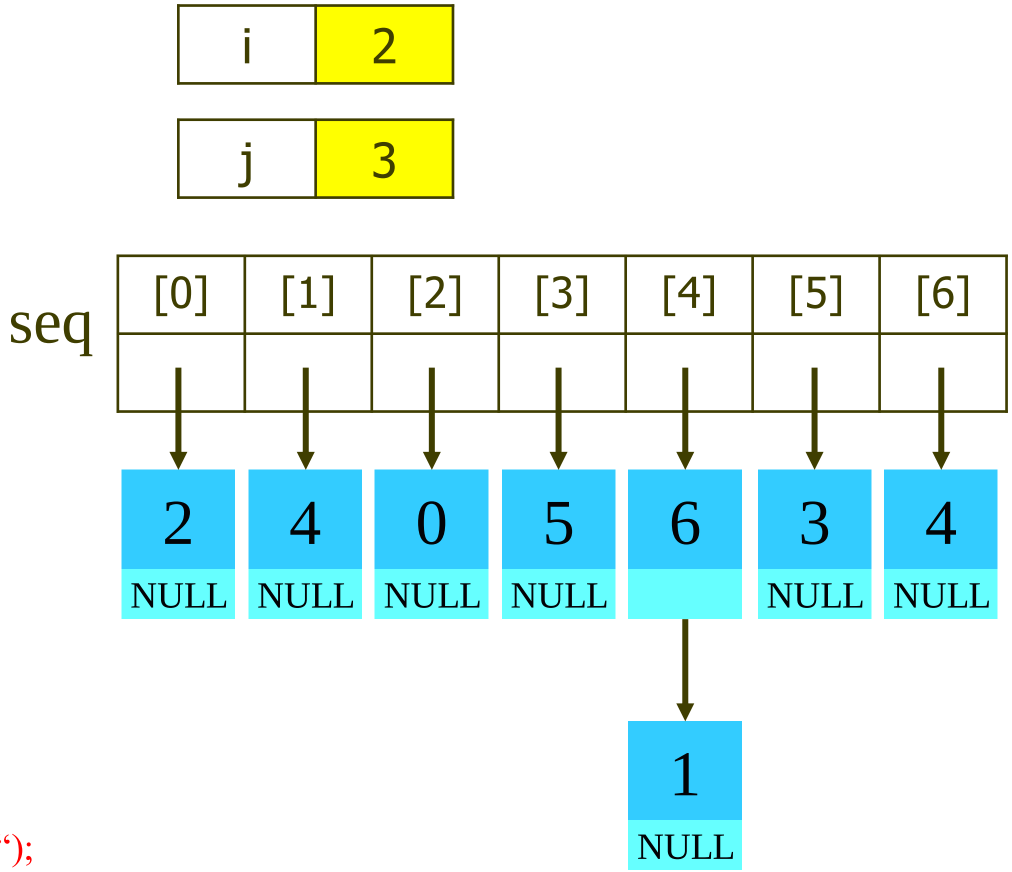
```
/*  Phase 1: Input the equivalence pairs :  */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf("Enter a pair of numbers (-1 -1 to quit): ");
        scanf("%d %d", &i, &j);
}
```



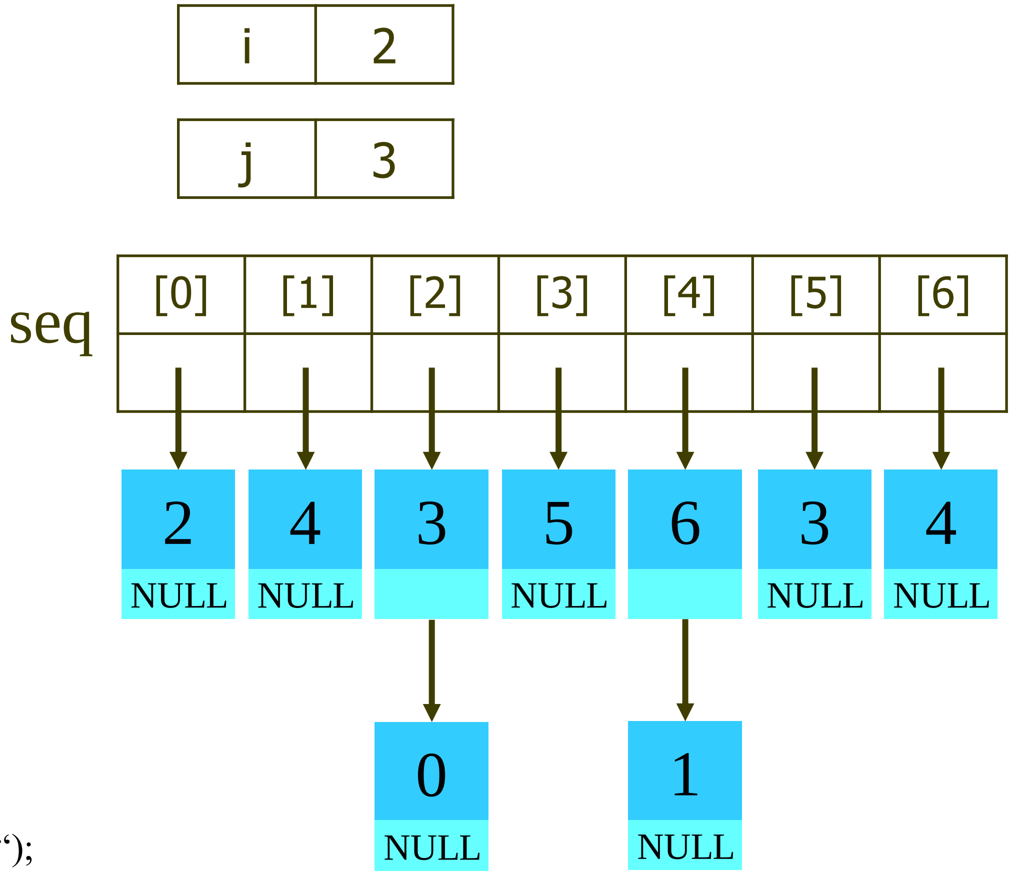
```
/*  Phase 1: Input the equivalence pairs :  */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf("Enter a pair of numbers (-1 -1 to quit): ");
        scanf("%d %d", &i, &j);
}
```



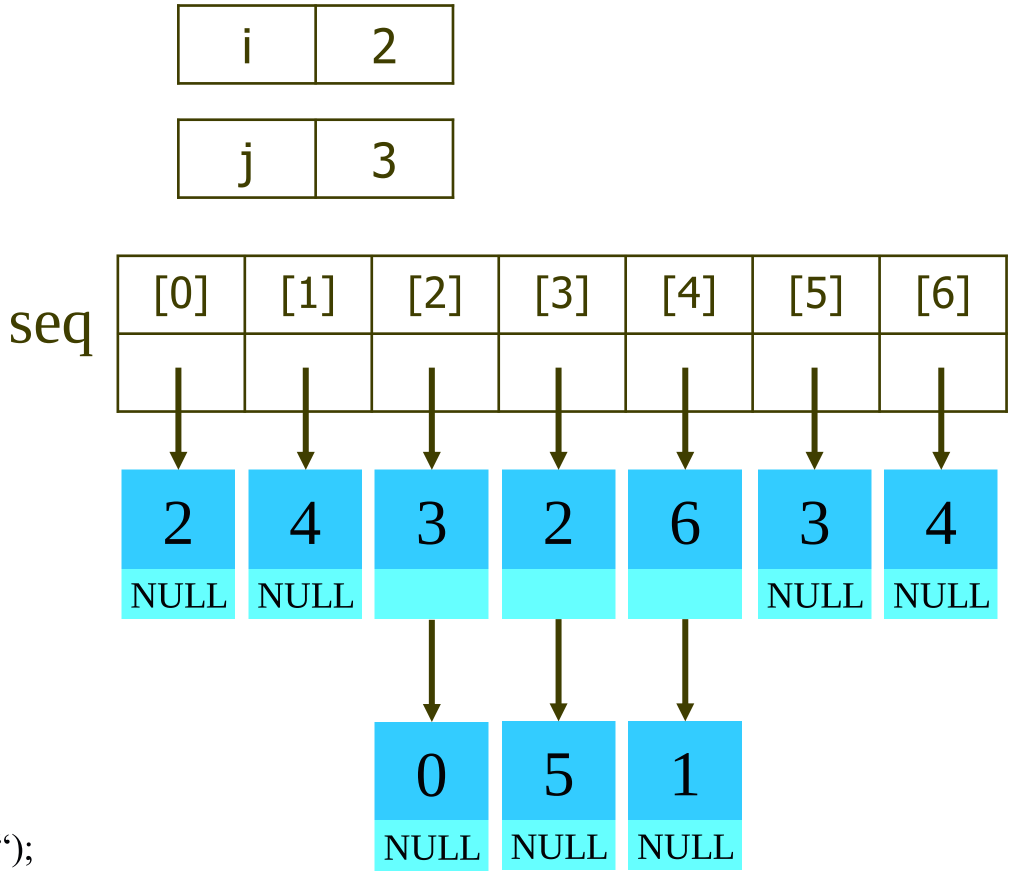
```
/*  Phase 1: Input the equivalence pairs :  */
printf(“Enter a pair of numbers (-1 -1 to quit): “);
scanf(“%d %d”, &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, “The memory is full\n”);
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, “The memory is full\n”);
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf(“Enter a pair of numbers (-1 -1 to quit): “);
        scanf(“%d %d”, &i, &j);
}
```



```
/*  Phase 1: Input the equivalence pairs :  */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf("Enter a pair of numbers (-1 -1 to quit): ");
        scanf("%d %d", &i, &j);
}
```

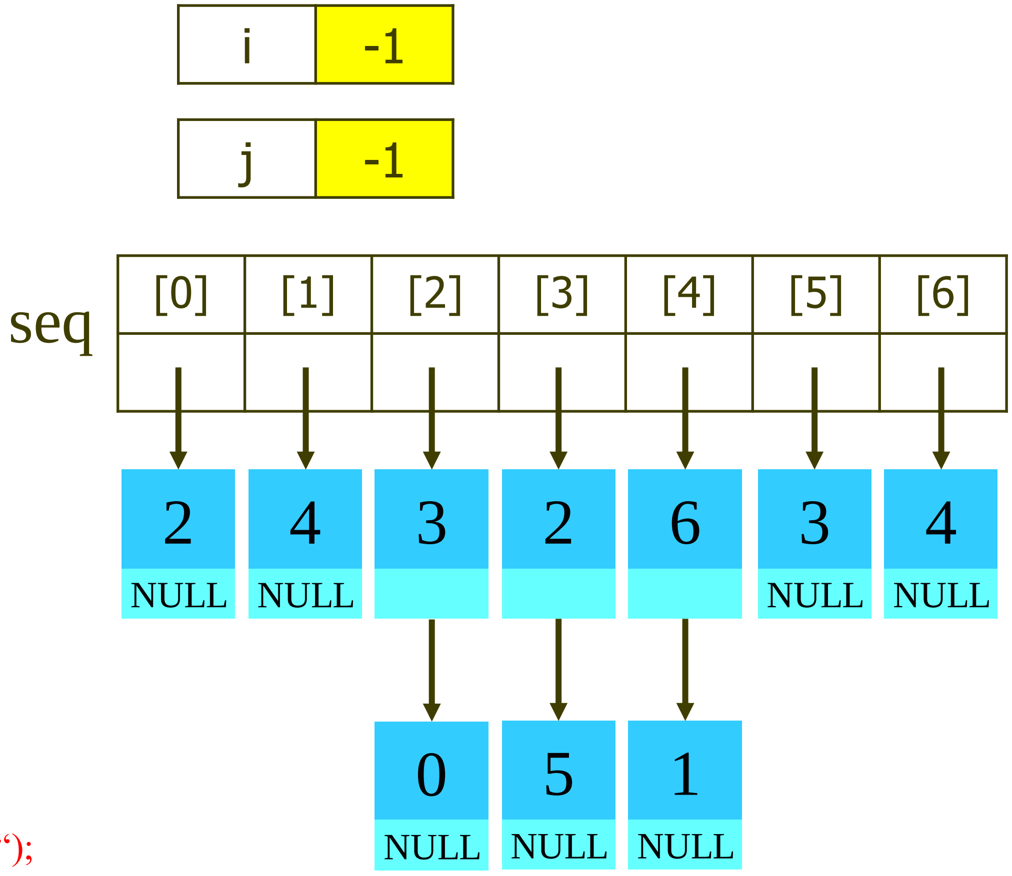


```
/*  Phase 1: Input the equivalence pairs :  */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf("Enter a pair of numbers (-1 -1 to quit): ");
        scanf("%d %d", &i, &j);
}
```

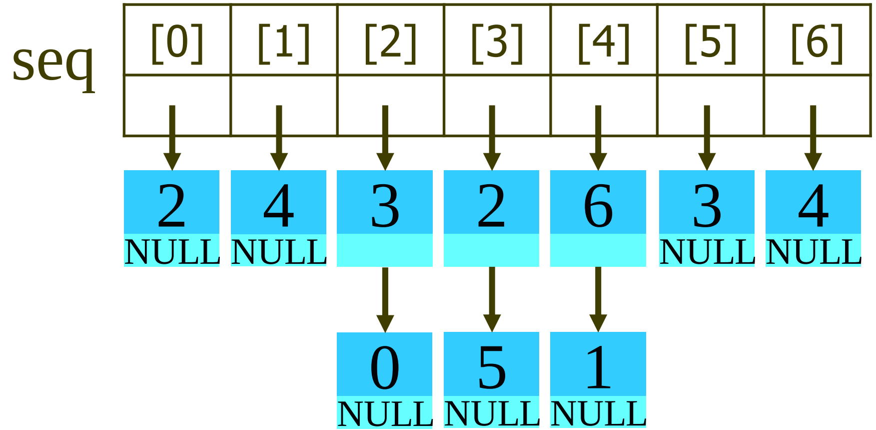


```
/* Phase 1: Input the equivalence pairs : */
printf("Enter a pair of numbers (-1 -1 to quit): ");
scanf("%d %d", &i, &j);
while (i >=0)   {
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = j;  x->link = seq[i];  seq[i] = x;
        x = (node_pointer)malloc(sizeof(struct node));
        if (IS_FULL(x))   {
                fprintf(stderr, "The memory is full\n");
                exit(1);
        }
        x->data = i;  x->link = seq[j];  seq[j] = x;
        printf("Enter a pair of numbers (-1 -1 to quit): ");
        scanf("%d %d", &i, &j);
}
```

→ break



```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {    /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

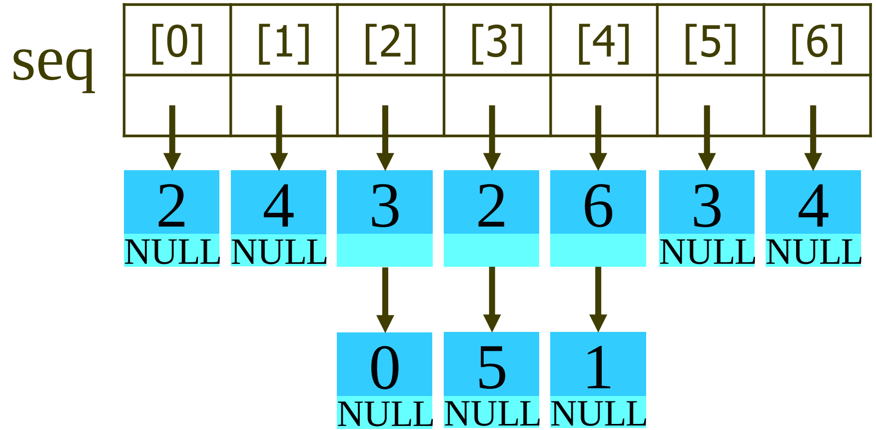
i		j	

x

top = NULL

/* output */

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
      if  (out[i])  {
            printf("\nNew Class : %5d", i);
            out[i] =FALSE;    /*   set class to false  */
            x = seq[i];  top = NULL;  /*  initialize stack  */
            for ( ; ; )   {    /*  find rest of class  */
                  while (x)   {      /*  process list  */
                        j = x->data;
                        if  (out[j])   {
                              printf("%5d", j);   out[j] = FALSE;
                              y = x->link;  x->link = top;  top = x;  x = y;
                        }
                        else  x = x->link;
                  }
                  if  (!top)   break;
                  x = seq[top->data];  top = top->link;   /* unstack */
            }
      }
}
```

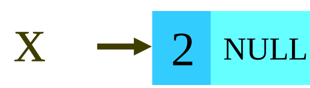


out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

i	0

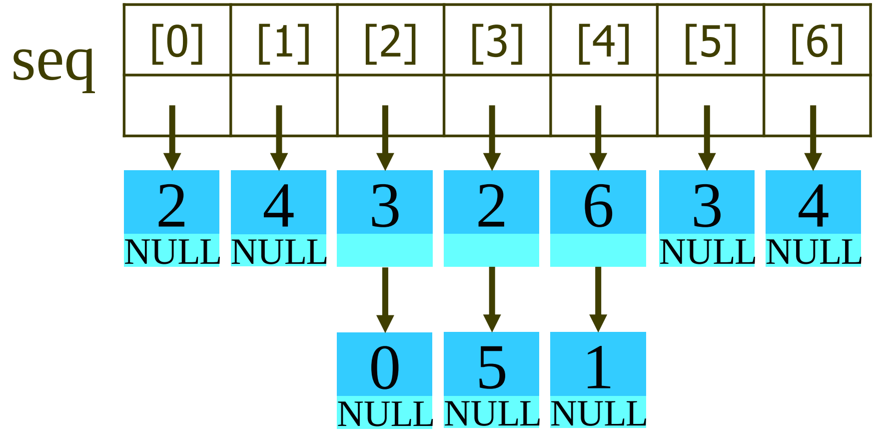
j	



top = NULL

/* output */
New Class : 0

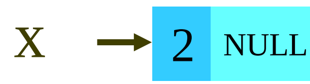
```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;   /*  initialize stack  */
                for ( ; ; )    {     /*  find rest of class  */
                        while (x)   {        /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE

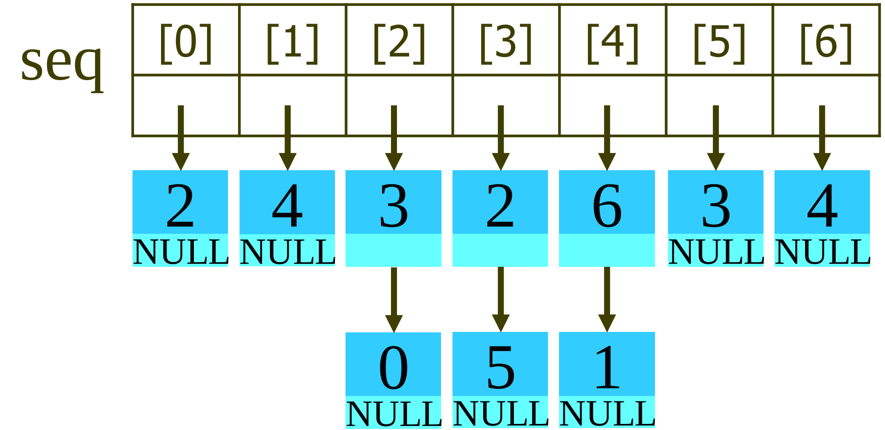
i	0	j	2



top = NULL

/* output */
New Class : 0

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {    /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



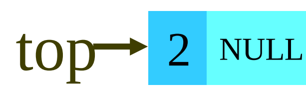
out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE

i	0

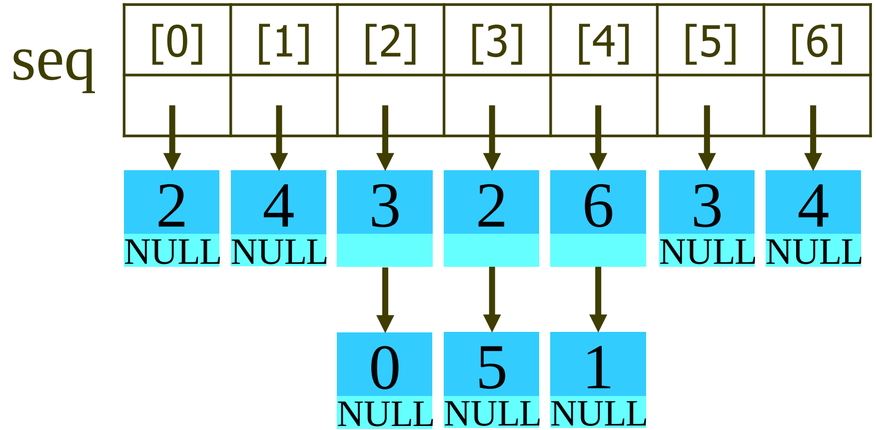
j	2

x = NULL



/* output */
New Class : 0 2

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {    /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



out

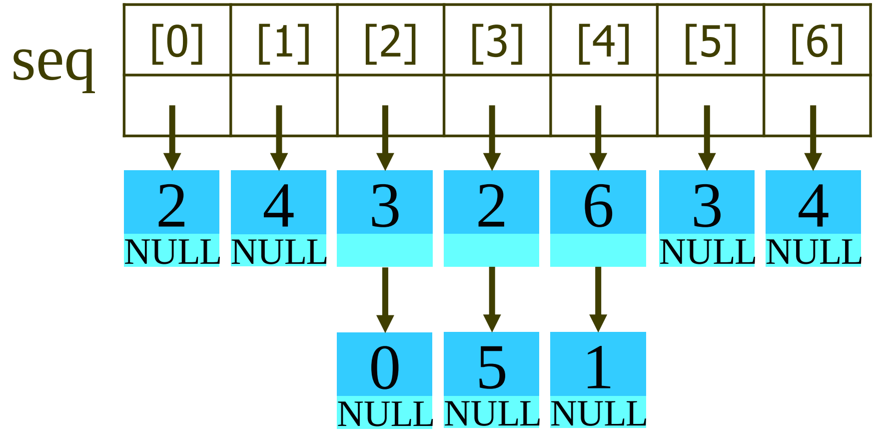
[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE

i	0	j	2

top = NULL

/* output */
New Class : 0 2

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {    /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



out

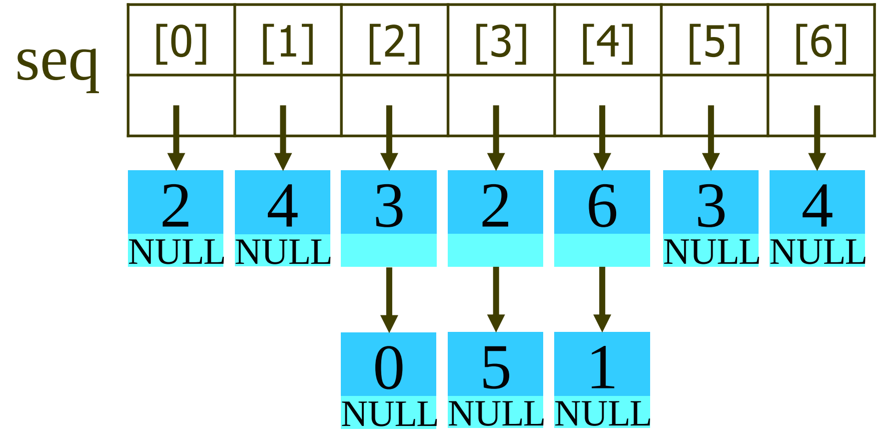
[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	TRUE	FALSE	TRUE	TRUE	TRUE	TRUE

i	0

j	3

top = NULL

/* output */
New Class : 0 2



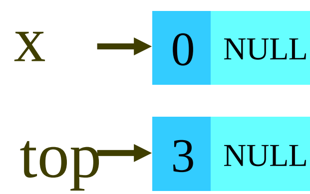

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
      if  (out[i])  {
            printf("\nNew Class : %5d", i);
            out[i] =FALSE;    /*   set class to false  */
            x = seq[i];  top = NULL;  /*  initialize stack  */
            for ( ; ; )   {    /*  find rest of class  */
                  while (x)   {       /*  process list  */
                        j = x->data;
                        if  (out[j])   {
                              printf("%5d", j);   out[j] = FALSE;
                              y = x->link;  x->link = top;  top = x;  x = y;
                        }
                        else  x = x->link;
                  }
                  if  (!top)   break;
                  x = seq[top->data];  top = top->link;   /* unstack */
            }
      }
}
```

out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE

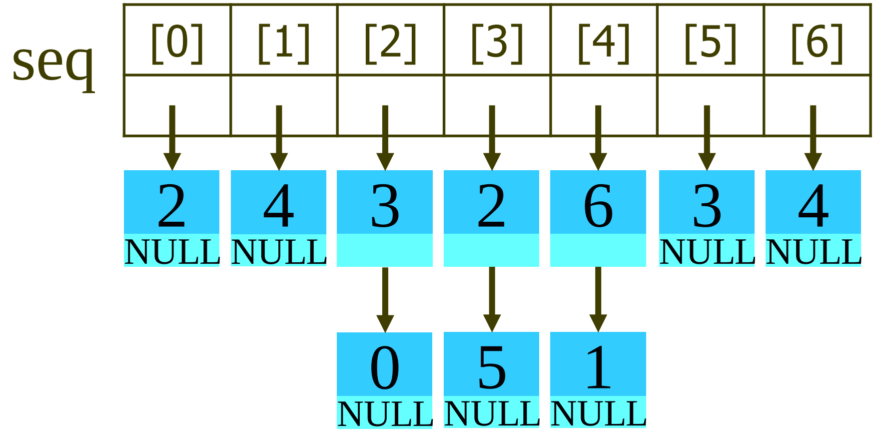
i	0

j	3



/* output */
New Class : 0 2 3

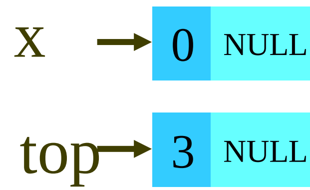
```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;   /*  initialize stack  */
                for ( ; ; )   {     /*  find rest of class  */
                        while (x)   {        /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);    out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



out

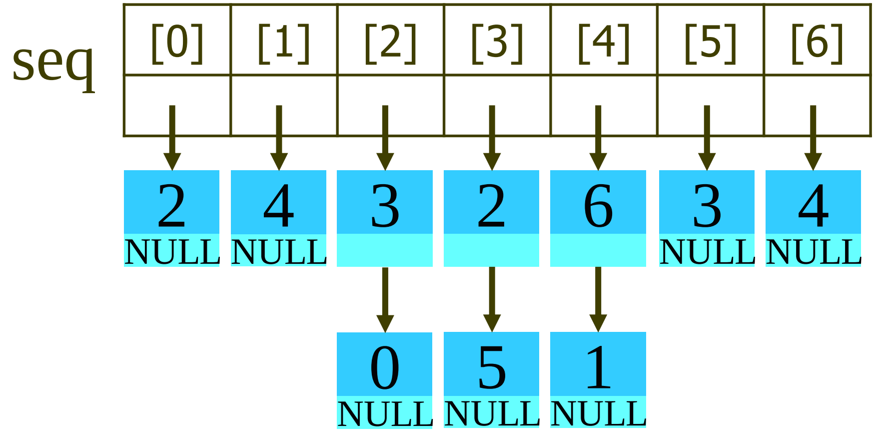
[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE

i	0	j	0



/* output */
New Class : 0 2 3

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {    /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```

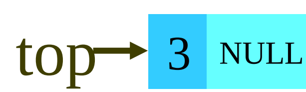


out	[0]	[1]	[2]	[3]	[4]	[5]	[6]
	FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE

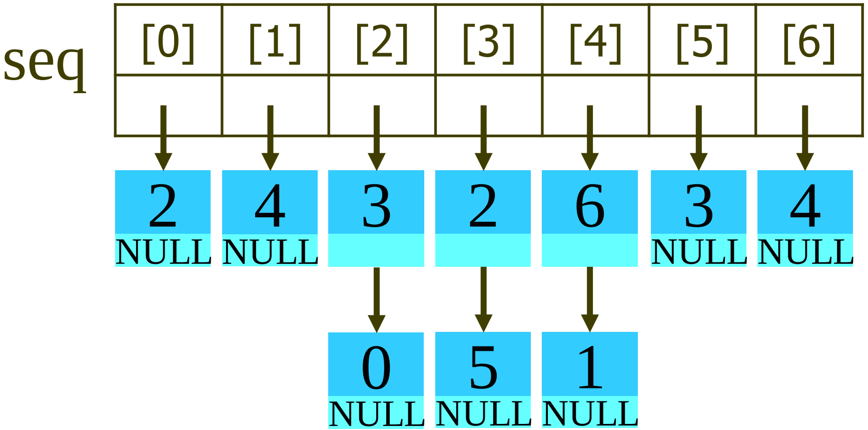
i	0

j	0

x = NULL



/* output */
New Class : 0 2 3




```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {     /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```

out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE

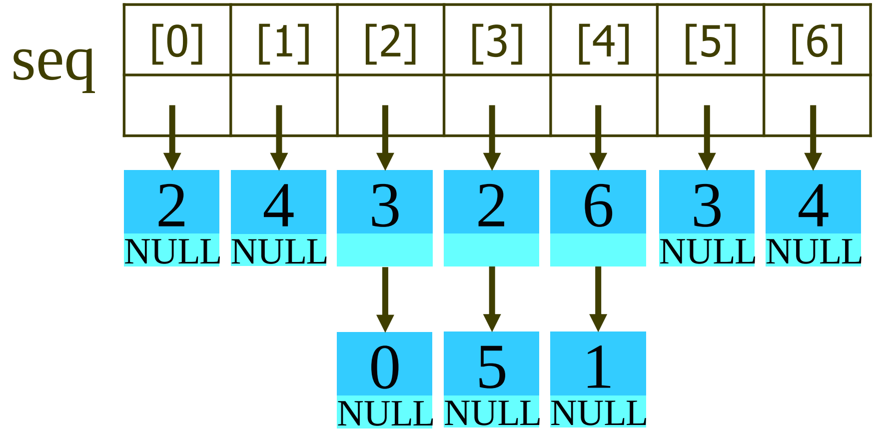
i	0

j	0

top = NULL

/* output */
New Class : 0 2 3

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {     /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



out

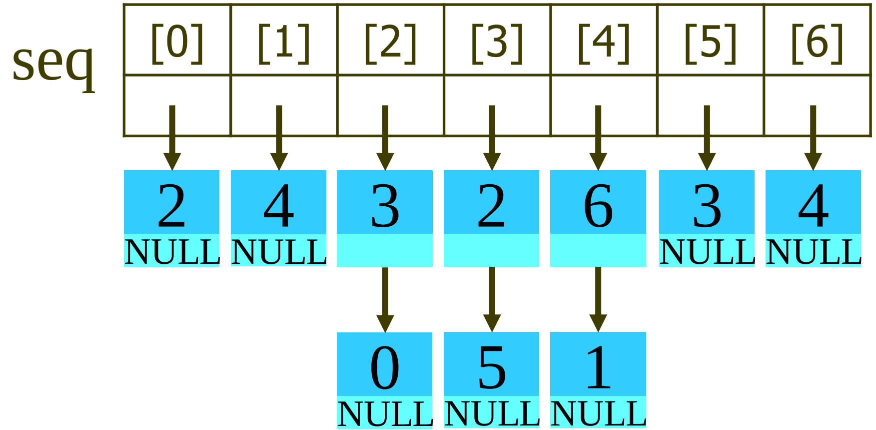
[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE

i	0	j	2

top = NULL

/* output */
New Class : 0 2 3

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {    /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```

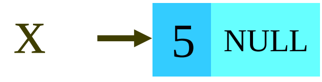


out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE

i	0

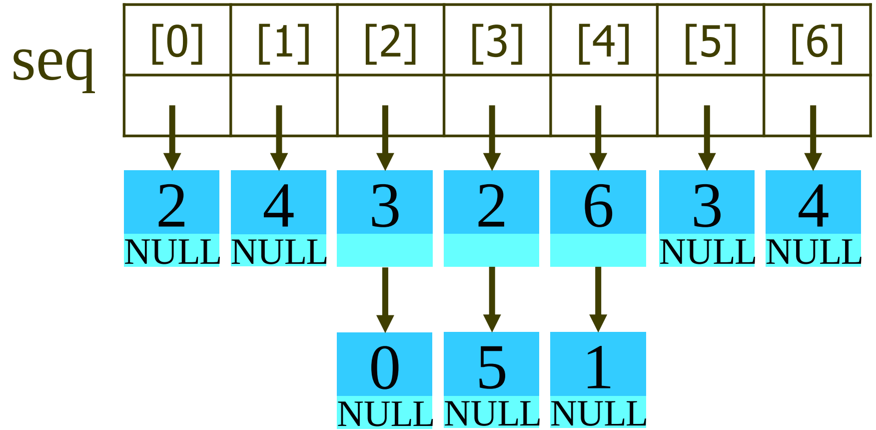
j	2



top = NULL

/* output */
New Class : 0 2 3

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {    /*  find rest of class  */
                        while (x)   {      /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```

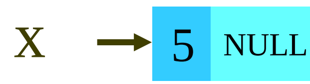


out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	TRUE	FALSE	FALSE	TRUE	TRUE	TRUE

i	0

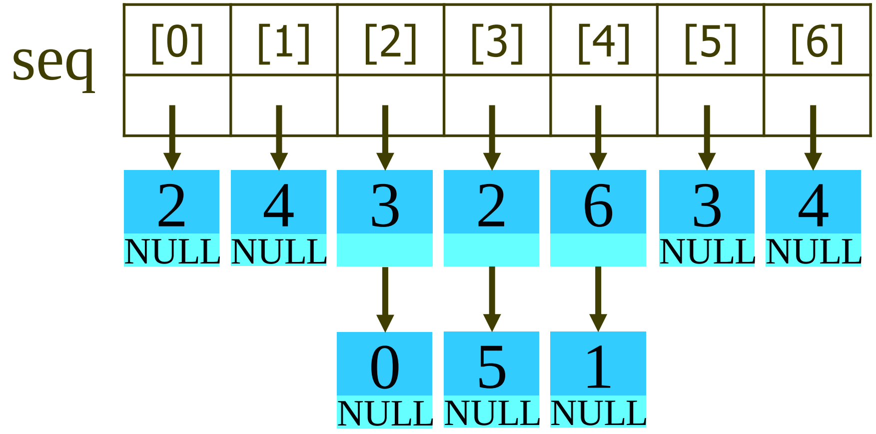
j	5



top = NULL

/* output */
New Class : 0 2 3

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;   /*  initialize stack  */
                for ( ; ; )   {     /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])    {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```

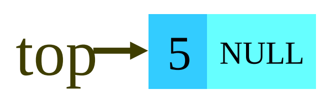


out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	TRUE	FALSE	FALSE	TRUE	FALSE	TRUE

i	0	j	5

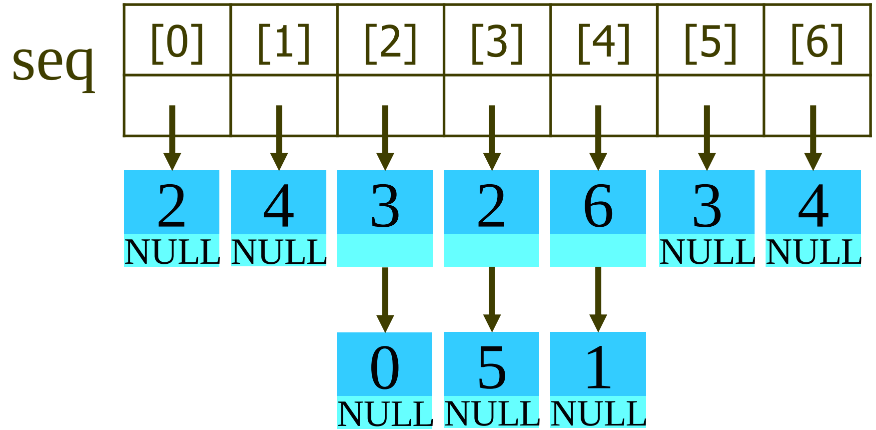
x = NULL



/* output */

New Class : 0 2 3 5

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {     /*  find rest of class  */
                        while (x)   {        /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```

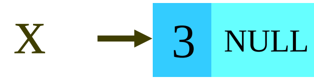


out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	TRUE	FALSE	FALSE	TRUE	FALSE	TRUE

i	0

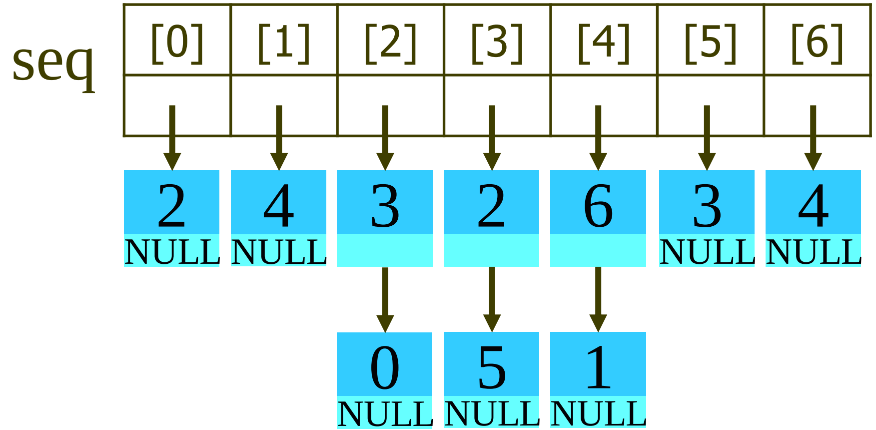
j	5



top = NULL

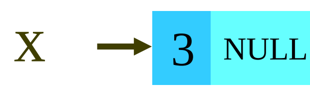
/* output */
New Class : 0 2 3 5

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;   /*  initialize stack  */
                for ( ; ; )   {    /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



out	[0]	[1]	[2]	[3]	[4]	[5]	[6]
	FALSE	TRUE	FALSE	FALSE	TRUE	FALSE	TRUE

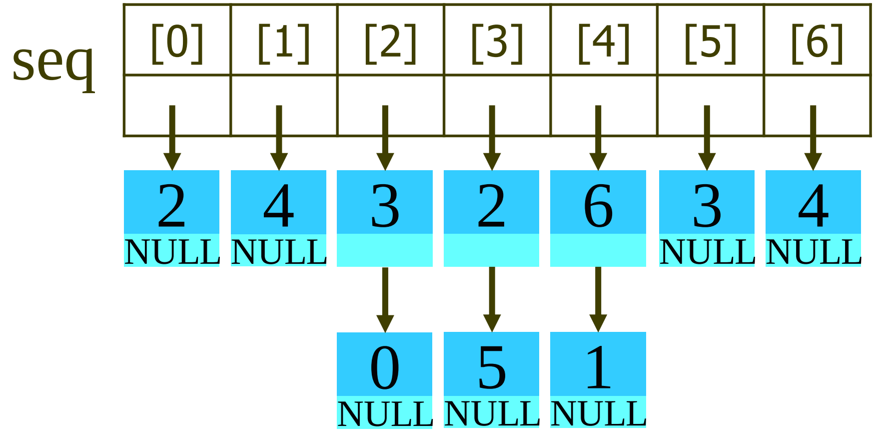
i	0	j	3



top = NULL

/* output */
New Class : 0 2 3 5

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {     /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	TRUE	FALSE	FALSE	TRUE	FALSE	TRUE

i	0	j	3

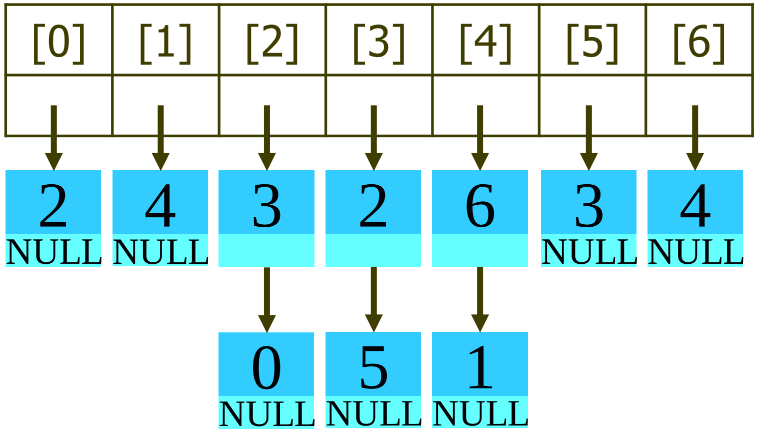
x = NULL

top = NULL

/* output */
New Class : 0 2 3 5

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {    /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```

seq



out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	TRUE	FALSE	FALSE	TRUE	FALSE	TRUE

i	0

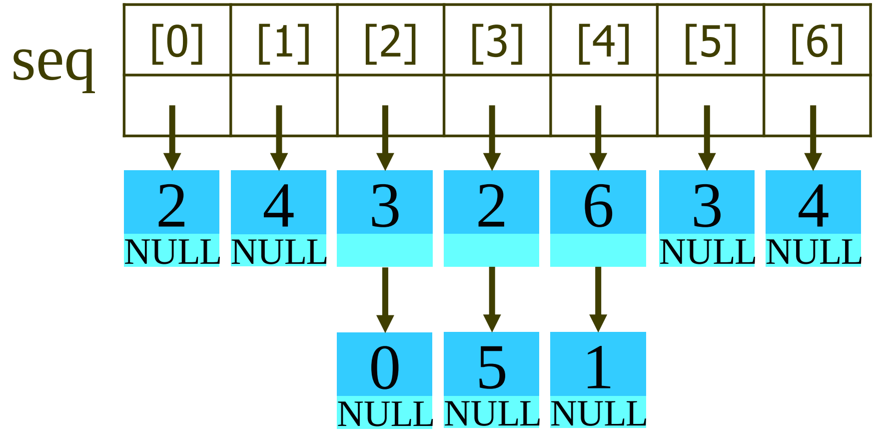
j	3

x = NULL

top = NULL

/* output */
New Class : 0 2 3 5

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;   /*  initialize stack  */
                for ( ; ; )   {     /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	FALSE	FALSE	FALSE	TRUE	FALSE	TRUE

