08: LARGE SAMPLE PROPERTIES

Rice 5.3, 8.5.2

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1 Sampling Distribution of a Statistic

Convergence in distribution

Let $X_1, ..., X_n$ be a sequence of random variables with CDFs $F_1, ..., F_n$ and let X be a random variable with CDF F. We say X_n converges in distribution to F if:

Sampling distribution of a statistic

Let $X_1, ..., X_n$ be a random sample with pdf $f_x(\theta)$. Let $T = h(X_1, ..., X_n, \theta)$. Then, the distribution of T (given θ) is called the *sampling distribution* of T.

Example:

Example:

2 Central Limit Theorem

Central Limit Theorem

For a large number n of iid observations $Y_i \sim f_y$, with mean μ and variance σ^2, the sampling distribution of $\bar{Y}$ is approximately:

Central Limit Theorem II

For a large number n of iid observations $Y_i \sim f_y$, with mean μ and variance σ^2, then for each fixed x:

3 Delta Method

Delta Method

Let $Y_1, Y_2, \ldots$ be a sequence of random variables, and let F^* be a continuous c.d.f. Let θ be a real number, and let $a_1, a_2, \ldots$ be a sequence of positive numbers that increase to ∞. Suppose that $a_n(Y_n - \theta)$ converges in distribution to F^*. Let α be a function with continuous derivative such that $\alpha'(\theta) \neq 0$. Then,

Example: Variance-stabilizing transformation

4 Large-Sample Properties of the MLE

1. MLE estimators are *sufficient*
2. MLE estimators are *invariant*
3. MLE estimators are *asymptotically unbiased.*
4. Under appropriate smoothness conditions of f_x, the MLE from an iid sample is *consistent.*
5. MLE estimators are *asymptotically efficient*: for large n, other estimators do not have smaller variance
6. Under smoothness conditions of f_x, the MLE has a *normal sampling distribution* for large samples

Sampling distribution of the MLE

Let $\hat{\theta} = h(Y)$ be the MLE for θ, where $Y \sim f_y(\theta)$.

Example:

5 Large-Sample Properties of the Bayes Estimator

The large-sample properties of the MLE generally extend to the Bayes estimator:

1. Bayes estimators are asymptotically unbiased
2. Bayes estimators are asymptotically efficient
3. Bayes estimators are consistent
4. Bayes estimators are sufficient
5. Bayes estimators have normal sampling distributions for large n