15: INFERENCE FOR SLR

Larsen & Marx 11.3

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1 Properties of MLEs for Simple Linear Regression

1. $\hat{\beta}_0$ and $\hat{\beta}_1$ are normal RV's
2. $\hat{\beta}_0$ and $\hat{\beta}_1$ are unbiased
3. $V(\hat{\beta}_1) = \frac{\sigma^2}{\sum(x_i - \bar{X})^2}$
4. $V(\hat{\beta}_0) = \frac{\sigma^2 \sum x_i^2}{n\sum(x_i - \bar{X})^2} = \sigma^2[\frac{1}{n} + \frac{\bar{x}^2}{\sum(x_i - \bar{x})^2}]$
5. $\hat{\beta}_1$, $\bar{Y}$ and $\hat{\sigma}^2$ are mutually independent
6. $\frac{n\hat{\sigma}^2}{\sigma^2} \sim \chi^2_{n-2}$
7. $S^2 = \frac{n}{n-2}\hat{\sigma}^2$ is an unbiased estimator for σ^2

Proof: ($\hat{\beta}_1$ is a normal RV)

Everything but the Y's are constant

$Y_i \sim N(\beta_0 + \beta_1 x_1, \sigma^2)$

$$\hat{\beta}_1 = \frac{n\sum x_i y_i - (\sum x_i)(\sum y_i)}{n(\sum x_i^2) - (\sum x_i)^2}$$

$$= \frac{\sum x_i y_i - \frac{1}{n}\sum x_i (\sum y_i)}{(\sum x_i^2) - \frac{1}{n}(\sum x_i)^2}$$

$$= \frac{\sum x_i y_i - \bar{x}\sum y_i}{(\sum x_i^2) - n\bar{x}^2}$$

a, b_i: constants

$$= \frac{\sum (x_i - \bar{x}) y_i}{(\sum x_i^2) - n\bar{x}^2} \rightarrow \frac{1}{a} \cdot \sum b_i Y_i \rightarrow \text{linear combination of a bunch of Normal RV's}$$

$\rightarrow \hat{\beta}_1$ is also normally distributed!

Proof: ($V(\hat{\beta}_1)$)

2 Inference for Simple Linear Regression

2.1 Inference for β_1

Test statistic for β_1

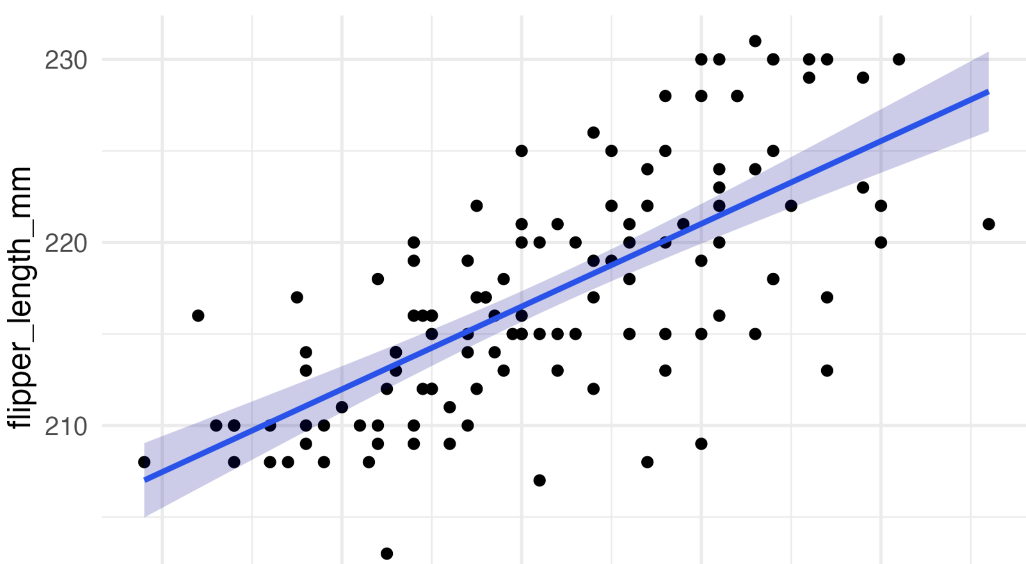
Let $(x_1, Y_1), (x_2, Y_2), ..., (x_n, Y_n)$ be a set of points satisfying $E(Y|X = x) = \beta_0 + \beta_1 x$ and let $S^2 = \frac{1}{n-2} \sum (Y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i))^2$. Then,
$T = \frac{\hat{\beta}_1 - \beta_1}{S / \sqrt{\sum (x_i - \bar{x})^2}}$

Proof:

Note: Hypothesis tests based on T are GLRTs!

2.2 Inference for σ^2

2.3 Inference for $E(Y|x)$



2.4 Inference for new Y_i's