Homework 06: Due 11/1

Stat061-F23

Prof Amanda Luby

Note: You should be able to start 1, 2, and 3a whenever. We'll cover 3b-c and 4 on Monday.

1. What confidence level would be associated with each of the following intervals? You can assume that σ is known.
   (a) $(\bar{Y} - 1.64\frac{\sigma}{\sqrt{n}}, \bar{Y} + 2.33\frac{\sigma}{\sqrt{n}})$
   (b) $(-\infty, \bar{Y} + \frac{\sigma}{\sqrt{n}})$
   (c) $(\bar{Y} - 1.96\frac{\sigma}{\sqrt{n}}, \bar{Y})$
2. In a simple random sample of 1500 voters, 53% said they planned to vote for a particular proposition, and 47% said they planned to vote against it. The estimated margin of victory for the proposition is thus 6%.
   (a) Find the sampling distribution for the margin of victory
   (b) Find a 95% confidence interval for the margin of victory. What do you conclude?

The following problems refers to a study of human temperatures. For $n = 64$ female subjects, the average temperature was $\bar{X} = 98.36$ with a sample standard deviation of $s = 0.68$. There were 24 women with temperatures of 98.6 or higher. Assume this is a representative sample and we can invoke the CLT.

3. For each of the following, find 95% and 99% confidence intervals for μ.
   (a) First, assume that the population standard deviation is known to be $\sigma = .7$
   (b) Now, place a prior distribution on μ where $\mu \sim N(98.6, 0.2^2)$. Find 95% and 99% posterior intervals for μ.
   (c) Provide a real-world conclusion about the intervals in (a) and (b).
4. Define θ to be the proportion of people with temperatures of 98.6 or higher.
   (a) Find 95% and 99% confidence intervals for θ using the conservative formula.
   (b) Now find the CI's using the large-sample formula
   (c) Now find 95% and 99% Bayesian posterior intervals assuming a uniform prior distribution (recall this is equivalent to a Beta(1,1) distribution).
   (d) Based on prior work, a collaborator strongly believes that the population proportion θ is close to 0.5. Choose a prior distribution for θ to capture this believe and recalculate the posterior intervals (you can use a distribution explorer to see different shapes of Beta distributions). How do they change from (c)?