

# 05: MAXIMUM LIKELIHOOD ESTIMATION

Stat250-S25

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## 1 WARM UP

**Example:** You are testing seeds from a new plant variety. You plant 10 seeds ( $n=10$ ) and observe that 3 of them successfully germinate ( $k=3$ ). Consider two hypotheses about the true germination probability ( $p$ ) for this variety:

Hypothesis A:

Hypothesis B:

Given you observed 3 germinations out of 10 seeds, which hypothesis (A or B) is more supported by the data?

In general, the *likelihood* of seeing  $X = 3$  given  $p$  is:

3 ways of finding the maximum:

1. Approximate solution graphically

```
germ_function <- function(p) {choose(10,3) * p^3 * (1-p)^7} # define function
ggplot() +
  geom_function(fun = germ_function) +
  xlim(c(0,1))
```



2. Find a numerical approximation

```
optimize(germ_function, interval = c(0,1), maximum = TRUE)
```

```
$maximum  
[1] 0.3000157
```

```
$objective  
[1] 0.2668279
```

3. Use calculus to find an exact maximum

Can we generalize to *any* data?

## 2 DEFINITIONS

### Likelihood function

Let  $f(x; \theta)$  denote the probability mass function for a discrete distribution with associated parameter  $\theta$ . Suppose  $X_1, X_2, \dots, X_n$  are a random sample from this distribution and  $x_1, x_2, \dots, x_n$  are the actual observed values. Then, the *likelihood function* of  $\theta$  is:

### Maximum likelihood estimate

A maximum likelihood estimate (MLE),  $\hat{\theta}_{MLE}$  is the value of  $\theta$  that maximizes the likelihood function, or equivalently, that maximizes the log-likelihood  $l(\theta) = \ln L(\theta)$

### Bernoulli Data Example

**Estimator**

**Estimate**

### 3 EXERCISE

Let  $X_1, \dots, X_n$  be an iid random sample from a distribution with PDF

$$f(x|\theta) = (\theta + 1)x^\theta, 0 \leq x \leq 1$$

1. Find the maximum likelihood estimator for a random sample of size  $n$  using calculus.
2. Suppose we observe a sample of size 5:  $\{.83, .49, .72, .57, .66\}$ . Find the maximum likelihood estimate and verify with a graph or numerical approximation