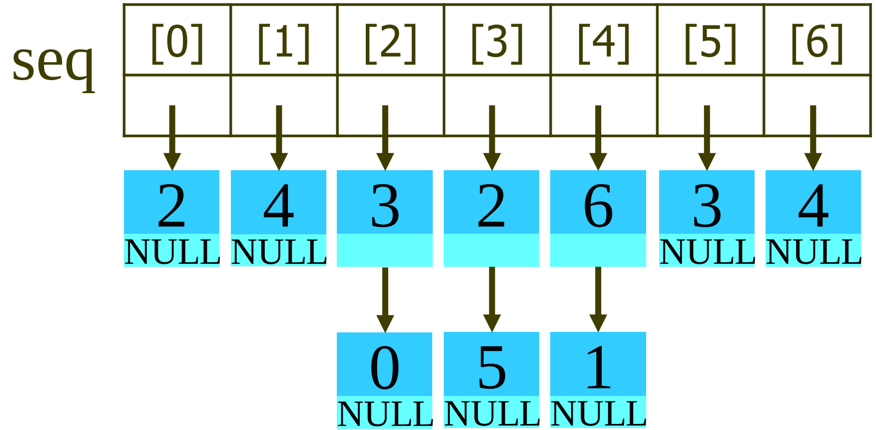
i	1	j	3

x = NULL

top = NULL

/* output */
New Class : 0 2 3 5
New Class : 1

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {     /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```

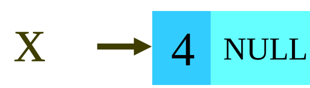


out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	FALSE	FALSE	FALSE	TRUE	FALSE	TRUE

i	1

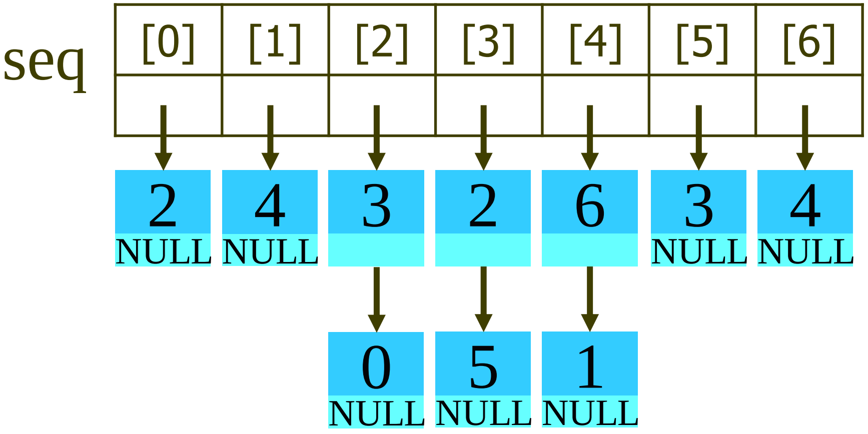
j	3



top = NULL

/* output */
New Class : 0 2 3 5
New Class : 1

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {    /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```

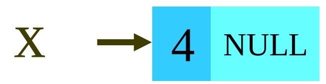


out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	FALSE	FALSE	FALSE	TRUE	FALSE	TRUE

i	1

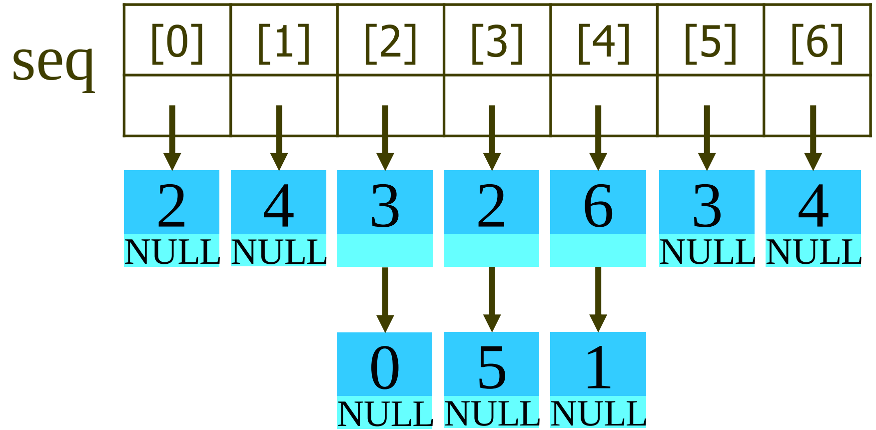
j	4



top = NULL

/* output */
New Class : 0 2 3 5
New Class : 1

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
      if  (out[i])  {
            printf("\nNew Class : %5d", i);
            out[i] =FALSE;   /*   set class to false  */
            x = seq[i];  top = NULL;  /*  initialize stack  */
            for ( ; ; )   {    /*  find rest of class  */
                  while (x)   {      /*  process list  */
                        j = x->data;
                        if  (out[j])   {
                              printf("%5d", j);   out[j] = FALSE;
                              y = x->link;  x->link = top;  top = x;  x = y;
                        }
                        else  x = x->link;
                  }
                  if  (!top)   break;
                  x = seq[top->data];  top = top->link;   /* unstack */
            }
      }
}
```



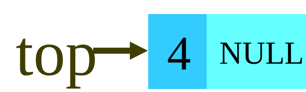
out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE

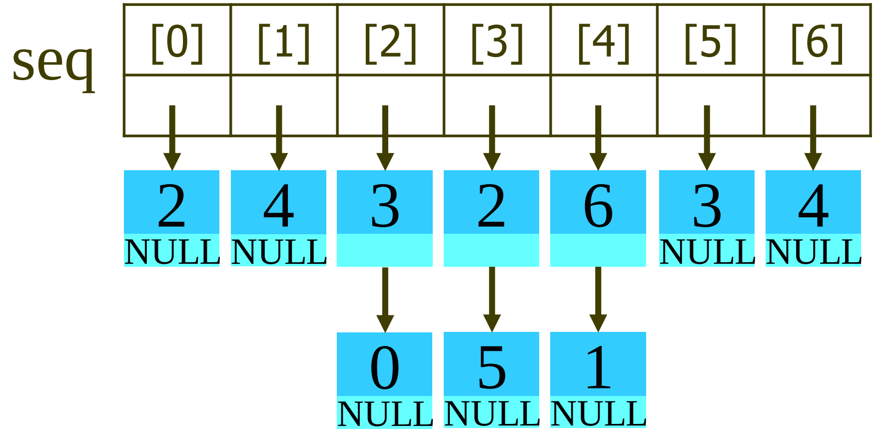
i	1

j	4

x = NULL



/* output */
New Class : 0 2 3 5
New Class : 1 4




```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {     /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```

out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE

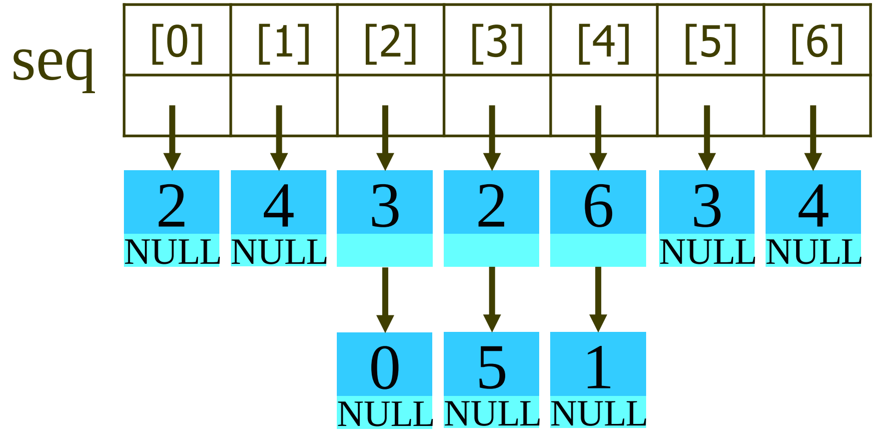
i	1

j	4

top = NULL

/* output */
New Class : 0 2 3 5
New Class : 1 4

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {     /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	TRUE

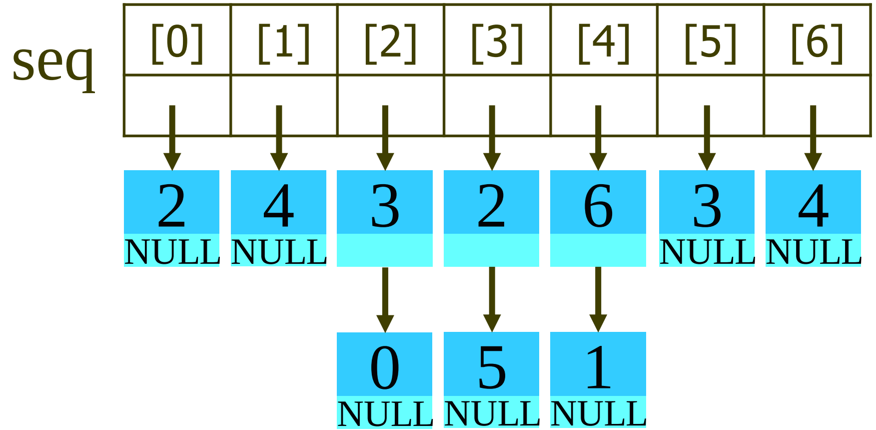
i	1

j	6

top = NULL

/* output */
New Class : 0 2 3 5
New Class : 1 4

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;   /*  initialize stack  */
                for ( ; ; )   {     /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```

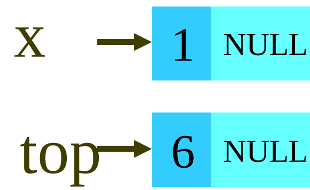


out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE

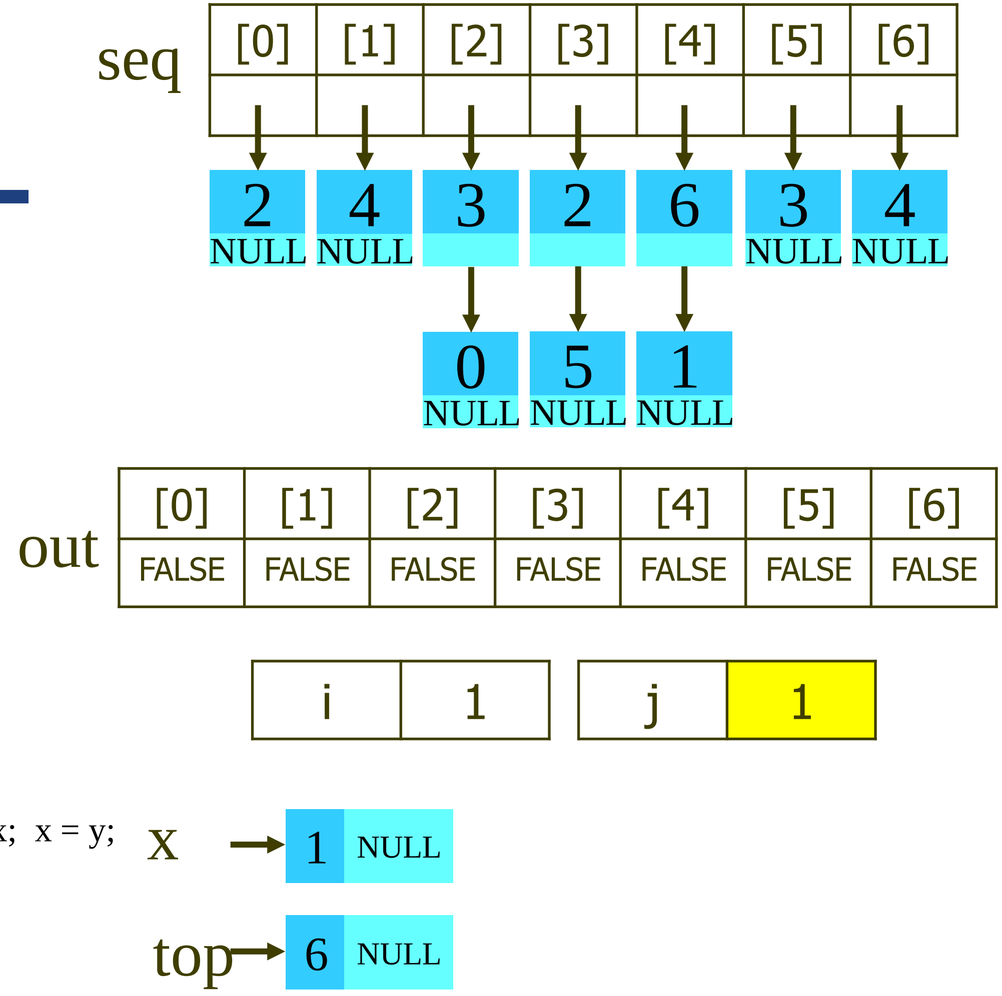
i	1

j	6



/* output */
New Class : 0 2 3 5
New Class : 1 4 6

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {     /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```

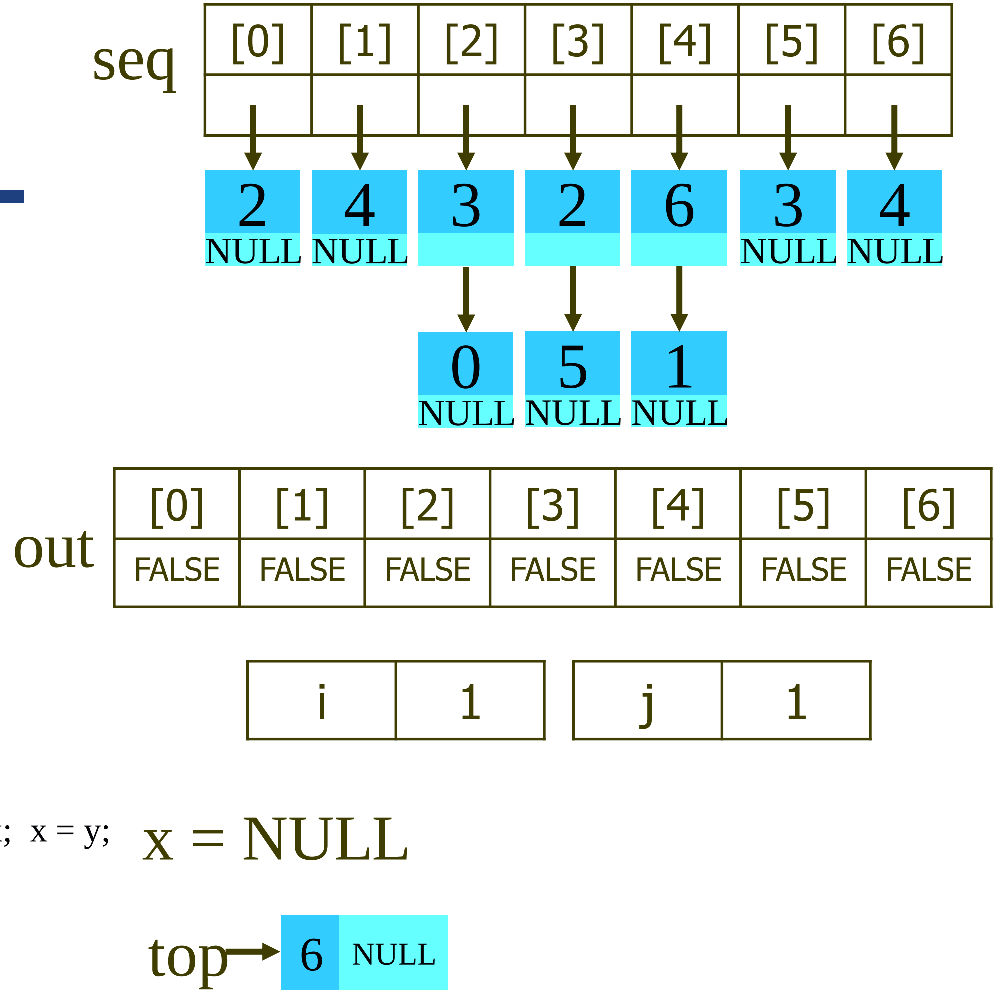


/* output */

New Class : 0 2 3 5

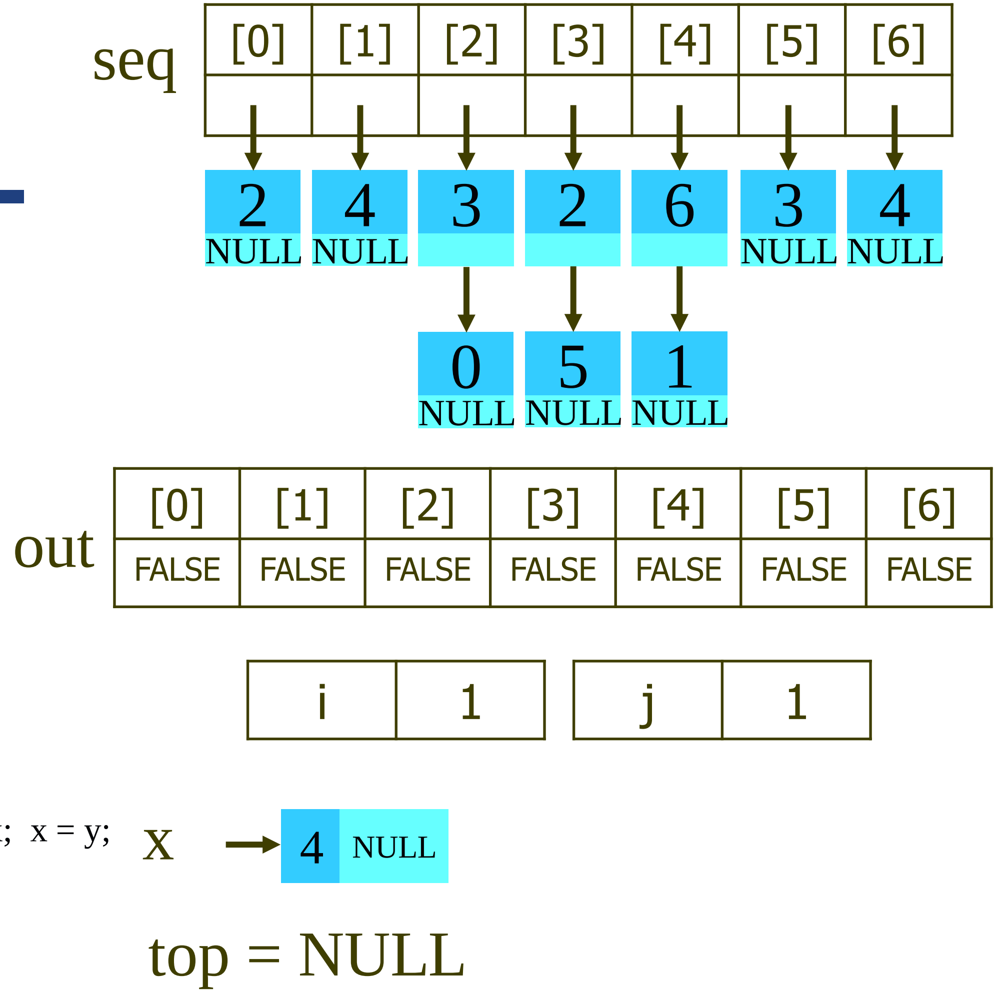
New Class : 1 4 6

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {    /*  find rest of class  */
                        while (x)   {      /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



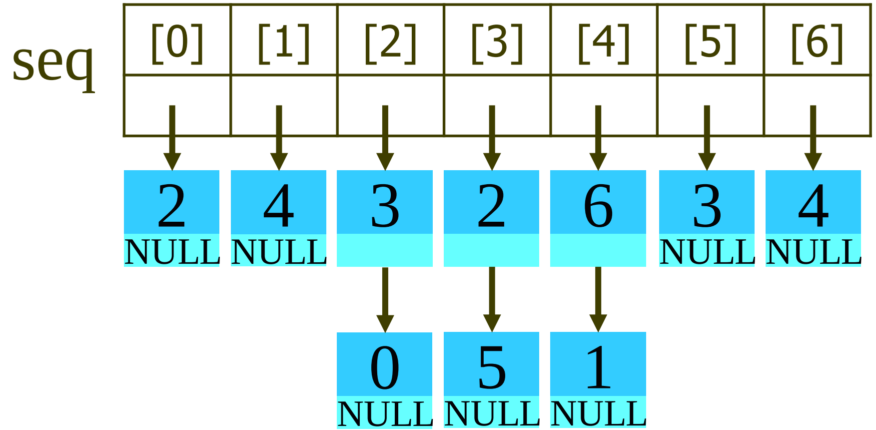
/* output */
New Class : 0 2 3 5
New Class : 1 4 6

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;   /*  initialize stack  */
                for ( ; ; )   {    /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



/* output */
New Class : 0 2 3 5
New Class : 1 4 6

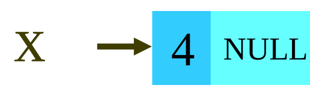
```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {     /*  find rest of class  */
                        while (x)   {        /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE

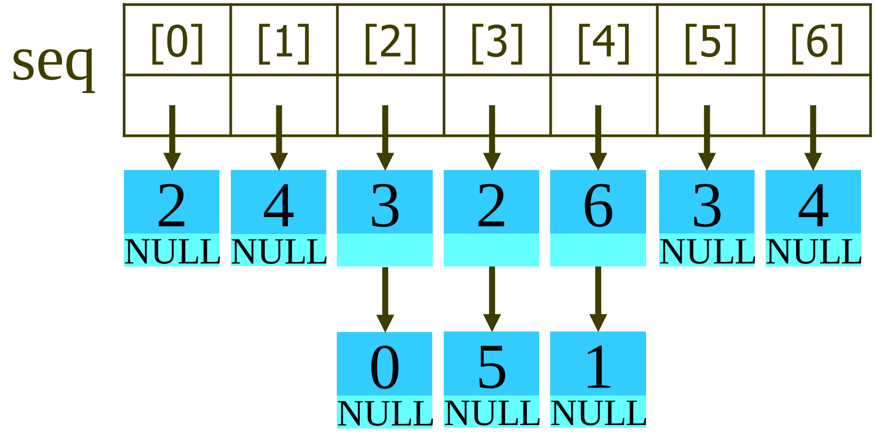
i	1	j	4



top = NULL

/* output */
New Class : 0 2 3 5
New Class : 1 4 6

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf("\nNew Class : %5d", i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;  /*  initialize stack  */
                for ( ; ; )   {    /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf("%5d", j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE

i	1

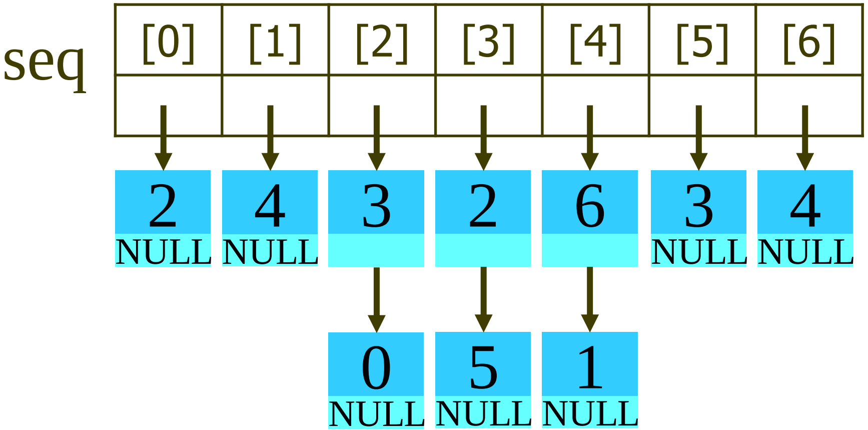
j	4

x = NULL

top = NULL

/* output */
New Class : 0 2 3 5
New Class : 1 4 6

```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
        if  (out[i])  {
                printf(“\nNew Class : %5d”, i);
                out[i] =FALSE;    /*   set class to false  */
                x = seq[i];  top = NULL;   /*  initialize stack  */
                for ( ; ; )   {    /*  find rest of class  */
                        while (x)   {       /*  process list  */
                                j = x->data;
                                if  (out[j])   {
                                        printf(“%5d”, j);   out[j] = FALSE;
                                        y = x->link;  x->link = top;  top = x;  x = y;
                                }
                                else  x = x->link;
                        }
                        if  (!top)   break;
                        x = seq[top->data];  top = top->link;   /* unstack */
                }
        }
}
```



out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE

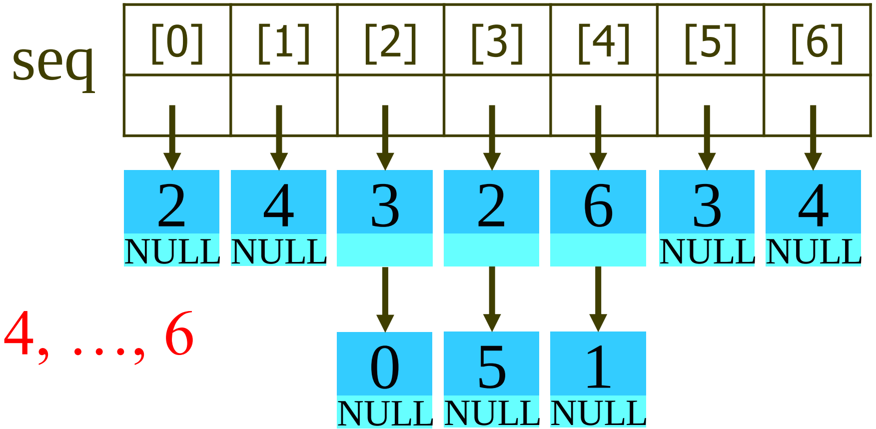
i	1

j	4

x = NULL

top = NULL

/* output */
New Class : 0 2 3 5
New Class : 1 4 6




```
/* Phase 2 : output the equivalence classes  */
for (i = 0; i < n; i++)  {
    if  (out[i])  {
        printf("\nNew Class : %5d", i);
        out[i] =FALSE;    /*   set class to false  */
        x = seq[i];  top = NULL;   /*  initialize stack  */
        for ( ; ; )   {    /*  find rest of class  */
            while (x)   {       /*  process list  */
                j = x->data;
                if  (out[j])   {
                    printf("%5d", j);   out[j] = FALSE;
                    y = x->link;  x->link = top;  top = x;  x = y;
                }
                else  x = x->link;
            }
            if  (!top)   break;
            x = seq[top->data];  top = top->link;   /* unstack */
        }
    }
}
```

→ i will increase to 3, 4, …, 6

out

[0]	[1]	[2]	[3]	[4]	[5]	[6]
FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE

i	2

j	4

x = NULL

top = NULL

/* output */
New Class : 0 2 3 5
New Class : 1 4 6

- Analysis of the equivalence program :
  - Initialization of *seq* and *out* takes O(n) time.
  - Each of Phase 1 and 2 takes O($m + n$) time
    where m is the number of pairs input.

  - Time complexity is O($m+n$) and space complexity is also O($m+n$).

  - In Chapter 5, we will look at an alternate solution that requires only O(n) space.

4.7 SPARSE MATRIX

- In Chapter 2, we considered a sequential representation of sparse matrices and implemented matrix operations.

- However, we found that when we performed matrix operations, the number of nonzero terms varied.
  → The sequential representation of sparse matrices suffered from the same inadequacies as the similar representation of polynomials.

- As we have seen previously, linked lists allow us to efficiently represent structures that vary in size, a benefit that also applies to sparse matrices.

- In our data representation, we represent each column of a sparse matrix as a circularly linked list with a head node. We use a similar representation for each row of a sparse matrix.

- **[Figure 4.17] Node structure for sparse matrices**

<table>
<tr><td colspan="2">next</td></tr>
<tr><td>down</td><td>right</td></tr>
</table>

(a) header node

<table>
<tr><td colspan="2">row</td><td colspan="2">col</td><td colspan="2">value</td></tr>
<tr><td colspan="3">down</td><td colspan="3">right</td></tr>
</table>

(a) entry node

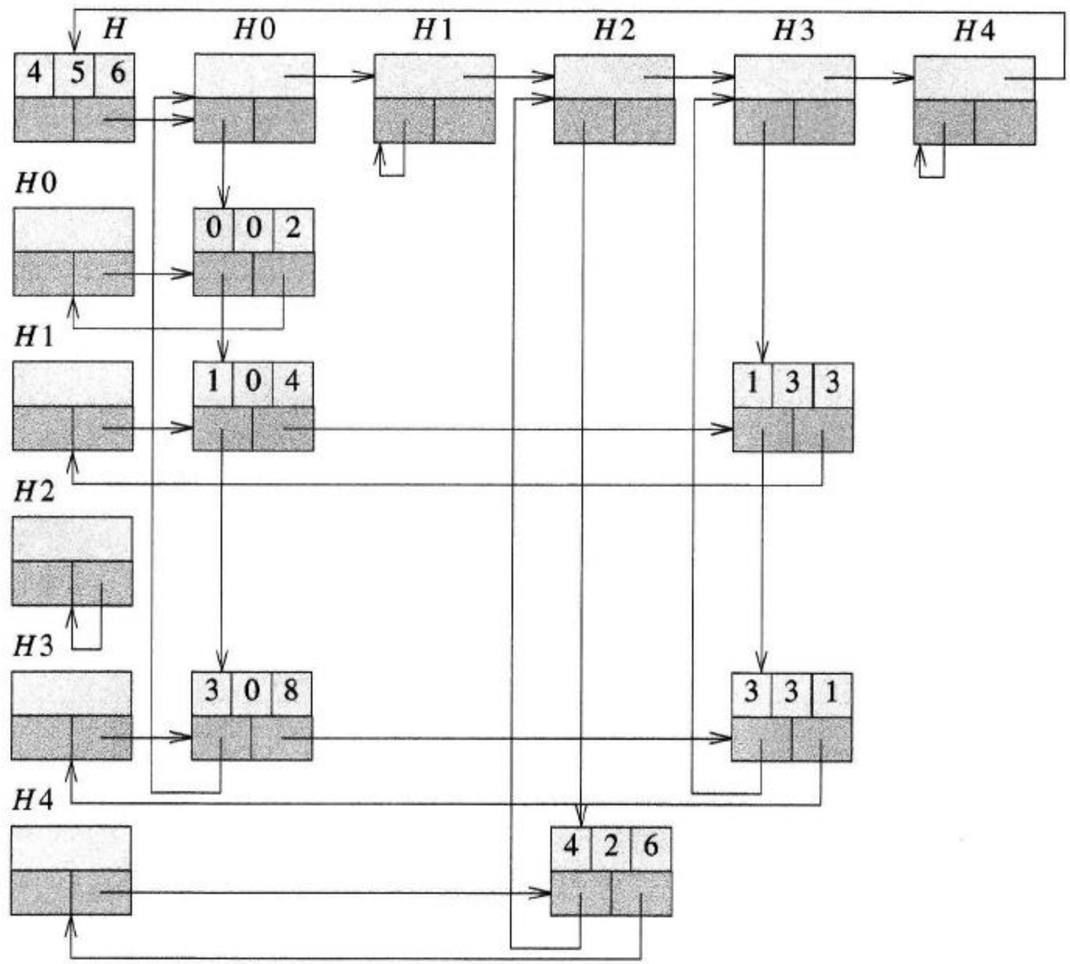
- Each node has a tag field, which we use to distinguish between header nodes and entry nodes.
- Each header node has three additional fields:
  down, right, and *next.*
- We use the *down* field to link into a column list and the *right* field to link into a row list.
- The *next* field links the header nodes together.

- Each entry node has five additional fields:
  row, col, down, right, value
- We use the *down* field to link to the next nonzero term in the same column and the *right* field to link to the next nonzero term in the same row.

- Each entry node is in two lists:
  a circular linked list for row and a circular linked list for column.
- Each head node is in three lists:
  a list of rows, a list of columns, and a list of head nodes.

- The list of head nodes also has a head node that has the same structure as an entry node.

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 4 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 8 & 0 & 0 & 1 \\ 0 & 0 & 6 & 0 \end{bmatrix}$$



```
#define MAX_SIZE 50 /* size of largest matrix */

typedef enum {head, entry} tagfield;
typedef  struct  matrix_node  *matrix_pointer;
typedef  struct  entry_node  {
      int row;
      int col;
      int value;
};
typedef struct matrix_node {
      matrix_pointer down;
      matrix_pointer right;
      tagfield tag;
      union {
            matrix_pointer next;
            struct entry_node entry;
      } u;
};
matrix_pointer hdnode[MAX_SIZE];
```

[Program 4.23] Read in a sparse matrix

```
matrix_pointer mread()
{ /* read in a matrix and set up its linked representation.
    An auxiliary global array hdnode is used */
    int num_rows, num_cols,num_terms, num_heads, i;
    int row, col, value, current_row;
    matrix_pointer temp, last, node;

    printf("Enter the number of rows, columns and number of nonzero terms: ");
    scanf("%d %d %d", &num_rows, &num_cols, &num_terms);
    num_heads = (num_cols > num_rows) ? num_cols : num_rows;
    /* set up header node for the list of header nodes */
    node = new_node(); node->tag = entry;
    node->u.entry.row = num_rows;
    node->u.entry.col = num_cols;
```

```
if (!num_heads) node->right = node;
else { /* initialize the header nodes */
        for (i=0; i<num_heads; i++) {
                temp = new_node();
                hdnode[i] = temp; hdnode[i]->tag = head;
                hdnode[i]->right = temp; hdnode[i]->u.next=temp;
        }
        current_row = 0;
        last = hdnode[0]; /* last node in current row */
        for (i=0; i<num_terms; i++) {
                printf("Enter row, column and value: ");
                scanf("%d %d %d", &row, &col, &value);
                if (row > current_row) { /* close current row */
                        last->right = hdnode[current_row];
                        current_row = row;  last = hdnode[row];
                }
                temp = new_node();  temp->tag = entry;
                temp->u.entry.row = row;  temp->u.entry.col = col;
                temp->u.entry.value = value;
                last->right = temp; /* link into row list */
                last = temp;
                hdnode[col]->u.next->down = temp; /* link into column list */
                hdnode[col]->u.next = temp;
        }
```

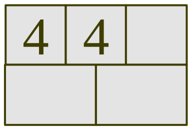
```
        /* close last row */
        last->right = hdnode[current_row];
        /* close all column lists */
        for (i=0; i<num_cols; i++)
            hdnode[i]->u.next->down = hdnode[i];
        /* link all header nodes together */
        for (i=0; i<num_heads-1; i++)
            hdnode[i]->u.next = hdnode[i+1];
        hdnode[num_heads-1]->u.next = node;
        node->right = hdnode[0];
    }
    return node;
}
```

```
/* input */
num_rows = 4
num_cols  = 4
num_terms = 4
```

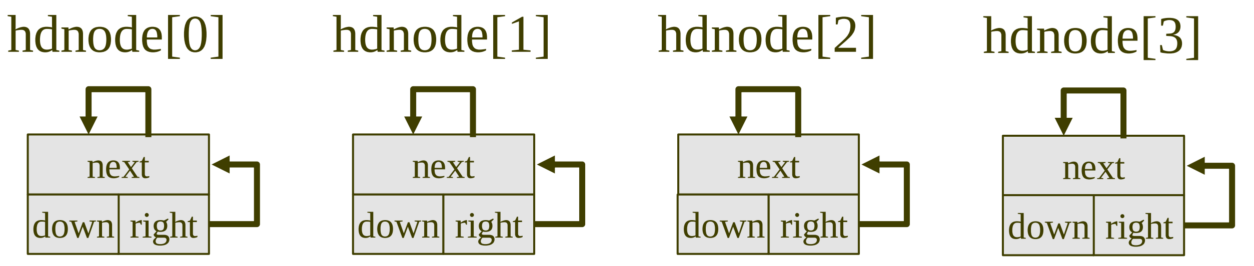
```
printf("Enter the number of rows, columns and number of nonzero terms: ");
scanf("%d %d %d", &num_rows, &num_cols, &num_terms);
num_heads = (num_cols > num_rows) ? num_cols : num_rows;
/* set up header node for the list of header nodes */
node = new_node(); node->tag = entry;
node->u.entry.row = num_rows;
node->u.entry.col = num_cols;
```

num_heads = 4

node

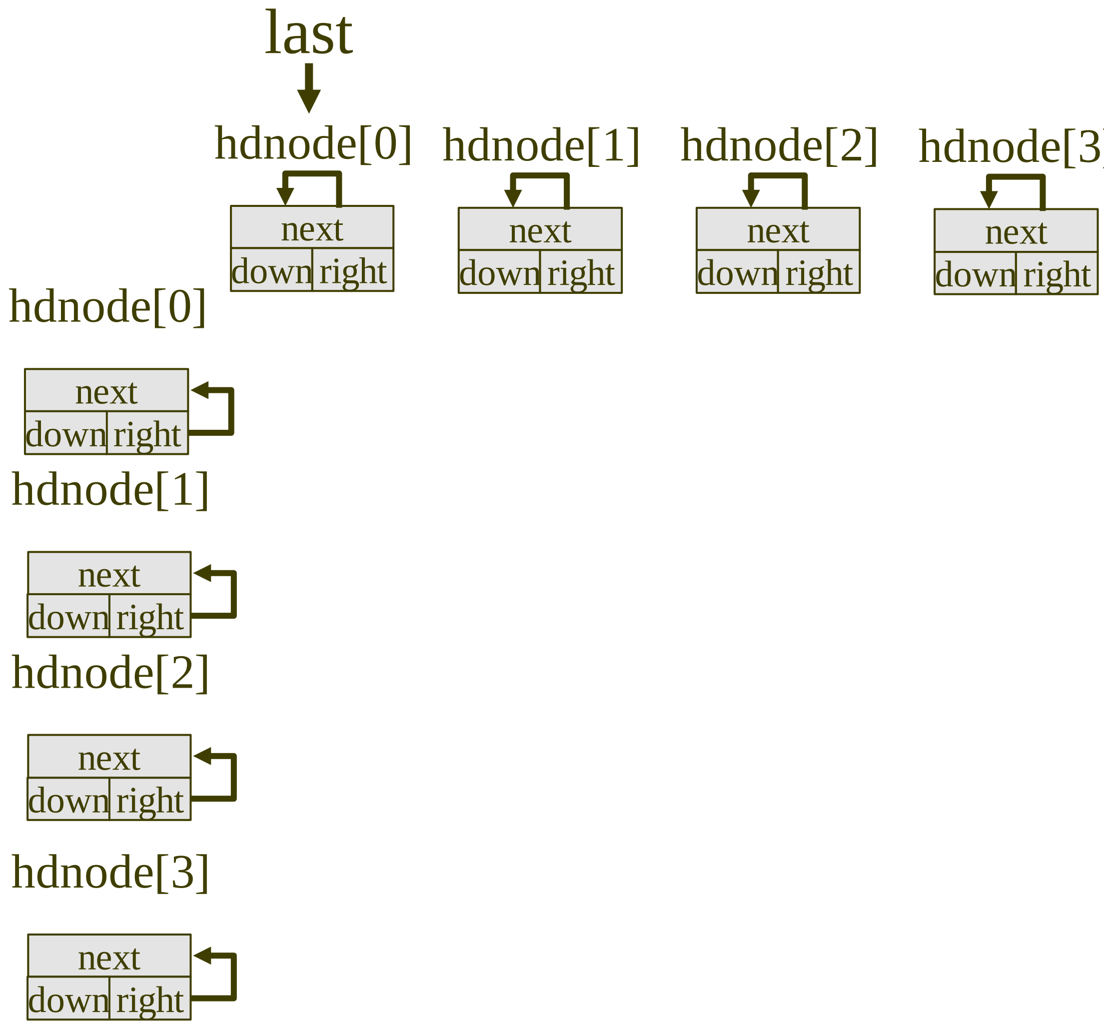


