

NOTES 10: SAMPLING DISTRIBUTIONS

Stat 120 | Fall 2025

Prof Amanda Luby

“Big picture” picture:

Quantity	Statistic	Parameter
Mean		
Proportion		
Standard Deviation		
Correlation		
Regression Coefficient		

Carleton publishes an “at a glance” page with some facts and figures about the student body:
<https://www.carleton.edu/about/carleton-at-a-glance/>

Some highlights:

- Geographic distribution:
 - Midwest 36.7%
 - West 21.7%
 - East 17.3%
 - South 10.9%
 - International 11.8%
 - Other 1.6%
- 34% BIPOC
- 12% are among the first generation in their families to attend college
- 61% graduated in the top 10% of their high school class

In a moment, we’re going to see one of these quantities for our class. Before we do, what is your *best guess* for each of these quantities?

Example: In this set-up, what is the:

- Population
- Sample
- Parameter

- Statistic

We know that our class will likely not have **exactly** 36.7% from the Midwest, but we probably wouldn't expect it to be 0% or 90%.

Sampling variability

We might start to ask ourselves, what if a *different* set of 32 students enrolled in this course?

First, we create a population.

```
# A tibble: 2,007 x 2
  student_id midwest
      <int> <chr>
1         193 Yes
2        1062 No
3         111 Yes
4        1111 No
5         533 Yes
6        1686 No
7        1776 No
8        1224 No
9         877 No
10        338 Yes
# i 1,997 more rows
```

Then, we take a random sample:

```
set.seed(100424)
sample1 <- carls |>
  sample_n(32)
sample1
```

```
# A tibble: 32 x 2
  student_id midwest
      <int> <chr>
1        1259 No
2         826 No
3         858 No
4        1905 No
5          40 Yes
6        1336 No
7        1607 No
8        1757 No
```

```

9         489 Yes
10        445 Yes
# i 22 more rows

```

and calculate the proportion of “yes” responses:

```

sample1 |>
  group_by(midwest) |>
  summarize(
    n = n()
  ) |>
  mutate(p_hat = n/sum(n))

```

```

# A tibble: 2 x 3
  midwest      n p_hat
  <chr>    <int> <dbl>
1 No         23 0.719
2 Yes         9 0.281

```

This isn’t *super* useful, but if we do it a bunch of times, we can start to see what a range of possible samples could look like. (Note: this code requires the {infer} package)

```

many_samples <- carls |>
  rep_sample_n(35, reps = 1000, replace = TRUE) |>
  group_by(replicate, midwest) |>
  summarize(
    n = n()
  ) |>
  mutate(p_hat = n/sum(n)) |>
  filter(midwest == "Yes")

many_samples

```

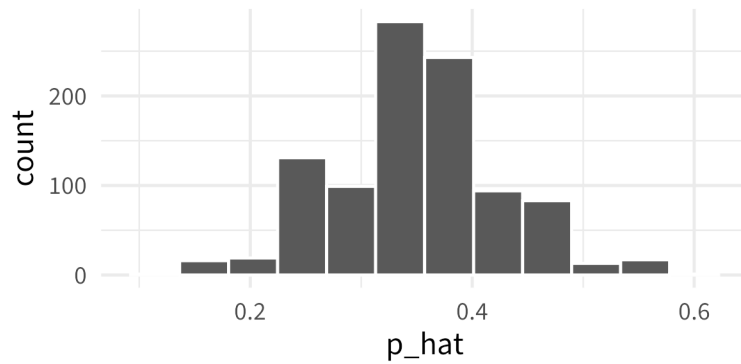
```

# A tibble: 1,000 x 4
# Groups:   replicate [1,000]
  replicate midwest      n p_hat
    <int> <chr>    <int> <dbl>
1         1 Yes         12 0.343
2         2 Yes         8 0.229
3         3 Yes        15 0.429
4         4 Yes        16 0.457
5         5 Yes        15 0.429
6         6 Yes        16 0.457
7         7 Yes        16 0.457
8         8 Yes        17 0.486
9         9 Yes        15 0.429

```

```
10      10 Yes      10 0.286
# i 990 more rows
```

Looking at this first few rows, we can start to get a sense of the range of possible sample proportions, but there are 990 rows that we can't see. Let's make a graph!



Example: Carleton Mission Statement

In your own words: provide explanations for:

Population distribution

Sample distribution

Sampling distribution