07: METHOD OF MOMENTS

Stat250 S25

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Theoretical Moment

Sample Moment

Method of Moments

Example: Suppose we observe $y_1, ..., y_n$ from $f_y(y|\theta) = \theta y^{\theta-1}$, $0 < y < 1$. Find $E(Y)$ and $\hat{\theta}_{MoM}$

A helpful fact

Central Moment

1 Exercises

Exercise 1: Let $X_1, \dots, X_n$ be an iid sample from a Unif$(0, \theta)$ distribution.

(a) Compute the first theoretical moment of this distribution

(b) Use (a) to derive the MoM estimator of θ

(c) Compute the MoM estimate if $X_1 = X_2 = X_3 = 1$ and $X_4 = 9$.

(d) Now compute the MLE estimate ($\hat{\theta}_{MLE} = X_{max}$) if $X_1 = X_2 = X_3 = 1$ and $X_4 = 9$.

(e) Which estimator, the MLE or MoM, do you think is better in this case? Why?

Exercise 2: A manufacturing facility knows that historically 2% of items are defective. Each day, they manufacture k items and record the number of defective items (so the data is $X_1, X_2, ..., X_n \sim \text{Binom}(k, .02)$. We are able to see results from n days $(x_1, x_2, ..., x_n)$, and our goal is to estimate k.

(a) Find $\hat{\theta}_{MoM}$ (Note that $E(X_i) = kp = .02k$ in this case)

(b) Now, suppose the rate of defective items is *unknown*. That is, $X_i \sim \text{Binom}(k, p)$ for $i = 1, ..., n$. Find the method of moments estimators for k and p. (Note that $E(X_i) = kp$ and $\text{Var}(X_i) = kp(1-p)$)

(c) Suppose we observe 2 days of results and the following outcomes are recorded. For each situation, give $\hat{k}_{MoM}$ and $\hat{p}_{MoM}$. Do you obtain reasonable estimates?

i. $X_1 = 1, X_2 = 1$

ii. $X_1 = 1, X_2 = 3$

iii. $X_1 = 1, X_2 = 5$

Exercise 3: Let $Y_1, ..., Y_n$ be a random sample from a normal distribution with unknown mean μ and variance σ^2.

(a) What are the first two theoretical moments of this distribution? (You do not need to derive them!)

(b) Find the method of moments estimators for μ and σ^2.

(c) When you use `var(x)` to compute the sample standard deviation of a variable, the standard deviation is computed with the formula $\frac{1}{n-1} \sum (X_i - \bar{X})^2$. Can you come up with a reason for why we might prefer this formula?