Homework 04: Due 10/4

Stat061-F23

Prof Amanda Luby

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1. Let $X_1, ..., X_n \sim Pois(\lambda)$ and let $\hat{\lambda} = \bar{X}$ be an estimator for λ (recall this is the MLE).
   (a) Find the Fisher Information for X_i
   (b) Find the Cramer-Rao Lower Bound
   (c) Find the variance of $\hat{\lambda}$ and show that it is an efficient estimator.

2. Prove the equivalence of
$$E[(\frac{\partial \ln f_y(y;\theta)}{\partial \theta})^2] = E(\frac{\partial^2 \ln f_y(y;\theta)}{\partial \theta})$$
used in Cramer-Rao. *Hint:* The "trick" in this proof is to differentiate $\int f_y(y)dy = 1$ with respect to θ, and deduce that $\int \frac{\partial \ln f_y}{\partial \theta} f_y dy = 1$.

3. When Y has a positively skewed distribution over the positive real line, statisticians often treat $\ln Y$ as having a $N(\mu, \sigma^2)$ distribution. Then Y has the *log-normal distribution* which has pdf for $y > 0$:
$$f(y;\mu,\sigma) = \frac{1}{y\sigma\sqrt{2\pi}} e^{\frac{-[\ln y - \mu]^2}{2\sigma^2}}$$
   (a) For n independent observations, find the MLE for μ and σ^2.
   (b) Find the approximate variance of $\hat{\mu}_{MLE}$
   (c) Using the invariance property of the MLE, find the MLE for the mean and variance of this distribution, which are $E(Y) = e^{\mu+\sigma^2/2}$ and $V(Y) = (e^{\sigma^2} - 1)E(Y)^2$.

4. TBA after Friday's class

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