```
if (!num_heads) node->right = node;
else { /* initialize the header nodes */
    for (i=0; i<num_heads; i++) {
        temp = new_node();
        hdnode[i] = temp; hdnode[i]->tag = head;
        hdnode[i]->right = temp; hdnode[i]->u.next=temp;
    }
```



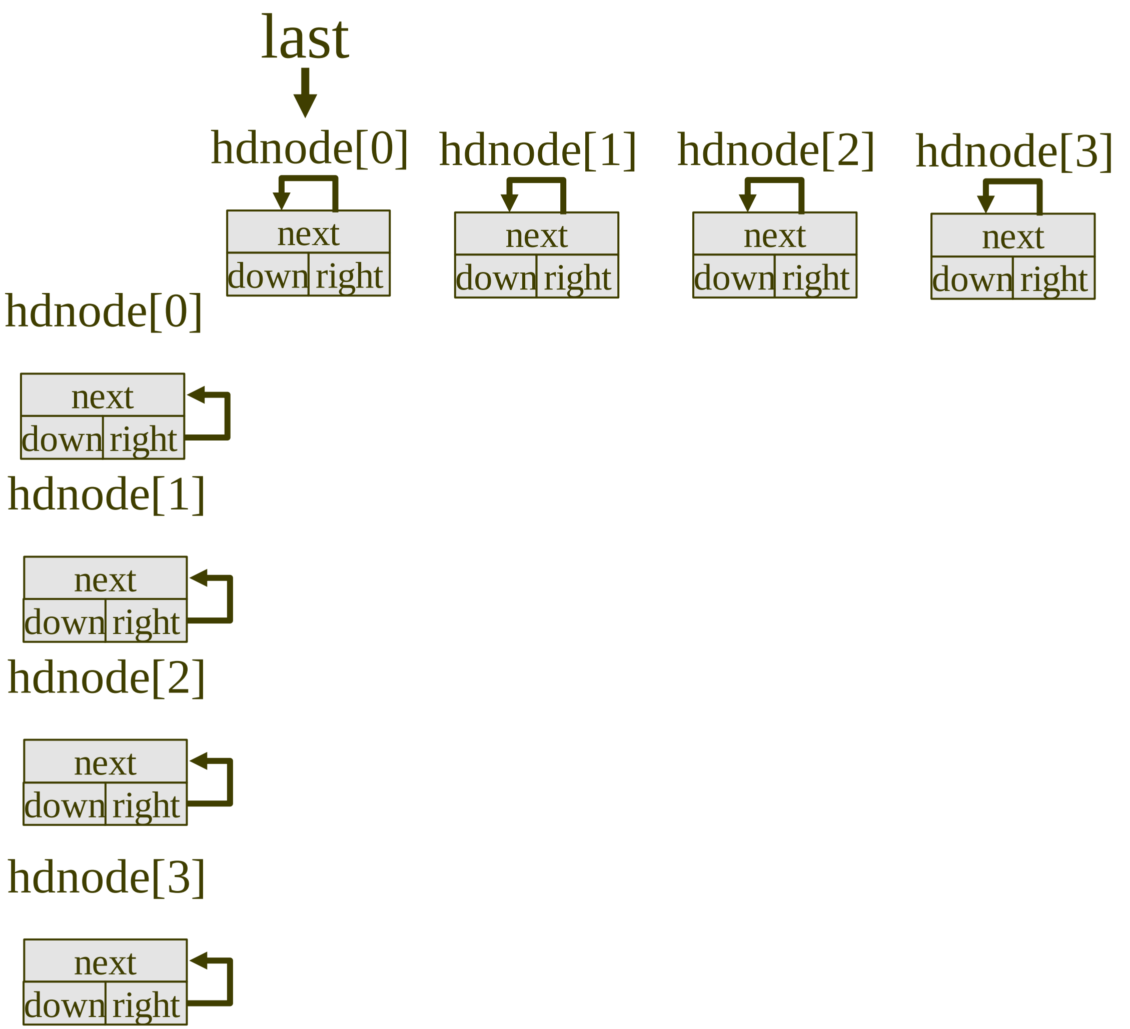
current_row	0	num_terms	4	i	
row		col		value	

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
	printf("Enter row, column and value: ");
	scanf("%d %d %d", &row, &col, &value);
	if (row > current_row) {
		last->right = hdnode[current_row];
		current_row = row;
		last = hdnode[row];
	}
	temp = new_node();
	temp->tag = entry;
	temp->u.entry.row = row;
	temp->u.entry.col = col;
	temp->u.entry.value = value;
	last->right = temp;
	last = temp;
	hdnode[col]->u.next->down = temp;
	hdnode[col]->u.next = temp;
}
```



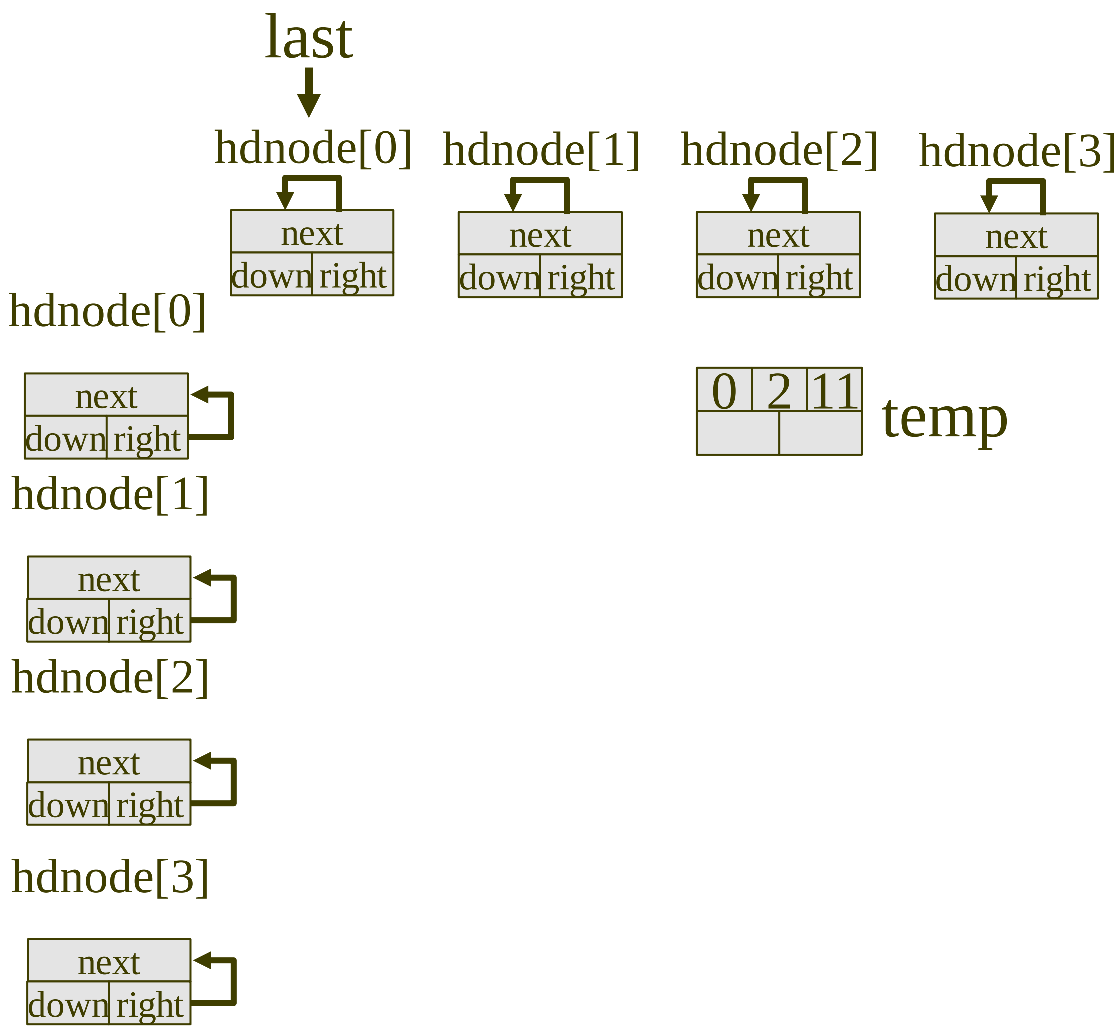
current_row	0	num_terms	4	i	0
row	0	col	2	value	11

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



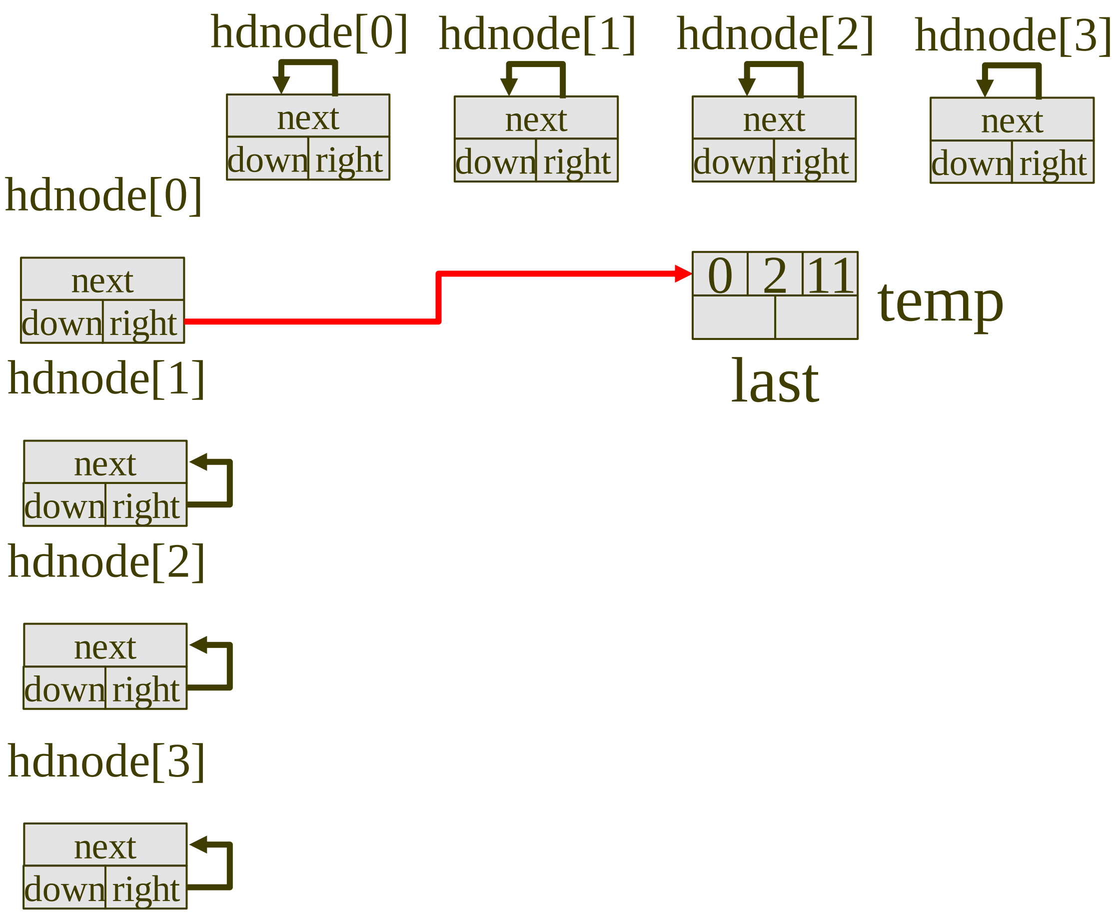
current_row	0	num_terms	4	i	0
row	0	col	2	value	11

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
    printf("Enter row, column and value: ");
    scanf("%d %d %d", &row, &col, &value);
    if (row > current_row) {
        last->right = hdnode[current_row];
        current_row = row;
        last = hdnode[row];
    }
    temp = new_node();
    temp->tag = entry;
    temp->u.entry.row = row;
    temp->u.entry.col = col;
    temp->u.entry.value = value;
    last->right = temp;
    last = temp;
    hdnode[col]->u.next->down = temp;
    hdnode[col]->u.next = temp;
}
```



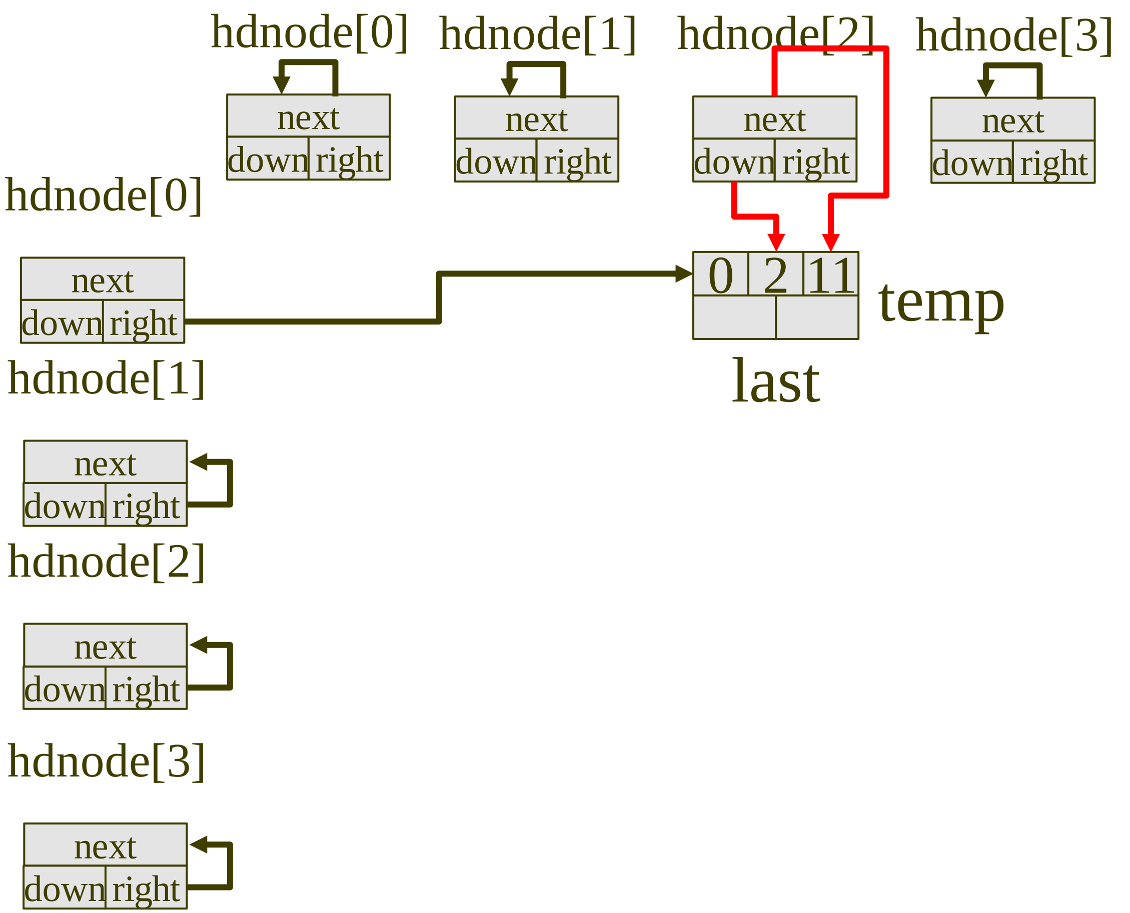
current_row	0	num_terms	4	i	0
row	0	col	2	value	11

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
        printf("Enter row, column and value: ");
        scanf("%d %d %d", &row, &col, &value);
        if (row > current_row) {
                last->right = hdnode[current_row];
                current_row = row;
                last = hdnode[row];
        }
        temp = new_node();
        temp->tag = entry;
        temp->u.entry.row = row;
        temp->u.entry.col = col;
        temp->u.entry.value = value;
        last->right = temp;
        last = temp;
        hdnode[col]->u.next->down = temp;
        hdnode[col]->u.next = temp;
}
```



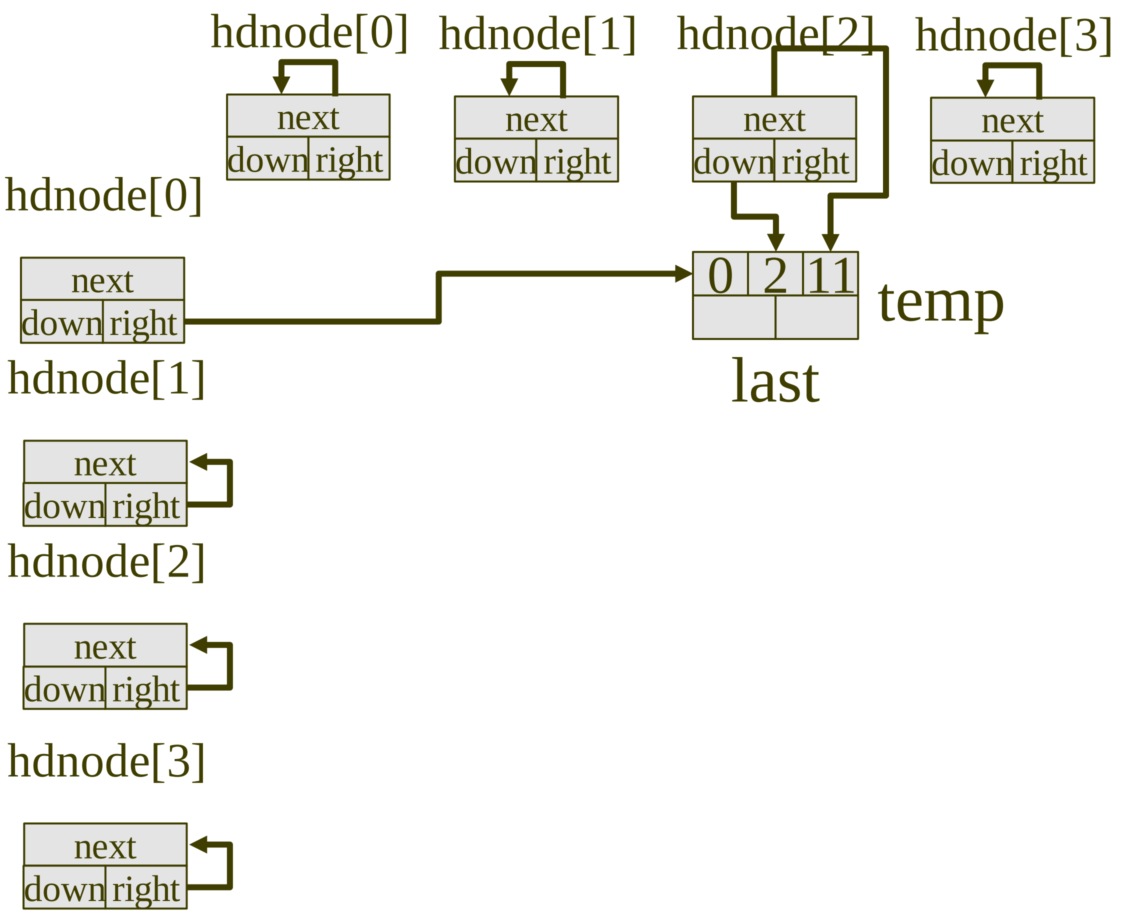
current_row	0	num_terms	4	i	0
row	0	col	2	value	11

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



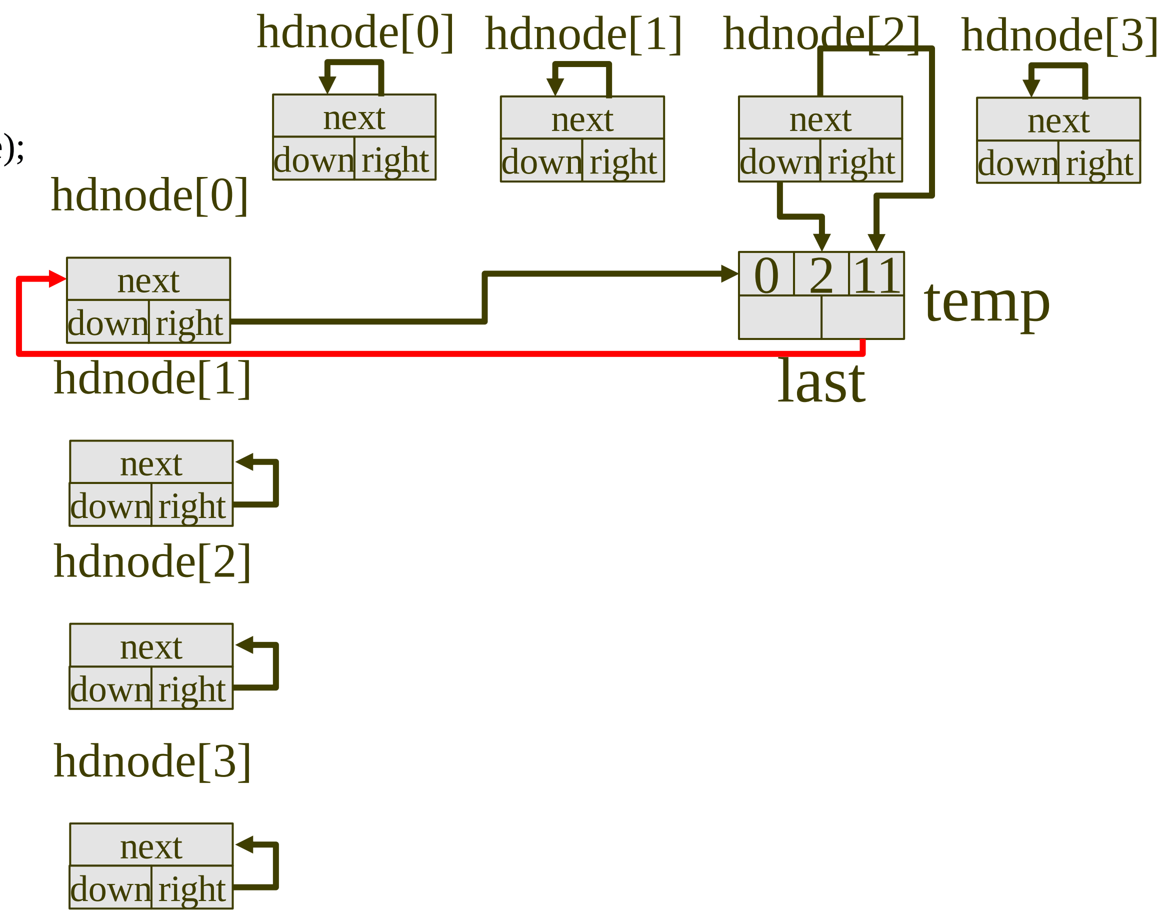
current_row	0	num_terms	4	i	1
row	1	col	0	value	12

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



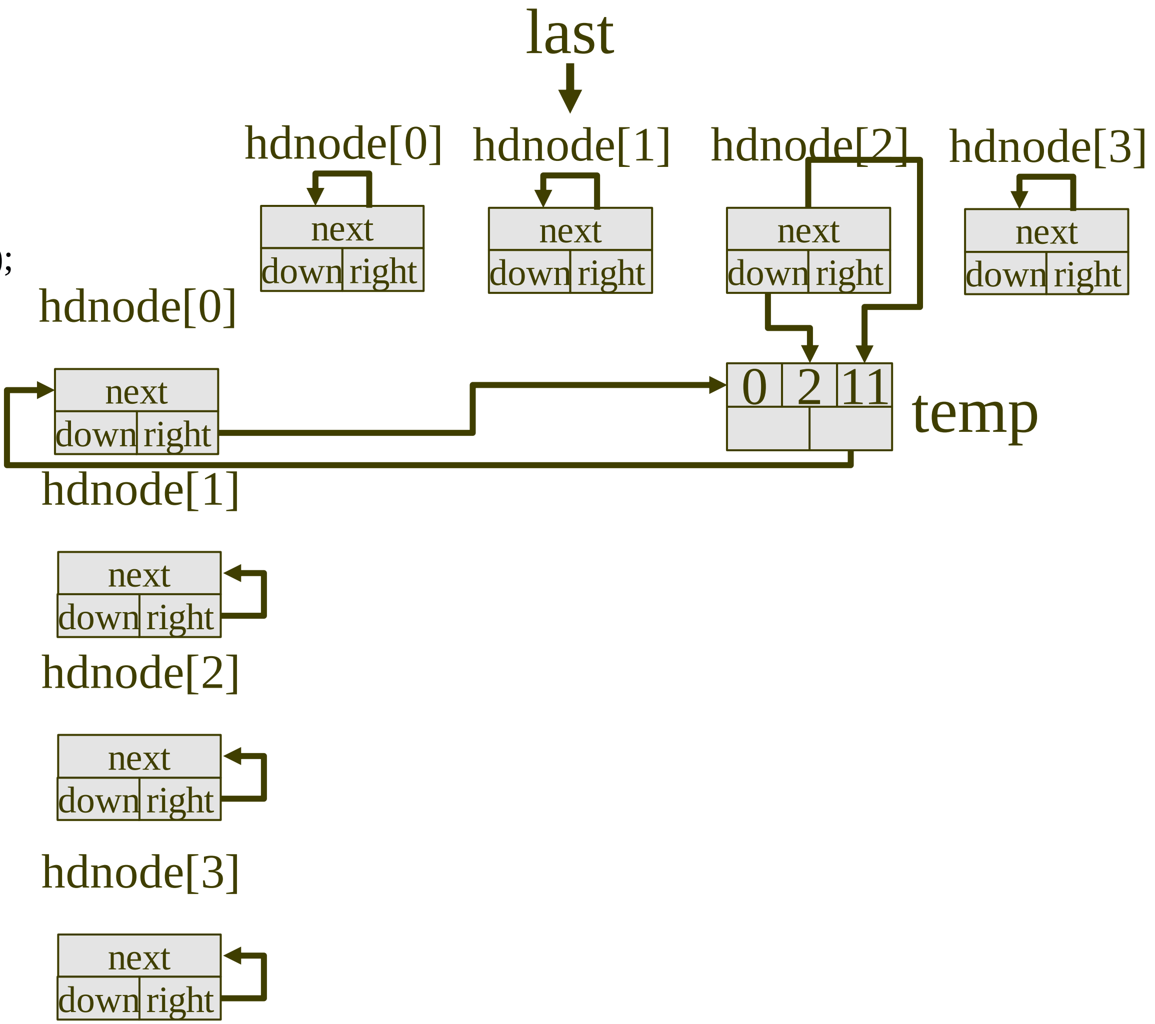
current_row	0	num_terms	4	i	1
row	1	col	0	value	12

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



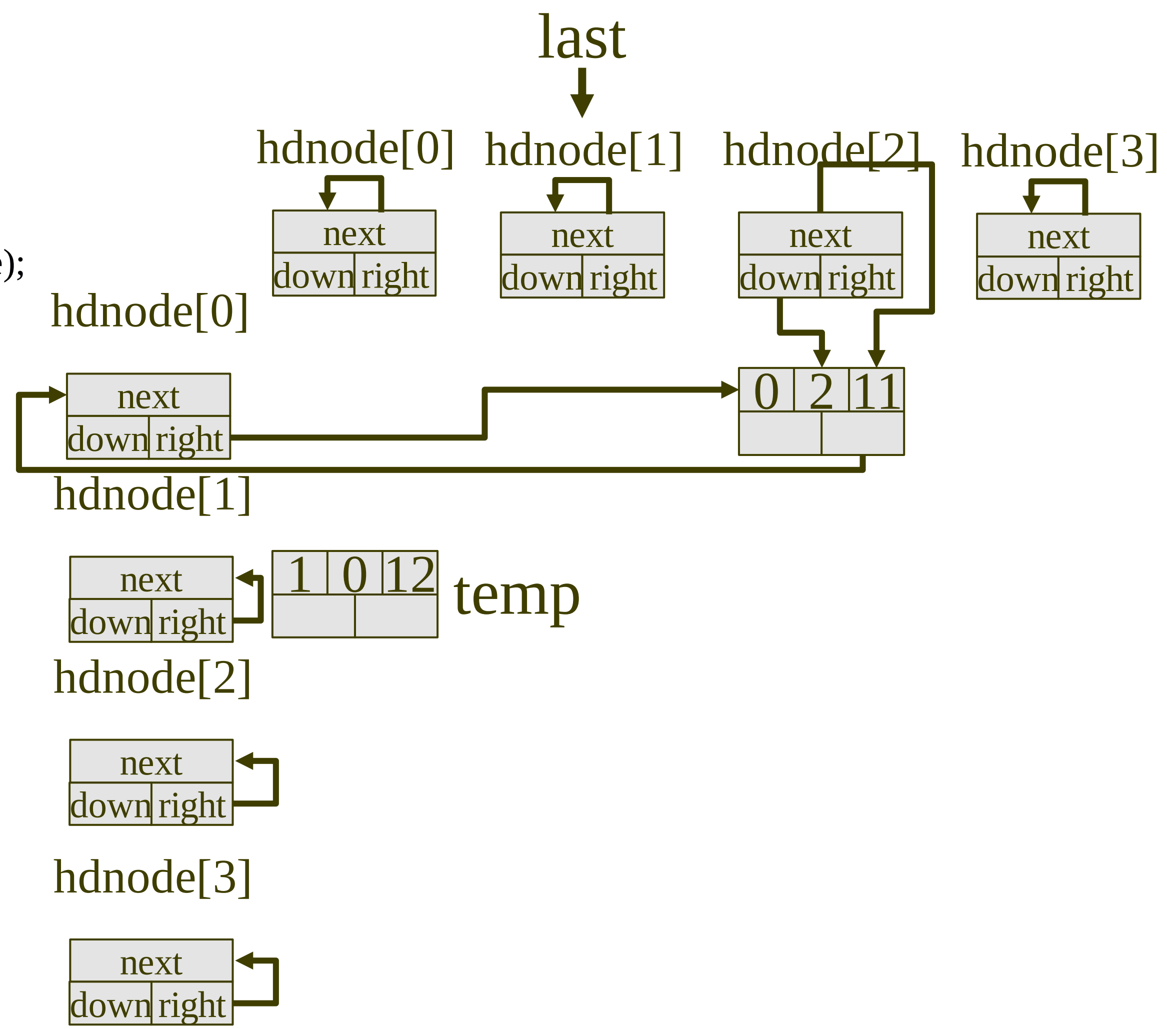
current_row	1	num_terms	4	i	1
row	1	col	0	value	12

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



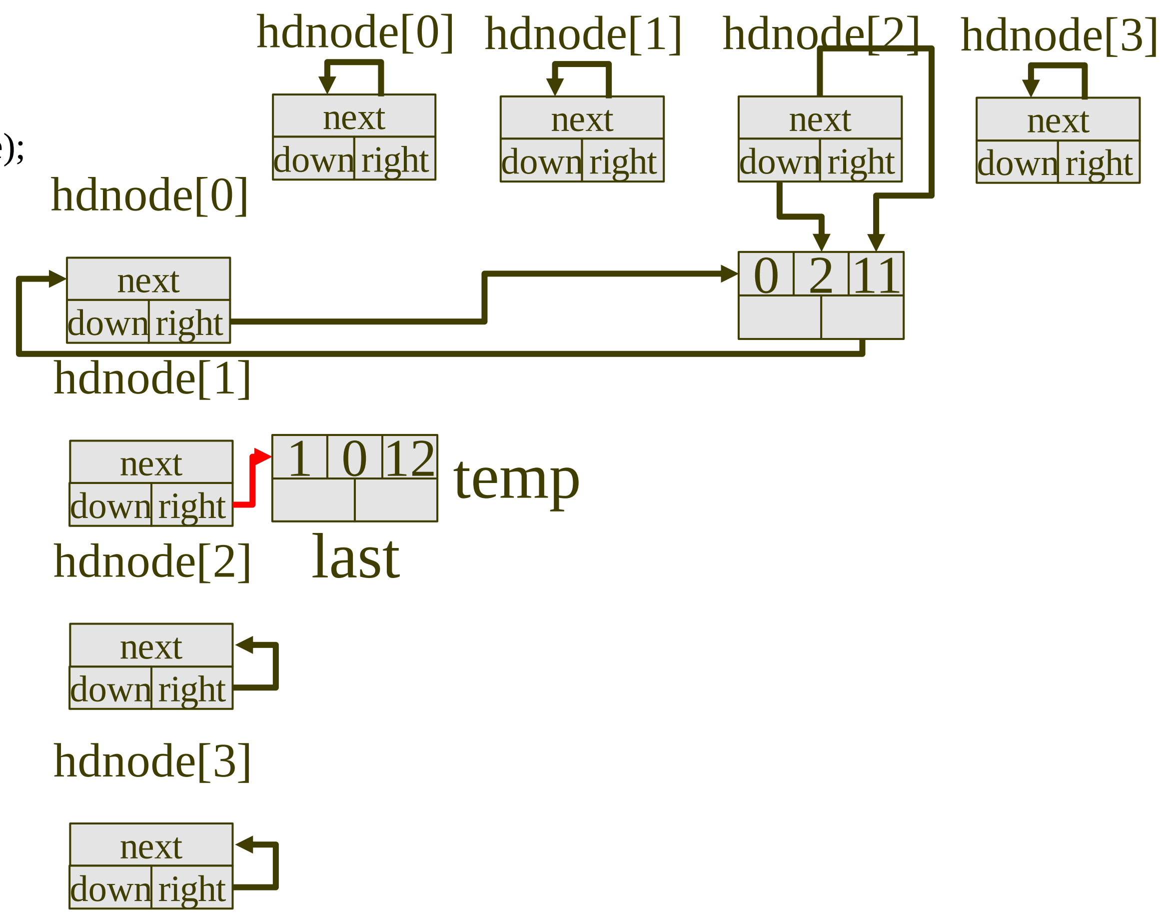
current_row	1	num_terms	4	i	1
row	1	col	0	value	12

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



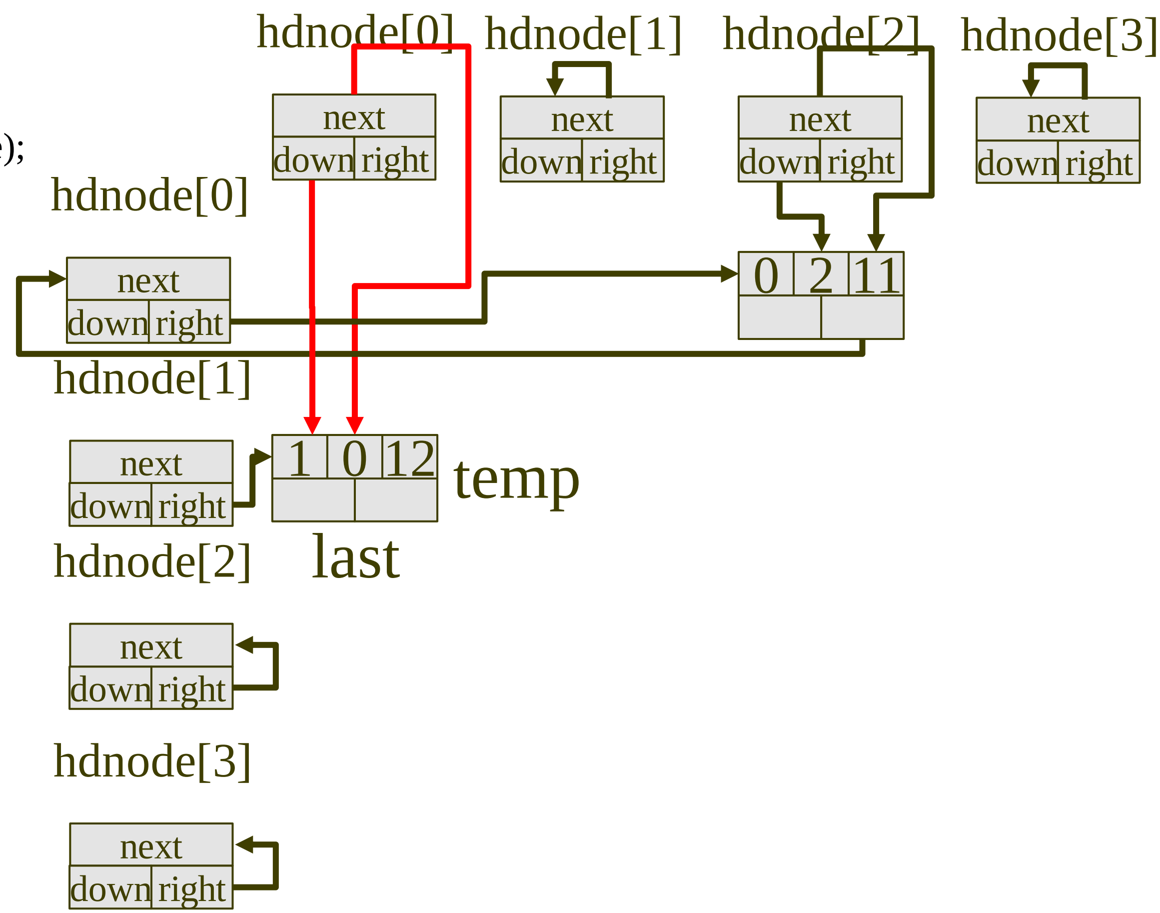
current_row	1	num_terms	4	i	1
row	1	col	0	value	12

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



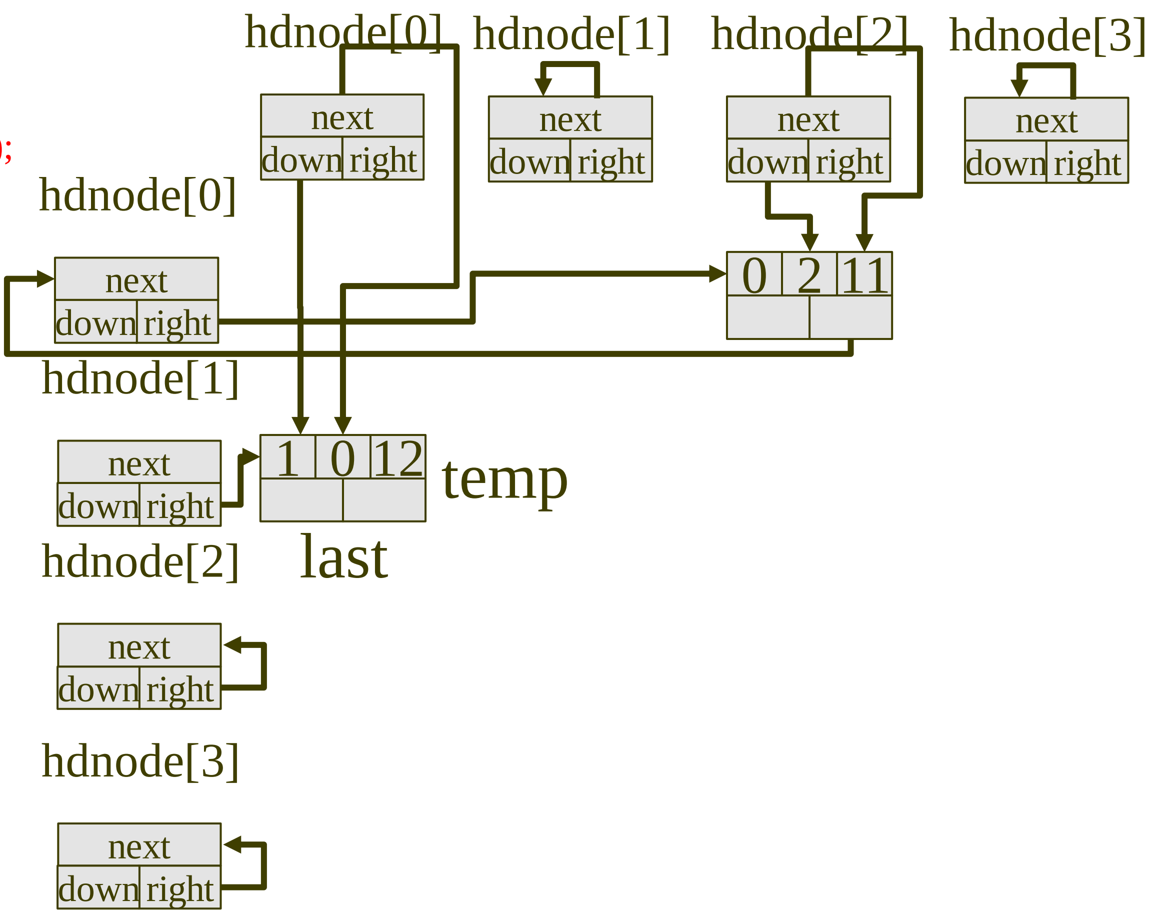
current_row	1	num_terms	4	i	1
row	1	col	0	value	12

