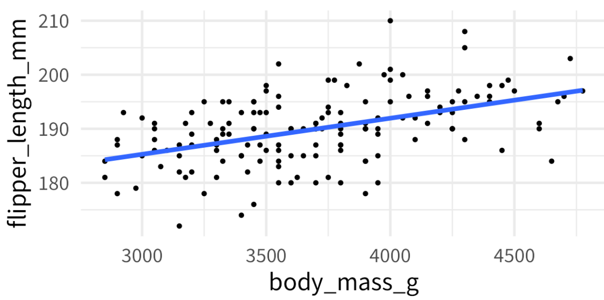
NOTES 24: INFERENCE FOR REGRESSION

Stat 120 | Fall 2025

Prof Amanda Luby

When we fit linear regression models earlier in this class, we used them to describe the relationship between the variables and interpreted the slope and the intercept as descriptions of the data. Now, we'd like to understand what the regression model can tell us about the relationship of the variables *in the population*.

We're going to return to the `palmerpenguins` dataset from earlier in the class and restrict our analysis today to `Adelie` penguins. We we want to predict `flipper_length_mm` using `body_mass_g`.



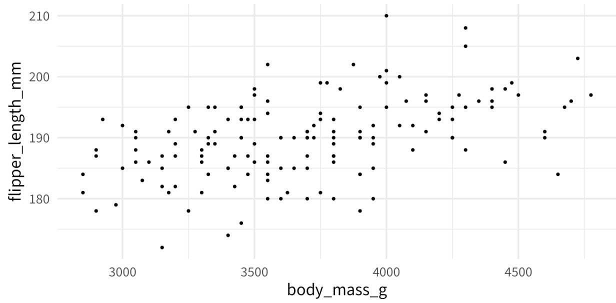

```
Call:
lm(formula = flipper_length_mm ~ body_mass_g, data = adelie

Coefficients:
(Intercept)  body_mass_g
   1.65e+02     6.68e-03
```

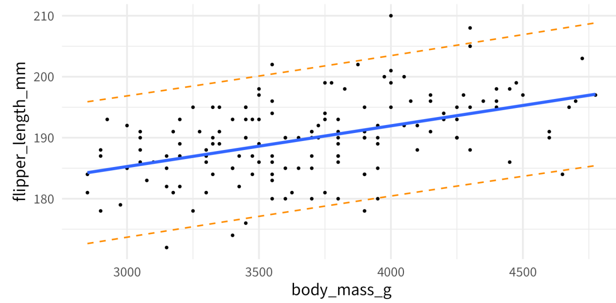
Warm up: Write out the regression equation and provide an interpretation for the slope

> **Note**
>
> The **population model** for the regression line is:

Idea:

To check if this population model is reasonable, use the "LINE" mnemonic:



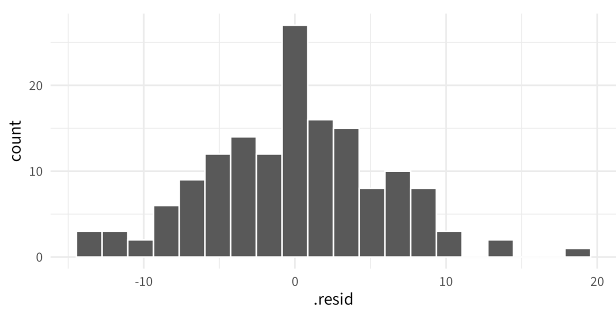


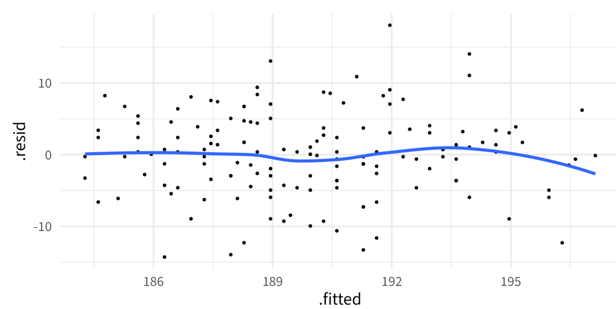
1. **L**________________________

2. **I**________________________

3. **N**________________________

4. **E**________________________





Note

The **sampling distribution** for b_1 is:

```r
summary(adelie_lm)
```

```

Call:
lm(formula = flipper_length_mm ~ body_mass_g, data = adelie)

Residuals:
    Min      1Q  Median      3Q     Max 
-14.277  -3.619   0.057   3.470  18.048 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1.65e+02   3.85e+00   42.93  < 2e-16 ***
body_mass_g 6.68e-03   1.03e-03    6.47  1.3e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5.8 on 149 degrees of freedom
  (1 observation deleted due to missingness)
Multiple R-squared:  0.219,	Adjusted R-squared:  0.214 
F-statistic: 41.8 on 1 and 149 DF,  p-value: 1.34e-09
```

> **Note**
>
> **CLT-based confidence interval for β_1:**
>
> $$b_1 \pm t^*_{n-2} \times SE_{b_1}$$

> **Note**
>
> **CLT-based test for $H_0 : \beta_1 = 0$:**
>
> $$t_{n-2} = \frac{b_1 - 0}{SE_{b_1}}$$

Example: Using the summary() output, along with the sampling distribution information above, how would you compute a 95% confidence interval for β_1?

1 Inference for Predictions (Ch 9.3)

Example: We measure a new Adelie penguin that weighs 4000g. What is (a) the predicted *mean* flipper length among all 4000g penguins and (b) the predicted flipper length for *any* 4000g penguin?

```r
augment(adelie_lm, newdata = tibble(body_mass_g = 4000))
```

```
# A tibble: 1 x 2
  body_mass_g .fitted
        <dbl>   <dbl>
1        4000    192.
```

```r
augment(adelie_lm, newdata = tibble(body_mass_g = 4000), interval = "confidence")
```

```
# A tibble: 1 x 4
  body_mass_g .fitted .lower .upper
        <dbl>   <dbl>  <dbl>  <dbl>
1        4000    192.   191.   193.
```

```r
augment(adelie_lm, newdata = tibble(body_mass_g = 4000), interval = "prediction")
```

```
# A tibble: 1 x 4
  body_mass_g .fitted .lower .upper
        <dbl>   <dbl>  <dbl>  <dbl>
1        4000    192.   180.   203.
```

We usually rely on R to make confidence and prediction intervals, but the formulas for these intervals are:

> **Note**
>
> A ________________________ **interval for the mean response** when the predictor is x^* is:
>
> $$\hat{y} \pm t^*_{n-2} \times s_\epsilon \sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{(n-1)s_x^2}}$$

> **Note**
>
> A ________________________ **interval for an individual response** when the predictor is x^* is:
>
> $$\hat{y} \pm t^*_{n-2} \times s_\epsilon \sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{(n-1)s_x^2}}$$