

# 07: METHOD OF MOMENTS

Stat250 S25

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## Theoretical Moment

## Sample Moment

## Method of Moments

**Example:** Suppose we observe  $y_1, \dots, y_n$  from  $f_y(y|\theta) = \theta y^{\theta-1}$ ,  $0 < y < 1$ . Find  $E(Y)$  and  $\hat{\theta}_{MoM}$

## A helpful fact

## Central Moment

### 1 Exercises

**Exercise 1:** Let  $X_1, \dots, X_n$  be an iid sample from a  $\text{Unif}(0, \theta)$  distribution.

- (a) Compute the first theoretical moment of this distribution
- (b) Use (a) to derive the MoM estimator of  $\theta$
- (c) Compute the MoM estimate if  $X_1 = X_2 = X_3 = 1$  and  $X_4 = 9$ .
- (d) Now compute the MLE estimate ( $\hat{\theta}_{MLE} = X_{max}$ ) if  $X_1 = X_2 = X_3 = 1$  and  $X_4 = 9$ .
- (e) Which estimator, the MLE or MoM, do you think is better in this case? Why?

**Exercise 2:** A manufacturing facility knows that historically 2% of items are defective. Each day, they manufacture  $k$  items and record the number of defective items (so the data is  $X_1, X_2, \dots, X_n \sim \text{Binom}(k, .02)$ ). We are able to see results from  $n$  days  $(x_1, x_2, \dots, x_n)$ , and our goal is to estimate  $k$ .

(a) Find  $\hat{\theta}_{MoM}$  (Note that  $E(X_i) = kp = .02k$  in this case)

(b) Now, suppose the rate of defective items is *unknown*. That is,  $X_i \sim \text{Binom}(k, p)$  for  $i = 1, \dots, n$ . Find the method of moments estimators for  $k$  and  $p$ . (Note that  $E(X_i) = kp$  and  $\text{Var}(X_i) = kp(1 - p)$ )

(c) Suppose we observe 2 days of results and the following outcomes are recorded. For each situation, give  $\hat{k}_{MoM}$  and  $\hat{p}_{MoM}$ . Do you obtain reasonable estimates?

i.  $X_1 = 1, X_2 = 1$

ii.  $X_1 = 1, X_2 = 3$

iii.  $X_1 = 1, X_2 = 5$

**Exercise 3:** Let  $Y_1, \dots, Y_n$  be a random sample from a normal distribution with unknown mean  $\mu$  and variance  $\sigma^2$ .

(a) What are the first two theoretical moments of this distribution? (You do not need to derive them!)

(b) Find the method of moments estimators for  $\mu$  and  $\sigma^2$ .

(c) When you use `var(x)` to compute the sample standard deviation of a variable, the standard deviation is computed with the formula  $\frac{1}{n-1} \sum (X_i - \bar{X})^2$ . Can you come up with a reason for why we might prefer this formula?