23: CHI-SQUARED TESTS

Stat250 S25

Prof Amanda Luby

Example: The General Social Survey (GSS) is a sociological survey used to collect data on demographic characteristics and attitudes of residents of the United States. We'll consider two survey questions:

- Compared with American families in general, would you say your family income is far below average, below average, average, above average, or far above average?
- Taken all together, how would you say things are these days—would you say that you are very happy, pretty happy, or not too happy?

happy	far below average	below average	average	above average	far above average	Total
not too happy	50	123	120	33	4	330
pretty happy	64	350	602	253	24	1293
very happy	39	121	319	190	25	694
Total	153	594	1041	476	53	2317

H_0

H_A

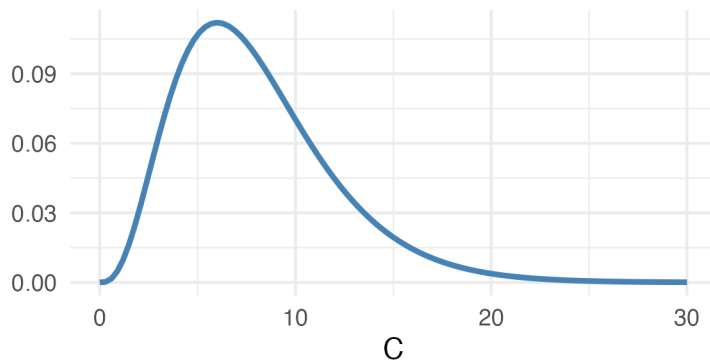
Expected table under H_0

happy	far below average	below average	average	above average	far above average
not too happy	21.79111	84.60078	148.2650	67.79456	7.548554
pretty happy	85.38153	331.48123	580.9292	265.63142	29.576608
very happy	45.82736	177.91800	311.8058	142.57402	15.874838

How was this table computed?

Test Statistic:

When H_0 is true, $C \dot{\sim}$ _____



3 ways to find p-values:

1. "By hand"

```
sum((observed - expected)^2/expected)
> [1] 128.0785

1-pchisq(sum((observed - expected)^2/expected), df = (3-1)*(5-1))
> [1] 0
```

2. `chisq.test` with vectors

```
chisq.test(happy2018$happy, happy2018$finrela)
>
>   Pearson's Chi-squared test
>
> data:  happy2018$happy and happy2018$finrela
> X-squared = 128.08, df = 8, p-value < 2.2e-16
```

3. `chisq.test` with table

```
chisq.test(observed)
>
>   Pearson's Chi-squared test
>
> data:  observed
> X-squared = 128.08, df = 8, p-value < 2.2e-16
```

Permutation test

1. Store the data in a table: one row per observation, one column per variable.
2. Calculate a test statistic for the original data.

3. **Repeat**

    - Randomly permute the rows in one of the columns.
    - Calculate the test statistic for the permuted data.

4. Calculate the p-value as the fraction of times the random statistics exceed the original statistic.

1. Drop any missing values

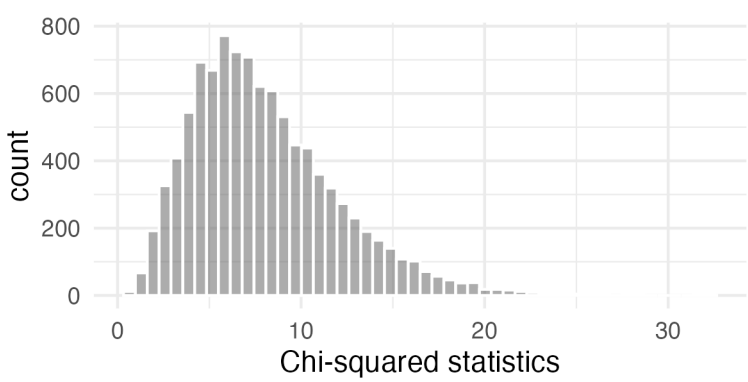
```
df <- happy2018 |> drop_na(happy, finrela)
```

2. Calculate the observed test statistic

```
observed <- chisq.test(happy2018$happy, happy2018$finrela)$statistic
```

3. Construct the permutation distribution

```
set.seed(55057)
N <- 10^4 - 1
result <- numeric(N)
for(i in 1:N) {
  finrela_perm <- sample(happy2018$finrela)
  result[i] <- chisq.test(happy2018$happy, finrela_perm)$statistic
}
```



4. Find p-value

```
(sum(result >= observed) + 1) / (N + 1)
> [1] 1e-04
```

Alternatively, use a shortcut to the permutation test:

```r
chisq.test(happy2018$happy, happy2018$finrela, simulate.p.value = TRUE)
>
>   Pearson's Chi-squared test with simulated p-value (based on 2000
>   replicates)
>
> data:  happy2018$happy and happy2018$finrela
> X-squared = 128.08, df = NA, p-value = 0.0004998
```

Caution!

The χ^2 distribution provides a reasonable approximation of the null distribution **as long as the sample size is "large enough"**

- "Cochran's rule:" All of the cells have **expected counts** > _____
- All expected counts are at least _____ and no more than 20% of cells have **expected counts** < _____

Use a permutation test if the expected counts aren't large enough

Example: Some people think that children who are the older ones in their class at school naturally perform better in sports and that these children then get more coaching and encouragement as they get older. Could that make a difference in who makes it to the professional level in sports? Below is the birth month of 1478 major league players born since 1975, along with the national birth percentage across the same years.

Month	1	2	3	4	5	6	7	8	9	10	11	12
Obs	137	121	116	121	126	114	102	165	134	115	105	122
Birth %	8%	7%	8%	8%	8%	8%	9%	9%	9%	9%	8%	9%

- Write out an appropriate null and alternative hypothesis
- Calculate the expected counts for each cell under the null hypothesis
- Compute the chi-square test statistic
- Compute the p-value
- Draw a conclusion in context

Likelihood Ratio test