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



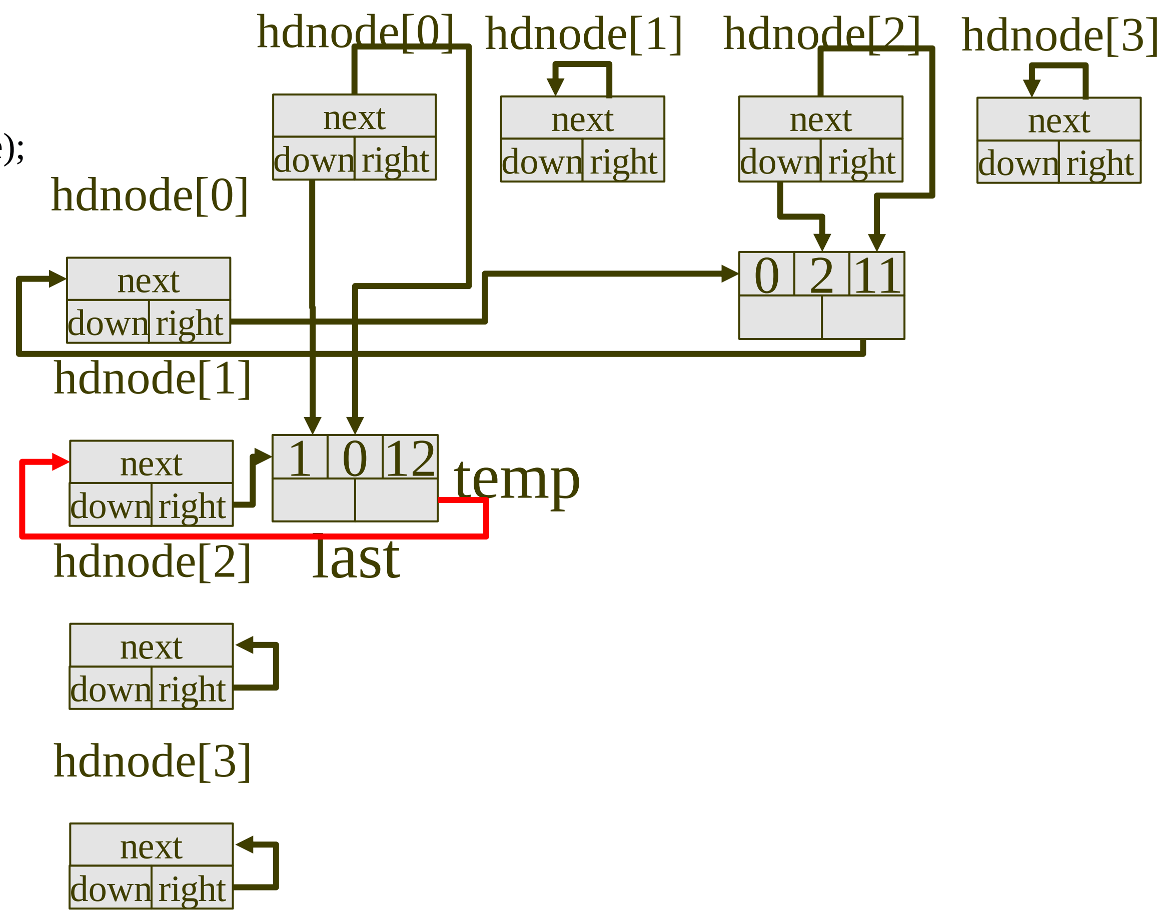
current_row	1	num_terms	4	i	2
row	2	col	1	value	-4

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



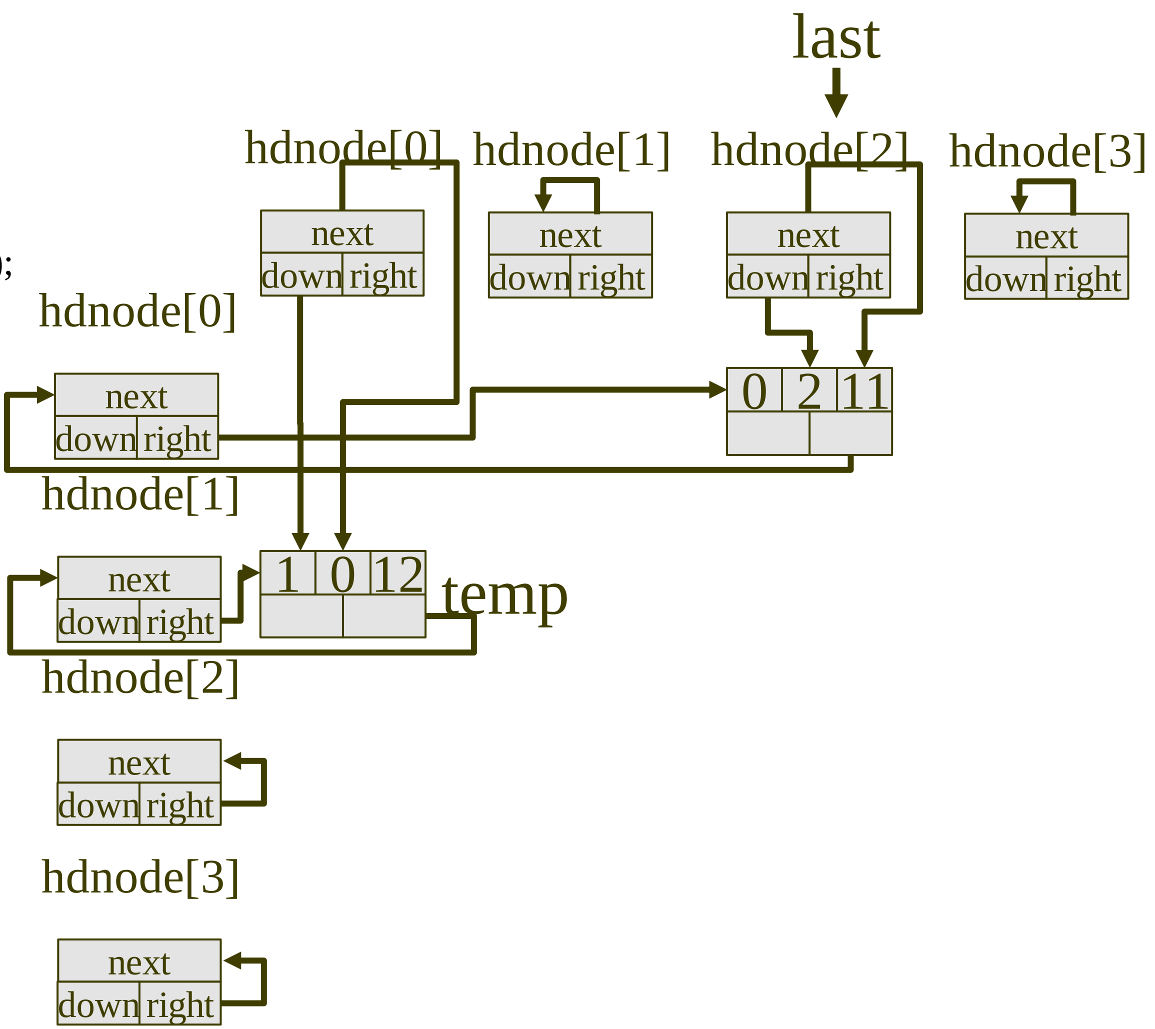
current_row	1	num_terms	4	i	2
row	2	col	1	value	-4

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



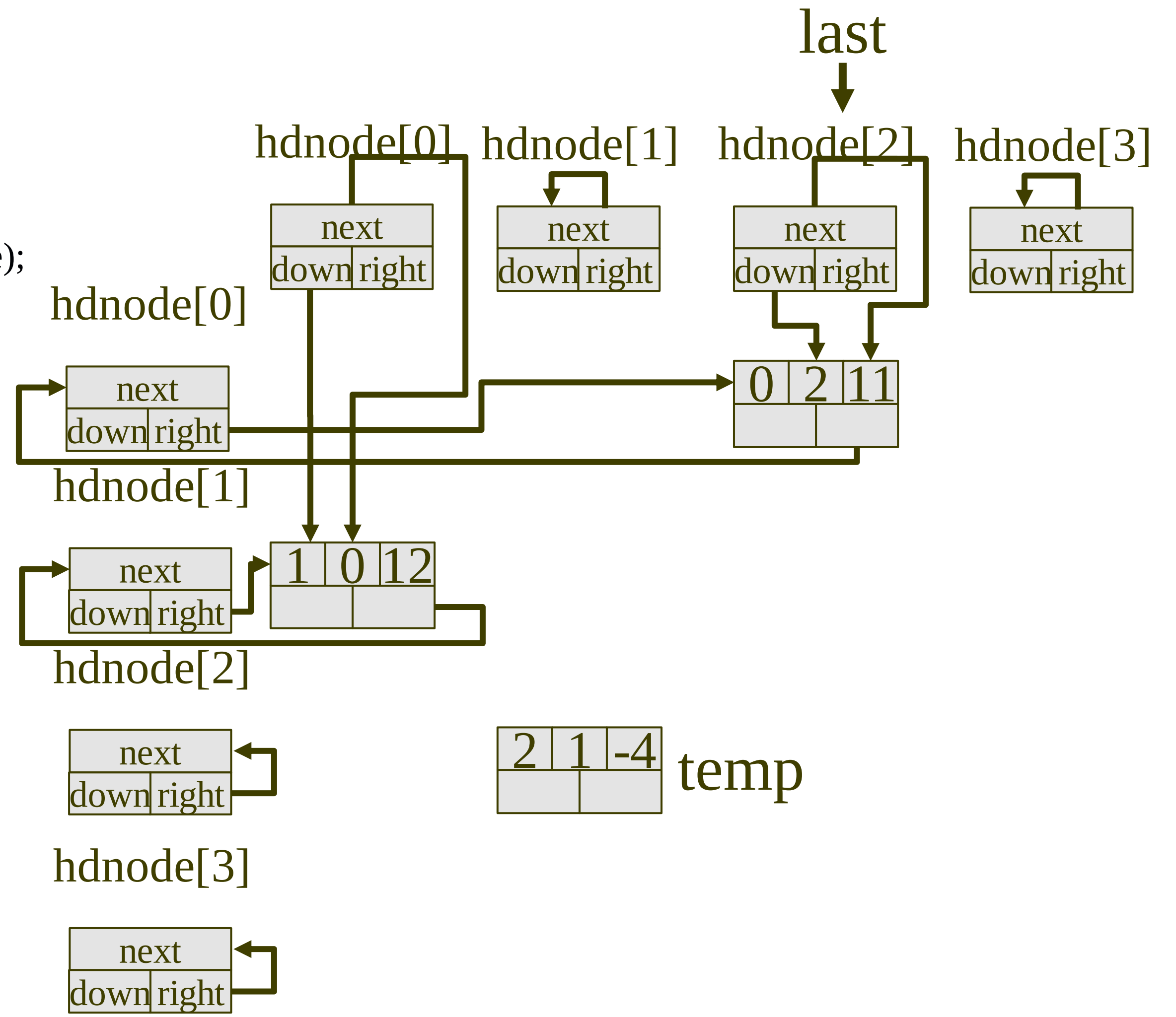
current_row	2	num_terms	4	i	2
row	2	col	1	value	-4

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
    printf("Enter row, column and value: ");
    scanf("%d %d %d", &row, &col, &value);
    if (row > current_row) {
        last->right = hdnode[current_row];
        current_row = row;
        last = hdnode[row];
    }
    temp = new_node();
    temp->tag = entry;
    temp->u.entry.row = row;
    temp->u.entry.col = col;
    temp->u.entry.value = value;
    last->right = temp;
    last = temp;
    hdnode[col]->u.next->down = temp;
    hdnode[col]->u.next = temp;
}
```



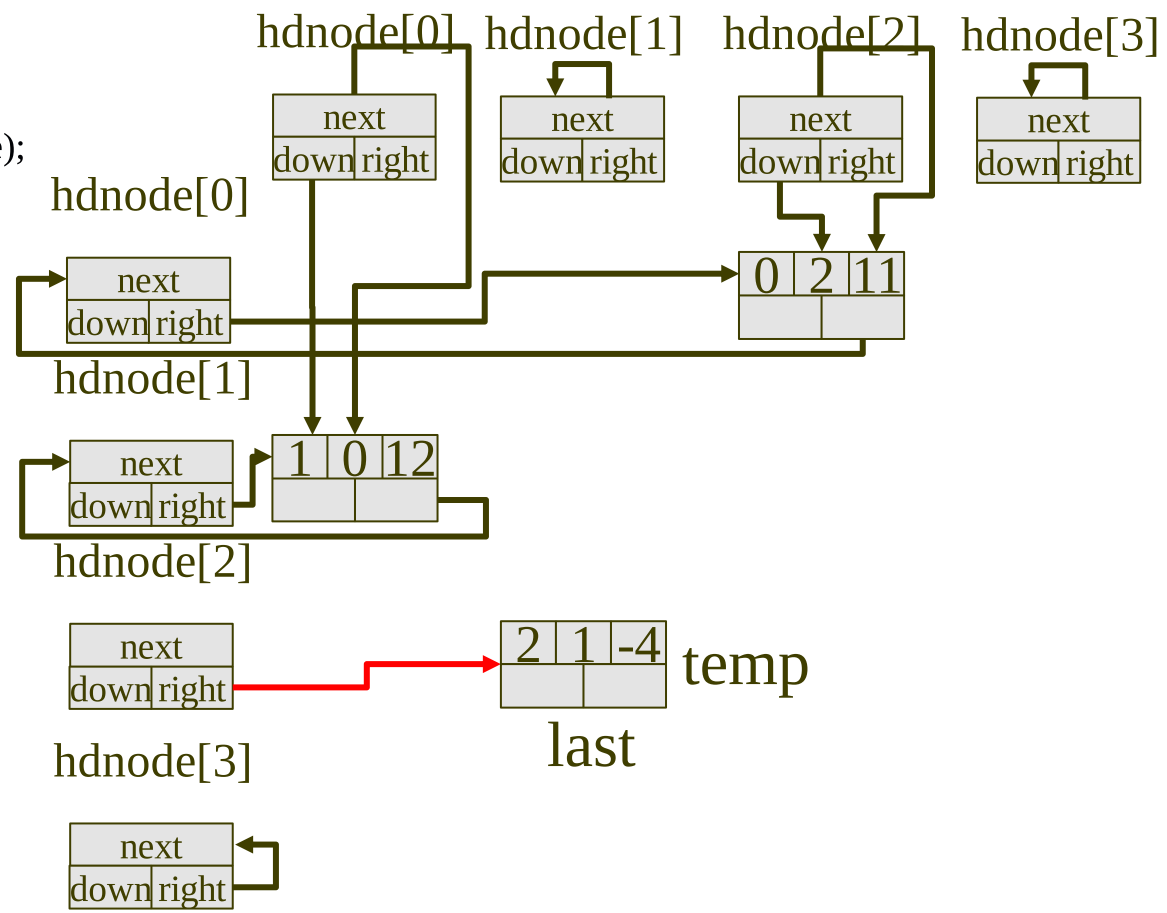
current_row	2	num_terms	4	i	2
row	2	col	1	value	-4

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



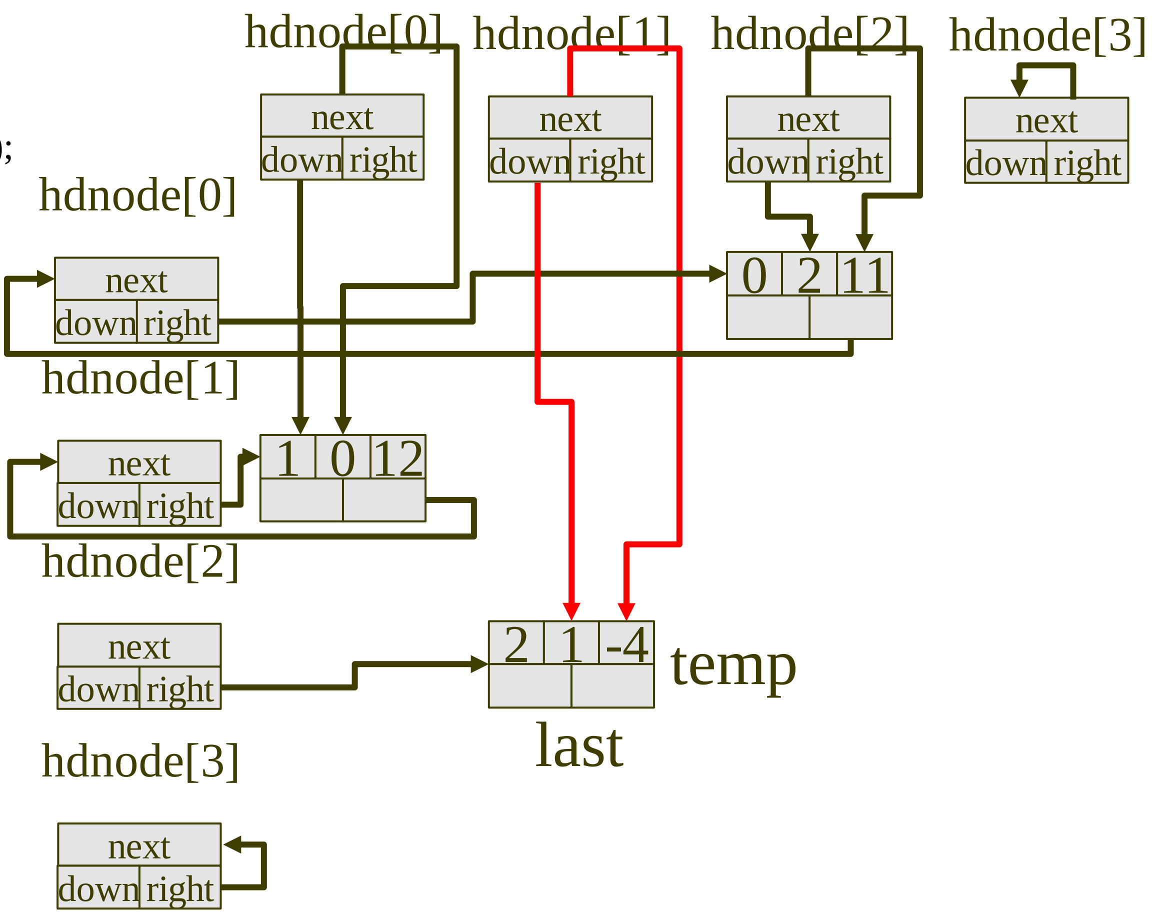
current_row	2	num_terms	4	i	2
row	2	col	1	value	-4

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



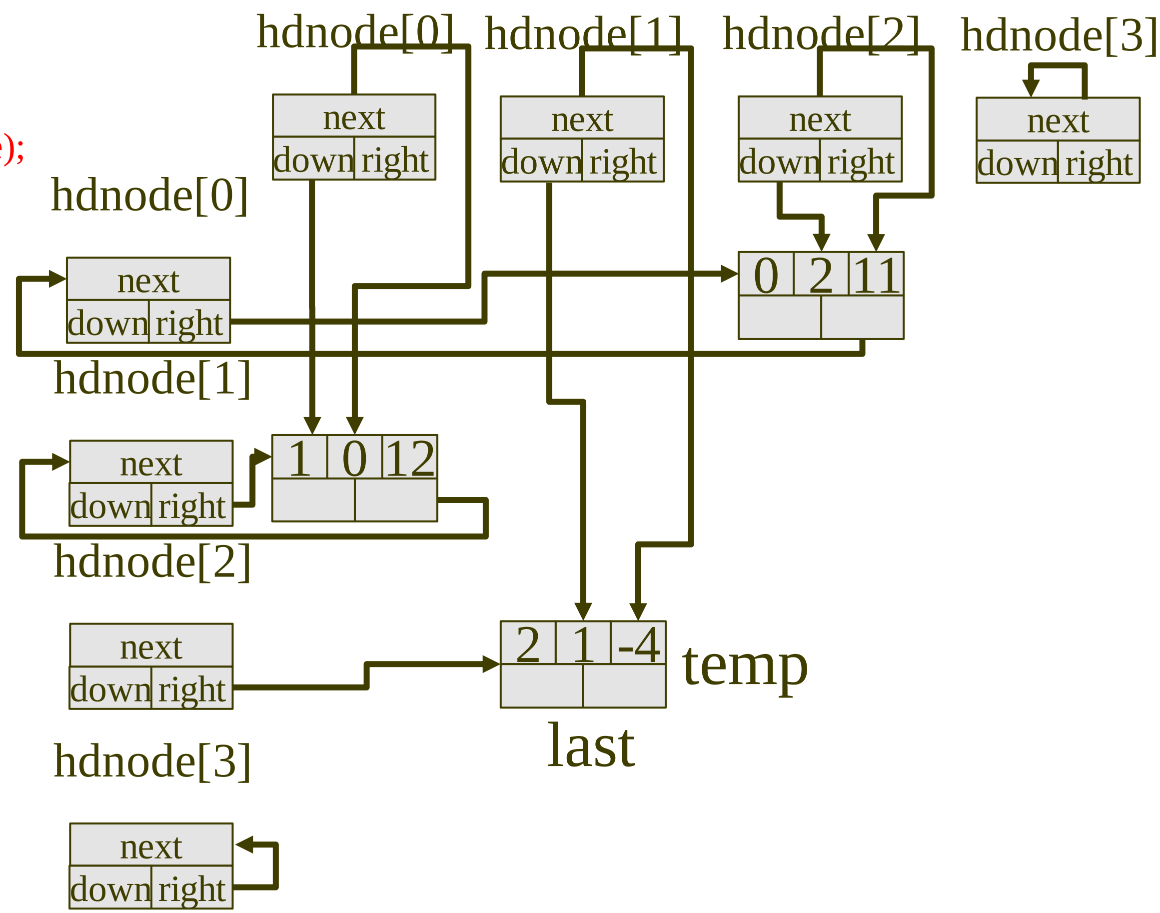
current_row	2	num_terms	4	i	2
row	2	col	1	value	-4

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



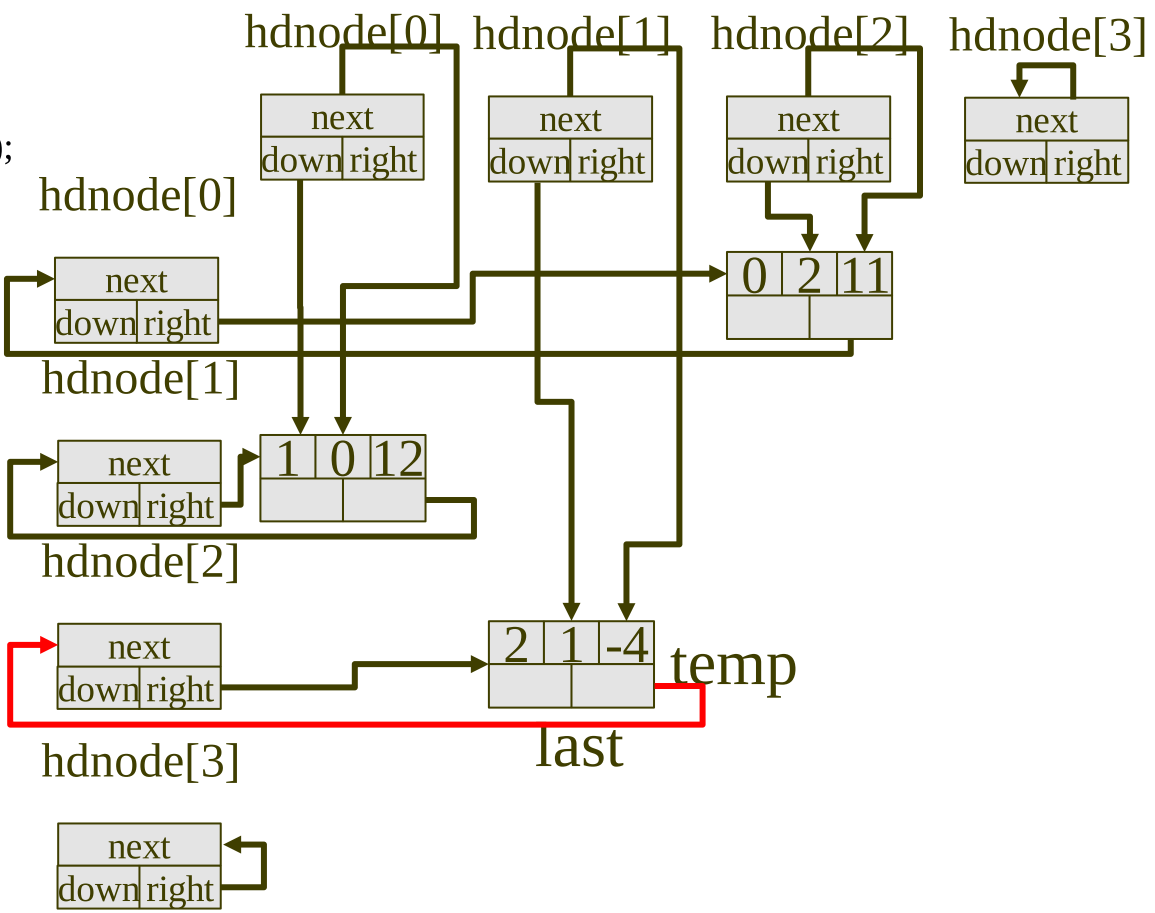
current_row	2	num_terms	4	i	3
row	3	col	3	value	-15

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



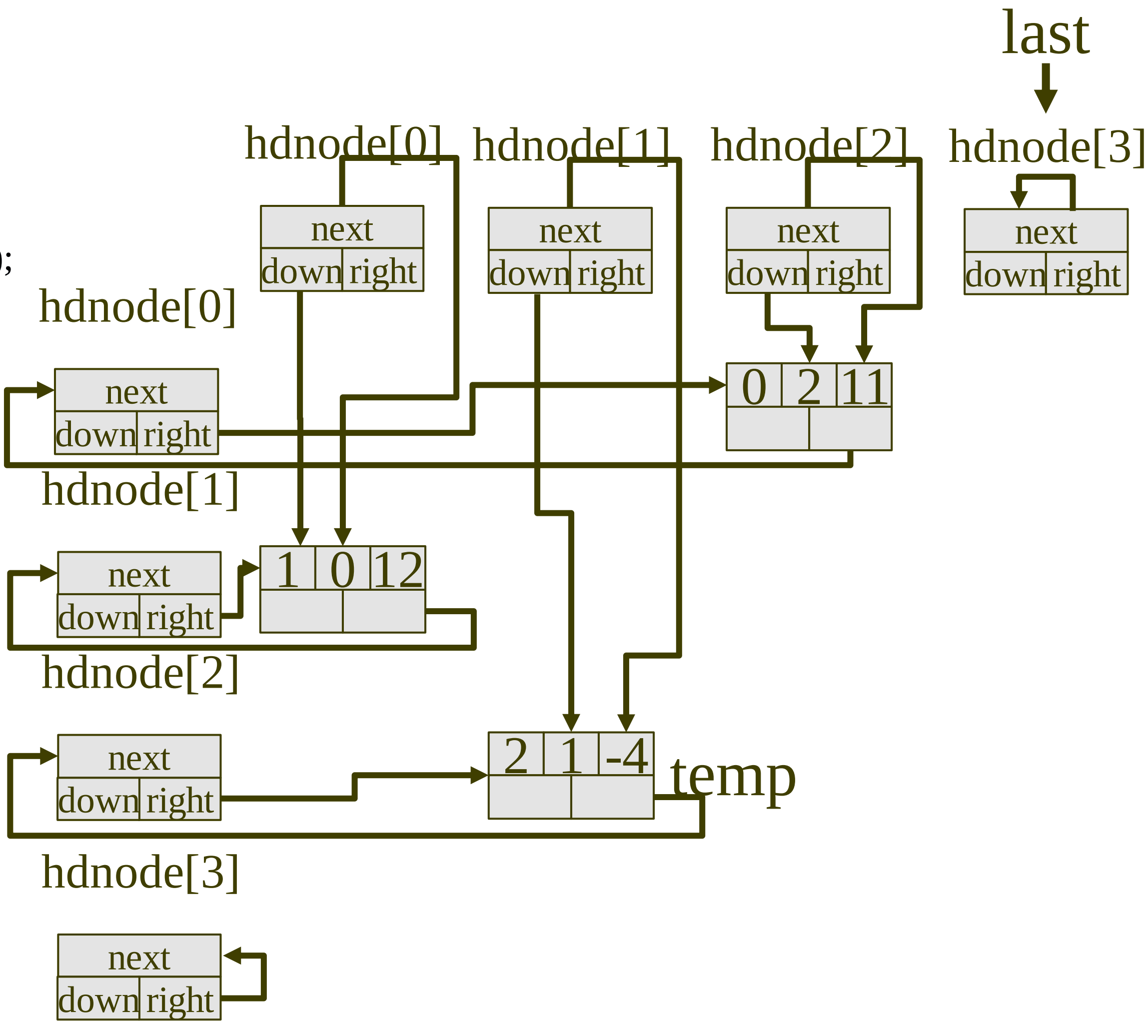
current_row	2	num_terms	4	i	3
row	3	col	3	value	-15

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



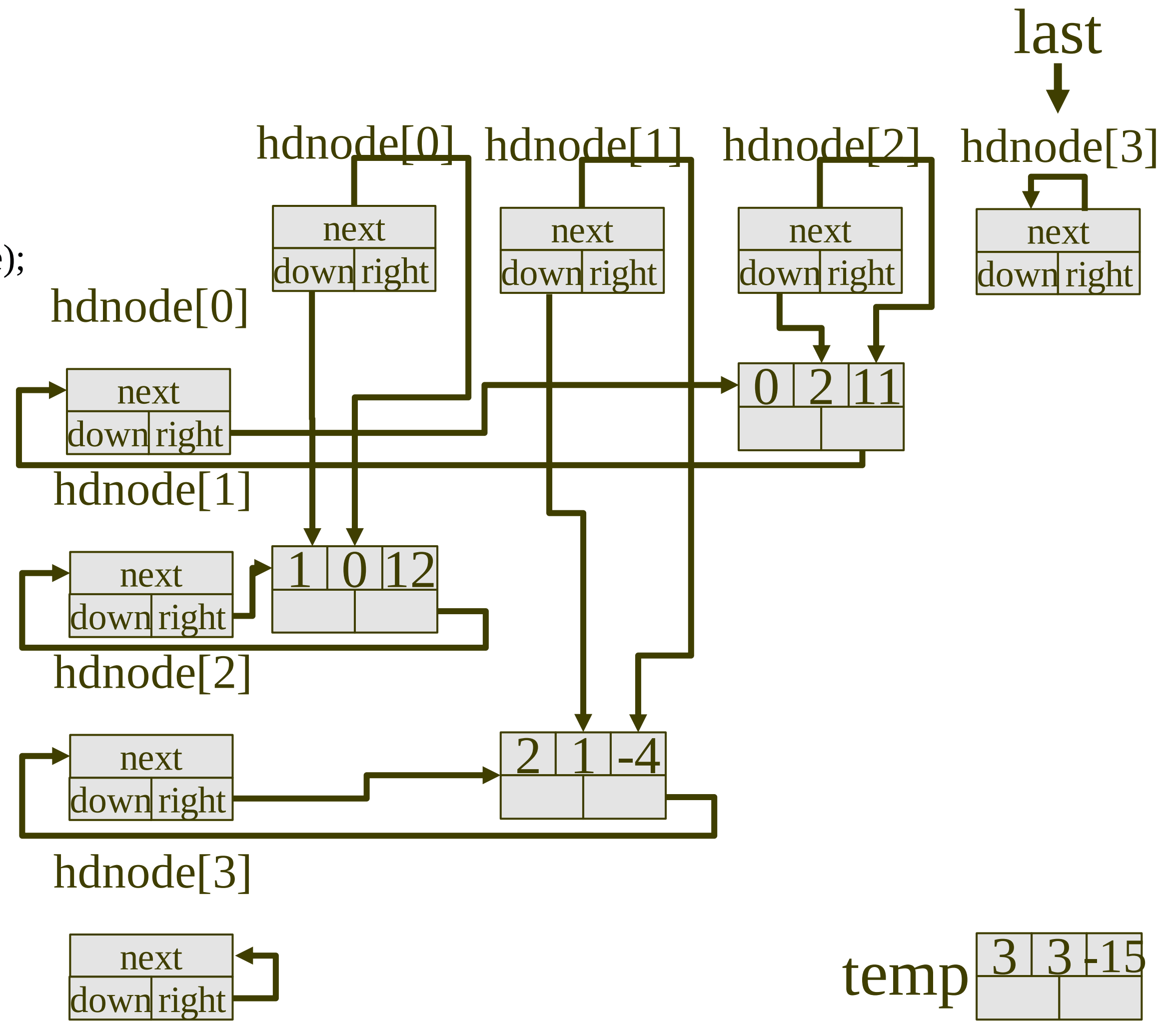
current_row	3	num_terms	4	i	3
row	3	col	3	value	-15

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



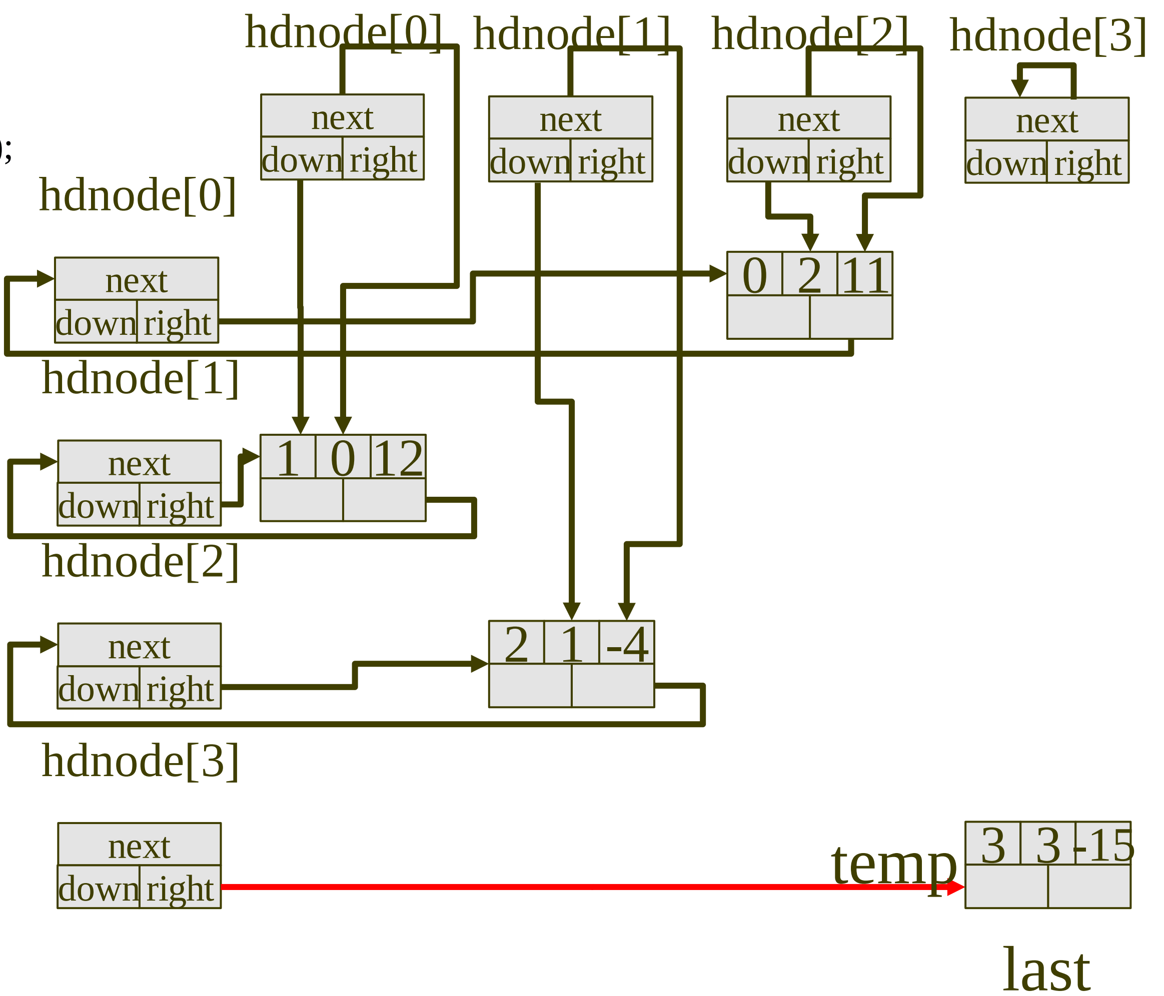
current_row	3	num_terms	4	i	3
row	3	col	3	value	-15

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
    printf("Enter row, column and value: ");
    scanf("%d %d %d", &row, &col, &value);
    if (row > current_row) {
        last->right = hdnode[current_row];
        current_row = row;
        last = hdnode[row];
    }
    temp = new_node();
    temp->tag = entry;
    temp->u.entry.row = row;
    temp->u.entry.col = col;
    temp->u.entry.value = value;
    last->right = temp;
    last = temp;
    hdnode[col]->u.next->down = temp;
    hdnode[col]->u.next = temp;
}
```



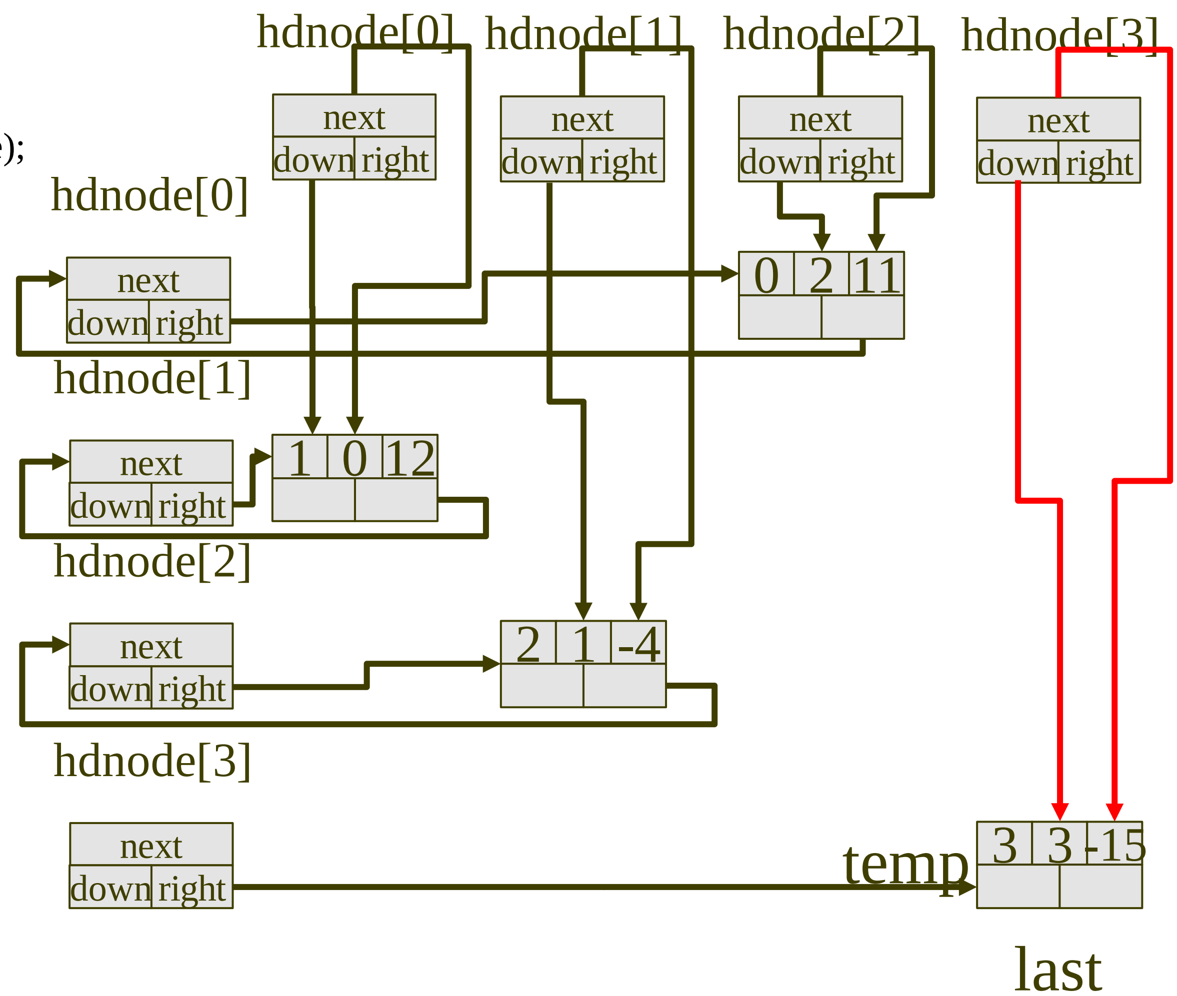
current_row	3	num_terms	4	i	3
row	3	col	3	value	-15

