06: MORE ON MAXIMUM LIKELIHOOD

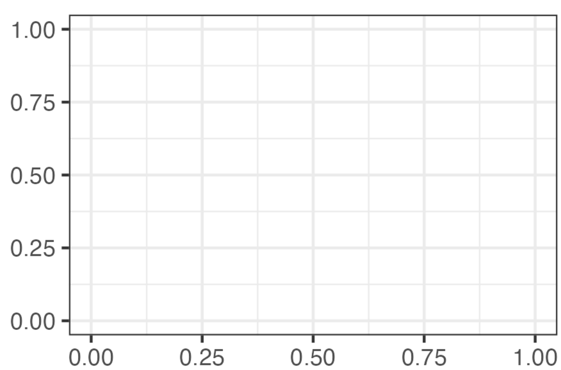
Stat250 S25

Prof Amanda Luby

1 Overview

2 More on the likelihood function and MLEs

Example: Recall that the likelihood function for n iid Bernoulli(θ) random variable is $L(\theta) = \theta^{\sum x_i}(1-\theta)^{n-\sum x_i}$



Scenario 1: 0 “Yes” responses

θ	0.0	0.2	0.4	0.6	0.8	1.0
$L(\theta)$						

Scenario 2: 6 “Yes” responses

θ	0.0	0.2	0.4	0.6	0.8	1.0
$L(\theta)$						

Exercise Let $X_1, \dots, X_n$ be an iid random sample from a distribution with PDF

$$f(x|\theta) = (\theta + 1)x^{\theta}, 0 \leq x \leq 1$$

$$L(\theta) =$$

$$\hat{\theta}_{MLE} =$$

Suppose we observe a sample of size 5: {.83, .49, .72, .57, .66}. Find the maximum likelihood estimate and verify with a graph or numerical approximation

3 Uniform distribution

Find the MLE for $Y_1, ..., Y_n \sim \text{Unif}(0, \theta)$

Exercise: Is the likelihood function a probability distribution? Why or why not?

4 Finding the MLE when more than one parameter is unknown

If the pdf or pmf that we're using has two or more parameters, say θ_1 and θ_2, finding MLEs for the θ_i's requires the solution of a sest of simultaneous equations. We would typically need to solve the following system:

$$\frac{\partial}{\partial \theta_1} \ln L(\theta_1, \theta_2) =$$

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2) =$$

Example: Suppose a random sample of size n is drawn from the two parameter normal pdf

$$f_y(y|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma}} \exp(\frac{-(y-\mu)^2}{2\sigma^2})$$

find the MLEs $\hat{\mu}$ and $\hat{\sigma}^2$