## Homework 03: Due 9/27 (completion based)

Stat061-F23

Prof Amanda Luby

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In HW1, we found the MOM and MLE estimates for each of the probability distributions below. For Q1-Q4, find (a) the posterior distribution for an iid sample of $X_1, X_2, \ldots, X_n$ and (b) the posterior mean (our Bayesian estimate).

1. The parameter $\lambda$ for a Poisson distribution where $P(X = k) = \frac{\lambda^k}{k!}e^{-\lambda}$ for $k = 0, 1, 2, \ldots$ and we assume the prior distribution for $\lambda$ is Gamma$(\alpha, \beta)$. (This should be a named distribution; be sure to specify the parameters)
2. The parameter $p$ in the Geometric distribution where $P(X = k) = p(1-p)^{k-1}$ for $k = 1, 2, 3, \ldots$ and we assume the prior distribution for $p$ is Beta$(a, b)$. (This should be a named distribution; be sure to specify the parameters)
3. The parameter $\alpha$ in the distribution with pdf $f(x|\alpha) = \frac{\Gamma(2\alpha)}{\Gamma(\alpha)^2}[x(1-x)]^{\alpha-1}$ where $x \in [0, 1]$ and we assume the prior distribution for $\alpha$ is Unif(0,1). (You will probably not be able to solve the integral to determine the posterior mean, but write out the integral you would have to solve.)
4. The parameter $\beta$ in the Pareto distribution with pdf $f(x|\beta) = \frac{\beta}{x^{\beta+1}}$ where $x > 1$ and we assume the prior distribution for $\beta$ is Gamma$(\alpha, \lambda)$. (This should be a named distribution; be sure to specify the parameters)
5. For a Binomial$(n, \pi)$ observation $y$, consider the Bayes estimator of $\pi$ using a Beta$(\alpha, \beta)$ prior distribution.
   (a) For large $n$, show that the posterior distribution of $\pi$ has approximate mean $\hat{\pi} = \frac{y}{n}$ (it also has approximate variance $\frac{\hat{\pi}(1-\hat{\pi})}{n}$. Relate this result to classical estimation.
   (b) Show that the MLE estimator is a limit of Bayes estimators, for a certain sequence of $\alpha = \beta$ values.
6. TBA after Friday's class
7. Wrap up lab activity
8. Review for quiz on Wednesday!