01: INTRODUCTION TO ESTIMATION

Stat061-F23

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The first topic we're going to cover in this class is *estimation*. That is, how to use observed sample data to estimate population parameters. A health-care study, for example, might want to estimate the proportion of people who have private health insurance and the mean annual cost for those who have it. Some studies assume a particular *parametric* family of probability distributions for a response variable and then estimate the parameters of that distribution in order to fit the distribution to the data.

This set of notes covers the basics of estimating a parameter by constructing an *estimator*, that yields a single number, called a *point estimate*.

Motivating Example: The 2018 General Social Survey asked "Do you believe there is a life after death?" For the 2,123 people interviewed, one point estimate for the *population* proportion of Americans who would respond yes is the sample proportion, which was 0.81.

1 Definitions and Notation

Before we get started, let's re-introduce ourselves to some key definitions from probability, add some new definitions, and introduce the notation that we'll use.

Parameter

Estimator

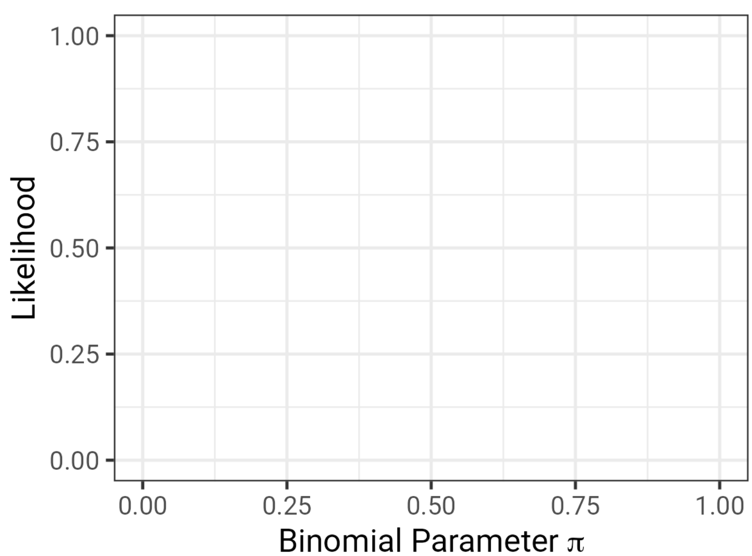
Estimate

Probability Density Function (PDF)

Probability Mass Function (PMF)

Likelihood function

Example: Binomial Distribution



Exercise: Is the likelihood function a probability distribution? Why or why not?

Exercise: Suppose a researcher is collecting n measurements of a continuous random variable Y, that they believe has the pdf $f_y(y;\theta) = \frac{1}{\theta^2} y e^{-y/\theta}, 0 < y < \infty, 0 < \theta < \infty$. What is the likelihood function?

2 Method of Maximum Likelihood

Example: Continuing the example from above, what value of θ would *maximize* $L(\theta)$?

Method of Maximum Likelihood

Exercise: If five data points have been recorded ($Y_i = 9.2, 5.6, 18.4, 12.1, 10.7$), what would the MLE from the previous example be?

2.1 Finding the MLE when more than one parameter is unknown

If the pdf or pmf that we're using has two or more parameters, say θ_1 and θ_2, finding MLEs for the θ_i's requires the solution of a set of simultaenous equations. We would typically need to solve the following system:

$$\frac{\delta \ln L(\theta_1, \theta_2)}{\delta \theta_1} = 0$$
$$\frac{\delta \ln L(\theta_1, \theta_2)}{\delta \theta_2} = 0.$$

Example: Suppose a random sample of size n is drawn from the two parameter normal pdf

$$f_y(y; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}(\frac{y-\mu}{\sigma})^2}.$$

Find the MLE's $\hat{\mu}$ and $\widehat{\sigma^2}$.

3 Method of Moments

A second procedure for finding estimators of parameters is the *method of moments*. This method is often more tractable than the method of maximum likelihood when the underlying probability distribution has multiple parameters.

Moment

Example: Suppose we draw n random variables from $f_y(y; \theta) = \theta y^{\theta - 1}$, $0 < y < 1$.

The Method of Moments

The method of moments is also especially helpful when we're working with named distributions (or functions of named distributions).

Exercise: Find the MoM estimators for μ and σ^2 for the two parameter normal pdf.

Some helpful "tricks" from probability:

1. Named distributions

2. Properties of expected value and variance

3. Law of the unconscious statistician (LOTUS)

4. Moment generating functions