

# 21: LIKELIHOOD RATIO TESTS

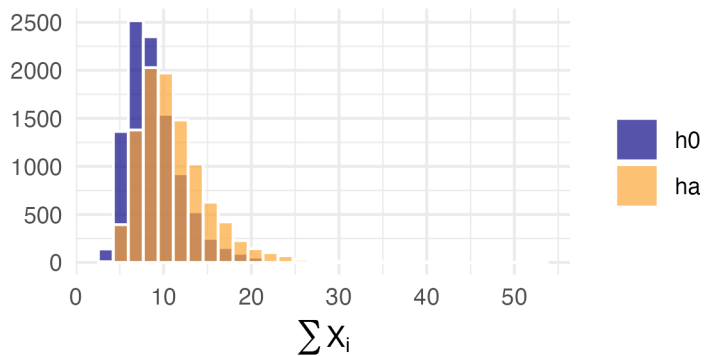
Stat250 S25

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Most of the inference that we've covered so far has been for limited settings (e.g. binomial or normal data) or based on simulation (e.g. bootstrap). Today, we're going to talk about some of the theoretical underpinnings for setting up more complicated tests.

**Example:** Suppose we observe  $X_1, \dots, X_9 \sim \text{Exp}(\theta)$  and are interested in testing  $H_0 : \theta = 9$  against  $H_A : \theta = 10$ .



Likelihood Ratio Test Statistic

### Neyman-Pearson Lemma

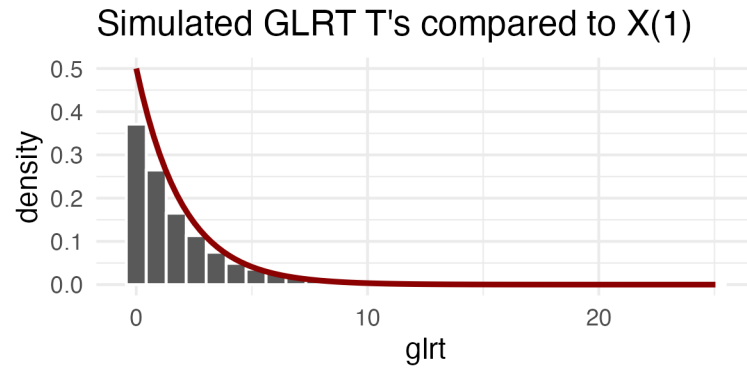
**Example:** Consider a sequence of  $n$  independent Bernoulli random variables  $X_1, \dots, X_n$ . We are interested in the most powerful test for  $H_0 : p = .4$  versus  $H_A : p = .3$ . Derive the form of the rejection region for this test directly. How would you define an  $\alpha$  level test in this case?

## 1 LRT for Composite Hypotheses

### Generalized Likelihood Ratio Test Statistic

**Example:** Suppose we observe  $X_1, \dots, X_9 \sim \text{Exp}(\theta)$  and are interested in testing  $H_0 : \theta \leq 8$  against  $H_A : \theta > 8$ .

### Wilk's Theorem



**Example:** Consider a sequence of  $n$  independent Bernoulli random variables  $X_1, \dots, X_n$ . We are interested in the most powerful test for  $H_0 : p = .4$  versus  $H_A : p < .4$ . Derive the form of the rejection region for this test directly. How would you define an  $\alpha$  level test in this case?