EXAM 02 REVIEW

Stat250 S25

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Solutions

Note: These solutions are provided solely for Carleton College students enrolled in Amanda Luby's Stat250 course in Spring 2025. Dissemination of this solution to people who are not registered for this course is not permitted and will be considered grounds for an Academic Dishonesty report for all individuals involved in the giving and receiving of the solution.

Example question: Suppose that $X_1, ..., X_n$ form a random sample from an exponential distribution with mean $1/\lambda$. The sum of iid $\text{Exp}(\lambda)$ random variables is $\text{Gamma}(n, \lambda)$. Based on this fact, it can be shown that $\frac{\lambda}{2} \sum X_i \sim \text{Gamma}(n, \frac{1}{2})$. Explain why $\frac{\lambda}{2} \sum X_i$ is a pivotal quantity and then derive a formula for a $(1 - \alpha)$ confidence interval for λ.

Example question: 20 subjects are enrolled in a weight loss program, and each person's weight loss over one month was recorded (in pounds). Mean weight loss was 4.5 pounds with a sample standard deviation of 6.3 pounds. Assume that each observation comes from a normal distribution with unknown mean μ and unknown variance σ^2. It may help to know that:

```
qt(.05, df = 19)
> [1] -1.729133
qt(.975, df = 19)
> [1] 2.093024
qt(.05, df = 25)
> [1] -1.708141
qt(.975, df = 25)
> [1] 2.059539
```

(a) Find a 95% confidence interval for μ

(b) What assumptions/conditions must hold for the interval in (a) to be valid?

Example question: A major credit card company is planning a new offer for their current card holders. To test the effectiveness of the campaign, the company sent out offers to a random sample of 50,000 cardholders. Of those, 1184 registered for the new offer. If the acceptance rate is 2% or less, the campaign won't be worth the expense.

Consider the below R output to help you answer this question

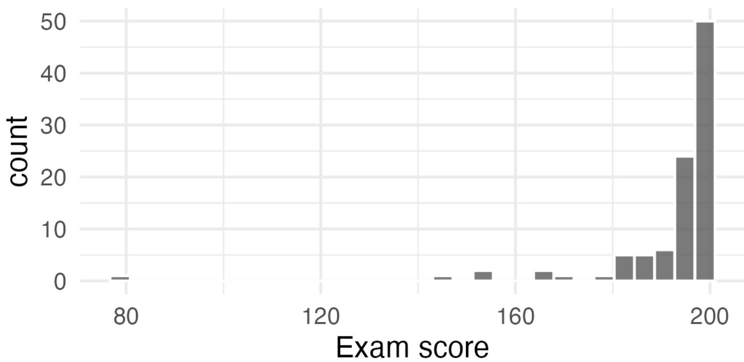
```
prop.test(x = 1184, n = 50000, conf.level = 0.9, alternative = "less")$conf
> [1] 0.00000000 0.02457739
> attr(,"conf.level")
> [1] 0.9
prop.test(x = 1184, n = 50000, conf.level = 0.9, alternative = "greater")$conf
> [1] 0.02281426 1.00000000
> attr(,"conf.level")
> [1] 0.9
prop.test(x = 1184, n = 50000, conf.level = 0.9, alternative = "two.sided")$conf
> [1] 0.02257725 0.02483475
> attr(,"conf.level")
> [1] 0.9
```

(a) Which confidence interval is most useful in exploring whether the campaign is worth the expense? Justify your answer.

(b) Will a 93% confidence interval be wider or narrower? How do you know?

Example question: The histogram below displays a random sample of 98 exam scores for graduating seniors in a certain country. We are interested in building a 90% confidence interval for μ, the average exam score.

(a) If we built a bootstrap percentile interval, formula-t confidence interval, and bootstrap-t confidence interval, would you expect to see nearly identical results? Why or why not?
(b) Which of the 3 intervals would you prefer in this scenario? Why?



Example Question: Subjects in the HELP (Health Evaluation and Linkage to Primary Care) study were asked about their depressive symptoms (CESD score) and homeless status (housed vs. homeless). Higher scores on the cesd measure indicate more depressive symptoms.

homeless	min	Q1	median	Q3	max	mean	sd	n
homeless	1.00	28.00	36.00	41.00	60.00	34.02	12.32	209
housed	3.00	23.00	32.00	40.00	58.00	31.84	12.62	244

(a) State the appropriate hypotheses to test whether there is a true difference between the CESD score for the groups based on homeless status.

(b) Calculate the appropriate test statistic (plug in completely, but you do not need to simplify).

(c) Explain how you would find the p-value for this test. Note that df = 443.365. (A randomly selected stat250 student should be able to follow your explanation and perform the calculation)

Example question: Suppose $X_1, ..., X_10$ are a random sample from a population with exponential distribution with $\lambda > 0$.

(a) Derive the most powerful test for $H_0 : \lambda = 7$ against $H_A : \lambda = 5$
(b) If we instead want to test $H_0 : \lambda \geq 7$ against $H_A : \lambda < 7$, explain how the most powerful test statistic would change

Example question: The time between arrivals are generally known to follow an exponential distribution. Let X_i be the time between arrivals at the TSA pre-check line at MSP Airport. Assume that $X_1, ..., X_5$ are a random sample from an exponential distribution with unknown parameter λ. We wish to test $H_0 : \lambda \geq .5$ against $H_A : \lambda < .5$ using the test statistic $Y = \sum X_i$. We will reject H_0 if $Y > c$, where c is chosen to give $\alpha = .05$.

(a) In this setting, $c = 18.3$. Show how this was found.
(b) Find the power of the test when $\lambda = .25$.

Example question: I am interested in testing whether there is a difference in the average number of hours slept per night among first-years, sophomores, and juniors. I conduct a random survey and obtain the following results.

year	min	Q1	median	Q3	max	mean	sd	n	missing
fy	5.31	7.18	7.69	8.19	9.53	7.70	0.74	66	0
soph	3.84	5.93	7.95	8.65	12.51	7.66	1.96	48	0
jr	5.36	7.05	7.75	8.70	10.04	7.86	1.25	25	0

(a) If I do the following tests at the $\alpha = .05$ level:

$H_0 : \mu_{fy} - \mu_{soph} = 0$ against $H_A : \mu_{fy} - \mu_{soph} \neq 0$

$H_0 : \mu_{fy} - \mu_{jr} = 0$ against $H_A : \mu_{fy} - \mu_{jr} \neq 0$

$H_0 : \mu_{soph} - \mu_{jr} = 0$ against $H_A : \mu_{soph} - \mu_{jr} \neq 0$

what is the probability I make at least one Type I error?

(b) What α level should I use for each test to obtain an overall Type I error rate of $\alpha^* = .05$ according to the Sidak correction? According to the Bonferroni?

(c) Which correction gives me greater power?