NOTES 11: CONFIDENCE INTERVALS + INTRO TO THE BOOTSTRAP

Stat 120 | Fall 2025

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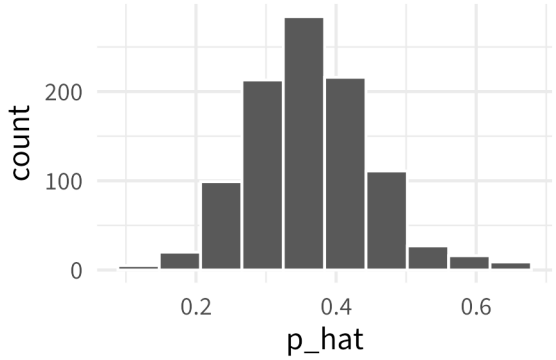
Let's begin with the class proportion from last time: 36.7% of Carleton students are "from the Midwest", while ____% of our class (who answered) are from the Midwest.



The mean and standard deviation of the sampling distribution are:

```
# A tibble: 1 x 2
   xbar     se
  <dbl>  <dbl>
1 0.366 0.0898
```

Standard Error:

Standard deviation of a statistic or a sampling distribution

In a random class of 32 students, how many would we expect to be from the Midwest?

Confidence Interval

An interval computed from sample data by a method that will capture the parameter for a specified percentage of all samples

95% confidence interval using the standard error:

Margin of Error

Example: Percent of each country with internet access (StatKey)

⚠ CI Misinterpretations

- A 95% CI contains 95% of the data from the population
- I am 95% sure that the mean of the sample will be in CI
- The probability that the parameter is in the CI is 95%

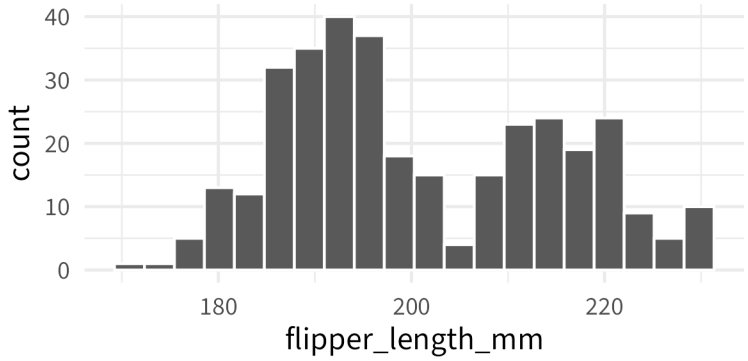
Structure of a CI:

1. I am ___ % confident
2. that the [population parameter in context]
3. is between ____ and ____ [units]

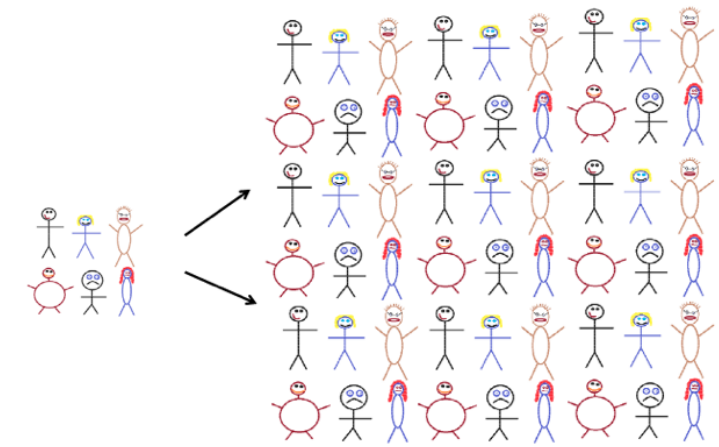
1 The Bootstrap

So far, we've used the *population* to generate samples and construct sampling distributions. For most situations, this is unrealistic.

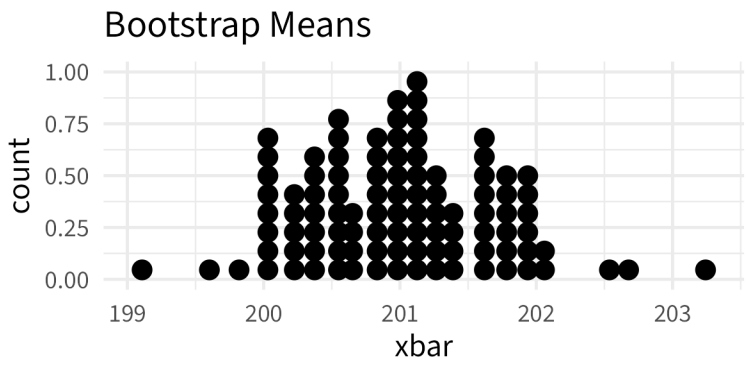
Example: Let's consider our `penguin` friends. I'm interested in finding a 95% confidence interval for the mean flipper length of *all* Palmer Archipelago penguins. The mean flipper length is 200.9152 and the standard deviation is 14.0617.



Idea:



Let's return to our penguins. I've made 100 bootstrap samples and found the mean for each one:



The mean of this distribution is 200.9947 and the standard deviation is 0.6988.

Bootstrap confidence interval:

The magic of the bootstrap is that it works for any statistic we can compute from our sample! We simply have to follow these steps:

1. Generate *bootstrap samples*:
    - Sample from the original sample with replacement
    - Use the same sample size as original sample
2. Compute the statistic of interest for each bootstrap sample
3. Collect the statistics for many bootstrap samples to create the *bootstrap distribution*
4. Treat the *bootstrap distribution* as the *sampling distribution* to estimate the standard error

⚠ What happens when the bootstrap distribution is not symmetric?