Homework 02: Due 9/20

Stat061-F23

Prof Amanda Luby

---

1. Define a new estimator $\hat{\theta}_4$ for the Uniform$(0, \theta)$ distribution as follows. Is $\hat{\theta}_4$ unbiased? You can give an intuitive justification or a mathematical one, and are welcome to use any results from class.

$$\hat{\theta}_4 = \begin{cases} 2\bar{X} & \text{if } \max\{X_i\} < 2\bar{X} \\ \max\{X_i\} & \text{otherwise} \end{cases}$$

2. Let $Y_1, Y_2, ...Y_n$ be a random sample of size n from the pdf $f_y(y) = \frac{1}{\theta}e^{-y/\theta}, y > 0$.
   (a) Show that $\hat{\theta}_1 = Y_1$, $\hat{\theta}_2 = \bar{Y}$, and $\hat{\theta}_3 = nY_{\text{min}}$ are all unbiased estimators for θ.
   (b) Find the variances of $\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3$. Comment on which estimator is most efficient.

3. Suppose $X_1, X_2, ..., X_n$ are iid from a Gamma(α, λ) distribution. That is, $f_x(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\lambda x}$. You can also use $E(X) = \frac{\alpha}{\lambda}$ and $V(X) = \frac{\alpha}{\lambda^2}$.
   (a) Write out the likelihood function, and show that it depends on the data values only through $\bar{X}$ and $\bar{X}_g = (\prod X_i)^{1/n}$ ($\bar{X}_g$ is the *geometric mean*).
   (b) When α and λ are both unknown, the MLE does not have a closed form solution. Instead, find the MoM estimates $\hat{\alpha}$ and $\hat{\lambda}$.
   (c) Are the MoM estimates unbiased?
   (d) Gamma random variables are the waiting times for Poisson occurrences. In sports, goals are often assumed to follow a Poisson process, which means that the waiting time for the 1st goal can be assumed to be a Gamma random variable. For the Swarthmore women's soccer team so far this year, the first goal in $n = 4$ games has occurred at $X_i = 11.01667, 3.05, 76.65, 24.1333$ minutes. Report $\hat{\alpha}$ and $\hat{\lambda}$ for these data. (Note that $\alpha = 1$ implies an exponential distribution, which would be the case if goals occur as a Poisson process)

4. TBA after Friday's class

5. TBA after Friday's class