

# 06: MORE ON MAXIMUM LIKELIHOOD

Stat250 S25

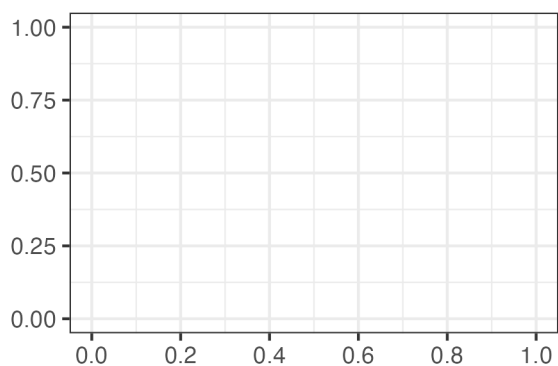
Prof Amanda Luby

---

## 1 Overview

## 2 More on the likelihood function and MLEs

**Example:** Recall that the likelihood function for  $n$  iid Bernoulli( $\theta$ ) random variable is  $L(\theta) = \theta^{\sum x_i} (1 - \theta)^{n - \sum x_i}$



Scenario 1: 0 “Yes” responses

| $\theta$    | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|-------------|-----|-----|-----|-----|-----|-----|
| $L(\theta)$ |     |     |     |     |     |     |

Scenario 2: 6 “Yes” responses

| $\theta$    | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|-------------|-----|-----|-----|-----|-----|-----|
| $L(\theta)$ |     |     |     |     |     |     |

**Exercise:** Is the likelihood function a probability distribution? Why or why not?

**Exercise** Let  $X_1, \dots, X_n$  be an iid random sample from a distribution with PDF  $f(x|\theta) = (\theta + 1)x^\theta, 0 \leq x \leq 1$

$$L(\theta) =$$

$$\hat{\theta}_{MLE} =$$

Suppose we observe a sample of size 5:  $\{.83, .49, .72, .57, .66\}$ . Find the maximum likelihood estimate and verify with a graph or numerical approximation

### 3 Uniform distribution

Find the MLE for  $Y_1, \dots, Y_n \sim \text{Unif}(0, \theta)$

## 4 Finding the MLE when more than one parameter is unknown

If the pdf or pmf that we're using has two or more parameters, say  $\theta_1$  and  $\theta_2$ , finding MLEs for the  $\theta_i$ 's requires the solution of a set of simultaneous equations. We would typically need to solve the following system:

$$\frac{\partial}{\partial \theta_1} \ln L(\theta_1, \theta_2) =$$

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2) =$$

**Example:** Suppose a random sample of size  $n$  is drawn from the two parameter normal pdf

$$f_y(y|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\left(\frac{y-\mu}{\sigma}\right)^2\right)$$

find the MLEs  $\hat{\mu}$  and  $\hat{\sigma}^2$