

## Homework 12

Stat061-F23

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1. For the simple linear regression case ( $y = \beta_0 + \beta_1 x + \epsilon$ ), show that  $\hat{\beta}_1 = r \frac{\hat{\sigma}_y}{\hat{\sigma}_x}$ .
2. Assuming the standard multiple linear model ( $Y = \beta X + \epsilon$ , where  $X$  is an  $n \times p$  design matrix):
  - (a) Show that  $\sigma^2 I = \Sigma_{\hat{y}} + \Sigma_{\epsilon}$
  - (b) Using (a), conclude that  $n\sigma^2 = \sum \text{Var}(\hat{Y}_i) + \sum \text{Var}(\epsilon_i)$
3. Consider a multiple linear regression problem with design matrix  $\mathbf{X}$  and observations  $\mathbf{Y}$ . Let  $\mathbf{X}_1$  be the matrix remaining when at least one column is *removed* from  $\mathbf{X}$ . (So  $\mathbf{X}_1$  is the design matrix for a linear regression on  $\mathbf{Y}$  but with fewer predictors). Show that  $R^2$  (non-adjusted) for the regression model calculated using design matrix  $\mathbf{X}$  is *at least as large* as the  $R^2$  for the regression model using design matrix  $\mathbf{X}_1$ .
4. Problem from Monday
5. Problem from Wednesday