Homework 12

Stat061-F23

Prof Amanda Luby

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1. For the simple linear regression case ($y = \beta_0 + \beta_1 x + \epsilon$), show that $\hat{\beta}_1 = r\frac{\hat{\sigma}_y}{\hat{\sigma}_x}$.
2. Assuming the standard multiple linear model ($Y = \beta X + \epsilon$, where X is an $n \times p$ design matrix):
   (a) Show that $\sigma^2 I = \Sigma_{\hat{y}} + \Sigma_{\hat{\epsilon}}$
   (b) Using (a), conclude that $n\sigma^2 = \sum Var(\hat{Y}_i) + \sum Var(\hat{\epsilon}_i)$
3. Consider a multiple linear regression problem with design matrix $\mathbf{X}$ and observations $\mathbf{Y}$. Let $\mathbf{X}_1$ be the matrix remaining when at least one column is *removed* from $\mathbf{X}$. (So $\mathbf{X}_1$ is the design matrix for a linear regression on $\mathbf{Y}$ but with fewer predictors). Show that R^2 (non-adjusted) for the regression model calculated using design matrix $\mathbf{X}$ is *at least as large* as the R^2 for the regression model using design matrix $\mathbf{X}_1$.
4. ~~Problem from Monday~~
5. Problem from Mon/Wed TBA