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
    printf("Enter row, column and value: ");
    scanf("%d %d %d", &row, &col, &value);
    if (row > current_row) {
        last->right = hdnode[current_row];
        current_row = row;
        last = hdnode[row];
    }
    temp = new_node();
    temp->tag = entry;
    temp->u.entry.row = row;
    temp->u.entry.col = col;
    temp->u.entry.value = value;
    last->right = temp;
    last = temp;
    hdnode[col]->u.next->down = temp;
    hdnode[col]->u.next = temp;
}
```



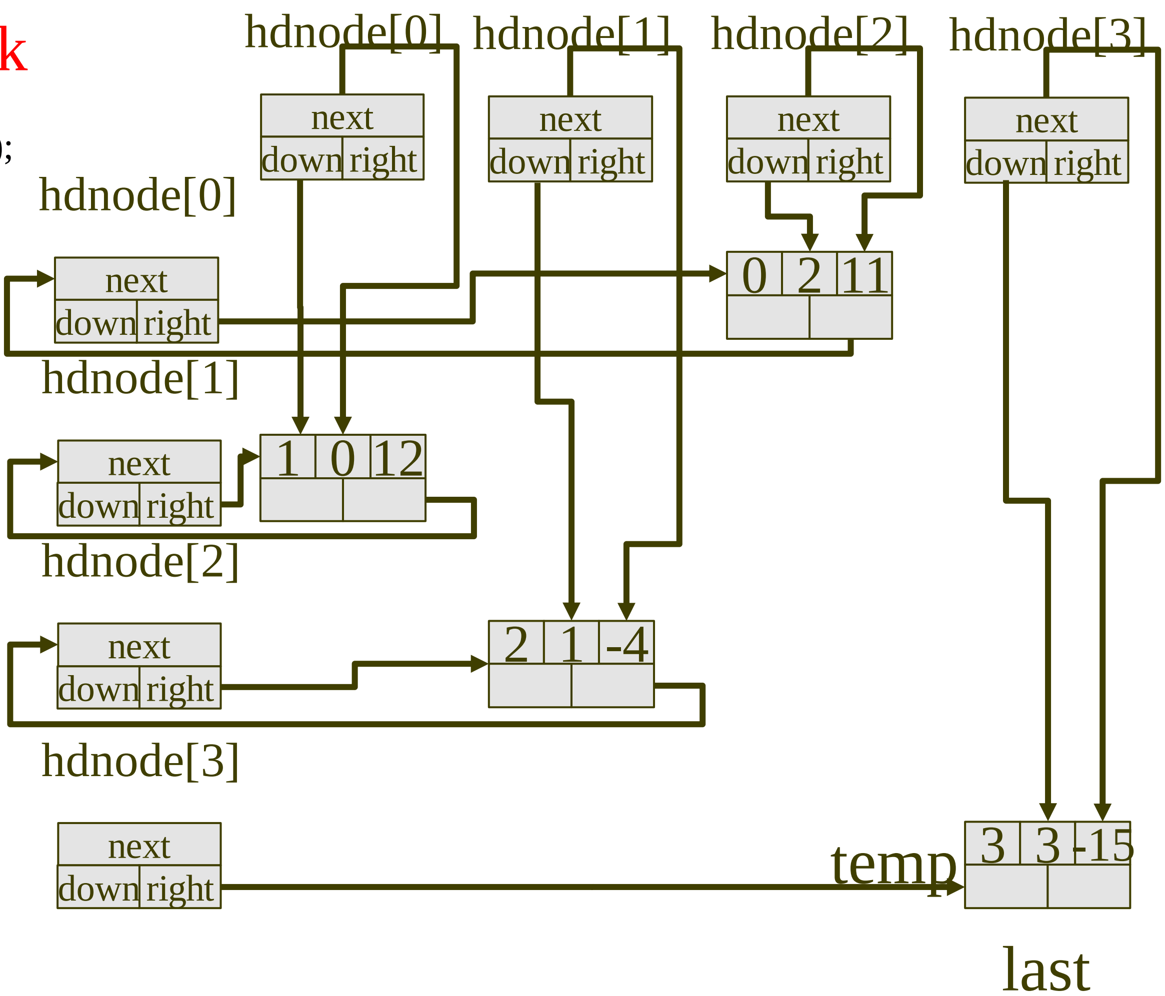
current_row	3	num_terms	4	i	3
row	3	col	3	value	-15

```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```

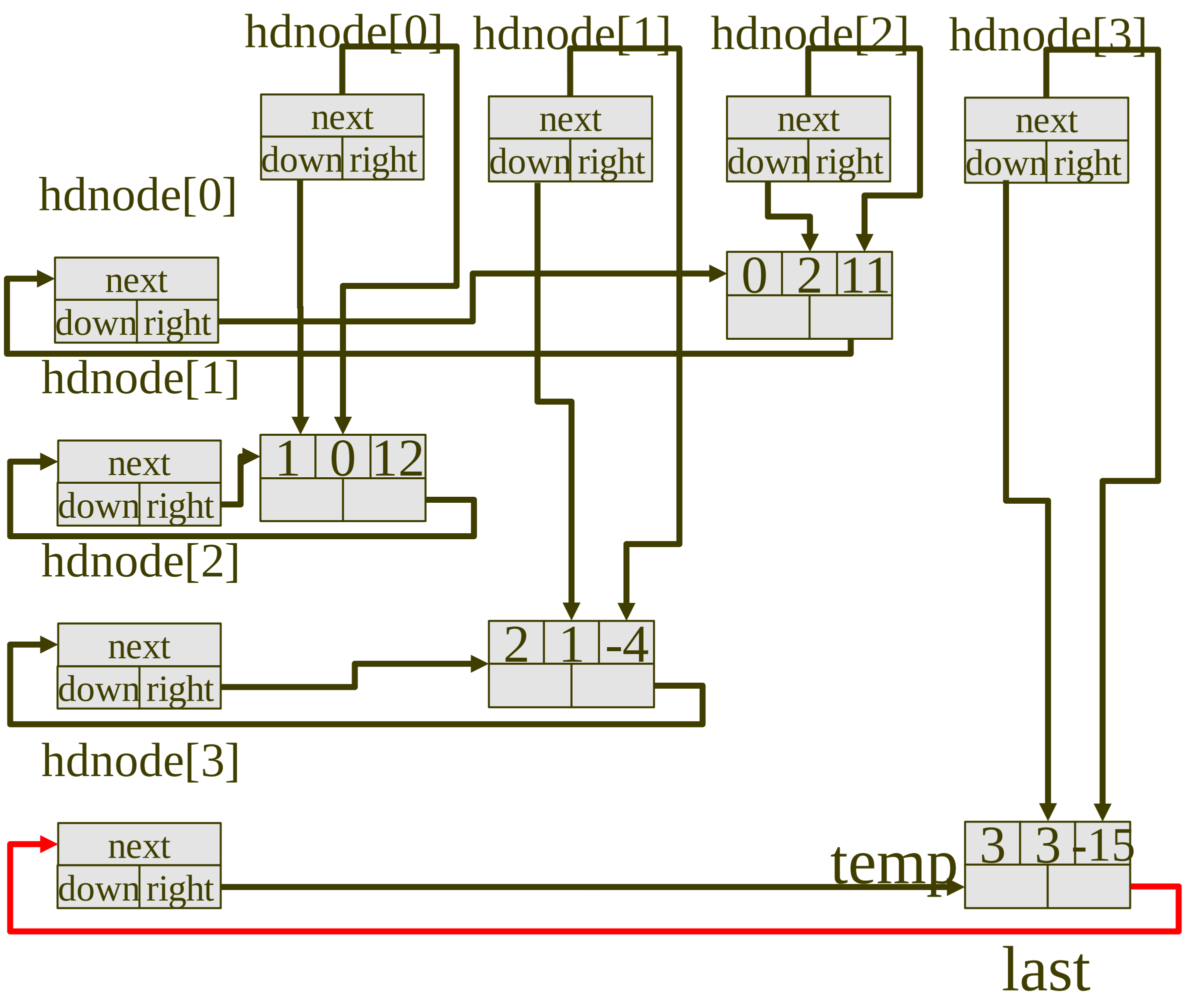


current_row	3	num_terms	4	i	4
row	3	col	3	value	-15

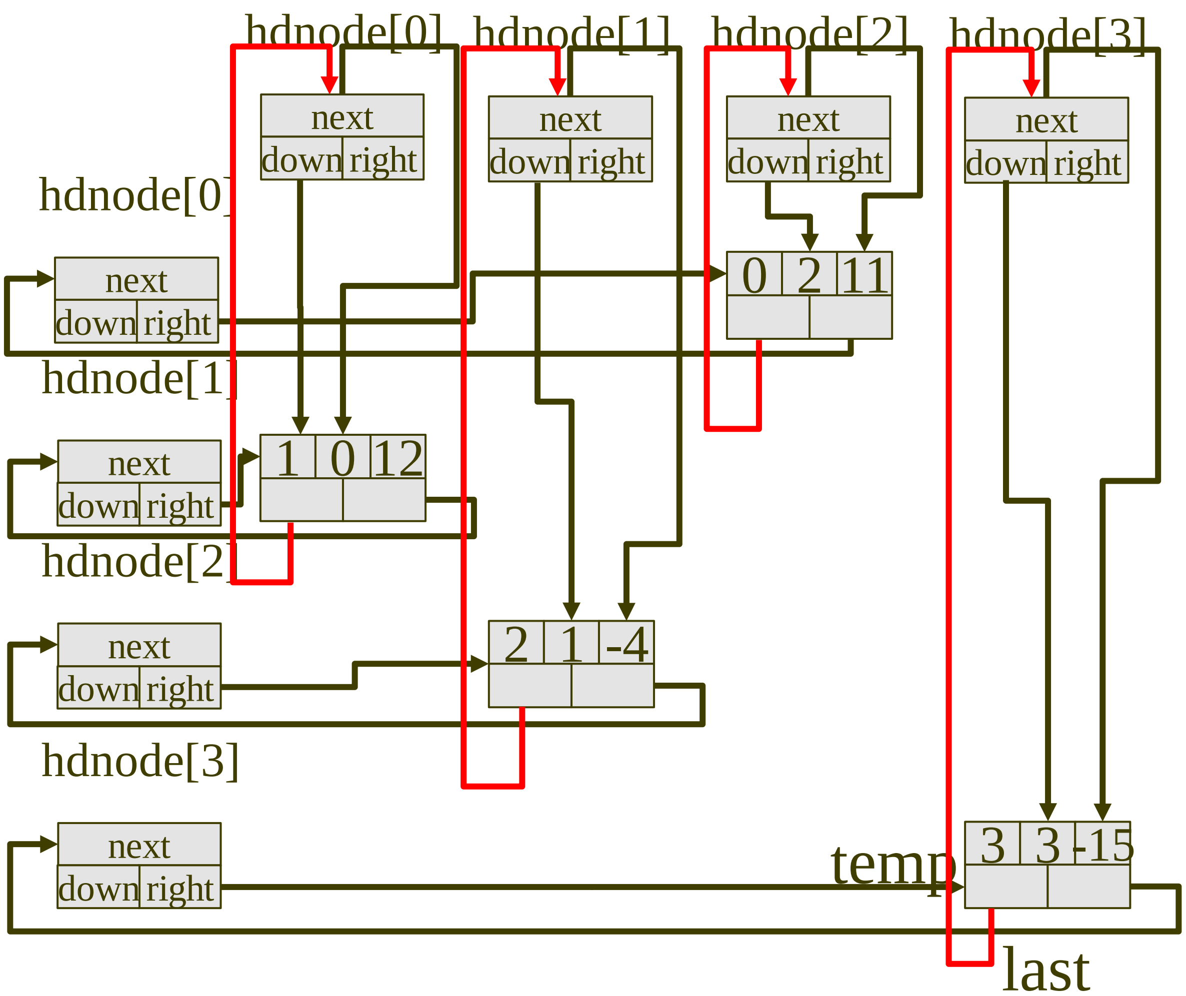
```
current_row = 0;
last = hdnode[0];
for (i=0; i<num_terms; i++) {          → break
      printf("Enter row, column and value: ");
      scanf("%d %d %d", &row, &col, &value);
      if (row > current_row) {
            last->right = hdnode[current_row];
            current_row = row;
            last = hdnode[row];
      }
      temp = new_node();
      temp->tag = entry;
      temp->u.entry.row = row;
      temp->u.entry.col = col;
      temp->u.entry.value = value;
      last->right = temp;
      last = temp;
      hdnode[col]->u.next->down = temp;
      hdnode[col]->u.next = temp;
}
```



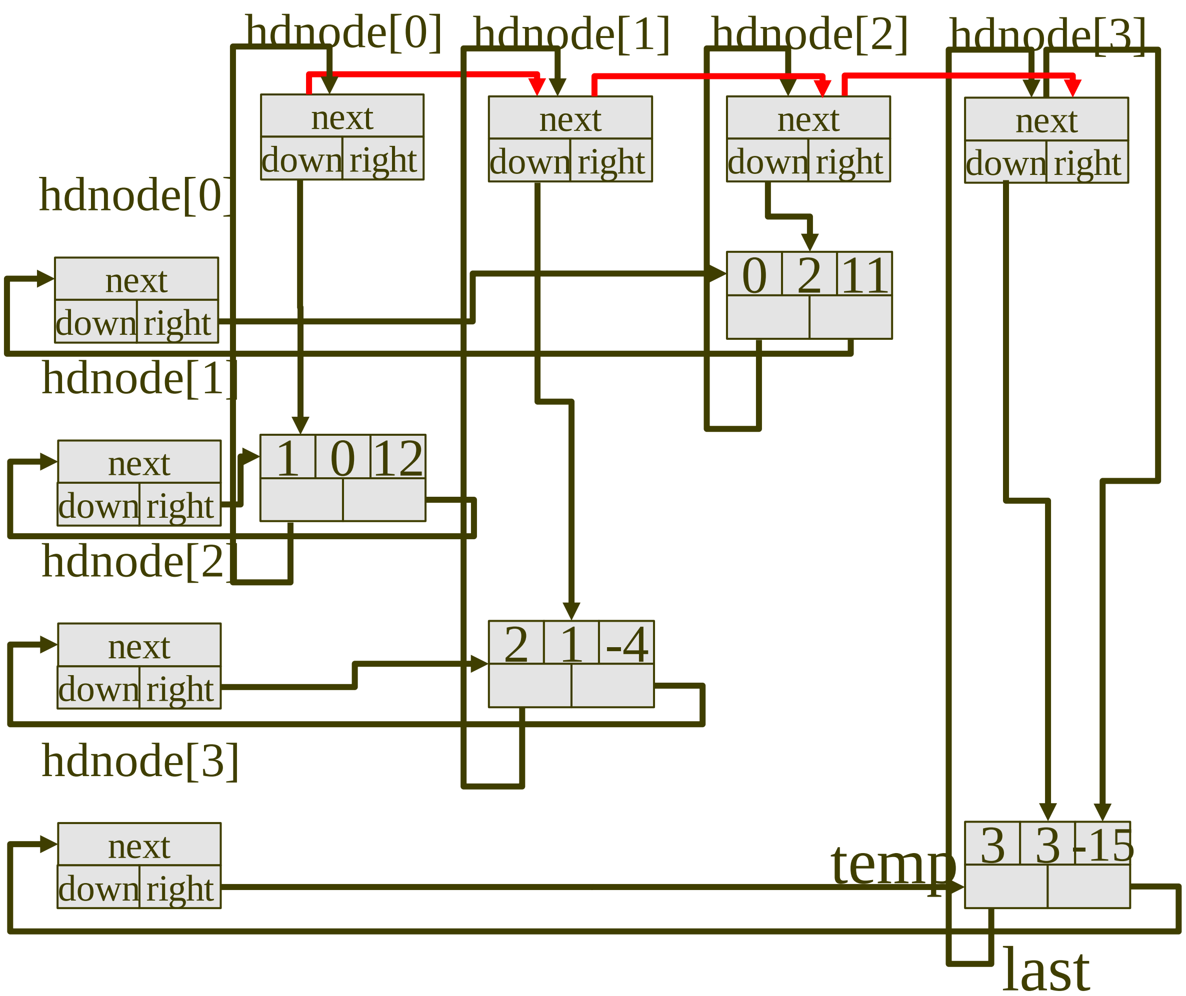
/* close last row */
last->right = hdnode[current_row];
/* close all column lists */
for (i=0; i<num_cols; i++)
    hdnode[i]->u.next->down = hdnode[i];
/* link all header nodes together */
for (i=0; i<num_heads-1; i++)
    hdnode[i]->u.next = hdnode[i+1];
hdnode[num_heads-1]->u.next = node;
node->right = hdnode[0];



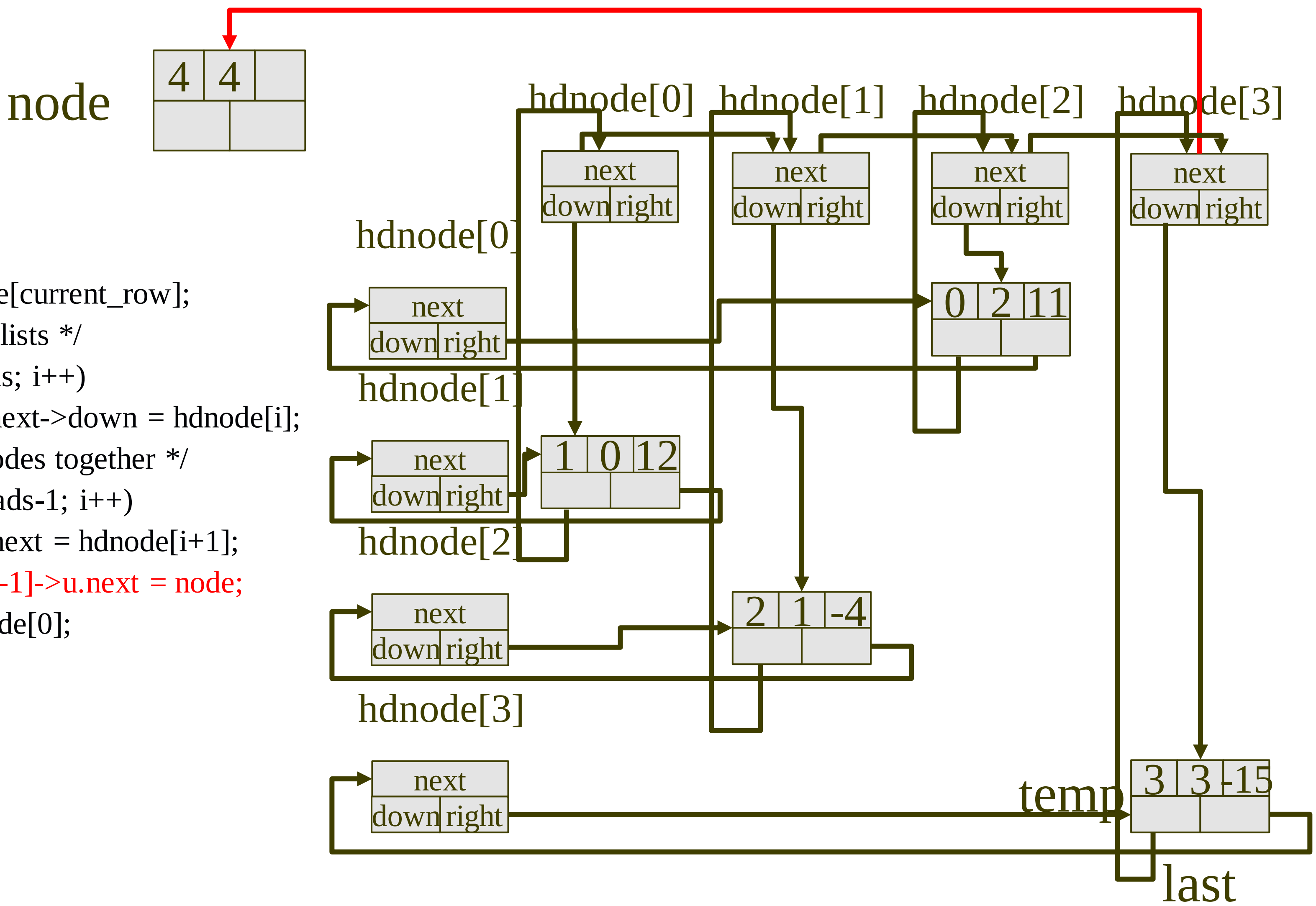
```
/* close last row */
last->right = hdnode[current_row];
/* close all column lists */
for (i=0; i<num_cols; i++)
      hdnode[i]->u.next->down = hdnode[i];
/* link all header nodes together */
for (i=0; i<num_heads-1; i++)
      hdnode[i]->u.next = hdnode[i+1];
hdnode[num_heads-1]->u.next = node;
node->right = hdnode[0];
```



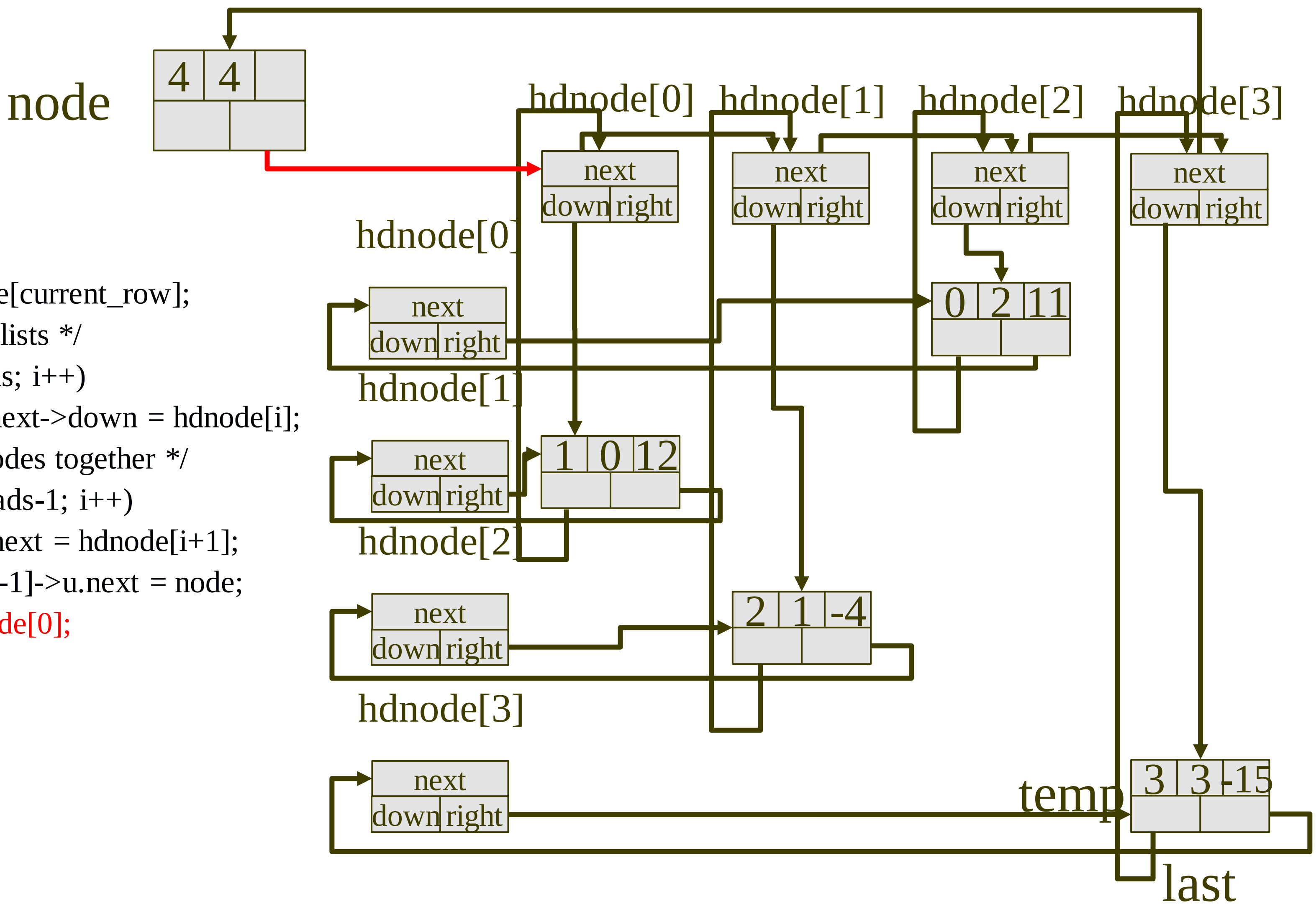
```
/* close last row */
last->right = hdnode[current_row];
/* close all column lists */
for (i=0; i<num_cols; i++)
	hdnode[i]->u.next->down = hdnode[i];
/* link all header nodes together */
for (i=0; i<num_heads-1; i++)
	hdnode[i]->u.next = hdnode[i+1];
hdnode[num_heads-1]->u.next = node;
node->right = hdnode[0];
```



/* close last row */
last->right = hdnode[current_row];
/* close all column lists */
for (i=0; i<num_cols; i++)
	hdnode[i]->u.next->down = hdnode[i];
/* link all header nodes together */
for (i=0; i<num_heads-1; i++)
	hdnode[i]->u.next = hdnode[i+1];
hdnode[num_heads-1]->u.next = node;
node->right = hdnode[0];



```
/* close last row */
last->right = hdnode[current_row];
/* close all column lists */
for (i=0; i<num_cols; i++)
      hdnode[i]->u.next->down = hdnode[i];
/* link all header nodes together */
for (i=0; i<num_heads-1; i++)
      hdnode[i]->u.next = hdnode[i+1];
hdnode[num_heads-1]->u.next = node;
node->right = hdnode[0];
```



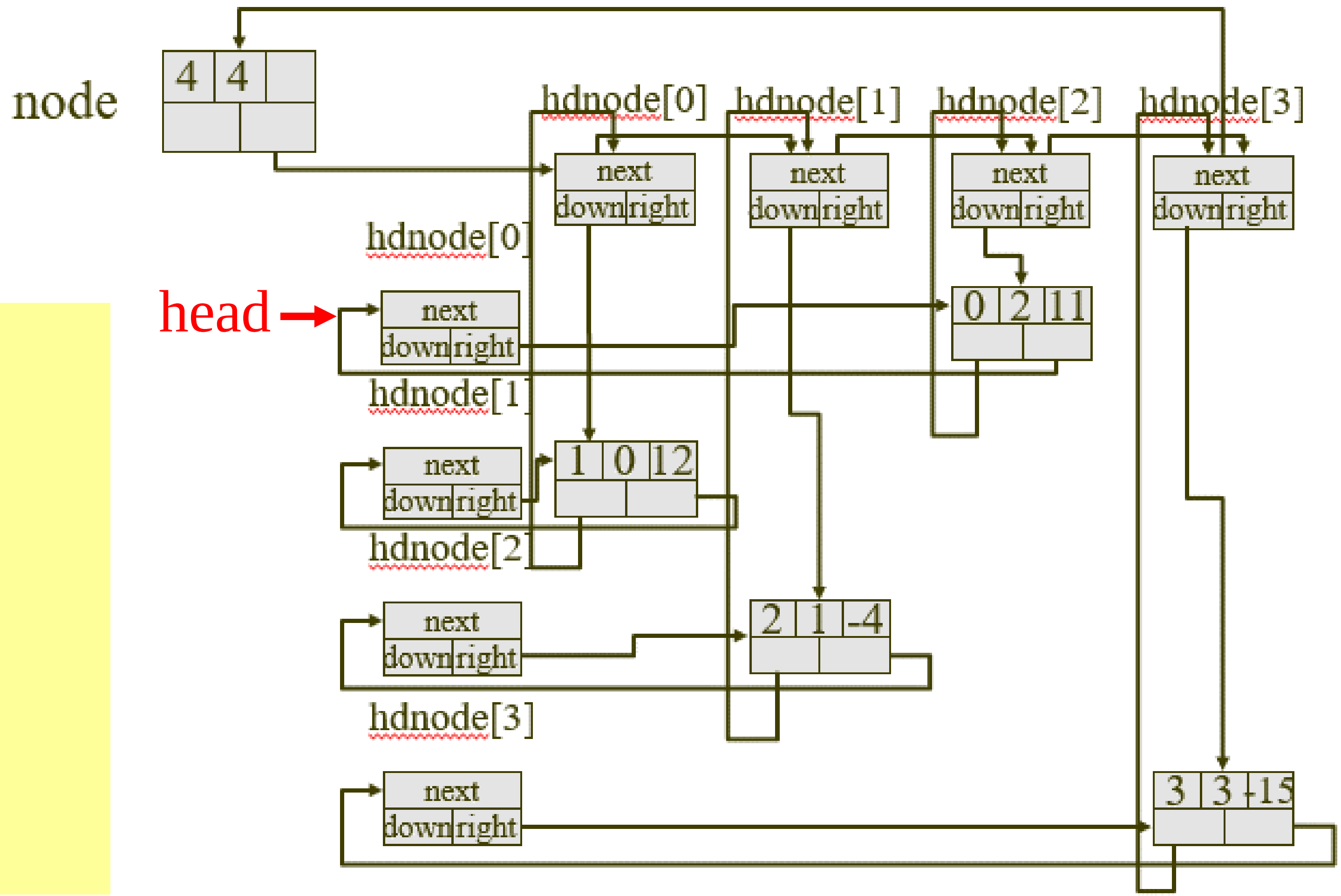
[Program 4.24] Write out a sparse matrix

```
void mwrite(matrix_pointer node)
{ /* print out the matrix in row major form */
    int i;
    matrix_pointer temp, head=node->right;
    /* matrix dimensions */
    printf("\n num_rows=%d, num_cols=%d\n", node->u.entry.row, node->u.entry.col);
    printf(" The matrix by row, column, and value: \n\n");
    for(i=0; i<node->u.entry.row; i++) {
        /* print out the entries in each row */
        for(temp=head->right; temp!=head; temp=temp->right)
            printf("%5d%5d%5d\n",
                    temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
        head = head->u.next; /* next row */
    }
}
```

```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
      /* print out the entries in each row */
      for(temp=head->right; temp!=head; temp=temp->right)
            printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
      head = head->u.next; /* next row */
}
```

i	0

/* Output */

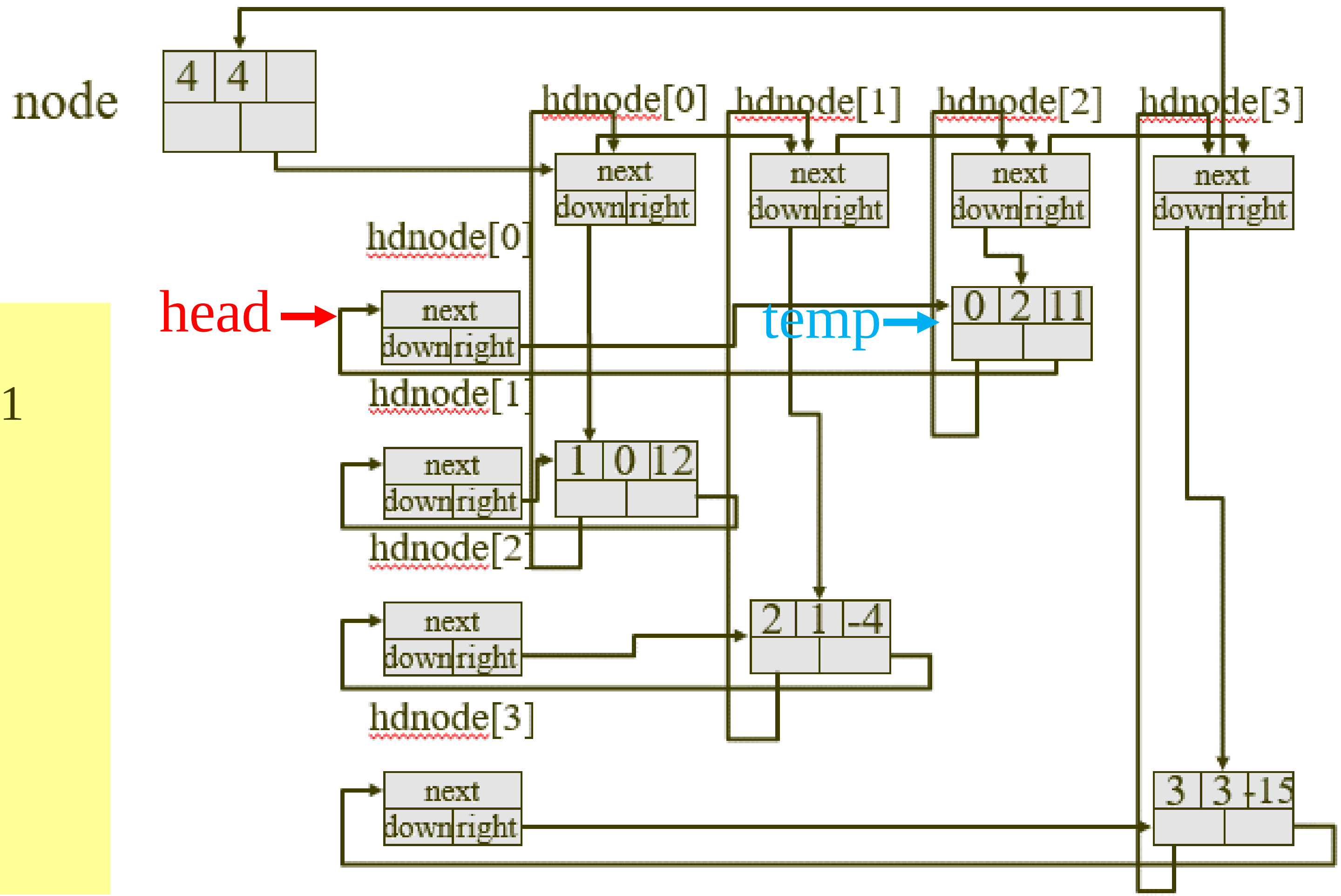


```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
    /* print out the entries in each row */
    for(temp=head->right; temp!=head; temp=temp->right)
        printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
    head = head->u.next; /* next row */
}
```

i	0

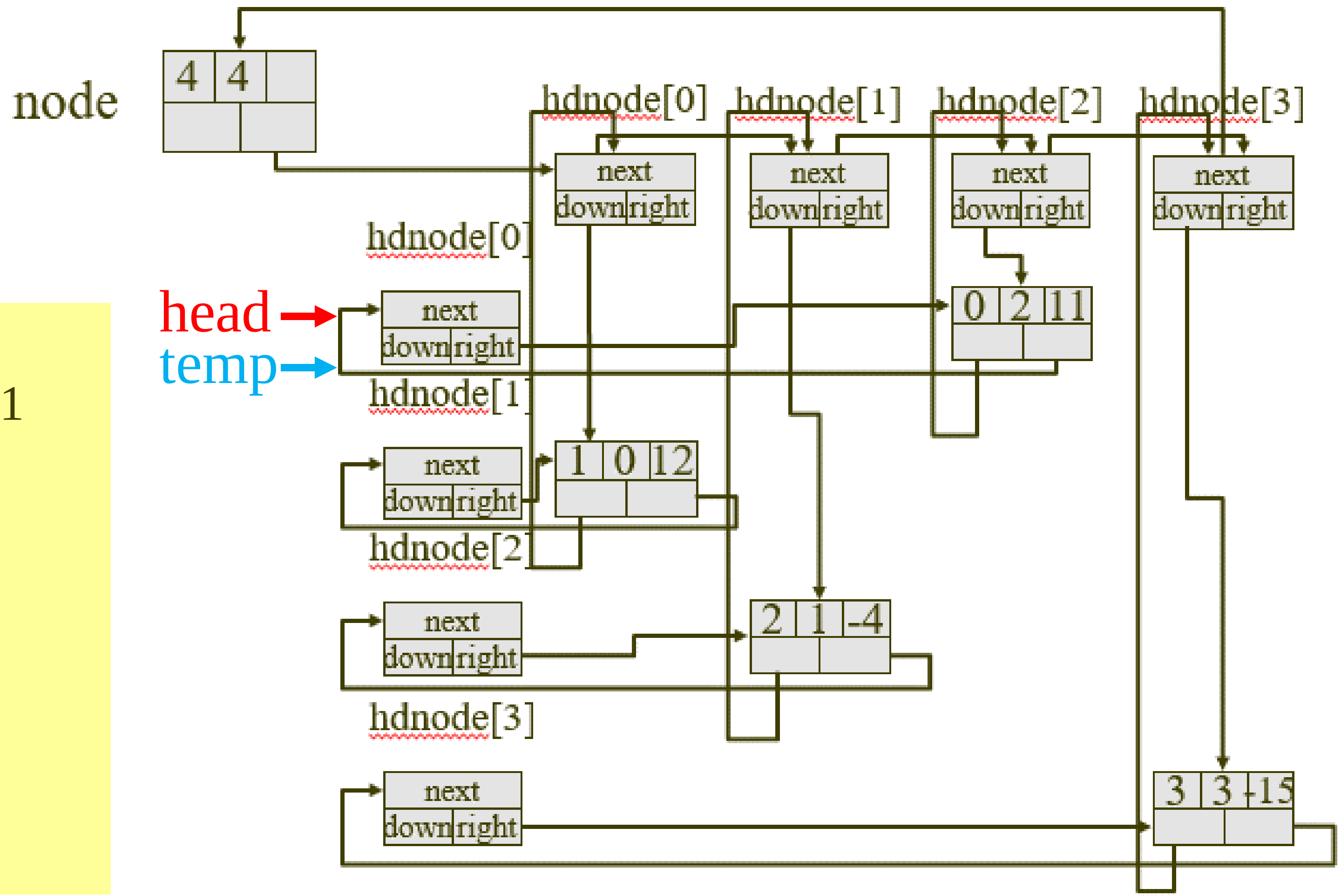
/* Output */

0 2 11



```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
    /* print out the entries in each row */
    for(temp=head->right; temp!=head; temp=temp->right) → break
        printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
    head = head->u.next; /* next row */
}
```

i	0



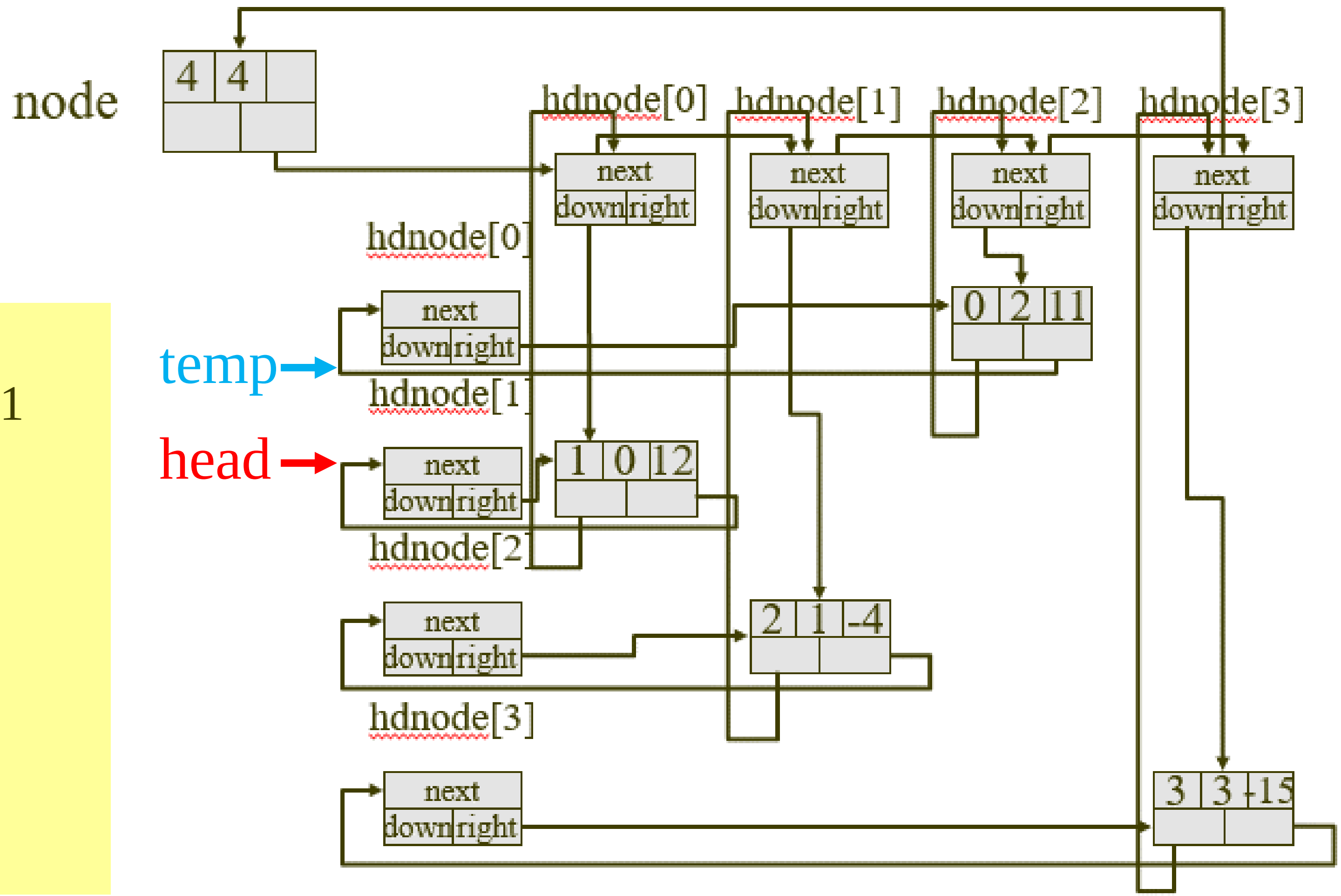
/* Output */
0 2 11

```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
    /* print out the entries in each row */
    for(temp=head->right; temp!=head; temp=temp->right)
        printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
    head = head->u.next; /* next row */
}
```

