NOTES 19: CLT-BASED INFERENCE FOR PROPORTIONS

Stat 120 | Fall 2025

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CLT: The Central Limit Theorem (CLT) tells us that if the sample size is big enough and the sample is random,

$$\bar{X} \sim N(\_\_\_\_\_, \_\_\_\_\_)$$

$$\hat{p} \sim N(\_\_\_\_\_, \_\_\_\_\_)$$

The general form for a confidence interval is:

Example: Finding z^*

- 95% confidence interval
- 68% confidence interval
- 99% confidence interval
- 90% confidence interval

1 How big is big enough?

Example 1: $\hat{p} = .5$

Example 1: $\hat{p} = .05$

> Rule of Thumb for Proportions:
>
> - Expected count in each category (Yes/No) should be > ____
> - $np > $ ____ and $n(1-p) > $ ____

2 How do we find the SE?

> Standard Error for Proportions:

Idea: As n gets bigger, the SE gets _____. If $\hat{p}$ is close to .5, the SE is _____ than if $\hat{p}$ is close to 0 or 1

Example: ESP Example Again

n = 14

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p_hat = 3/14
n = 14
SE_p = sqrt((p_hat*(1-p_hat))/n)
z_score = (p_hat - .2)/SE_p
p_val = pnorm(z_score, lower.tail = FALSE)
p_val
```

```
[1] 0.4482
```

n=1400:

n=14000:

Big Picture Picture