Quiz02 Review

Stat061-F23

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1. Suppose that $X_1, ..., X_n \sim Poisson(\theta)$.
   (a) Find *two* method of moments estimators
   (b) Find the MLE of θ
   (c) Find $I(\theta)$
   (d) Is $\bar{X}$ the MVUE of θ? How can you tell?

2. Suppose $X_1, ..., X_n \sim Gamma(\alpha, \frac{1}{2})$.
   (a) Find a sufficient statistic for α.
   (b) Show that a method of moments estimator for α is $\hat{\alpha}_{MoM} = 2\bar{X}$
   (c) The sum of Gamma random variables with the same rate parameter is also a Gamma Random variable. That is, $\sum X_i \sim Gamma(n\alpha, \frac{1}{2})$. Compute the exact mean and variance of $\hat{\alpha}_{MoM}$.
   (d) Now, find the asymptotic variance of $\hat{\alpha}_{MoM}$
   (e) Find the MSE of $\hat{\alpha}_{MoM}$
   (f) Let $\hat{\alpha}_{MVUE}$ be the minimum-variance unbiased estimator of α. Is MSE($\hat{\alpha}_{MoM}$) $<, \leq, =, \geq, >$ MSE($\hat{\alpha}_{MVUE}$). Explain how you can tell.

3. Suppose $X_1, ..., X_n$ are iid from $f_x(x) = \lambda^2 x^{-3} e^{-\lambda/x}$ for $x > 0, \lambda > 0$.
   (a) Write out the likelihood function
   (b) Show that $T(X) = \sum X_i^{-1}$ is a sufficient statistic for λ.
   (c) Find the MLE $\hat{\lambda}_{MLE}$
   (d) What is the asymptotic distribution of $\hat{\lambda}_{MLE}$
   (e) Now, let $\theta = \log \lambda$. Find the MLE for θ.

4. **True or False:** If the statement is true, provide a brief explanation why. If the statement is false, cite a counterexample or explain how the statement could be corrected to make it true.
   (a) The MLE is always unbiased
   (b) It follows from the law of large numbers that if the sample size n is large enough, the sample mean $\bar{X}$ is equal to the population mean μ.
   (c) Let $X_1, ..., X_n \sim f_x(\theta)$. By the law of large numbers, the sample mean $\bar{X}$ is consistent for θ.
   (d) Let $X_1, ..., X_n$ be a random sample from a population with mean θ and variance σ^2. Suppose that $I(\theta) = \frac{1}{\sigma^2}$. $\bar{X}$ is the MVUE.
   (e) If $\hat{\theta}$ meets the Cramer-Rao Lower Bound, it is an efficient estimator.