18: INTRO TO GENERALIZED LINEAR MODELS

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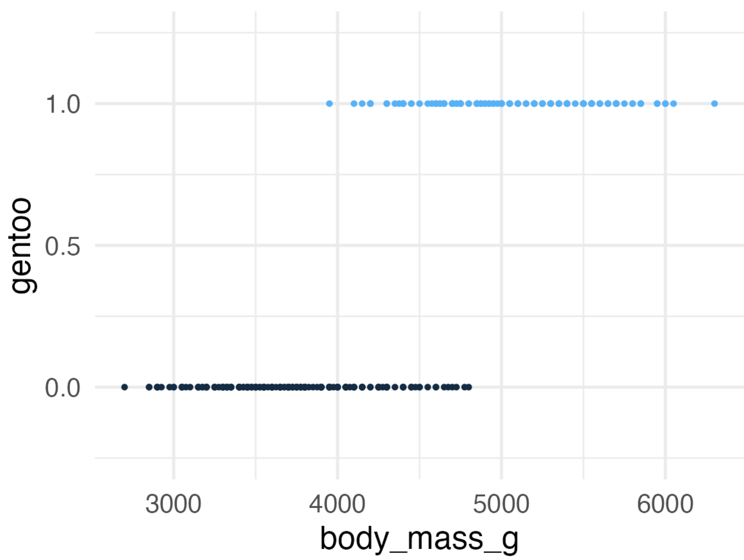
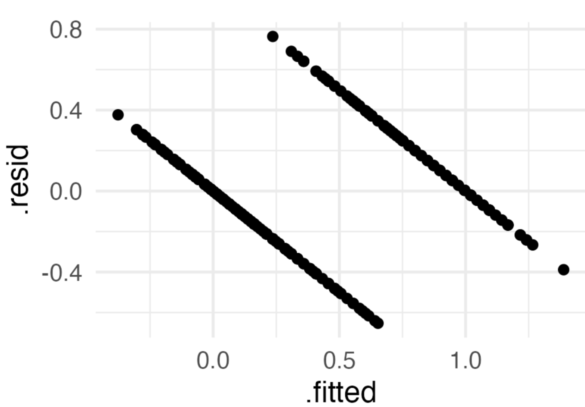
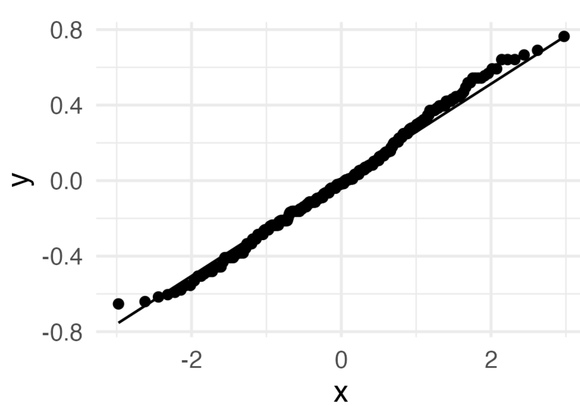
Let's start with our dear old `penguins` friends. The full dataset contains information about three different species of penguins. Rather than understanding the relationship between `body_mass` and `flipper_length`, we might instead be interested in how `body_mass` is related to species. In this case, we'll treat species as either `Gentoo` or `Not Gentoo`



On first glance, it looks like we could go ahead and fit a linear regression model for this problem:

```
              Estimate Std. Error  t value Pr(>|t|)
(Intercept)   -1.7006      0.0799 -21.2771        0
body_mass_g    0.0005      0.0000  26.2408        0
```





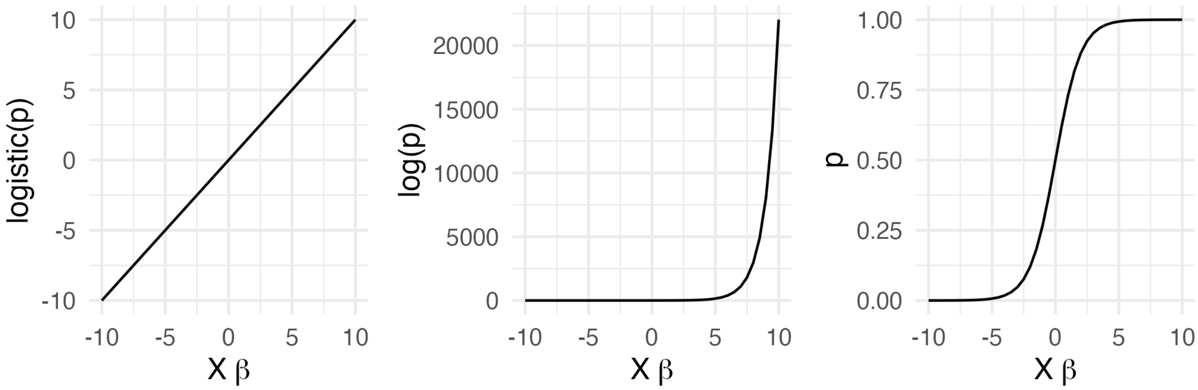
Let's list some reasons why this approach is not ideal:

What distribution does `gentoo` have? A better approach would be to start there.

1 Logistic Regression

Logistic Regression Model

Solving for p, this gives:



1.1 Maximum Likelihood Estimation

Now that we have the structure of the model, we have to think about how to estimate the β's. Recall that the likelihood function for a n Bernoulli random variables is:

$$l(p) = \sum y_i \ln p + (1 - y_i) \ln(1 - p)$$

But, since we now have an X variable, $p = p(x_i)$

Sampling distribution of logistic regression coefficients

```r
gentoo_mod = glm(gentoo ~ body_mass_g,
                 data = penguins,
                 family = "binomial")
summary(gentoo_mod)
```

```

Call:
glm(formula = gentoo ~ body_mass_g, family = "binomial", data = penguins)

Coefficients:
              Estimate Std. Error z value Pr(>|z|)
(Intercept) -2.842e+01  3.609e+00  -7.873 3.46e-15 ***
body_mass_g  6.371e-03  8.131e-04   7.835 4.69e-15 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 446.80  on 341  degrees of freedom
Residual deviance: 117.85  on 340  degrees of freedom
  (2 observations deleted due to missingness)
AIC: 121.85

Number of Fisher Scoring iterations: 7
```

1.2 Interpretation of coefficients

2 Generalized Linear Models

We've now seen two different settings for regression. If X is a vector of predictors and $Y \in \mathbb{R}$, we have assumed a linear model:

and if $Y \in \{0, 1\}$, we assumed a logistic model:

In both settings, we are assuming that a transformation of the conditional expectation is a linear function of X: