Group Work 03

```
library(bayesrules) # R package for our textbook
library(tidyverse) # Collection of packages for tidying and plotting data
library(janitor) # Helper functions like tidy and tabyl
```

> **ℹ Note**
>
> No group problems from Friday

1 BR Exercise 7.6: Proposing a new location

In each situation below, complete Step 1 of the Metropolis-Hastings algorithm. That is, starting from teh given current chain value $\lambda^{(i)} = \lambda$ and with `set.seed(84735)`, use the given proposal model to draw a λ' proposal value for the next value in the chain $\lambda^{(i+1)}$

(a) $\lambda = 4.6, \lambda'|\lambda \sim N(\lambda, 2^2)$
(b) $\lambda = 2.1, \lambda'|\lambda \sim N(\lambda, 7^2)$
(c) $\lambda = 8.9, \lambda'|\lambda \sim Unif(\lambda - 2, \lambda + 2)$
(d) $\lambda = 1.2, \lambda'|\lambda \sim Unif(\lambda - 0.5, \lambda + 0.5)$
(e) $\lambda = 7.7, \lambda'|\lambda \sim Unif(\lambda - 3, \lambda + 3)$
(f) Discuss why these proposal models "make sense"

2 BR Exercise 7.7: Calculate the acceptance probability

Suppose that a Markov chain is currently at $\lambda^{(i)} = 2$ and the proposal for $\lambda^{(i+1)}$ is $\lambda' = 2.1$. For each pair of unnnormalized pdf $f(\lambda)L(\lambda|y)$ and proposal $q(\lambda'|\lambda)$, calculate the acceptance probability α used in Step 2 of the Metropolis-Hastings algorithm.

(a) $f(\lambda)L(\lambda|y) = \lambda^{-1}, \lambda'|\lambda \sim N(\lambda, 2^2)$ with pdf $q(\lambda'|\lambda)$
(b) $f(\lambda)L(\lambda|y) = e^{3\lambda}, \lambda'|\lambda \sim N(\lambda, .5^2)$ with pdf $q(\lambda'|\lambda)$
(c) $f(\lambda)L(\lambda|y) = e^{-1.9\lambda}, \lambda'|\lambda \sim Unif(\lambda - .3, \lambda + .3)$ with pdf $q(\lambda'|\lambda)$

(d) $f(\lambda)L(\lambda|y) = e^{-\lambda^4}, \lambda'|\lambda \sim Exp(\lambda)$ with pdf $q(\lambda'|\lambda)$

(e) For which of these scenarios is there a 100% acceptance probability? Explain why we'd certainly want to accept λ in these scenarios

3 BR 7.14

Consider a Gamma-Poisson model in which rate λ has a Gamma(1,0.1) prior and you observe one Poisson data point, $Y = 4$. In this exercise, you will simulate the posterior model of λ using an independence sampler.

(a) Which of the following would make the most reasonable proposal model to use in your independence sampler for λ: Normal, Beta, or Exponential? Hint: Recall that $\lambda > 0$.

(b) Using the proposal model that you identified in part a, simulate a tour of 1000 λ values. Tune your algorithm until you are satisfied and produce a trace plot of your final results.

(c) This is a situation where we can derive the exact posterior model of λ. What is it?

(d) Plot a histogram of your tour and overlay the exact model. How good is your Markov chain approximation?

4 Independence Sampler for truncated normal prior

Metropolis-Hastings and Independence Samplers are powerful when we *don't* have a closed-form solution. Suppose we instead use a truncated normal model for λ. This pdf *looks* like a normal distribution, but is truncated at lower and/or upper bounds. The code chunk below loads the {truncnorm} package, which gives you access to `dtruncnorm`, `rtruncnorm`, etc, and plots a truncated normal with parameters $\mu = 0$, $\sigma = 1$ and $a = 0$.

(a) Test different values of `a`, `mean`, and `sd` to get a sense of what each parameter does. Provide explanations of each.

(b) Tune the truncated normal prior to cover a similar range as the Gamma distribution above

(c) Modify your independence sampler from the previous question to use a truncated normal prior distribution. Tune your algorithm until you are satisfied and produce a trace plot of your final results.

(d) Compare your posterior to the posterior from the last question.

```
library(truncnorm)

ggplot(data = data.frame(x = c(-4, 4)), aes(x = x)) +
  stat_function(fun = dtruncnorm,
                args = list(a = 0, mean = 0, sd = 1))
```