i	0

/* Output */

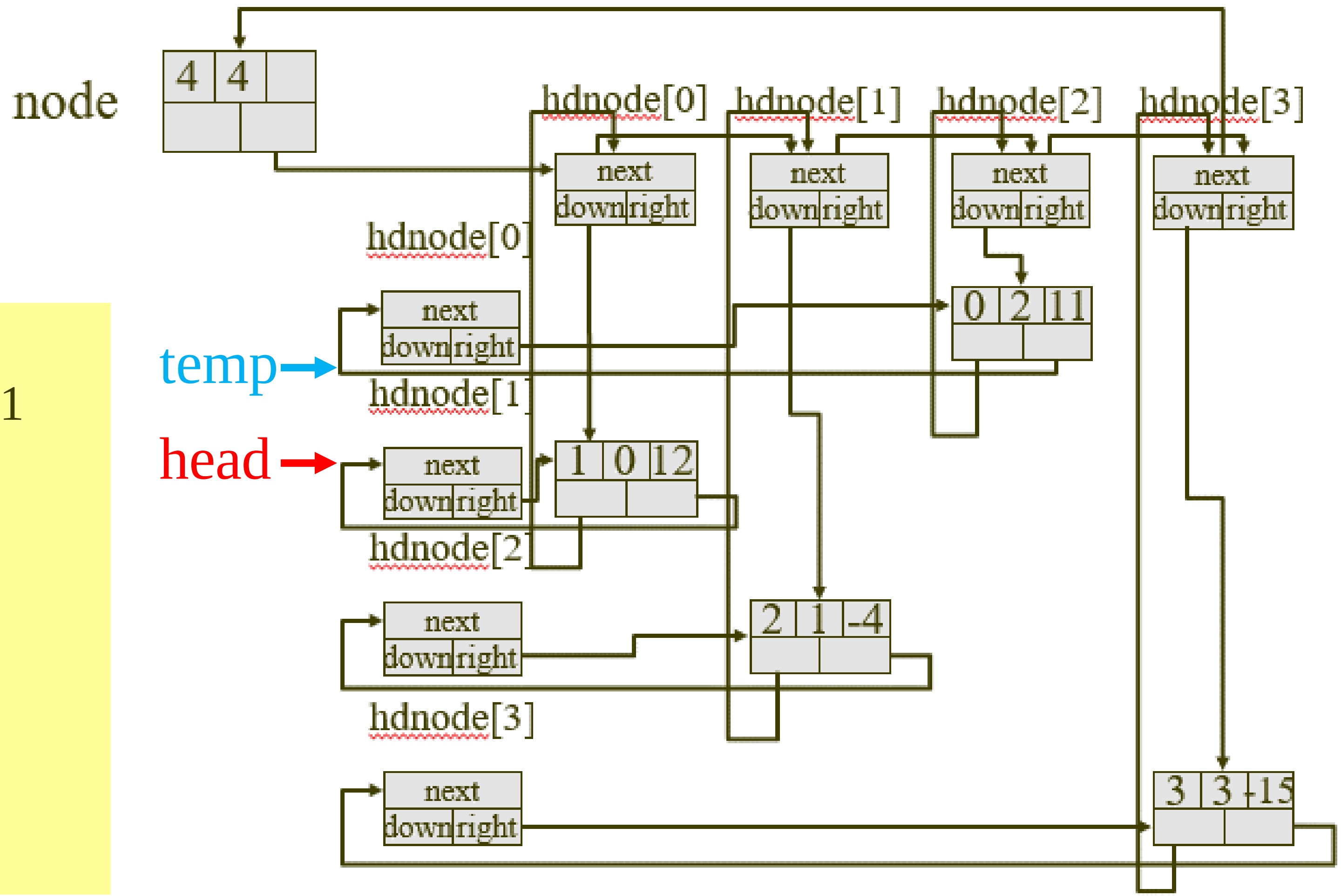
0 2 11



```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
      /* print out the entries in each row */
      for(temp=head->right; temp!=head; temp=temp->right)
            printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
      head = head->u.next; /* next row */
}
```

i	1

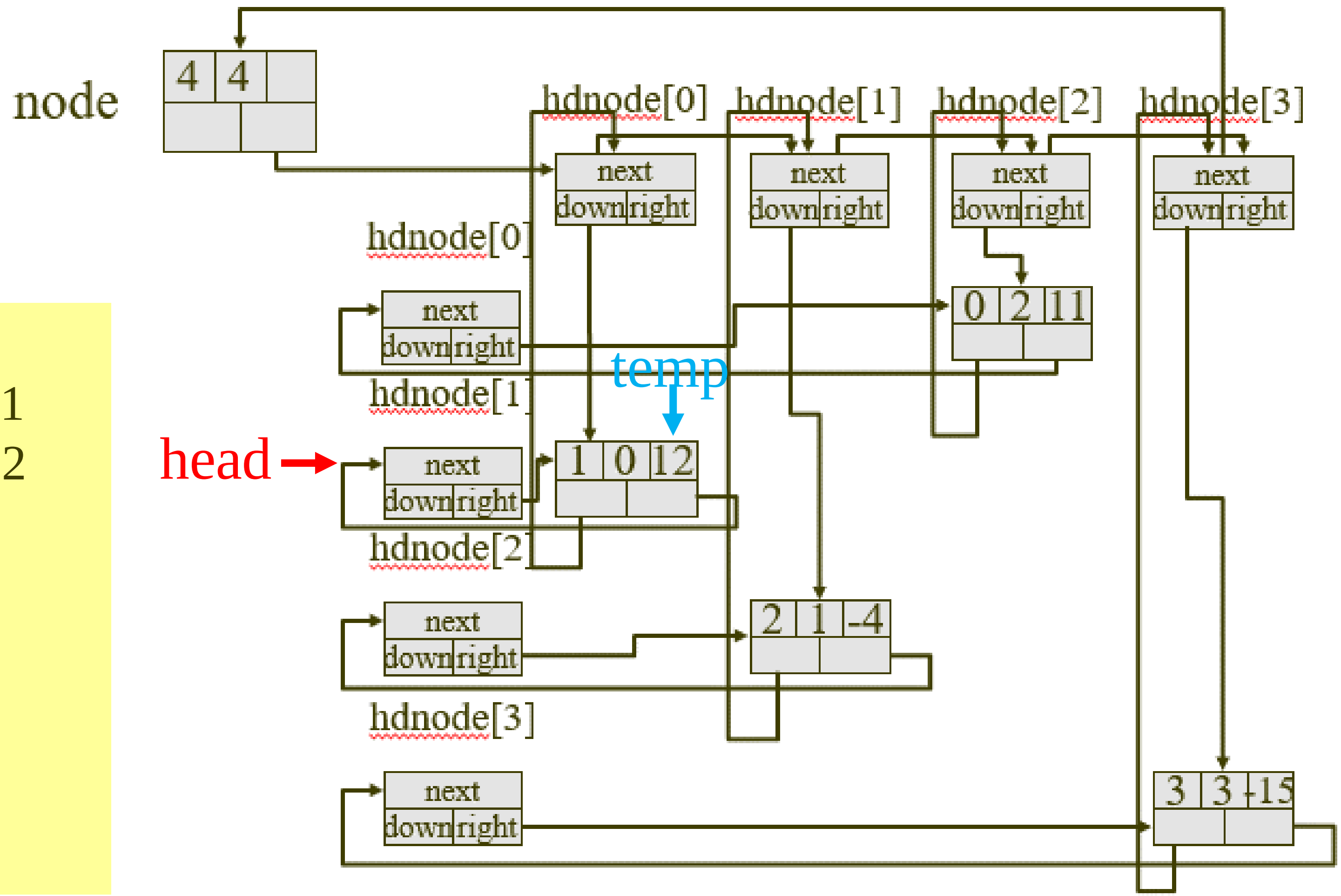
/* Output */
0 2 11



```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
    /* print out the entries in each row */
    for(temp=head->right; temp!=head; temp=temp->right)
        printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
    head = head->u.next; /* next row */
}
```

i	1

```
/* Output */
0       2       11
1       0       12
```

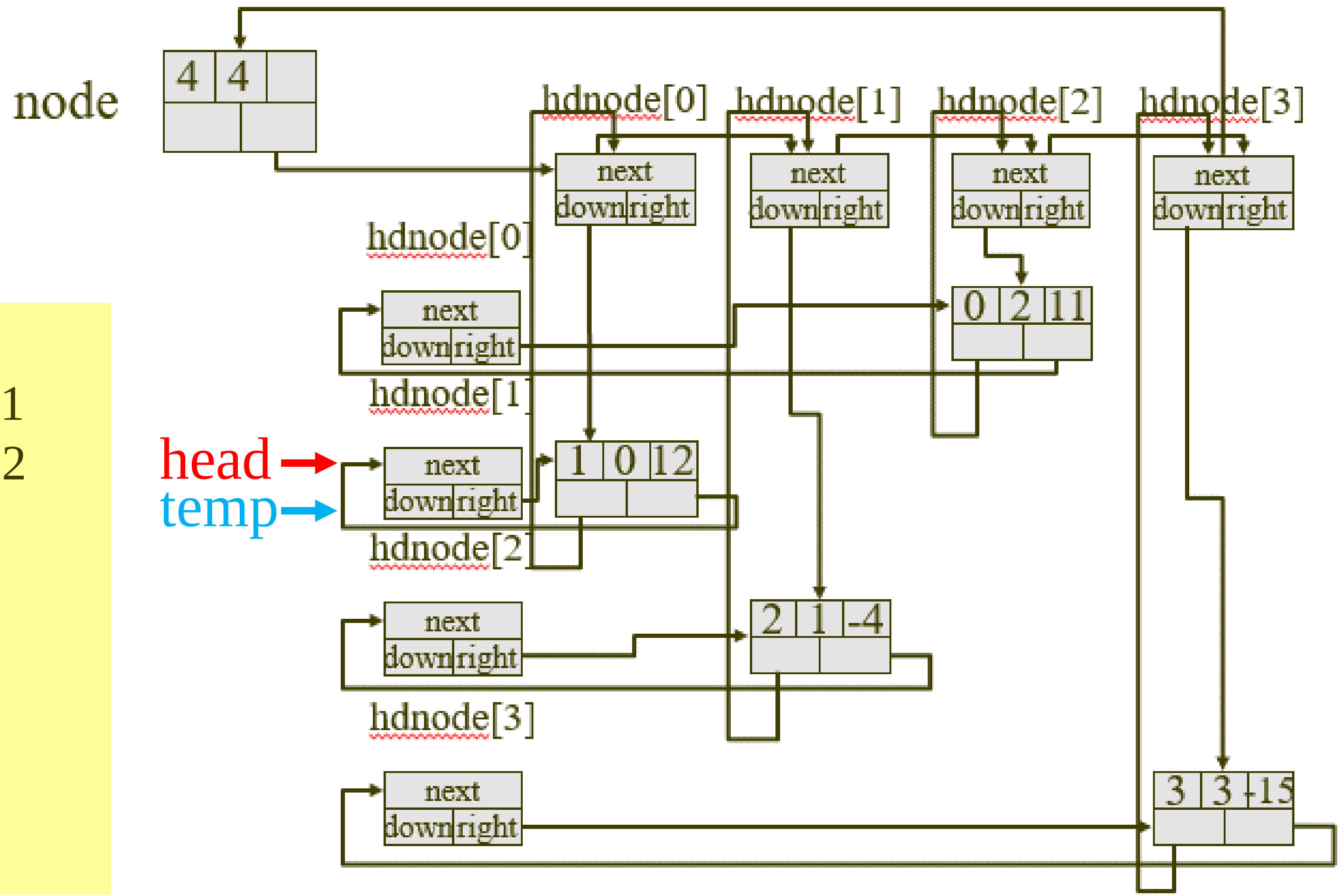


```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
    /* print out the entries in each row */
    for(temp=head->right; temp!=head; temp=temp->right)  → break
        printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
    head = head->u.next; /* next row */
}
```

i	1

/* Output */

0	2	11
1	0	12



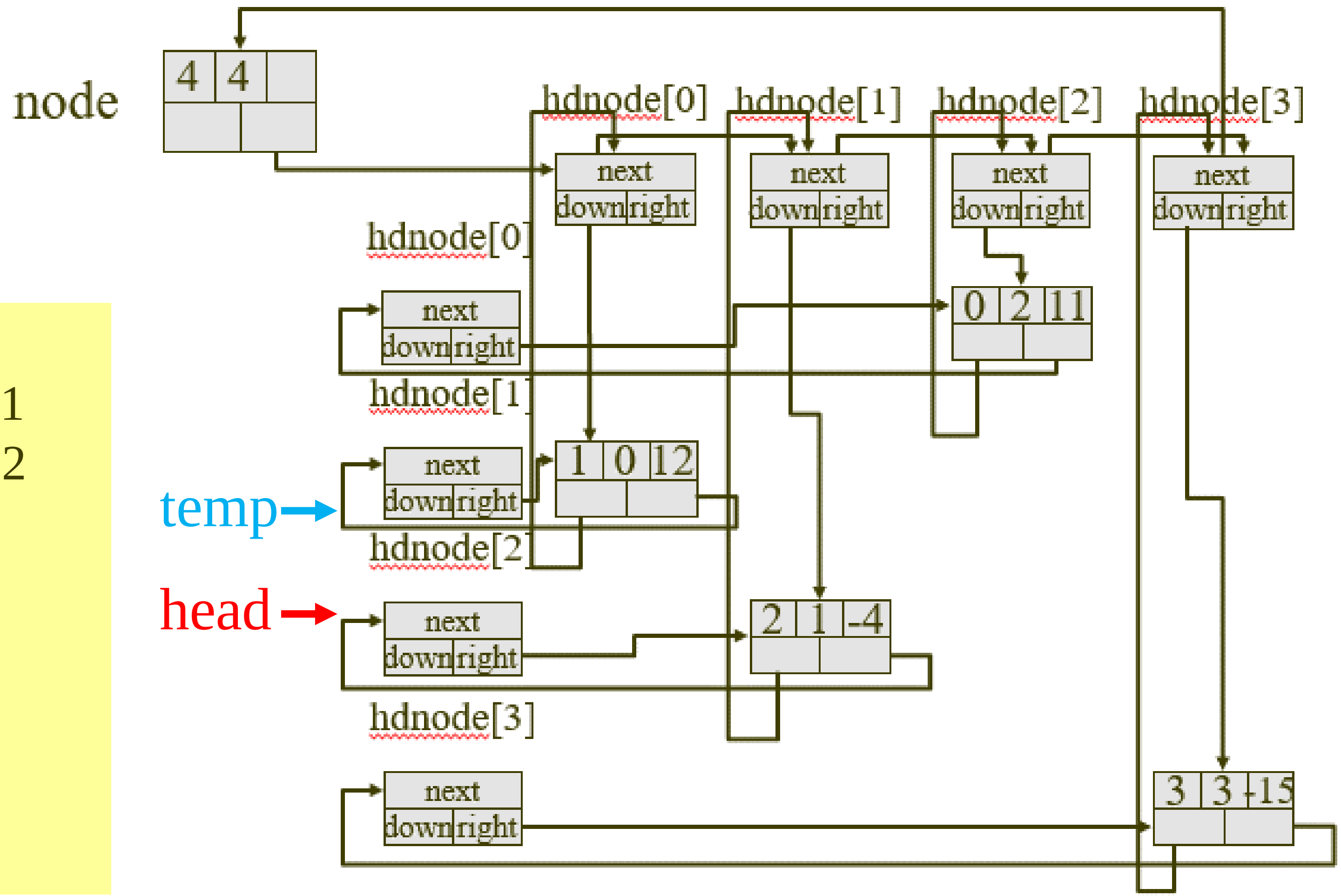
```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
    /* print out the entries in each row */
    for(temp=head->right; temp!=head; temp=temp->right)
        printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
    head = head->u.next; /* next row */
}
```

i	1

/* Output */

0 2 11

1 0 12

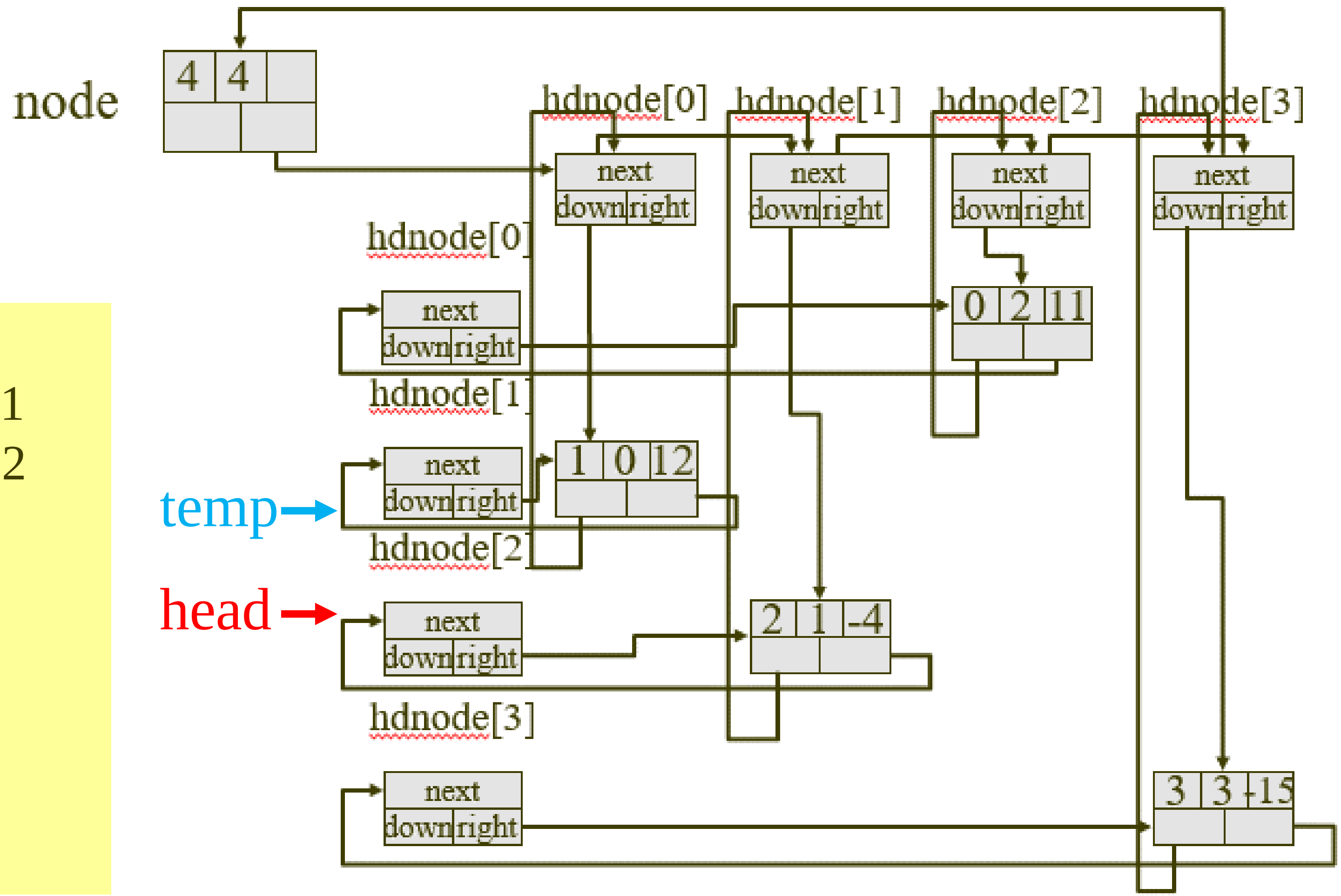


```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
    /* print out the entries in each row */
    for(temp=head->right; temp!=head; temp=temp->right)
        printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
    head = head->u.next; /* next row */
}
```

i	2

/* Output */

0	2	11
1	0	12

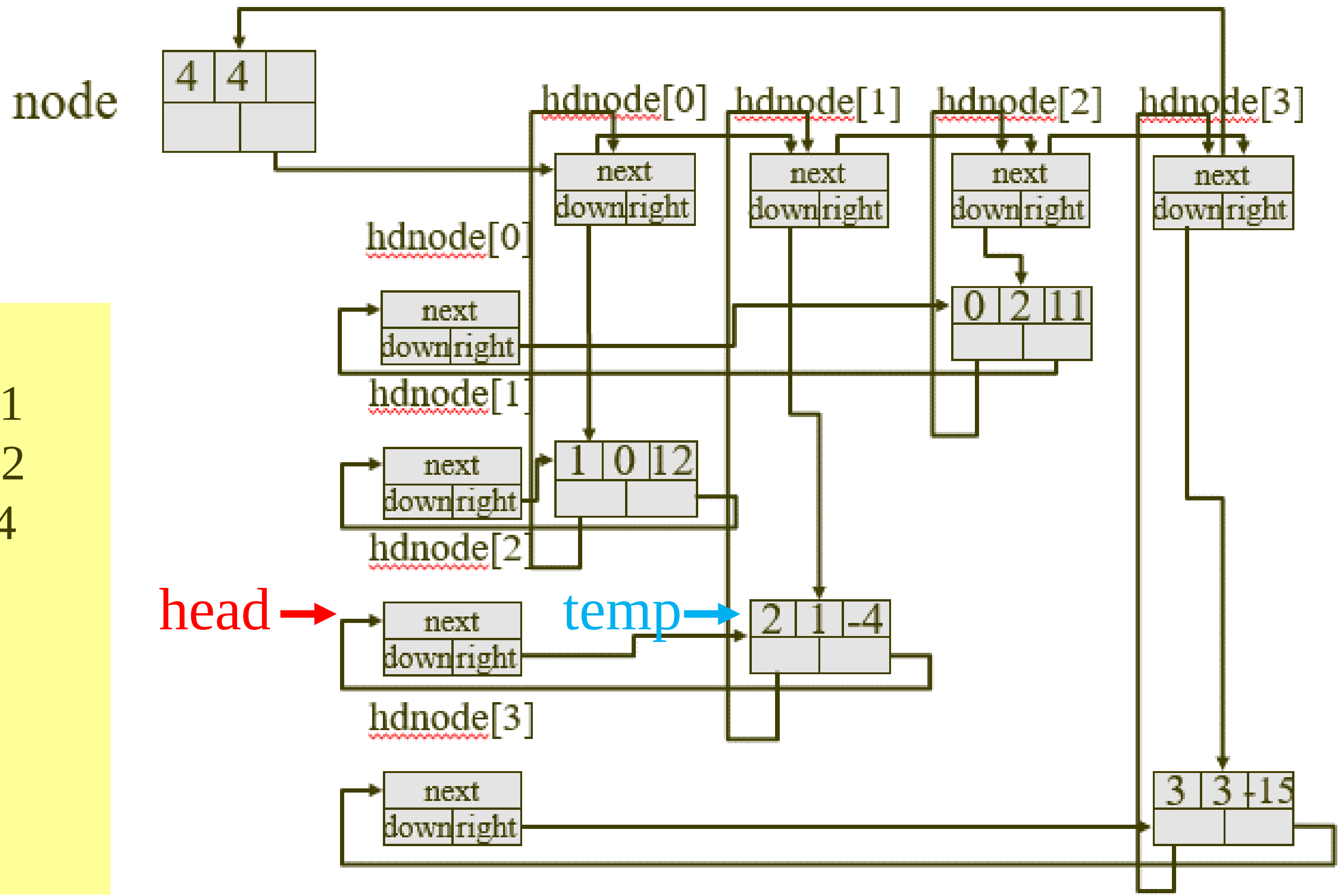


```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
    /* print out the entries in each row */
    for(temp=head->right; temp!=head; temp=temp->right)
        printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
    head = head->u.next; /* next row */
}
```

i	2

/* Output */

0	2	11
1	0	12
2	1	-4

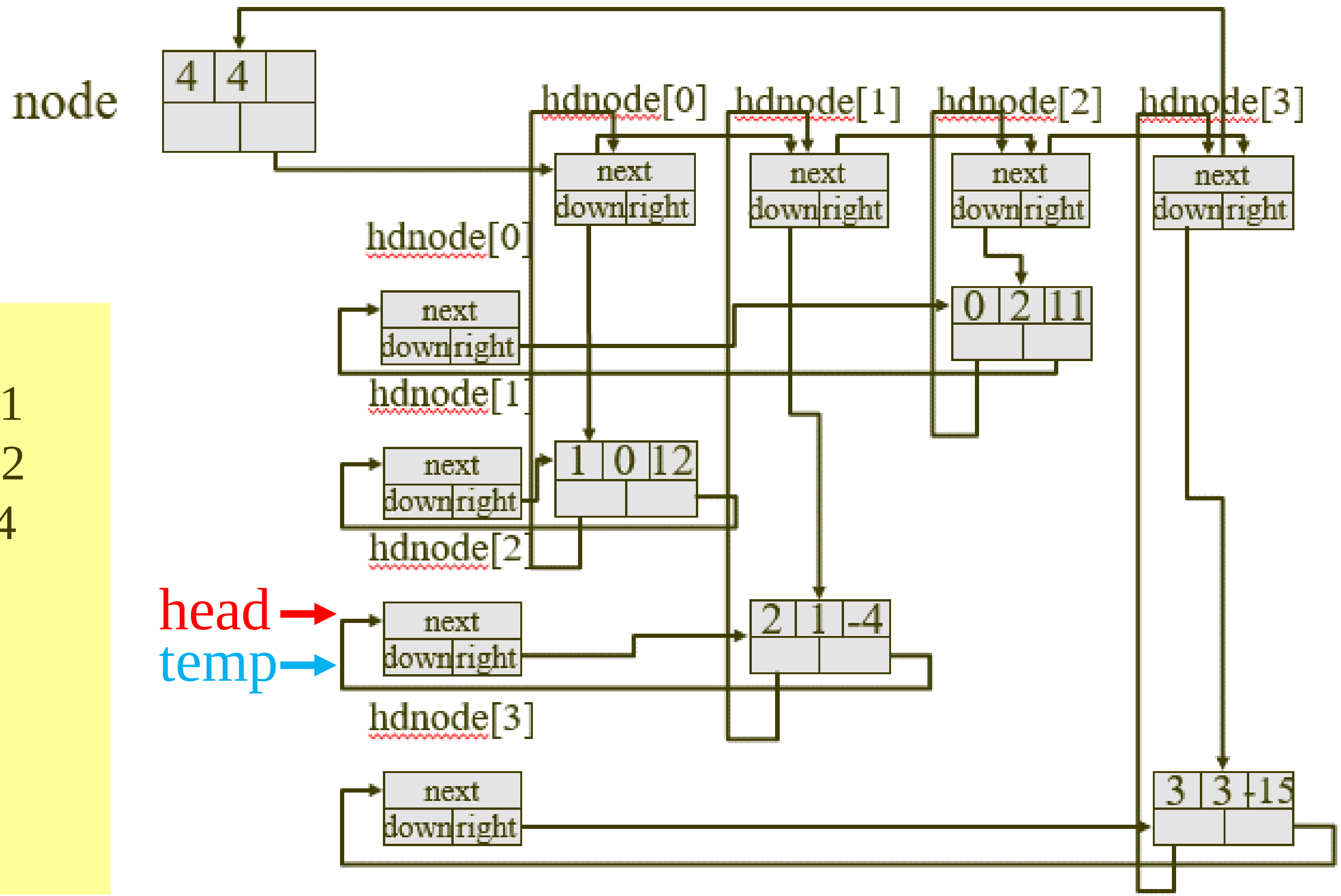


```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
    /* print out the entries in each row */
    for(temp=head->right; temp!=head; temp=temp->right)   → break
        printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
    head = head->u.next; /* next row */
}
```

i	2

/* Output */

0	2	11
1	0	12
2	1	-4

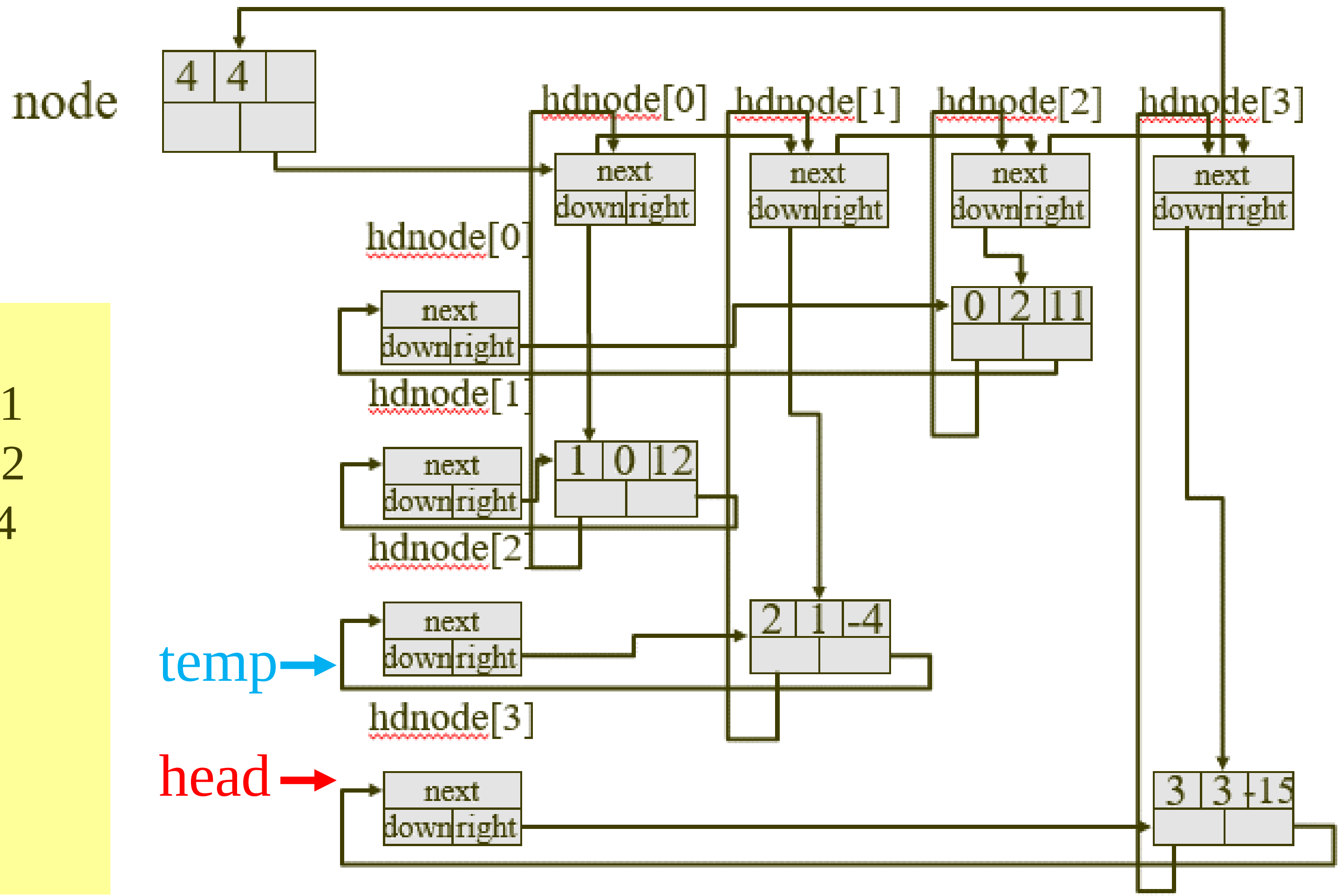


```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
    /* print out the entries in each row */
    for(temp=head->right; temp!=head; temp=temp->right)
        printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
    head = head->u.next; /* next row */
}
```

i	2

/* Output */

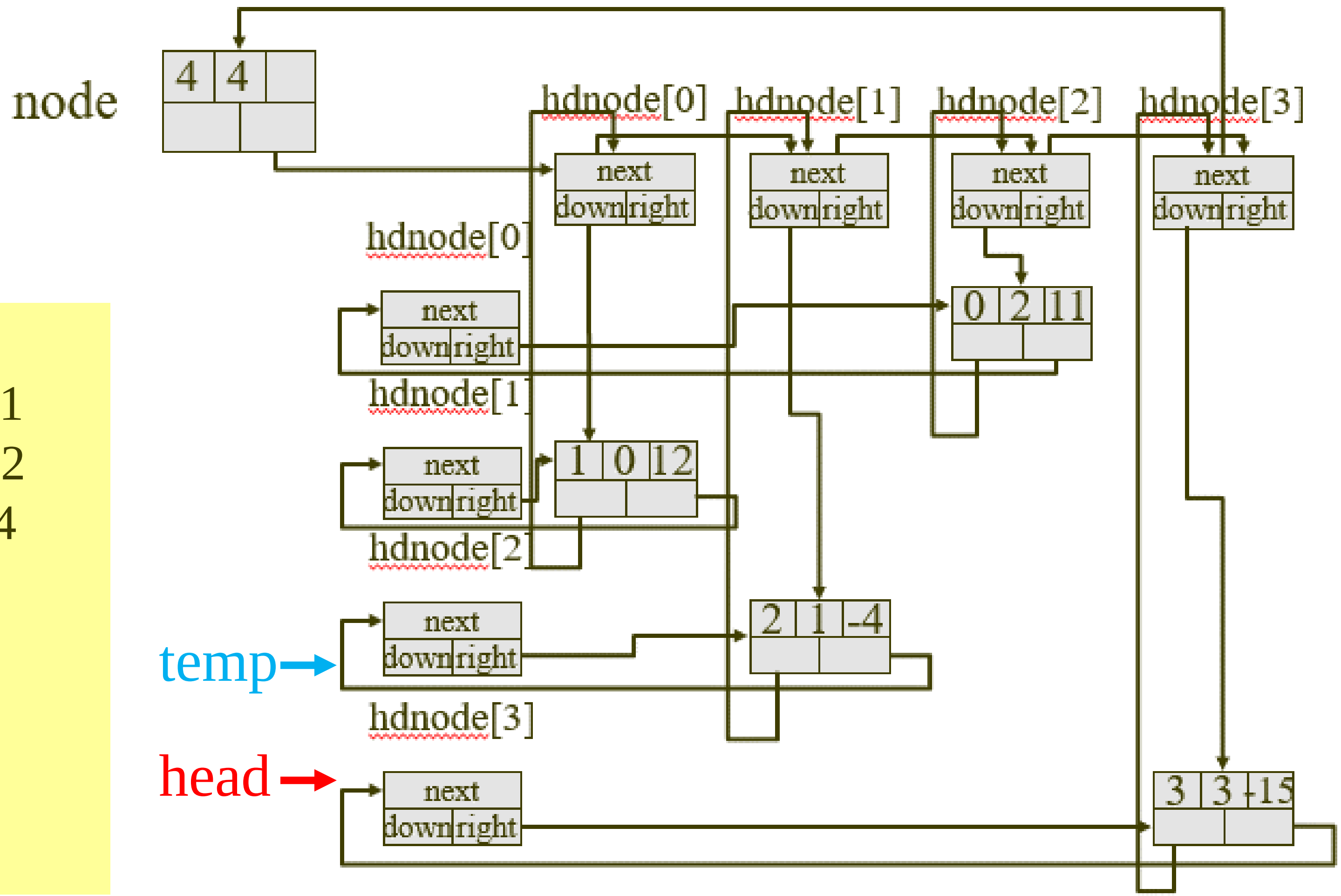
0 2 11
1 0 12
2 1 -4



```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
      /* print out the entries in each row */
      for(temp=head->right; temp!=head; temp=temp->right)
            printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
      head = head->u.next; /* next row */
}
```

i	3

```
/* Output */
0     2     11
1     0     12
2     1     -4
```

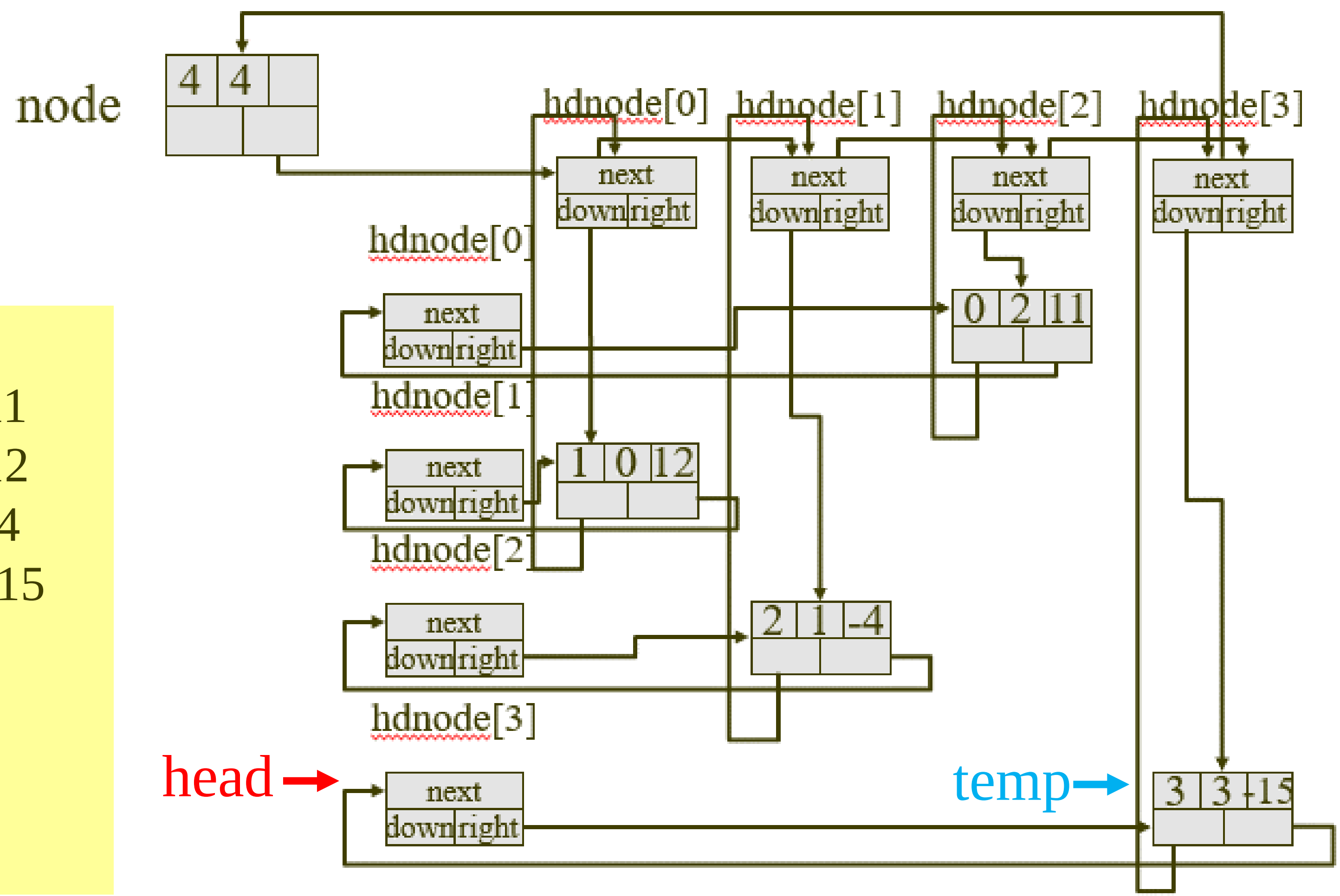


```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
    /* print out the entries in each row */
    for(temp=head->right; temp!=head; temp=temp->right)
        printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
    head = head->u.next; /* next row */
}
```

i	3

/* Output */

0	2	11
1	0	12
2	1	-4
3	3	-15

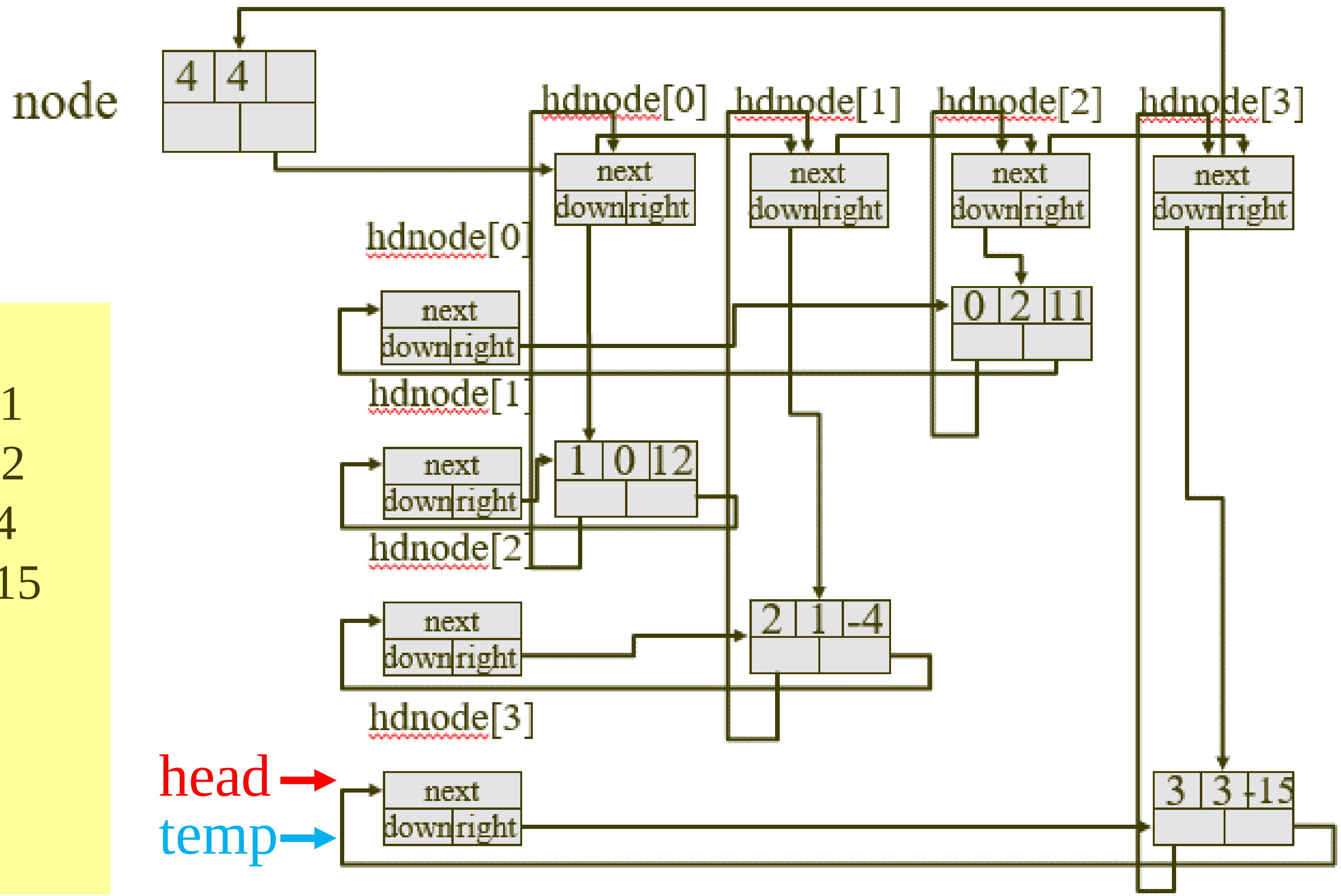


```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
    /* print out the entries in each row */
    for(temp=head->right; temp!=head; temp=temp->right) → break
        printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
    head = head->u.next; /* next row */
}
```

