

# Individual HW05

Your Name Here

```
library(bayesrules) # R package for our textbook
library(tidyverse) # Collection of packages for tidying and plotting data
library(janitor) # Helper functions like tidy and tabyl
library(rstan) # for MCMC
library(bayesplot) # for plotting
```

## 1 BR Exercise 8.7

For each situation, find the appropriate credible interval using the “middle” approach

- (a) A 99% credible interval of  $\lambda$  where  $\lambda|y \sim \text{Gamma}(1,5)$
- (b) A 95% credible interval of  $\mu$  where  $\mu|y \sim N(10, 2^2)$
- (c) An 80% credible interval of  $\mu$  where  $\mu|y \sim N(-3, 1^2)$

## 2 BR Exercise 8.8

There’s more than one approach to constructing a 95% credible interval. The “middle 95%” approach reports the range of the middle 95% of the posterior density, from the 2.5th to the 97.5th percentile. The “highest posterior density” approach reports the 95% of posterior values with the highest posterior densities.

- (a) Let  $\lambda|y \sim \text{Gamma}(1,5)$ . Construct the 95% highest posterior density credible interval for  $\lambda$ . Represent this interval on a sketch of the posterior pdf. *Hint:* The sketch itself will help you identify the appropriate CI. Do not try to find a solution that will generalize to any posterior density, just focus on this specific density.
- (b) Repeat part a using the middle 95% approach.
- (c) Compare the two intervals from parts a and b. Are they the same? If not, how do they differ and which is more appropriate here?
- (d) Let  $\mu|y \sim N(-13, 2^2)$ . Construct the 95% highest posterior density credible interval for  $\mu$ .
- (e) Repeat part d using the middle 95% approach.

- (f) Compare the two intervals from parts d and e. Are they the same? If not, why not?

### 3 BR Exercise 8.21

The loon is a species of bird common to the Ontario region of Canada. Let  $\lambda$  denote the typical number of loons observed by a birdwatcher across a 100-hour observation period. To learn about  $\lambda$ , we'll utilize bird counts  $(Y_1, \dots, Y_n)$  collected in  $n$  different outings.

- (a) Explain which Bayesian model is appropriate for this analysis: Beta-Binomial, Gamma-Poisson, or Normal-Normal.
- (b) Your prior understanding is that the typical rate of loon sightings is 2 per 100 hours with a standard deviation of 1 per 100-hours. Specify an appropriate prior model for  $\lambda$  and explain your reasoning.
- (c) The `loons` data in the `{bayesrules}` package contains loon counts in different 100-hour observation periods. How many data points do we have and what's the average loon count per 100 hours?
- (d) In light of your prior and data, calculate and interpret a (middle) 95% posterior credible interval for  $\lambda$ . NOTE: You'll first need to specify your posterior model of  $\lambda$

### 4 BR Exercise 8.22

Let's continue our analysis of  $\lambda$ , the typical rate of loon sightings in a 100-hour observation period. You hypothesize that birdwatchers should anticipate a rate of less than 1 loon per observation period.

- (a) State this as a formal hypothesis test (using  $H_0$ ,  $H_a$ , and  $\lambda$  notation)
- (b) What decision might you make about these hypotheses utilizing the credible interval from the previous exercise?
- (c) Calculate and interpret the posterior probability that your hypothesis is true.
- (d) Putting this together, explain your conclusion about  $\lambda$

### 5 8.24: Posterior predictive distribution of Loon sightings (modified slightly)

- (a) Simulate the posterior model of  $\lambda$ , the typical rate of loon sightings per observation period, with `{rstan}` using 4 chains and 10000 iterations per chain. Perform some MCMC diagnostics to confirm that your simulation has stabilized.

- (b) Use your MCMC simulation to approximate the posterior predictive model of  $Y'$ , the number of loons that a birdwatcher will spy in their *next* observation period. Construct a visualization of this model.
- (c) Summarize your observations/conclusions of the posterior predictive model of  $Y'$
- (d) Approximate the probability that the birdwatcher observes 0 loons in their next observation period.

## 6 8.13: Posterior predictive Normal-Normal with calculus

Let  $Y = y$  be an observed data point from a  $N(\mu, \sigma^2)$  model. Further, suppose  $\sigma$  is known and that  $\mu$  is unknown with a  $N(\theta, \tau^2)$  prior.

- (a) Identify the posterior pdf of  $\mu$  given the observed data,  $f(\mu|y)$
- (b) Let  $Y' = y'$  be the value of a *new* data point. Identify the conditional PDF of  $Y'$  given  $\mu$ ,  $f(y'|\mu)$
- (c) Identify the posterior predictive pdf of  $Y'$ ,  $f(y'|y)$
- (d) Suppose  $y = -10$ ,  $\sigma = 3$ ,  $\theta = 0$  and  $\tau = 1$ . Specify and sketch the posterior predictive pdf of  $Y'$