Homework 09: Due 11/15

Stat061-F23

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1. Suppose we observe $X_1, ..., X_n \sim \text{Bernoulli}(p)$ and we wish to test the hypothesis $H_0 : p = p_0$ against $H_1 : p \neq p_0$.
   (a) What is the sampling distribution of $\hat{p}$?
   (b) What is the sampling distribution of $\hat{p}$ under H_0?
   (c) Show that the acceptance region (the set of test statistics where we fail to reject H_0) corresponds to a $(1 - \alpha)$ confidence interval for $\hat{p}$.
2. Let $Y_1, ..., Y_n \sim N(\mu, 1)$. (*Note:* since σ is known, you should not need the t-distribution)
   (a) Derive the form of the GLRT for testing $H_0 : \mu = \mu_0$ versus $H_1 : \mu \neq \mu_0$.
   (b) Now, find the GLRT if $H_1 : \mu = \mu_1$. How does the rejection region depend on μ_1?
3. Let $X_1, ..., X_n \sim \text{Exp}(\theta)$. Suppose that we wish to test $H_0 : \theta \geq \theta_0$ against $H_1 : \theta < \theta_0$. Let $X = \sum X_i$ and let δ_c be the test that rejects H_0 if $X \geq c$.
   (a) Find $\pi(\theta|\delta_c)$ and argue that it is a decreasing function of θ.
   (b) Find c such that δ_c has size α_0
   (c) Let $\theta_0 = 2, n = 1$, and $\alpha_0 = 0.1$. Find the precise form of δ_c and sketch its power function.
4. TBA (will likely include R)
5. TBA (will include R)