

# 03: BAYESIAN ESTIMATION

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## 1 Bayes Theorem

**Bayesian statistics** is a set of techniques that are based on inverse probabilities calculated using Bayes' theorem. Relative to "classical techniques" (MoM and MLE), Bayesian estimation provides a way to incorporate "prior knowledge" into the estimation of parameters.

**Example:**

Classical Statistics	Bayesian Statistics
<i>Probability</i> refers to limiting relative frequencies. Probabilities are objective properties of the real world.	<i>Probability</i> describes a degree of belief, not a limiting frequency. As such, we can make probability statements about lots of things, not just data which are subject to random variation. For example, I might say that “the probability that Albert Einstein drank a cup of tea on August 1, 1948 is .35”. This does not refer to any limiting frequency. It reflects my strength of belief that the proposition is true.
<i>Parameters</i> are fixed, unknown constants, and the data we observe is random. Because they are constant, no useful probability statements can be made about parameters.	<i>Parameters</i> are random, and the data that we observe are fixed. We can therefore make probability statements about parameters.
Statistical procedures should be designed to have well-defined long-run frequency properties. For example, a 95% confidence interval should capture the true value of the parameter at least 95% of the time.	We make inferences about a parameter $\theta$ by producing a probability distribution for $\theta$ . Inferences, such as point estimates and interval estimates, may then be extracted from this distribution.

Bayesian inference is a controversial approach because it inherently embraces a subjective notion of probability. The field of statistics generally puts more emphasis on frequentist methods although Bayesian methods definitely have a presence.

## 2 Bayesian Inference

### 1. Prior distribution:

### 2. Statistical model for data:

### 3. Posterior distribution:

### Posterior distribution

Let  $W$  be a statistic dependent on parameter  $\theta$ . Call its pdf  $f_W(w|\theta)$ . Assume that  $\theta$  is the value of a random variable  $\Theta$ , whose prior distribution is denoted  $p_\Theta$  if discrete and  $f_\Theta$  if continuous. The *posterior distribution* of  $\Theta$  given  $W = w$  is:

#### 4. Posterior mean:

**Example:** Let  $X_1, \dots, X_n \sim \text{Bernoulli}(p)$  and suppose that  $p$  has the prior distribution  $p \sim \text{Beta}(\alpha, \beta)$ .

### Conjugate prior

**Example:** Let  $X_1, \dots, X_n \sim N(\theta, \sigma^2)$  and suppose we take  $\theta \sim N(a, b^2)$ . For simplicity, let's assume  $\sigma^2$  is known.