07: METHOD OF MOMENTS

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Theoretical Moment
Sample Moment
1
Method of Moments
Wethou of Moments
Example: Suppose we observe $y_1,,y_n$ from $f_y(y \theta)=\theta y^{\theta-1}, 0< y<1.$ Find $E(Y)$ and $\hat{\theta}_{MoM}$
f(y) = f(y) = f(y)
A helpful fact

1 Exercises

Exercise 1: Let $X_1,...,X_n$ be an iid sample from a $\mathrm{Unif}(0,\theta)$ distribution.

(a) Compute the first theoretical moment of this distribution

(b) Use (a) to derive the MoM estimator of θ

- (c) Compute the MoM estimate if $X_1=X_2=X_3=1$ and $X_4=9$.
- (d) Now compute the MLE estimate ($\hat{\theta}_{MLE}=X_{max}$) if $X_1=X_2=X_3=1$ and $X_4=9.$

(e) Which estimator, the MLE or MoM, do you think is better in this case? Why?

Exercise 2: A manufacturing facility knows that historically 2% of items are defective. Each day, they manufacture k items and record the number of defective items (so the data is $X_1, X_2, ..., X_n \sim \operatorname{Binom}(k, .02)$. We are able to see results from n days $(x_1, x_2, ..., x_n)$, and our goal is to estimate k.

(a) Find $\hat{\theta}_{MoM}$ (Note that $E(X_i)=kp=.02k$ in this case)

(b) Now, suppose the rate of defective items is $\mathit{unknown}$. That is, $X_i \sim \mathrm{Binom}(k,p)$ for i=1,...,n. Find the method of moments estimators for k and p. (Note that $E(X_i) = kp$ and $\mathrm{Var}(X_i) = kp(1-p)$)

(c) Suppose we observe 2 days of results and the following outcomes are recorded. For each situation, give \hat{k}_{MoM} and \hat{p}_{MoM} . Do you obtain reasonable estimates?

i.
$$X_1 = 1, X_2 = 1$$

ii.
$$X_1 = 1, X_2 = 3$$

iii.
$$X_1 = 1, X_2 = 5$$

Exercise 3: Let $Y_1,...,Y_n$ be a random sample from a normal distribution with unknown mean μ and variance σ^2 .

(a) What are the first two theoretical moments of this distribution? (You do not need to derive them!)

(b) Find the method of moments estimators for μ and $\sigma^2.$

(c) When you use $\operatorname{var}(\mathbf{x})$ to compute the sample standard deviation of a variable, the standard deviation is computed with the formula $\frac{1}{n-1}\sum (X_i-\bar{X})^2$. Can you come up with a reason for why we might prefer this formula?