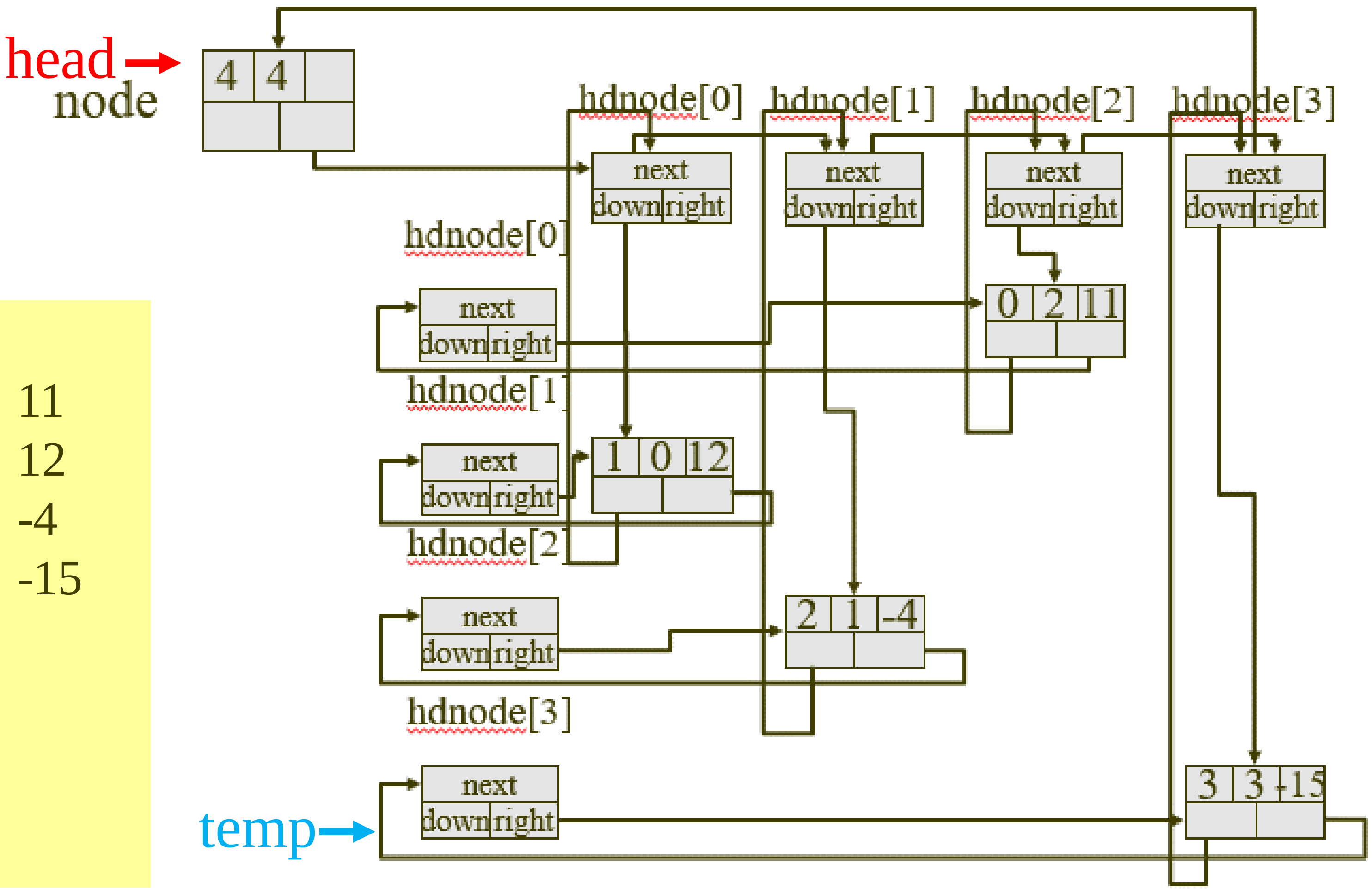
i	3

/* Output */

0	2	11
1	0	12
2	1	-4
3	3	-15



```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {
    /* print out the entries in each row */
    for(temp=head->right; temp!=head; temp=temp->right)
        printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
    head = head->u.next; /* next row */
}
```



i	3

/* Output */

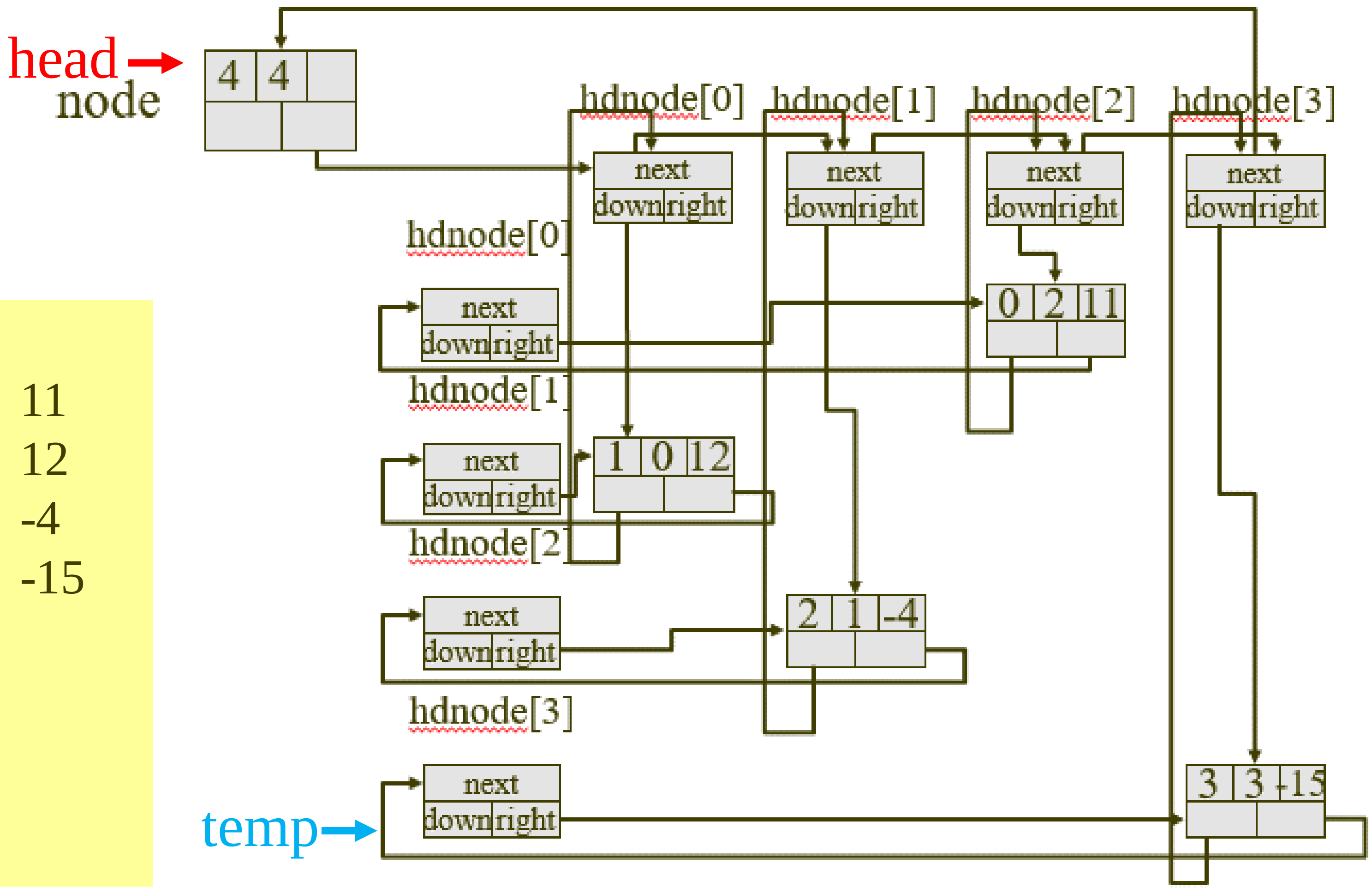
0	2	11
1	0	12
2	1	-4
3	3	-15

```
matrix_pointer temp, head=node->right;
for(i=0; i<node->u.entry.row; i++) {    → break
    /* print out the entries in each row */
    for(temp=head->right; temp!=head; temp=temp->right)
        printf("%5d%5d%5d\n", temp->u.entry.row, temp->u.entry.col, temp->u.entry.value);
    head = head->u.next; /* next row */
}
```

i	4

/* Output */

0	2	11
1	0	12
2	1	-4
3	3	-15

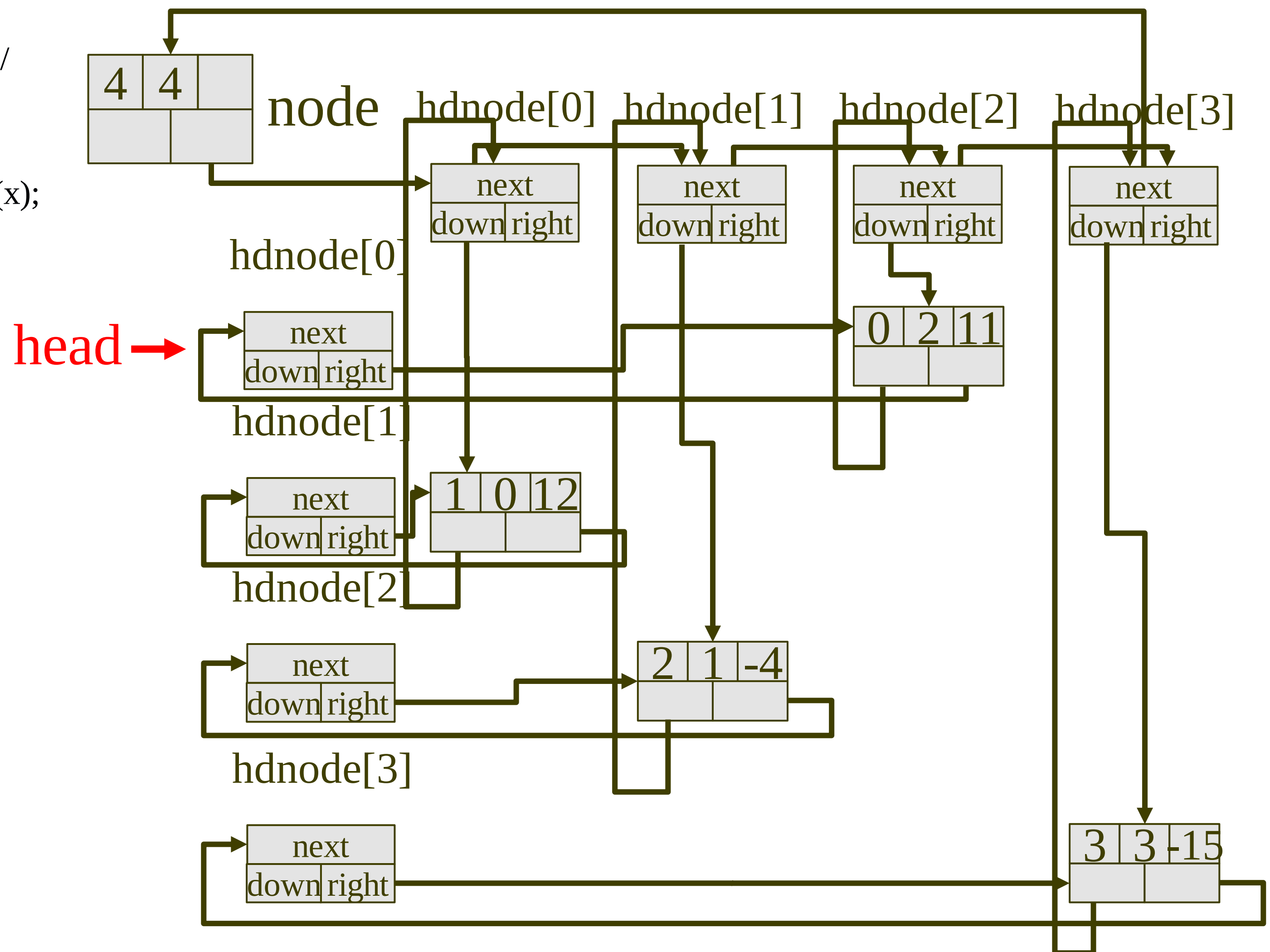


- **[Program 4.25] Erase a sparse matrix**

```
void merase(matrix_pointer *node)
{ /* erase the matrix, return the nodes to the heap */
      matrix_pointer x,y, head = (*node)->right;
      int i;
      /* free the entry and header nodes by row */
      for (i=0; i<(*node)->u.entry.row; i++) {
            y = head->right;
            while (y != head) {
                  x = y;  y = y->right;  free(x);
            }
            x = head;  head = head->u.next;  free(x);
      }
      /* free remaining head nodes */
      y = head;
      while (y != *node) {
            x = y;  y = y->u.next;  free(x);
      }
      free(*node);  *node = NULL;
}
```

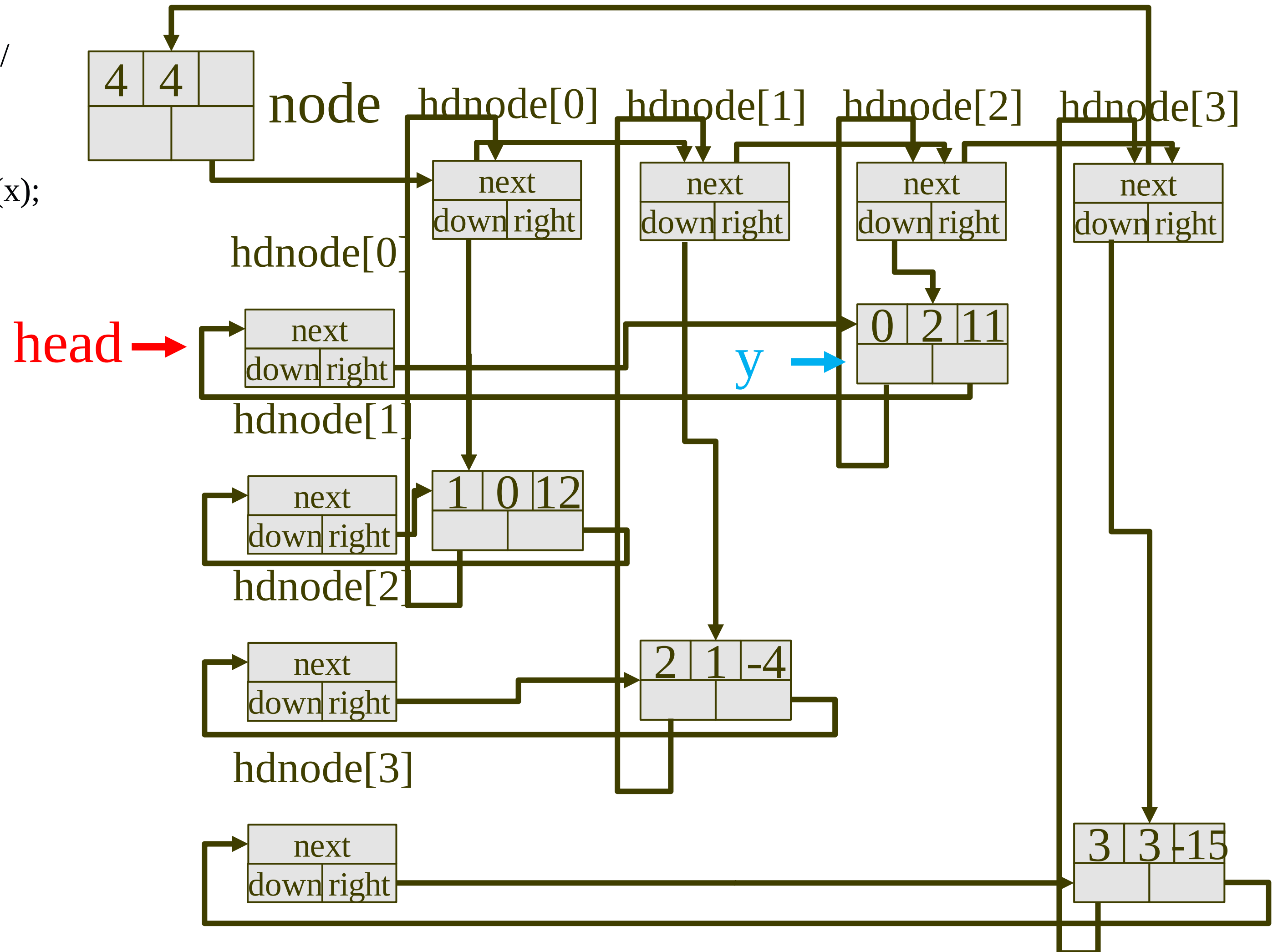
```
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {
        y = head->right;
        while (y != head) {
                x = y;  y = y->right;   free(x);
        }
        x = head;  head = head->u.next;   free(x);
 }
 /* free remaining head nodes */
 y = head;
 while (y != *node) {
        x = y;  y = y->u.next;   free(x);
 }
free(*node);   *node = NULL;
```

i	0



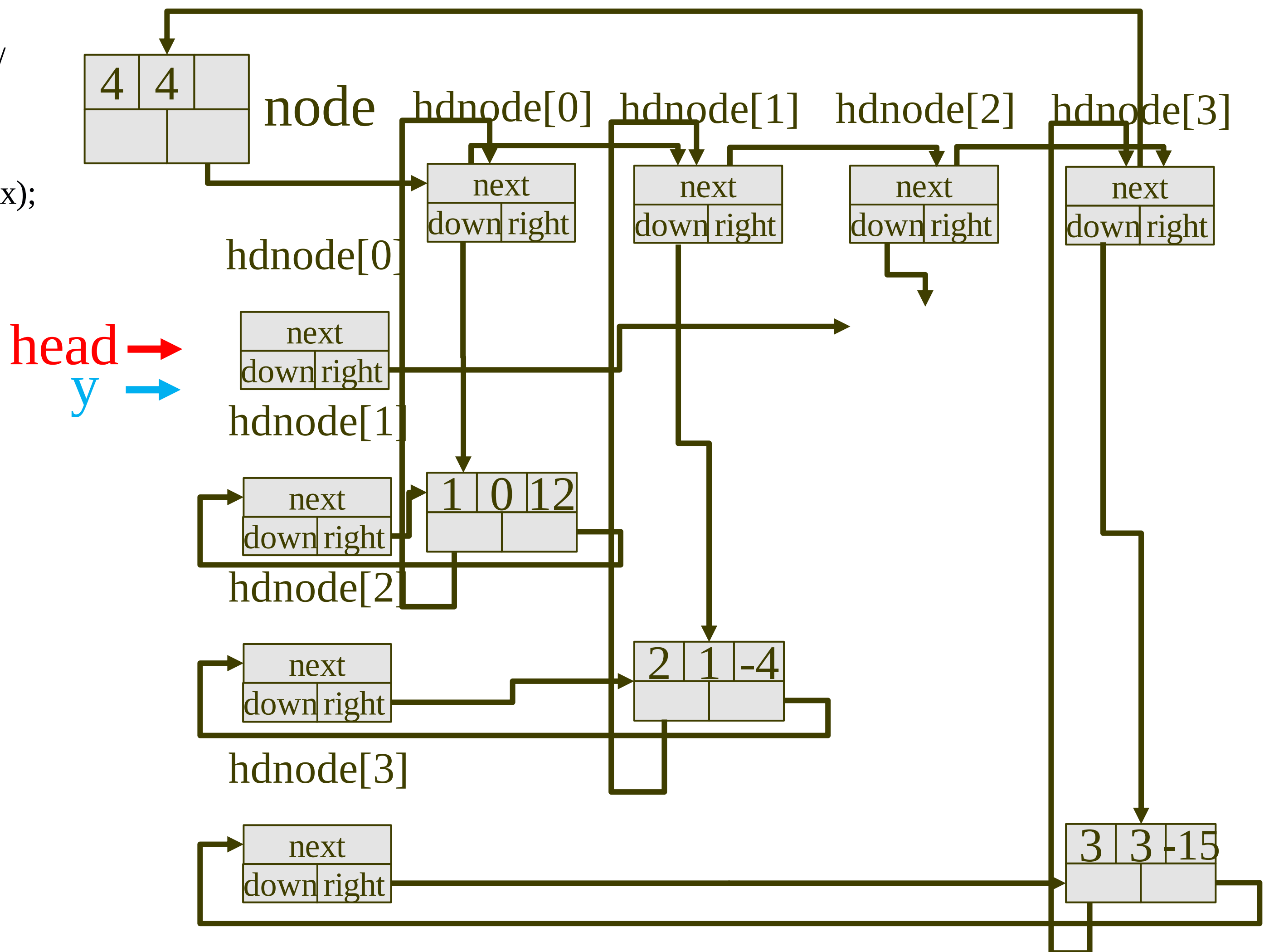
```
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {
	y = head->right;
	while (y != head) {
		x = y;  y = y->right;  free(x);
	}
	x = head;  head = head->u.next;  free(x);
 }
 /* free remaining head nodes */
 y = head;
 while (y != *node) {
	x = y;  y = y->u.next;  free(x);
 }
free(*node);  *node = NULL;
```

i	0



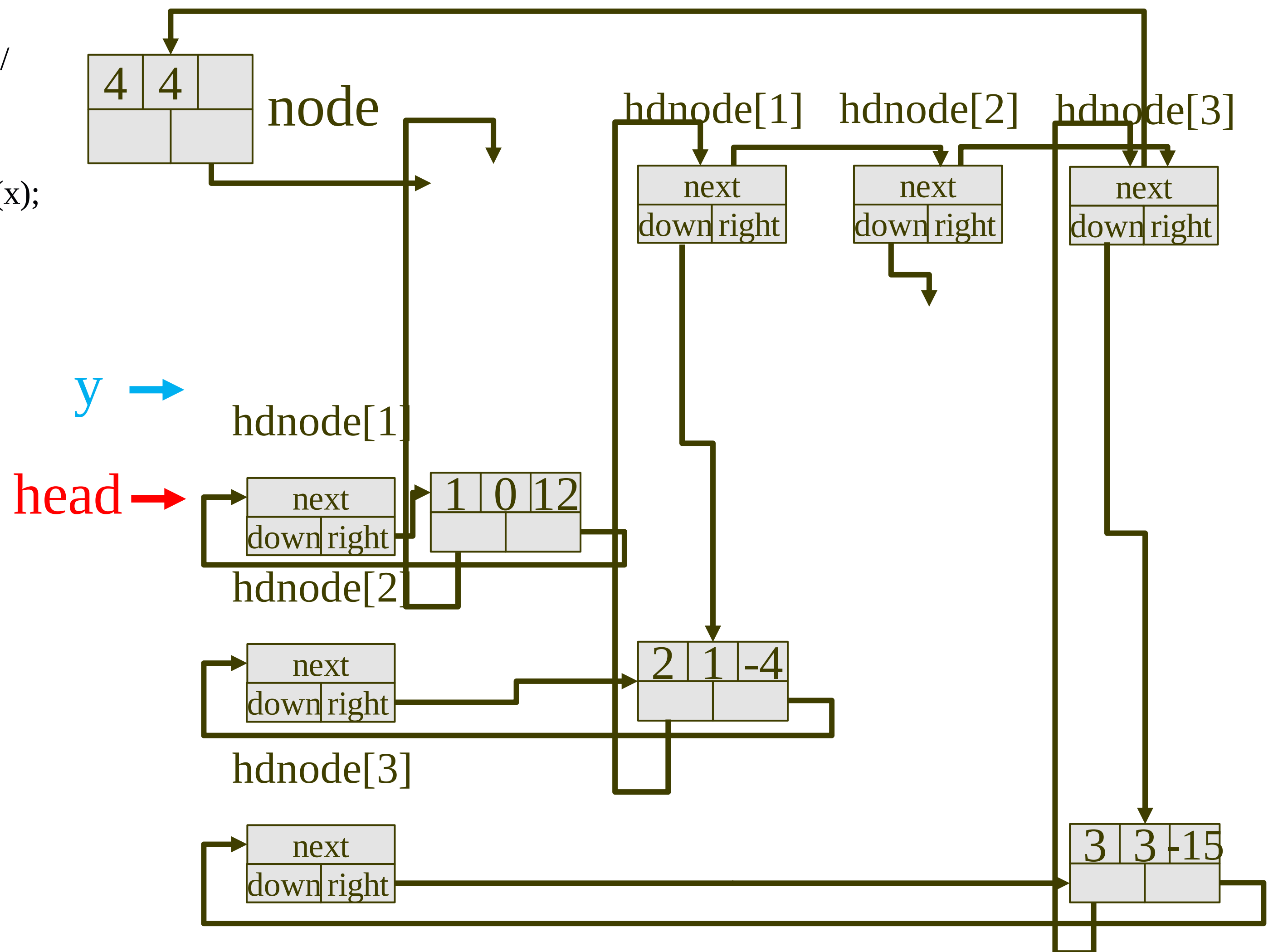
```
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {
        y = head->right;
        while (y != head) {
                x = y;  y = y->right;  free(x);
        }
        x = head;  head = head->u.next;  free(x);
  }
 /* free remaining head nodes */
 y = head;
 while (y != *node) {
        x = y;  y = y->u.next;  free(x);
 }
free(*node);  *node = NULL;
```

i	0



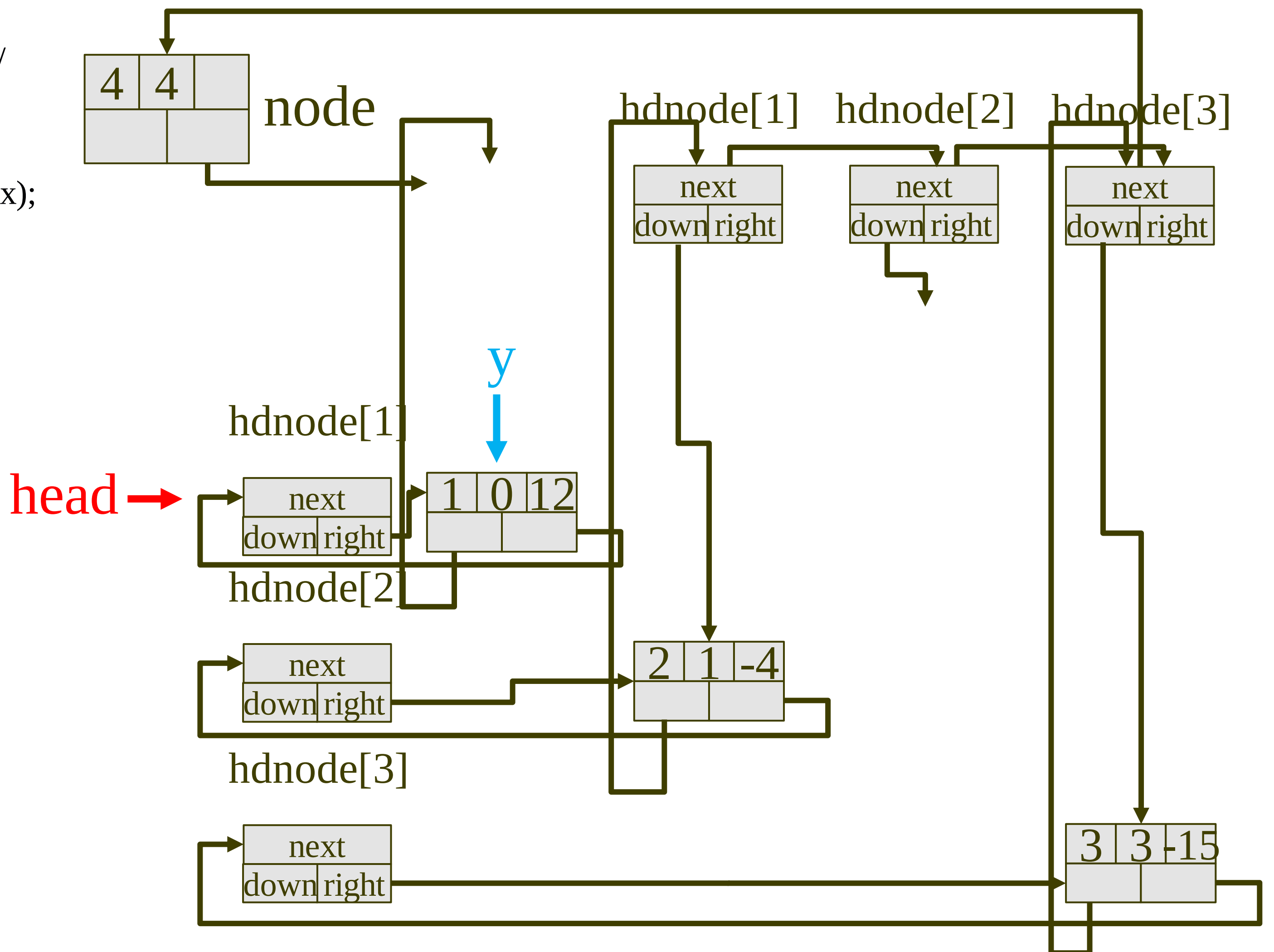
```
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {
        y = head->right;
        while (y != head) {  → break
                x = y;  y = y->right;  free(x);
        }
        x = head;  head = head->u.next;  free(x);
  }
  /* free remaining head nodes */
  y = head;
  while (y != *node) {
        x = y;  y = y->u.next;  free(x);
  }
 free(*node);  *node = NULL;
```

i	0



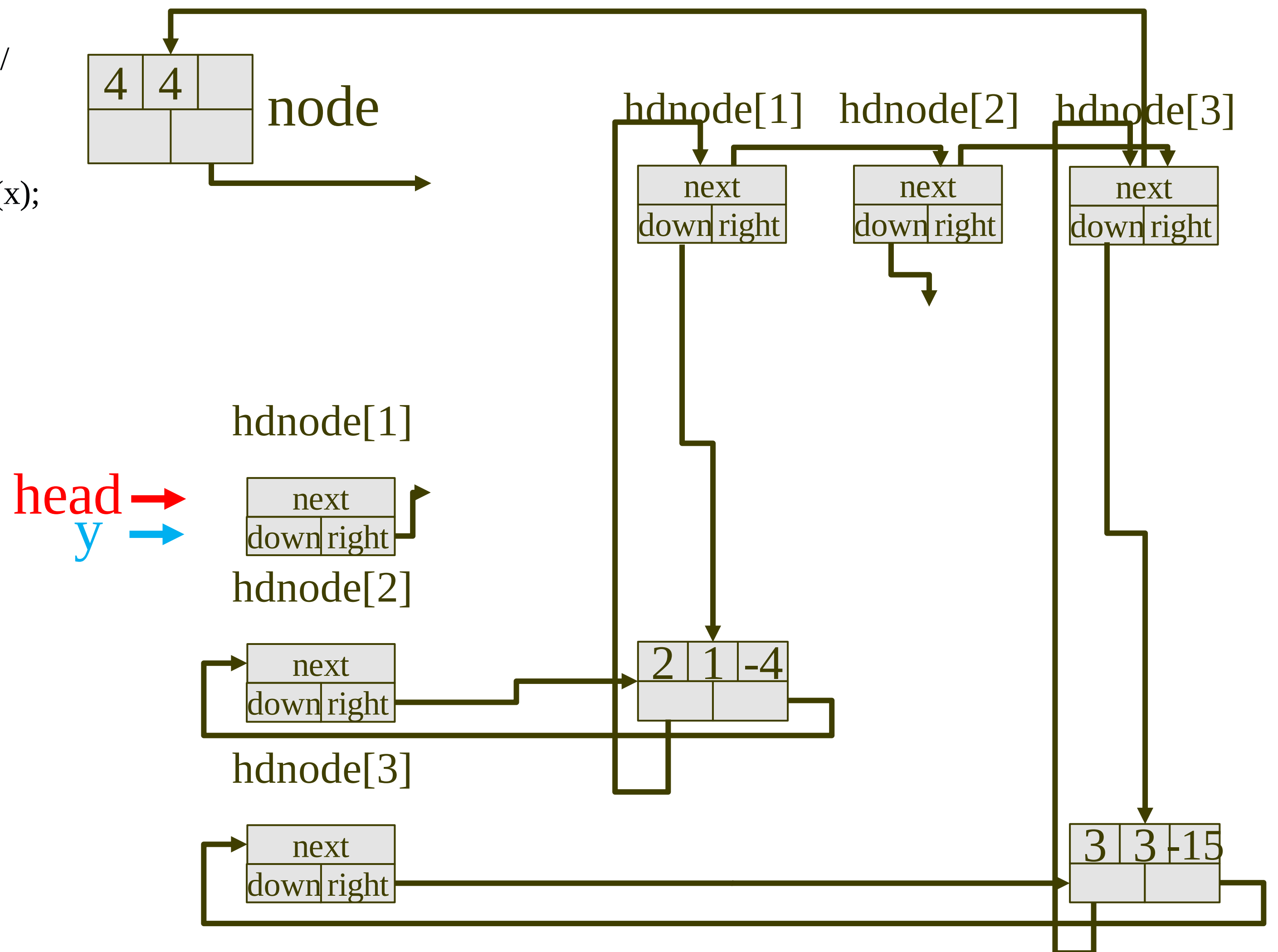
```
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {
      y = head->right;
      while (y != head) {
            x = y;  y = y->right;   free(x);
      }
      x = head;  head = head->u.next;   free(x);
  }
 /* free remaining head nodes */
 y = head;
 while (y != *node) {
      x = y;  y = y->u.next;   free(x);
 }
free(*node);  *node = NULL;
```

i	1



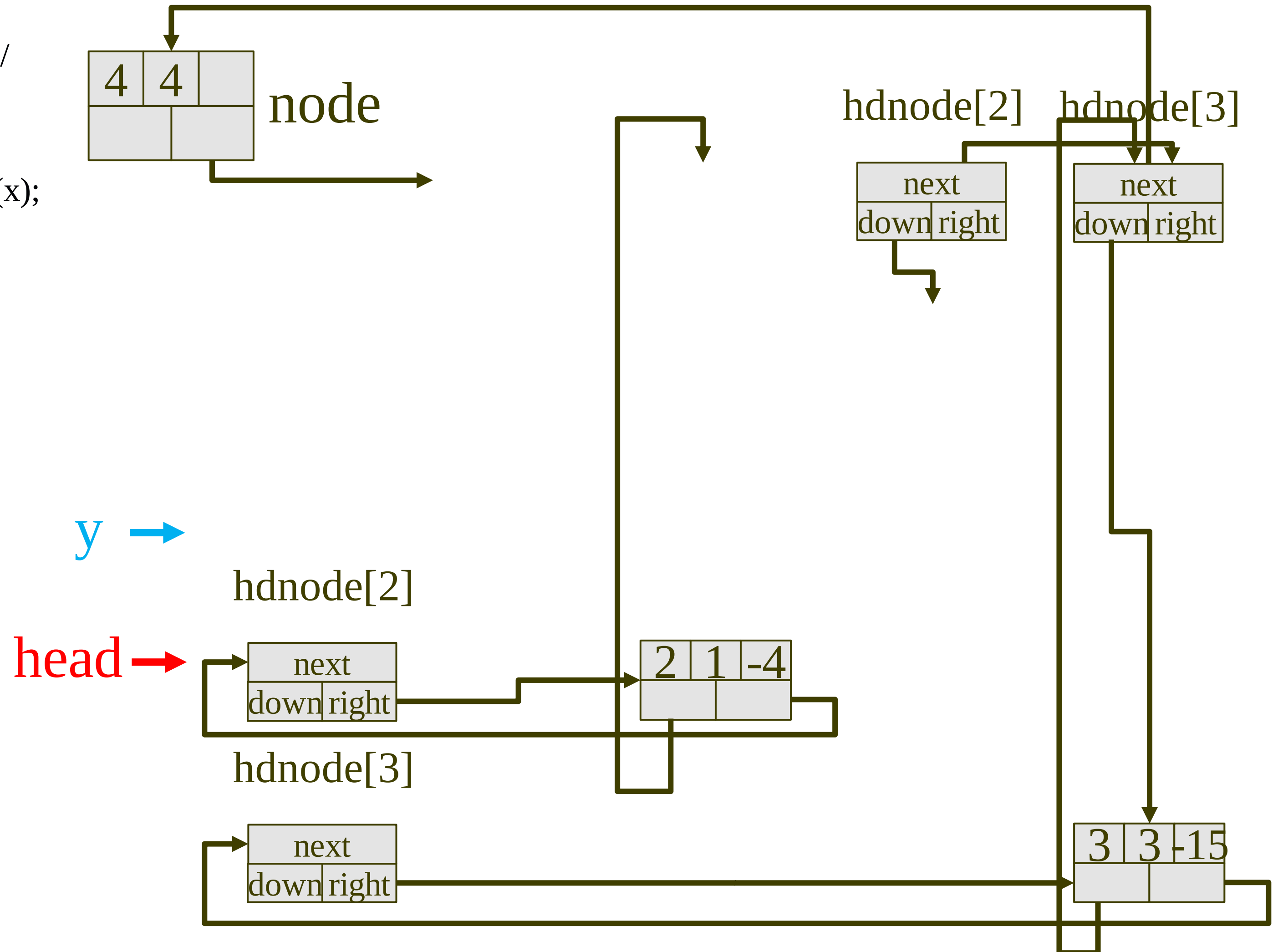
```
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {
      y = head->right;
      while (y != head) {
            x = y;  y = y->right;  free(x);
      }
      x = head;  head = head->u.next;  free(x);
  }
 /* free remaining head nodes */
 y = head;
 while (y != *node) {
      x = y;  y = y->u.next;  free(x);
 }
free(*node);  *node = NULL;
```

