

# NOTES 20: CLT-BASED INFERENCE FOR MEANS

Stat 120 | Fall 2025

Prof Amanda Luby

---

CLT: The Central Limit Theorem (CLT) tells us that if the sample size is big enough and the sample is random,

$$\bar{X} \sim N(\text{---}, \text{---})$$

Example: Finding  $z^*$  with a table. What could you say about the value of the p-value for a z-score of 2.917?

percentage	percentile (qnorm(percentage))
90%	1.3
95%	1.6
97.5%	2.0
99%	2.3
99.5%	2.6



## 1 How big is big enough?

Rule of Thumb for Means:

## 2 How do we find the SE?

Standard Error for Means:

Idea: As  $n$  gets bigger, the SE gets \_\_\_\_\_. If \_\_\_\_\_ is small, the SE is also \_\_\_\_\_.

Example: Florida Lakes

$H_0$  :

$H_A$  :

$\bar{X} =$

$s =$

Test stat:

p-value:

Example: Guinness Beer Acidity

$H_0$  :

$H_A$  :

$\bar{X} =$

$s =$

Test stat:

p-value:

p-value (t-distribution):

### 3 The t-distribution



- When we divide by \_\_\_\_\_, the test stat has a t-distribution instead of a  $N(0,1)$
- The t-distribution depends on the “degrees of freedom” (\_\_\_\_\_)
- When  $df$  is \_\_\_\_\_, t-distribution has “heavier tails” than  $N(0,1)$
- When  $df$  is \_\_\_\_\_, the t-distribution is approximately equal to  $N(0,1)$

Example: Florida Lakes (again)

```
t = -1.75  
pt(t, df = 52)
```

```
[1] 0.04301
```

### 4 Summary

- Test stat for means: \_\_\_\_\_
- SE: \_\_\_\_\_
- Can “safely” use the CLT if \_\_\_\_\_
- If \_\_\_\_\_, we can still use the CLT if there are no outliers or extreme skew
- t-distribution is better to use, but for large sample sizes it will be close to the normal distribution
- Percentage of t-distribution below  $t$ -score: `pt(t-score, df = n-1)`
- Percentile  $t^*$  for a specific percentage: `qt(percentage, df = n-1)`

## 5 Group Problems

1. (Adapted from Exercise 6.128)

Plastic microparticles contaminate shorelines. Much of the pollution comes from washing fleece clothing. In a recent study, washing a fleece garment discharged on average  $\bar{X} = 290$  fibers per liter of wastewater. The standard deviation was  $s = 87.6$  fibers and the sample size was  $n = 120$ .

- (a) What is the estimated standard error of the average number of fibers discharged per liter of wastewater when washing a fleece garment?
- (b) The table below gives some percentiles of the  $t_{119}$  distribution. Use this information to construct a 99% confidence interval for the population mean. Interpret the interval in context.

percentage	percentile (qnorm(percentage))
90%	1.3
95%	1.6
97.5%	2.0
99%	2.3
99.5%	2.6

- (c) What sample size would we need if we wanted this interval to be no wider than 20?