18: CLASSICAL HYPOTHESIS TESTS

Stat250 S25

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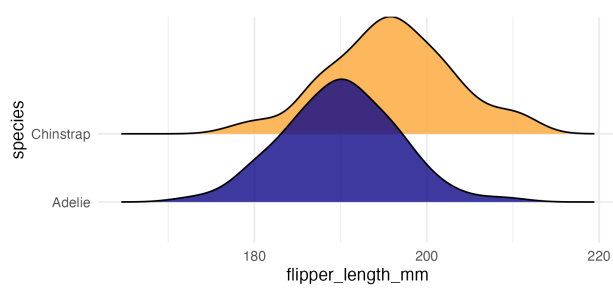
1 Logic of Testing

1. Formulate two competing **hypotheses** about the population: the *null hypothesis* and the *alternative hyptohesis*
2. Calculate a **test statistic** that summarizes the relevant information
3. Look at the **behavior of the test statistic** assuming that the null hypothesis is true
4. **Compare** the observed test statistic to the expected behavior (strength of evidence against the null)
5. State a **conclusion** in context.

2 Permutation Test Recap

Example: We are interested in whether there is a difference in the average flipper length between Adelie and Chinstrap penguins.



species	n	mean	sd
Adelie	151	189.95	6.54
Chinstrap	68	195.82	7.13

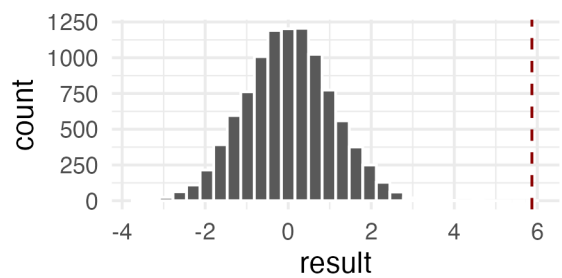
1. Pool the $n_1 + n_2$ data values
2. Draw a sample of size n_1 without replacement, assign those values to Group 1. Assign the remaining n_2 values to Group 2.
3. Calculate the test statistic comparing the samples from the resampled groups.
4. Repeat steps 2 and 3 until we have enough samples.
5. Estimate the p-value as the proportion of times the observed test statistic exceeds the original (observed) test statistic: p-value = $\frac{\text{\# statistics that exceed the original+1}}{\text{\# of statistics in the distribution+1}}$

```r
N <- 10^4 - 1 # Number of permutations to do
sample_size <- nrow(penguins_subset) # Sample size for each permutation (same as data)
x <- penguins_subset$flipper_length_mm # data vector

result <- numeric(N) # Create an empty vector to store results
for (i in 1:N){
  index <- sample(sample_size, 68, replace = FALSE) # Sample indices for group 1
  result[i] <- mean(x[index]) - mean(x[-index]) # Compute differences between groups
}

sum(result >= observed + 1)/(N+1)
```

```
[1] 0
```



3 T-Test for a difference in means

Instead of conducting a permutation test, we can instead **assume** that (a) our data comes from a normal population or (b) that the CLT applies.

$$\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim t_{df}$$

Example: Use the summary table from p1 to perform a t-test for $H_0 : \mu_{\text{Chinstrap}} - \mu_{\text{Adelie}} = 0$ against $H_A : \mu_{\text{Chinstrap}} - \mu_{\text{Adelie}} \neq 0$

```r
t.test(x_vector, y_vector)
t.test(numeric_vector ~ group_vector, data = dataset_name)
```

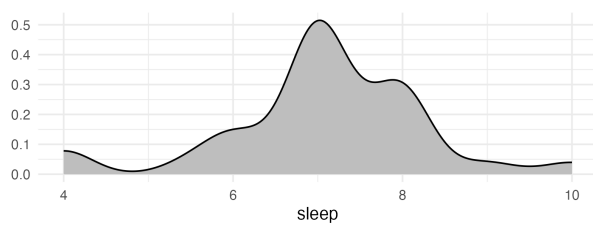
```r
t.test(flipper_length_mm ~ species, data = penguins_subset)
```

```
    Welch Two Sample t-test

data:  flipper_length_mm by species
t = -5.7804, df = 119.68, p-value = 6.049e-08
alternative hypothesis: true difference in means between group Adelie and group Chinstrap is not equal to 0
95 percent confidence interval:
 -7.880530 -3.859244
sample estimates:
   mean in group Adelie mean in group Chinstrap
               189.9536                195.8235
```

4 T-Test for one mean

Example: Do Carls sleep less than 8 hours per night? A sample of Stat 120 students answered the question: "On average, how many hours of sleep do you get on a weeknight?"



n	30.00
mean	7.07
sd	1.23
median	7.00
min	4.00
max	10.00

Set up a t-test for this question. Include **assumptions**, **hypotheses**, **test statistic**, **distribution of the test statistic** under the null hypothesis, and **p-value computation**.

```r
t.test(~sleep, data = survey, mu = 8, alternative = "less")
```

```
    One Sample t-test

data:  sleep
t = -4.1565, df = 29, p-value = 0.0001306
alternative hypothesis: true mean is less than 8
95 percent confidence interval:
     -Inf 7.448201
sample estimates:
mean of x
 7.066667
```

5 Hypothesis tests for binomial data

Example: Do Americans support a national health plan? A Kaiser Family Foundation poll for a random sample of US adults in 2019 found that 79% of Democrats, 55% of Independents, and 24% of Republicans supported a generic "National Health Plan." There were 347 Democrats, 298 Republicans, and 617 Independents surveyed. A political pundit on TV claims that a majority of Independents support a National Health Plan. Do these data provide strong evidence to support this type of statement?

Assume:

Hypotheses:

Test Statistic:

Null distribution:

```r
sum(dbinom(339:617, 617, .5))
```

```
[1] 0.007822546
```

```r
binom.test(x = 339, n = 617, p = 0.5, alternative = "greater")
```

```
data:  339 out of 617
number of successes = 339, number of trials = 617, p-value = 0.007823
alternative hypothesis: true probability of success is greater than 0.5
95 percent confidence interval:
 0.5155507 1.0000000
sample estimates:
probability of success
             0.5494327
```

CLT Assumption:

Approx null distribution:

Large-sample test statistic:

```r
prop.test(x = 339, n = 617, p = 0.5, alternative = "greater")
```

```
    1-sample proportions test with continuity correction

data:  339 out of 617
X-squared = 5.8347, df = 1, p-value = 0.007857
alternative hypothesis: true p is greater than 0.5
95 percent confidence interval:
 0.5155287 1.0000000
sample estimates:
        p
0.5494327
```

Is n large enough to use the CLT?

- Many textbooks suggest _____ and _____
- Our textbook suggest _____ and _____
- Otherwise, use the exact binomial test