i	1



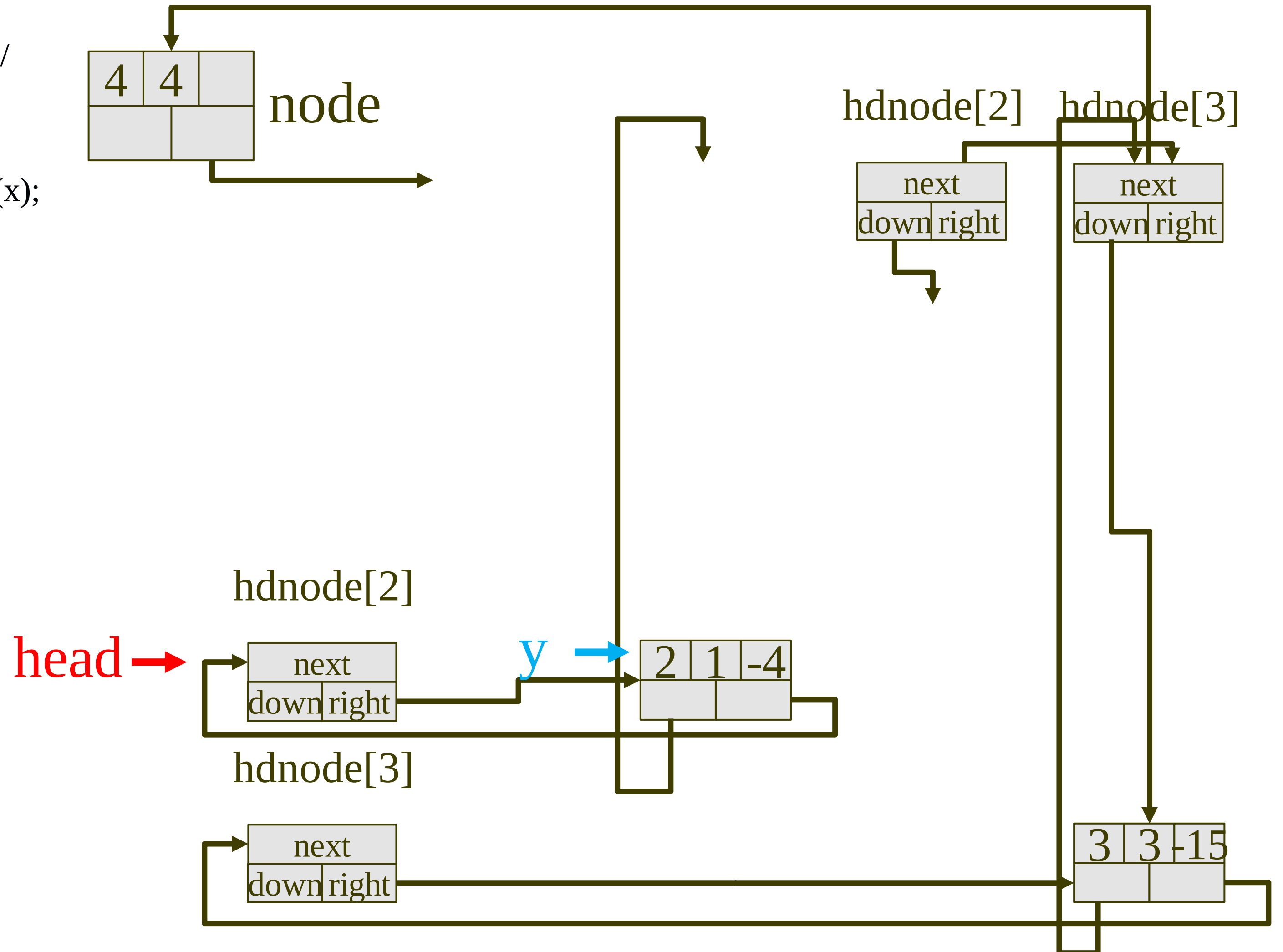
```
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {
    y = head->right;
    while (y != head) {  → break
        x = y;  y = y->right;  free(x);
    }
    x = head;  head = head->u.next;  free(x);
 }
 /* free remaining head nodes */
 y = head;
 while (y != *node) {
    x = y;  y = y->u.next;  free(x);
 }
free(*node);  *node = NULL;
```

i	1



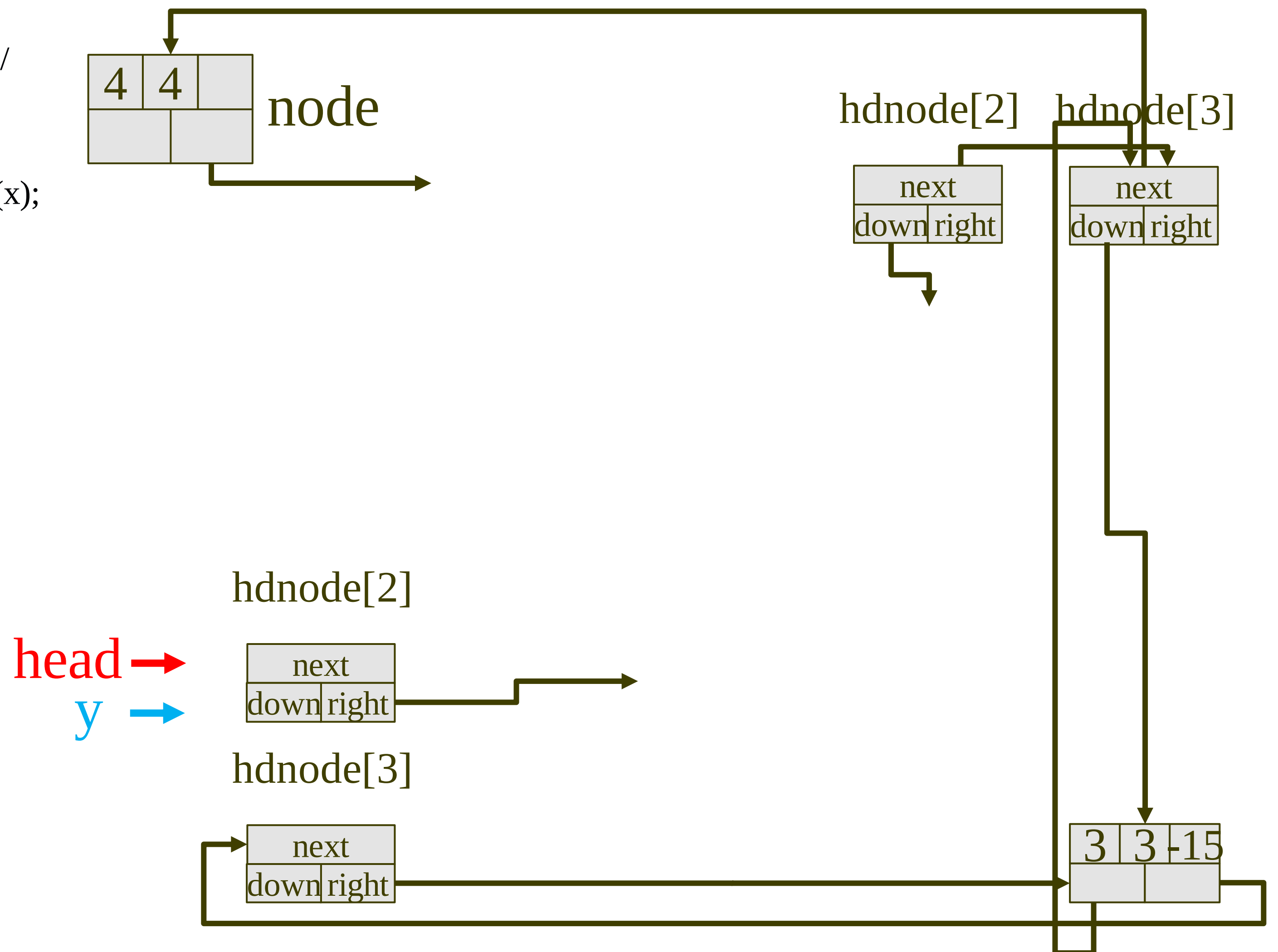
```
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {
      y = head->right;
      while (y != head) {
            x = y;  y = y->right;  free(x);
      }
      x = head;  head = head->u.next;  free(x);
 }
 /* free remaining head nodes */
 y = head;
 while (y != *node) {
      x = y;  y = y->u.next;  free(x);
 }
free(*node);  *node = NULL;
```

i	2



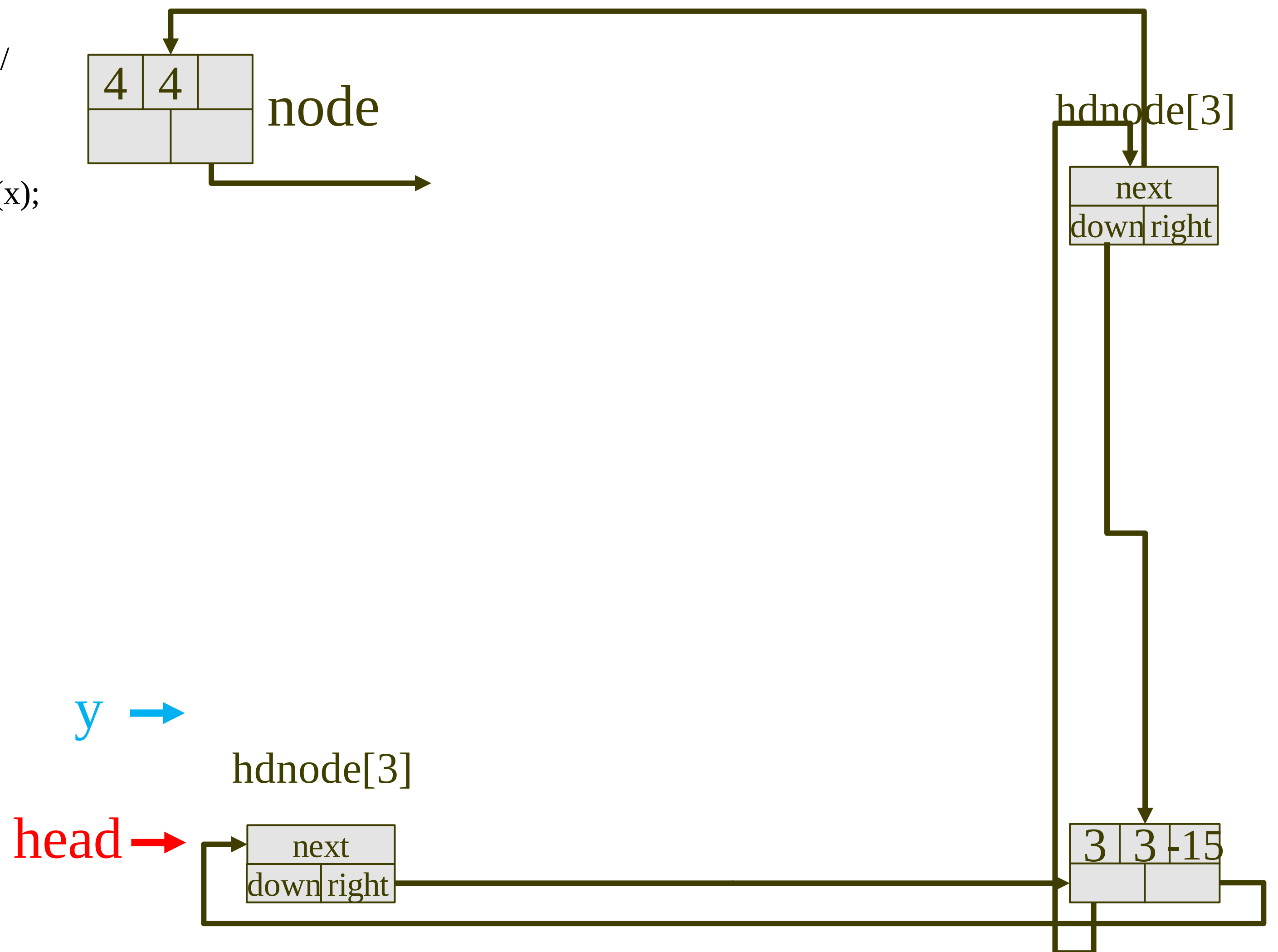
```
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {
        y = head->right;
        while (y != head) {
                x = y;  y = y->right;  free(x);
        }
        x = head;  head = head->u.next;  free(x);
  }
  /* free remaining head nodes */
  y = head;
  while (y != *node) {
        x = y;  y = y->u.next;  free(x);
  }
 free(*node);  *node = NULL;
```

i	2



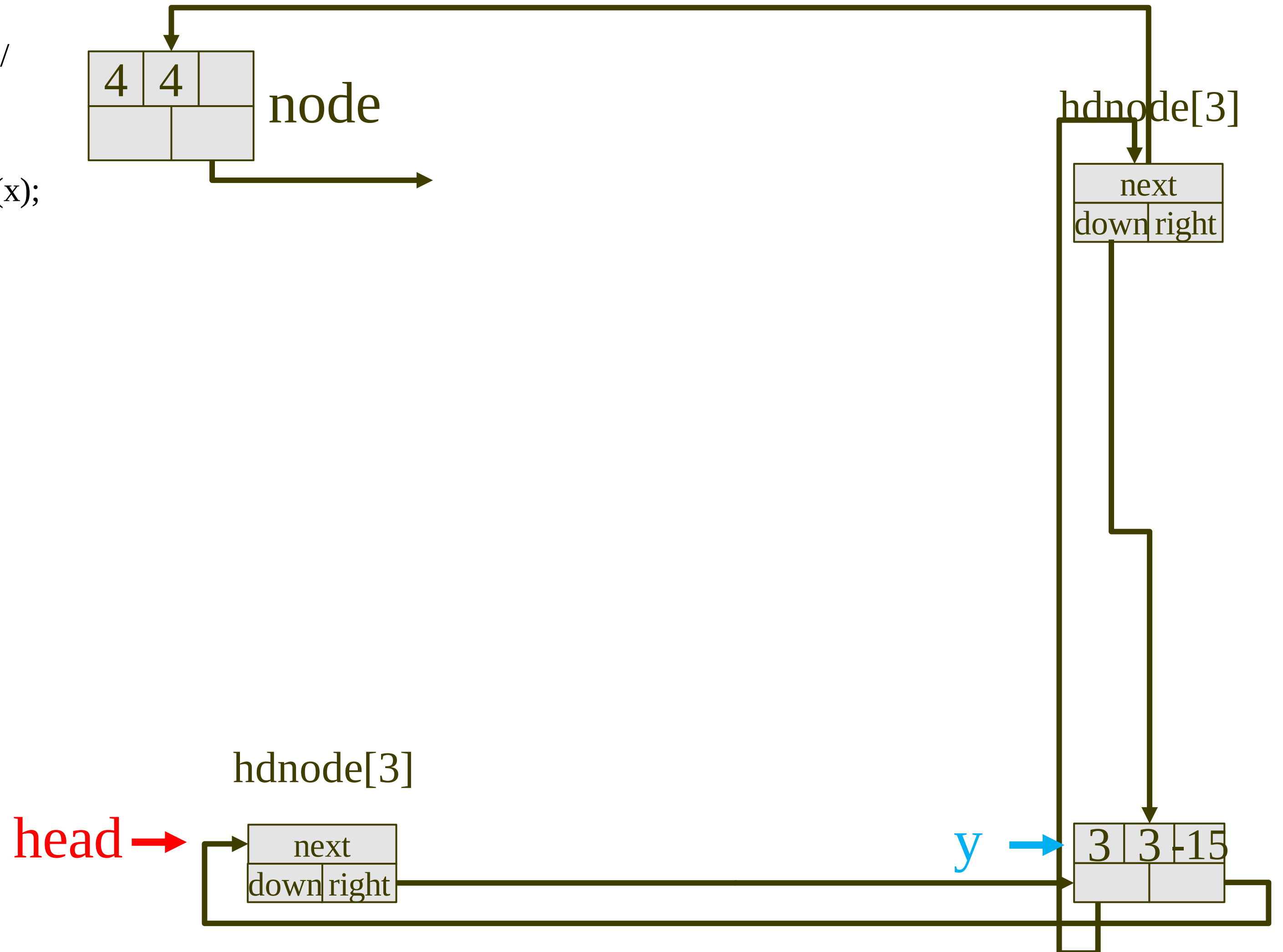
```c
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {
        y = head->right;
        while (y != head) {
                x = y;  y = y->right;  free(x);
        }
        x = head;  head = head->u.next;  free(x);
  }
 /* free remaining head nodes */
 y = head;
 while (y != *node) {
        x = y;  y = y->u.next;  free(x);
 }
 free(*node);  *node = NULL;
```

i	2



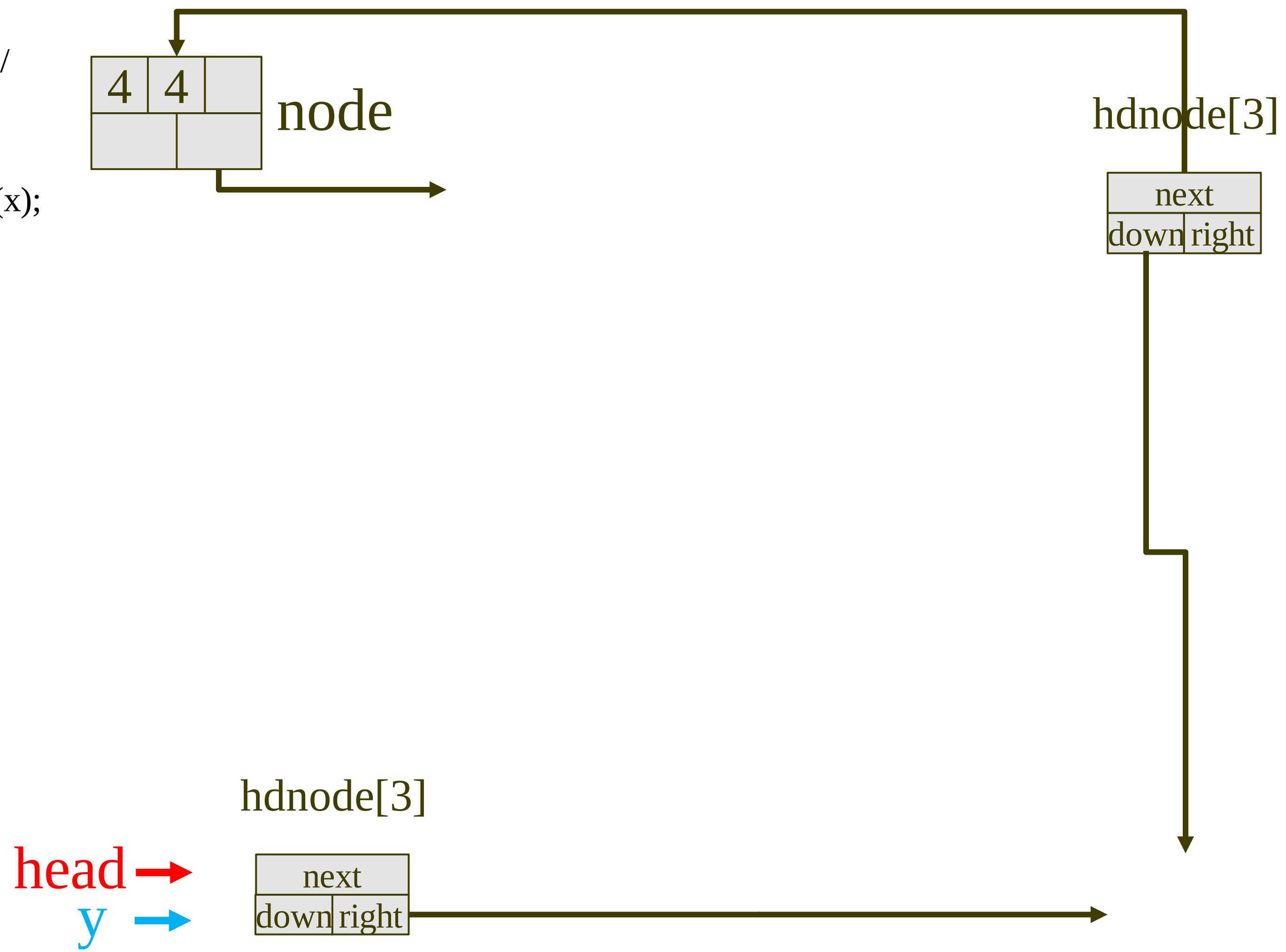
```
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {
        y = head->right;
        while (y != head) {
                x = y;  y = y->right;  free(x);
        }
        x = head;  head = head->u.next;  free(x);
 }
 /* free remaining head nodes */
 y = head;
 while (y != *node) {
        x = y;  y = y->u.next;  free(x);
 }
free(*node);  *node = NULL;
```

i	3

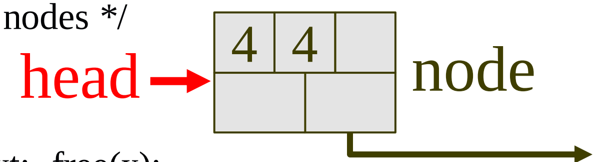


```
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {
        y = head->right;
        while (y != head) {
                x = y;  y = y->right;  free(x);
        }
        x = head;  head = head->u.next;  free(x);
  }
 /* free remaining head nodes */
 y = head;
 while (y != *node) {
        x = y;  y = y->u.next;  free(x);
 }
 free(*node);  *node = NULL;
```

i	3



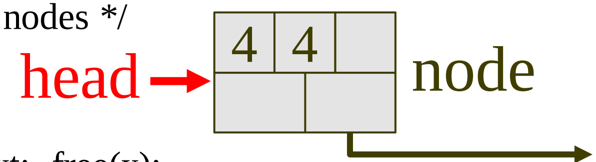
```
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {
      y = head->right;
      while (y != head) {   → break
            x = y;  y = y->right;  free(x);
      }
      x = head;  head = head->u.next;  free(x);
  }
 /* free remaining head nodes */
 y = head;
 while (y != *node) {
      x = y;  y = y->u.next;  free(x);
 }
 free(*node);  *node = NULL;
```



i	3

y →

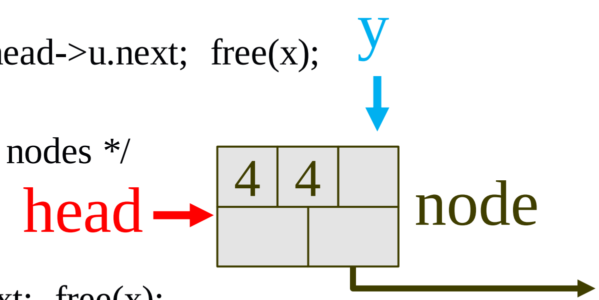
```
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {   → break
        y = head->right;
        while (y != head) {
                x = y;  y = y->right;  free(x);
        }
        x = head;  head = head->u.next;  free(x);
  }
 /* free remaining head nodes */
 y = head;
 while (y != *node) {
        x = y;  y = y->u.next;  free(x);
 }
 free(*node);  *node = NULL;
```



i	4

y →

```
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {
        y = head->right;
        while (y != head) {
                x = y;  y = y->right;  free(x);
        }
        x = head;  head = head->u.next;  free(x);
  }
 /* free remaining head nodes */
 y = head;
 while (y != *node) {
        x = y;  y = y->u.next;  free(x);
 }
free(*node);  *node = NULL;
```



i	4

```
head = (*node)->right;
for (i=0; i<(*node)->u.entry.row; i++) {
        y = head->right;
        while (y != head) {
                x = y;  y = y->right;  free(x);
        }
        x = head;  head = head->u.next;  free(x);
  }
  /* free remaining head nodes */
  y = head;
  while (y != *node) {
        x = y;  y = y->u.next;  free(x);
  }
 free(*node);  *node = NULL;
```

node = NULL

i	4

- **Analysis of *mread* : [Program 4.23]**

  $O(\max\{num\_rows, num\_cols\} + num\_terms)$

  $= O(num\_rows + num\_cols + num\_terms)$.

- **Analysis of *mwrite* : [Program 4.24]**

  $O(num\_rows + num\_terms)$.

- **Analysis of *merase* : [Program 4.25]**

  $O(num\_rows + num\_cols + num\_terms)$.

4.8 DOUBLY LINKED LISTS

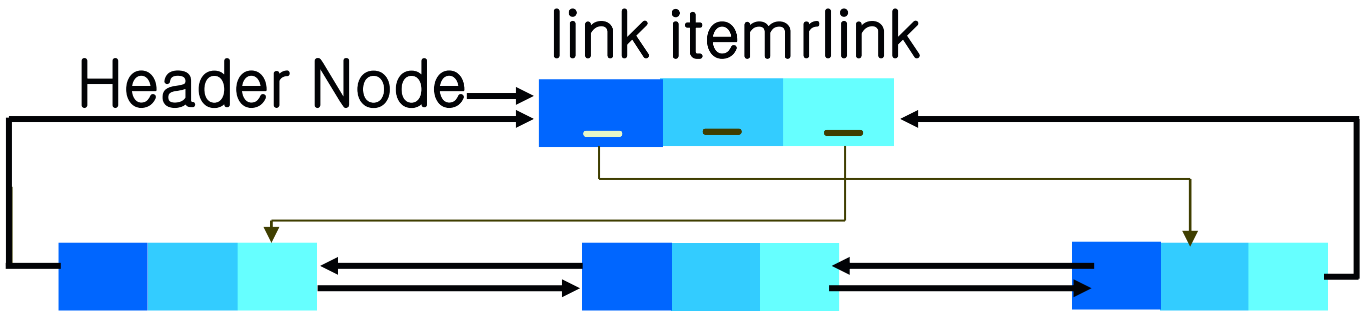
- Singly linked lists pose problems because we can move only in the direction of the links.

- Whenever we have a problem that requires us to move in either direction, it is useful to have doubly linked lists.

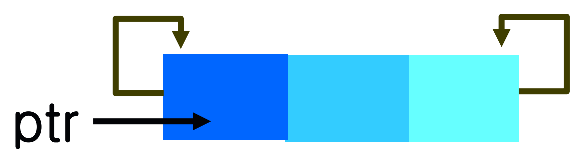
- The necessary declarations are :

```
typedef struct node *node_pointer;
typedef struct node {
    node_pointer llink;
    element item;
    node_pointer rlink;
};
```

- A doubly linked list may or may not be circular.
- **[Figure 4.21] Doubly linked circular list with header node**



- **[Figure 4.22] Empty doubly linked circular list with header node**



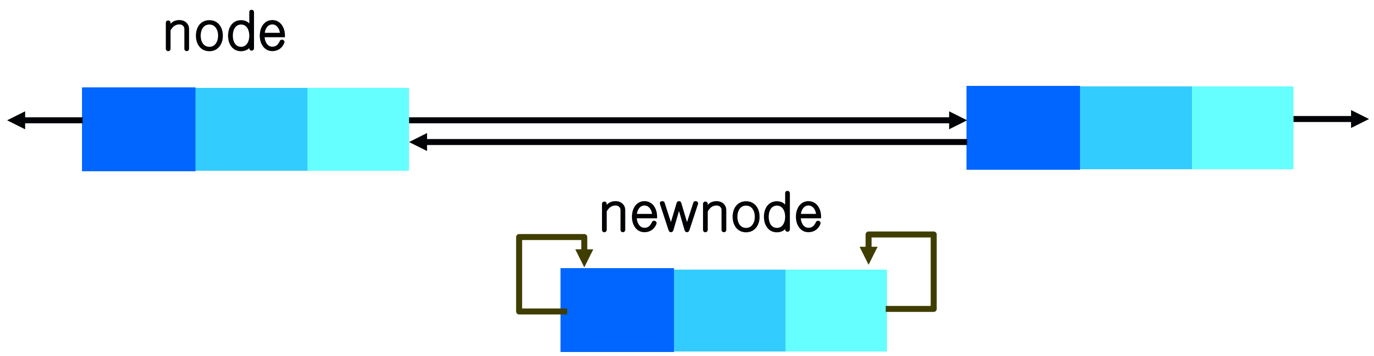
- Now suppose that *ptr* points to any node in a doubly linked list.

  Then :

  ptr == ptr->llink->rlink == ptr->rlink->llink

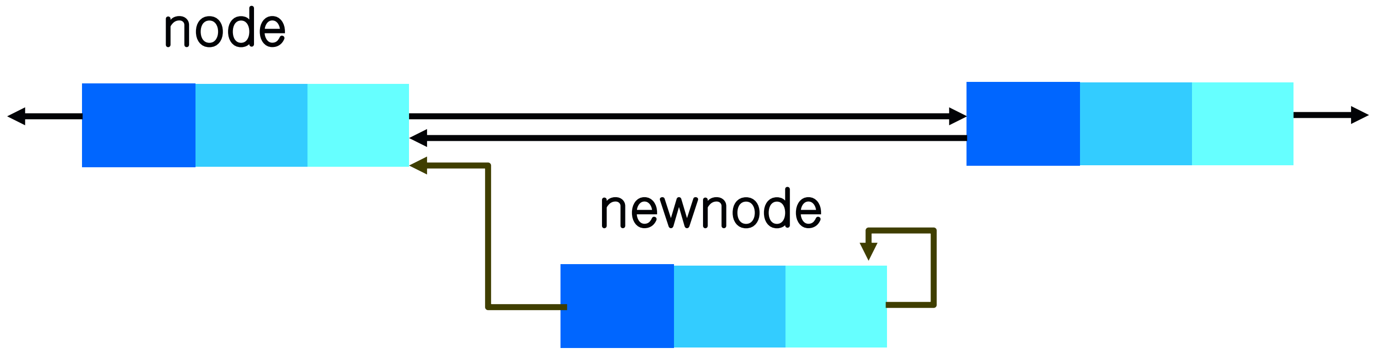
[Program 4.28] Insertion into a doubly linked circular list

```
void dinsert(node_pointer node, node_pointer newnode)
{
    /* insert newnode to the right of node */
    newnode->llink = node;
    newnode->rlink = node->rlink;
    node->rlink->llink = newnode;
    node->rlink = newnode;
}
```



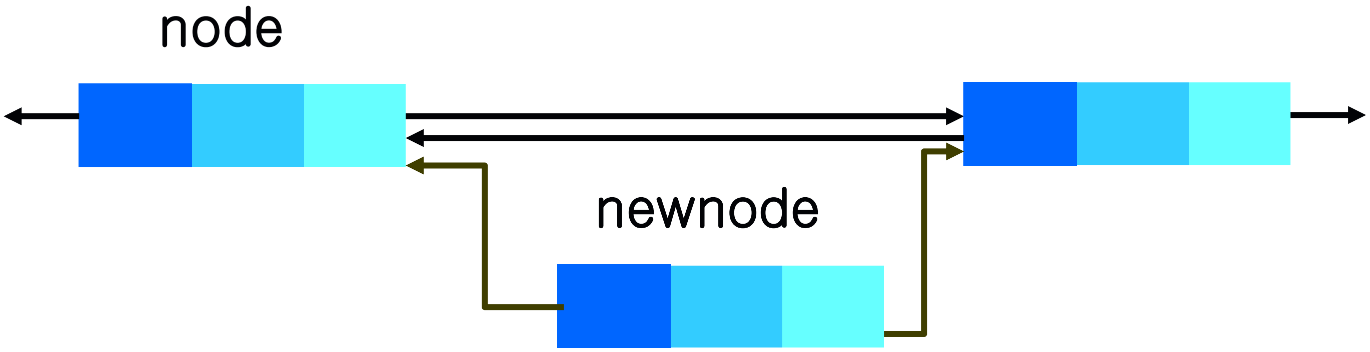
[Program 4.28] Insertion into a doubly linked circular list

```
void dinsert(node_pointer node, node_pointer newnode)
{
    /* insert newnode to the right of node */
    newnode->llink = node;
    newnode->rlink = node->rlink;
    node->rlink->llink = newnode;
    node->rlink = newnode;
}
```



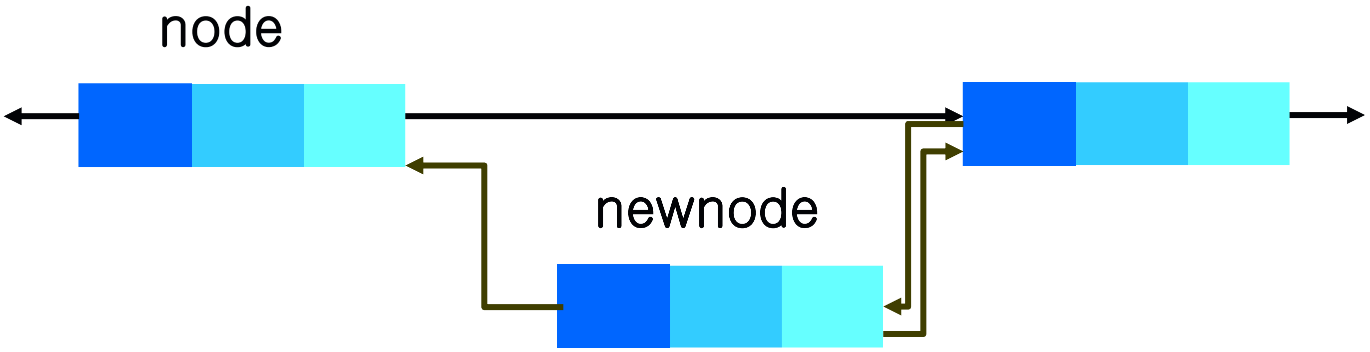
[Program 4.28] Insertion into a doubly linked circular list

```
void dinsert(node_pointer node, node_pointer newnode)
{
    /* insert newnode to the right of node */
    newnode->llink = node;
    newnode->rlink = node->rlink;
    node->rlink->llink = newnode;
    node->rlink = newnode;
}
```



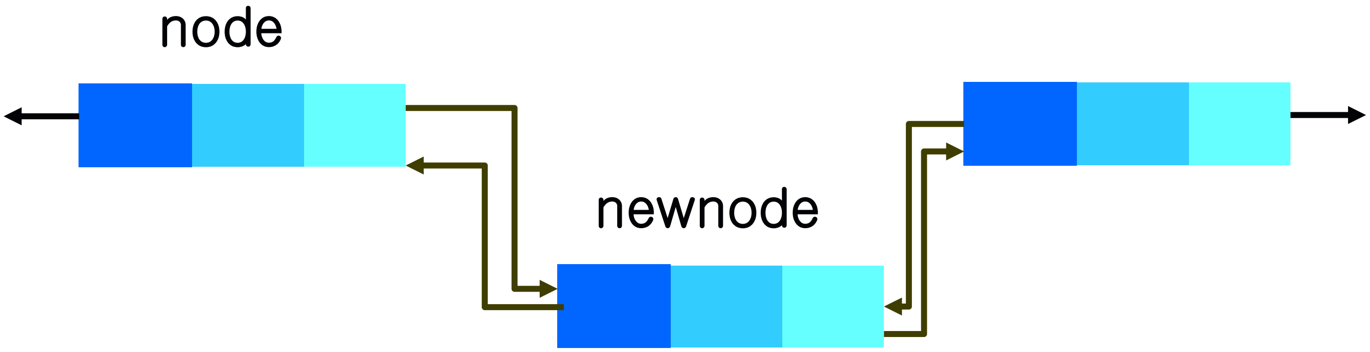
[Program 4.28] Insertion into a doubly linked circular list

```
void dinsert(node_pointer node, node_pointer newnode)
{
    /* insert newnode to the right of node */
    newnode->llink = node;
    newnode->rlink = node->rlink;
    node->rlink->llink = newnode;
    node->rlink = newnode;
}
```



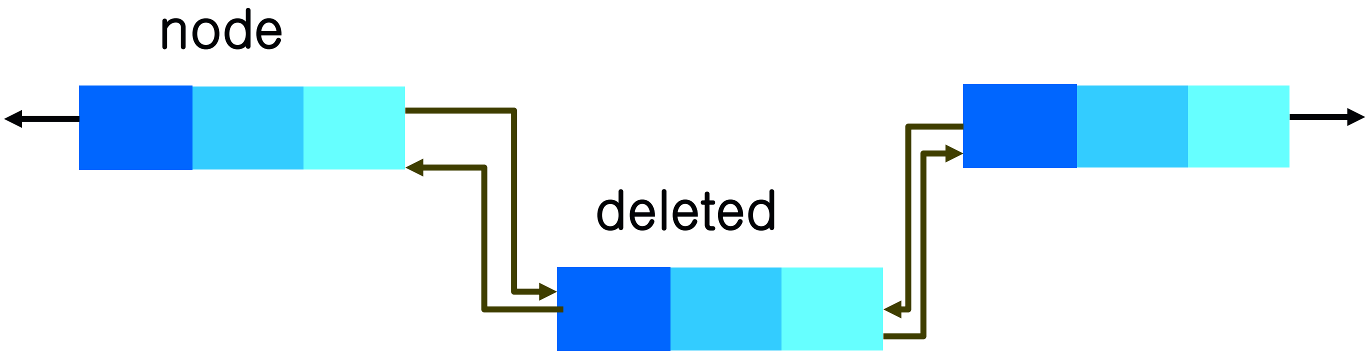
[Program 4.28] Insertion into a doubly linked circular list

```
void dinsert(node_pointer node, node_pointer newnode)
{
    /* insert newnode to the right of node */
    newnode->llink = node;
    newnode->rlink = node->rlink;
    node->rlink->llink = newnode;
    node->rlink = newnode;
}
```



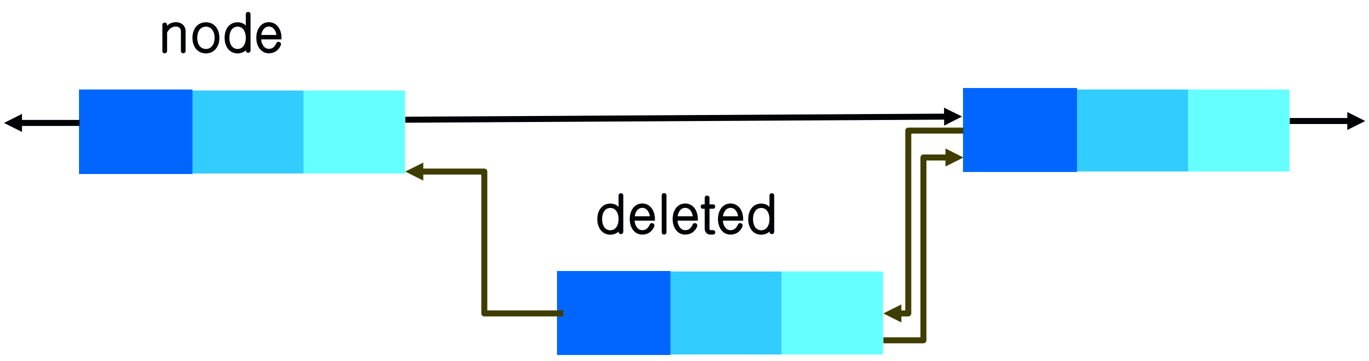
[Program 4.27] Deletion from a doubly linked circular list

```
void ddelete(node_pointer node,  node_pointer deleted)  {
    /* delete from the doubly linked list  */
    if (node == deleted)
        printf("Deletion of head node not permitted.\n");
    else  {
        deleted->llink->rlink = deleted->rlink;
        deleted->rlink->llink = deleted->llink;
        free(deleted);
    }
}
```



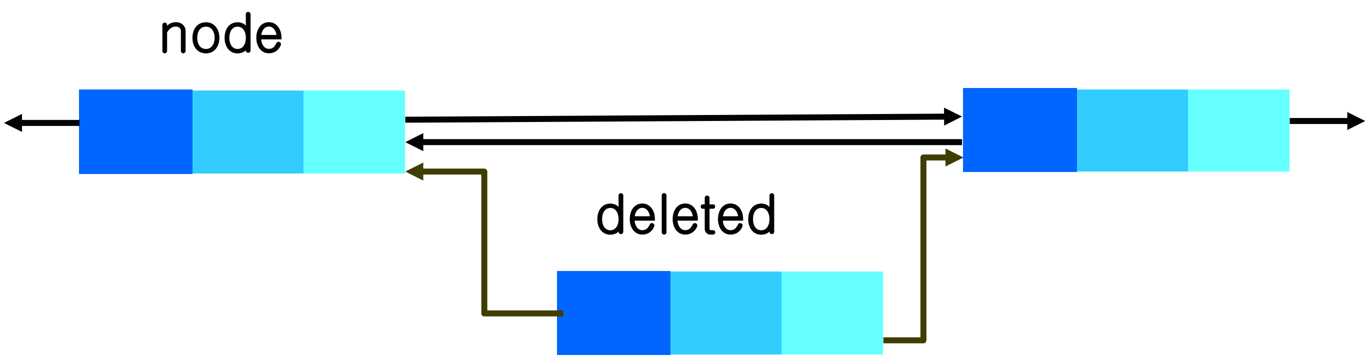
[Program 4.27] Deletion from a doubly linked circular list

```
void ddelete(node_pointer node,  node_pointer deleted)  {
    /* delete from the doubly linked list  */
    if (node == deleted)
        printf("Deletion of head node not permitted.\n");
    else  {
        deleted->llink->rlink = deleted->rlink;
        deleted->rlink->llink = deleted->llink;
        free(deleted);
    }
}
```



- **[Program 4.27] Deletion from a doubly linked circular list**

```
void ddelete(node_pointer node,  node_pointer deleted)  {
    /* delete from the doubly linked list  */
    if (node == deleted)
        printf("Deletion of head node not permitted.\n");
    else  {
        deleted->llink->rlink = deleted->rlink;
        deleted->rlink->llink = deleted->llink;
        free(deleted);
    }
}
```



- **[Program 4.27] Deletion from a doubly linked circular list**

```
void ddelete(node_pointer node,  node_pointer deleted)  {
    /* delete from the doubly linked list  */
    if (node == deleted)
        printf("Deletion of head node not permitted.\n");
    else  {
        deleted->llink->rlink = deleted->rlink;
        deleted->rlink->llink = deleted->llink;
        free(deleted);
    }
}
```