Named Probability Distributions

Discrete Probability Distributions

pmf: $p(y)$ cdf: $F(y) = \sum_{z=-\infty}^{y} p(z)$

$0 \leq p(y) \leq 1$; $\sum_{y=-\infty}^{\infty} p(y) = 1$

$P(Y = y) = p(y)$; $P(a \leq Y \leq b) = \sum_{a}^{b} p(y)$

Binomial – $Y \sim \mathrm{Binom}(n, p)$

$p(y) = \frac{n!}{y!(n-y)!} p^y (1-p)^{n-y}$, $y \in [0, n]$, $p \in [0, 1]$

$\mathbb{E}[Y] = np$

$\mathbb{V}[Y] = np(1-p)$

$m(t) = [pe^t + (1-p)]^n$

Geometric – $Y \sim \mathrm{Geom}(p)$

$p(y) = (1-p)^{y-1} p$, $y \in [1, \infty)$, $p \in [0, 1]$

$\mathbb{E}[Y] = 1/p$

$\mathbb{V}[Y] = (1-p)/p^2$

$m(t) = \frac{p}{1-qe^t}$

Hypergeometric – $Y \sim \mathrm{HG}(N, K, n)$

$p(y = k) = \frac{\binom{K}{k}\binom{N-k}{n-k}}{\binom{N}{n}}$, $k \in \{\max(0, n + K - N), \ldots, \min(n, K)\}$, $K \leq N; n \leq N$

$\mathbb{E}[Y] = \frac{nK}{N}$

$\mathbb{V}[Y] = \frac{nK(N-K)(N-n)}{N^2(N-1)}$

Negative Binomial – $Y \sim \mathrm{NBinom}(r, p)$

$P(Y = k) = \binom{r+k-1}{r-1} p^r (1-p)^k$, $k \in [r, \infty)$, $r \in \mathbb{Z}^+$, $p \in [0, 1]$

$\mathbb{E}[Y] = rq/p$

$\mathbb{V}[Y] = rq/p^2$

$m(t) = (\frac{p}{1-qe^t})^r$ for $qe^t < 1$

Poisson – $Y \sim \mathrm{Poi}(\lambda)$

$p(y) = \frac{\lambda^y}{y!} e^{-\lambda}$, $y \in [0, \infty)$;

$\mathbb{E}[Y] = \mathbb{V}[Y] = \lambda$

$m(t) = e^{\lambda(e^t - 1)}$

Continuous Probability Distributions

pdf: $f(y) = \frac{\mathrm{d}}{\mathrm{d}y}(y)$ cdf: $F(y) = \int_{-\infty}^{y} f(z)\,\mathrm{d}z$

$f(y) \geq 0$; $\int_{-\infty}^{\infty} f(y)\,\mathrm{d}y = 1$; $P(Y = y) = 0$

$P(a \leq Y \leq b) = \int_{a}^{b} f(y)\,\mathrm{d}y = F(b) - F(a)$

Uniform – $Y \sim \mathrm{Uniform}(a, b)$

$f(y) = (b-a)^{-1}$, $y \in [a, b]$

$\mathbb{E}[Y] = (a+b)/2$

$\mathbb{V}[Y] = (b-a)^2/12$

$m(t) = (e^{bt} - e^{at})/[t(b-a)]$

Normal – $Y \sim \mathrm{N}(\mu, \sigma^2)$

$f(y) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(y-\mu)^2/2\sigma^2}$ $y \in (-\infty, \infty)$, $\mu \in \mathbb{R}$, $\sigma \in \mathbb{R}^+$

$\mathbb{E}[Y] = \mu$;

$\mathbb{V}[Y] = \sigma^2$

$m(t) = \exp(\mu t + t^2\sigma^2/2)$

If $Y \sim N(\mu, \sigma)$, then $Z = (Y - \mu)/\sigma$; $Z \sim \mathrm{N}(0,1)$.

$P(Y \leq y) = \Phi\left(\frac{y-\mu}{\sigma}\right) = \Phi(z)$ (non-analytic function)

Exponential – $Y \sim \mathrm{Exponential}(\lambda)$

$f(y) = \lambda e^{-\lambda y}$, $y \in [0, \infty)$, $\lambda \in \mathbb{R}^+$

$\mathbb{E}[Y] = 1/\lambda$

$\mathbb{V}[Y] = 1/\lambda^2$

$m(t) = \frac{\lambda}{\lambda - t}$ for $t < \lambda$

Gamma – $Y \sim \mathrm{Gamma}(\alpha, \beta)$

$f(y) = y^{\alpha-1} e^{-y/\beta} / [\beta^\alpha \Gamma(\alpha)]$, $y \in [0, \infty)$, $\alpha \in \mathbb{R}^+$, $\beta \in \mathbb{R}^+$

$\Gamma(\alpha) = \int_0^\infty y^{\alpha-1} e^{-y} dy = (\alpha - 1)\Gamma(\alpha - 1)$

If n is a positive integer, $\Gamma(n) = (n-1)!$

$\mathbb{E}[Y] = \alpha\beta$

$V[Y] = \alpha\beta^2$

$m(t) = (1 - \beta t)^{-\alpha}$

$\alpha = 1 \Rightarrow$ **exponential distribution**

$\beta = 2$, $\alpha = \nu/2$, $\nu \in \mathbb{Z}^+ \Rightarrow$ **chi-square distribution**

Beta – $Y \sim \mathrm{Beta}(\alpha, \beta)$

$f(y) = y^{\alpha-1}(1-y)^{\beta-1}/B(\alpha, \beta)$, $y \in [0, 1]$, $\alpha \in \mathbb{R}^+$, $\beta \in \mathbb{R}^+$

$B(\alpha, \beta) = \Gamma(\alpha)\Gamma(\beta)/\Gamma(\alpha + \beta)$

$\mathbb{E}[Y] = \alpha/(\alpha + \beta)$

$V[Y] = \alpha\beta/[(\alpha+\beta)^2(\alpha+\beta+1)]$