NOTES 20: CLT-BASED INFERENCE FOR MEANS

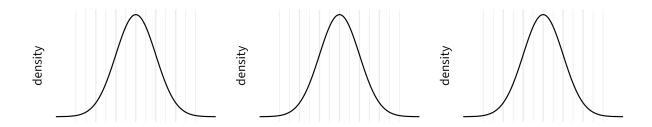
Stat 120 | Fall 2025 Prof Amanda Luby

CLT: The Central Limit Theorem (CLT) tells us that if the sample size is big enough and the sample is random,

$$\bar{X} \sim N(\underline{\hspace{1cm}},\underline{\hspace{1cm}})$$

Example: Finding z^* with a table. What could you say about the value of the p-value for a z-score of 2.917?

percentage	percentile (qnorm(percentage))
90%	1.3
95%	1.6
97.5%	2.0
99%	2.3
99.5%	2.6



1 How big is big enough?

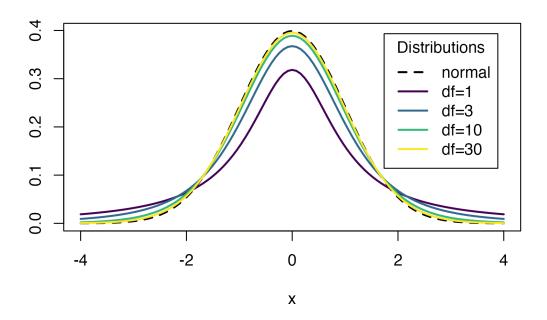
Note

Rule of Thumb for Means:

2 How do we find the SE?

i Note		
Standard Error for Means:		
Idea: As n gets bigger, the SE gets	If	is small, the SE is also
·		
Example: Florida Lakes		
H_0 :		
H_A :		
$\bar{X} =$		
s =		
Test stat:		
p-value:		
p-value.		
Example: Guinness Beer Acidity		
H_0 :		
$H_A:$		
$ar{X} =$		
s =		
Test stat:		
p-value:		
p-value (t-distribution):		
1		

3 The t-distribution



- When we divide by _____, the test stat has a t-distribution instead of a N(0,1)
- The t-distribution depends on the "degrees of freedom" (______)
- When df is ______, t-distribution has "heavier tails" than N(0,1)
- When df is _____, the t-distribution is approximately equal to N(0,1)

Example: Florida Lakes (again)

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t = -1.75
pt(t, df = 52)
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[1] 0.04301

4 Summary

- Test stat for means: ______
- SE.
- Can "safely" use the CLT if ______
- If _____, we can still use the CLT if there are no outliers or extreme skew
- t-distribution is better to use, but for large sample sizes it will be close to the normal distribution
- Percentage of t-distribution below t-score: pt(t-score, df = n-1)
- Percentile t^* for a specific percentage: qt(percentage, df = n-1)

5 Group Problems

1. (Adapted from Exercise 6.128)

Plastic microparticles contaminate shorelines. Much of the pollution comes from washing fleece clothing. In a recent study, washing a fleece garment discharged on average $\bar{X}=290$ fibers per liter of wastewater. The standard deviation was s=87.6 fibers and the sample size was n=120.

- (a) What is the estimated *standard error* of the average number of fibers discharged per liter of wastewater when washing a fleece garment?
- (b) The table below gives some percentiles of the t_{119} distribution. Use this information to construct a 99% confidence interval for the population mean. Interpret the interval in context.

percentage	percentile (qnorm(percentage))
90%	1.3
95%	1.6
97.5%	2.0
99%	2.3
99.5%	2.6

(c) What sample size would we need if we wanted this interval to be no wider than 20?