

06: MORE ON MAXIMUM LIKELIHOOD

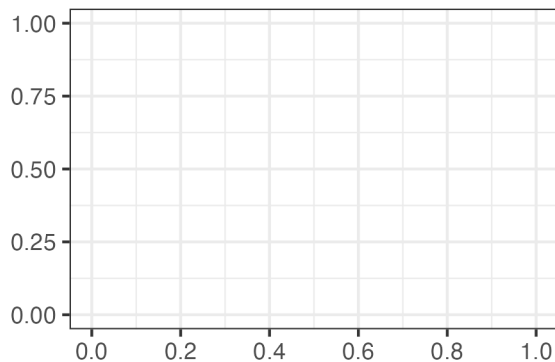
Stat250 S25

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1 Overview

2 More on the likelihood function and MLEs

Example: Recall that the likelihood function for n iid Bernoulli(θ) random variable is $L(\theta) = \theta^{\sum x_i} (1 - \theta)^{n - \sum x_i}$



Scenario 1: 0 “Yes” responses

| θ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|-------------|-----|-----|-----|-----|-----|-----|
| $L(\theta)$ | | | | | | |

Scenario 2: 6 “Yes” responses

| θ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
|-------------|-----|-----|-----|-----|-----|-----|
| $L(\theta)$ | | | | | | |

Exercise: Is the likelihood function a probability distribution? Why or why not?

Exercise Let X_1, \dots, X_n be an iid random sample from a distribution with PDF $f(x|\theta) = (\theta + 1)x^\theta, 0 \leq x \leq 1$

$$L(\theta) =$$

$$\hat{\theta}_{MLE} =$$

Suppose we observe a sample of size 5: $\{.83, .49, .72, .57, .66\}$. Find the maximum likelihood estimate and verify with a graph or numerical approximation

3 Uniform distribution

Find the MLE for $Y_1, \dots, Y_n \sim \text{Unif}(0, \theta)$

4 Finding the MLE when more than one parameter is unknown

If the pdf or pmf that we're using has two or more parameters, say θ_1 and θ_2 , finding MLEs for the θ_i 's requires the solution of a set of simultaneous equations. We would typically need to solve the following system:

$$\frac{\partial}{\partial \theta_1} \ln L(\theta_1, \theta_2) =$$

$$\frac{\partial}{\partial \theta_2} \ln L(\theta_1, \theta_2) =$$

Example: Suppose a random sample of size n is drawn from the two parameter normal pdf

$$f_y(y|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\left(\frac{y-\mu}{\sigma}\right)^2\right)$$

find the MLEs $\hat{\mu}$ and $\hat{\sigma}^2$