19: ERRORS + POWER

Stat250 S25

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Example: A high school was chosen to participate in the evaluation of a new geometry and algebra curriculum. In the recent past, the school's students were considered "typical", receiving scores on standardized tests that were very close to the nationwide average. In the year of the study, 86 sophomores were randomly selected to participate in a special set of classes that integrated geometry and algebra. Those students averaged 502 on the SAT-I math exam; the nationwide average was 494 with a standard deviation of 124.

Administrator A thinks that an average above 500 indicates a big enough improvement to warrant a change in curriculum, but Administrator B thinks that the average should be above 600. If there was actually no improvement, what's the probability we come to the wrong conclusion under Administrator A and B's cutoffs?

Decision Rule

Mapping to Z-scores:

Critical Region

Setting a significance level:

1 Types of Errors

In any hypothesis test procedure, there are two ways we can be wrong: we can (1) conclude H_0 is true when H_1 is actually true, or we can (2) conclude H_0 is false when H_0 is actually true.

	H_0 True	H_1 True
Reject H_0	Type I Error	Correct
Fail to reject H_0	Correct	Type II Error

Type I vs Type II Error tradeoff

Effect size and Type II error

SE and Type II Error

Power

Example: If the true μ in the Math curriculum example is 498 (so H_0 is false), what is the power of the test of Administrator A? What if true μ is 510?

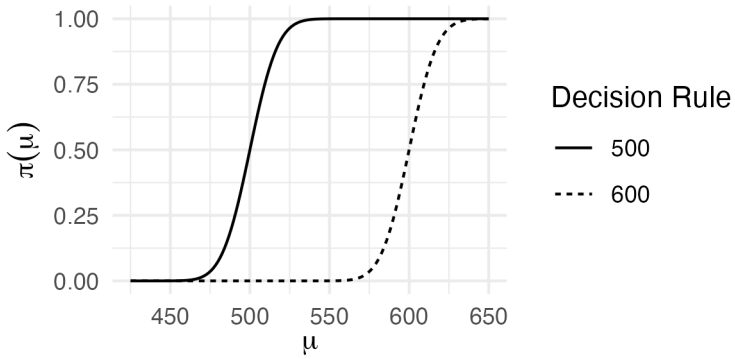
2 Power Function

Power Function

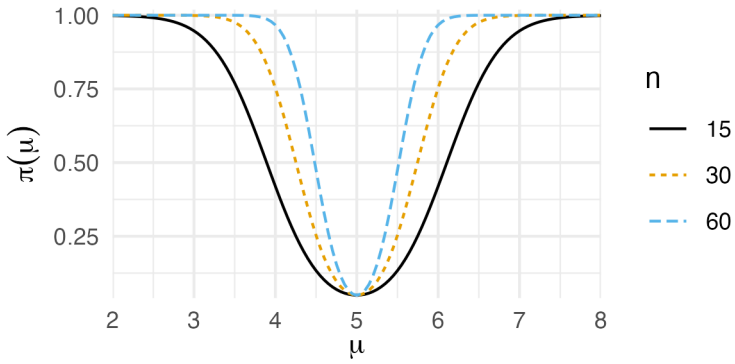
Note: What does an ideal power function look like?

- if $\theta = \theta_0$
- if $\theta \in \Omega_A$

Example: Math Curriculum Example



Example: One sample t-test for $H_0 : \mu = 5$ vs. $H_a : \mu \neq 5$ at the $\alpha = .05$ level (assuming $s = 2$).



Example: Let $Y_i \sim N(\mu, 52)$. We wish to test $H_0 : \mu = 7$ vs. $H_A : \mu > 7$ at the $\alpha = 0.05$ level. What is the smallest sample size such that the test has power at least .80 when $\mu = 8$?