Individual HW05

Your Name Here

```
library(bayesrules) # R package for our textbook
library(tidyverse) # Collection of packages for tidying and plotting data
library(janitor) # Helper functions like tidy and tabyl
library(rstan) # for MCMC
library(bayesplot) # for plotting
```

1 BR Exercise 8.7

For each situation, find the appropriate credible interval using the "middle" approach

- (a) A 99% credible interval of λ where $\lambda | y \sim \text{Gamma}(1,5)$
- (b) A 95% credible interval of μ where $\mu|y \sim N(10, 2^2)$
- (c) An 80% credible interval of μ where $\mu|y \sim N(-3, 1^2)$

2 BR Exercise 8.8

There's more than one approach to constructing a 95% credible interval. The "middle 95%" approach reports the range of the middle 95% of the posterior density, from the 2.5th to the 97.5th percentile. The "highest posterior density" approach reports the 95% of posterior values with the highest posterior densities.

- (a) Let $\lambda | y \sim \text{Gamma}(1,5)$. Construct the 95% highest posterior density credible interval for λ . Represent this interval on a sketch of the posterior pdf. *Hint:* The sketch itself will help you identify the appropriate CI. Do not try to find a solution that will generalize to any posterior density, just focus on this specific density.
- (b) Repeat part a using the middle 95% approach.
- (c) Compare the two intervals from parts a and b. Are they the same? If not, how do they differ and which is more appropriate here?
- (d) Let $\mu|y \sim N(-13, 2^2)$. Construct the 95% highest posterior density credible interval for μ .
- (e) Repeat part d using the middle 95% approach.

(f) Compare the two intervals from parts d and e. Are they the same? If not, why not?

3 BR Exercise 8.21

The loon is a species of bird common to the Ontario region of Canada. Let λ denote the typical number of loons observed by a birdwatcher across a 100-hour observation period. To learn about λ , we'll utilize bird counts $(Y_1,...,Y_n)$ collected in n different outings.

- (a) Explain which Bayesian model is appropriate for this analysis: Beta-Binomial, Gamma-Poisson, or Normal-Normal.
- (b) Your prior understanding is that the typical rate of loon sightings is 2 per 100 hours with a standard deviation of 1 per 100-hours. Specify an appropriate prior model for λ and explain your reasoning.
- (c) The loons data in the {bayesrules} package contains loon counts in different 100-hour observation periods. How many data points do we have and what's the average loon count per 100 hours?
- (d) In light of your prior and data, calculate and interpret a (middle) 95% posterior credible interval for λ . NOTE: You'll first need to specify your posterior model of λ

4 BR Exercise 8.22

Let's continue our analysis of λ , the typical rate of loon sightings in a 100-hour observation period. You hypothesize that birdwatchers should anticipate a rate of less than 1 loon per observation period.

- (a) State this as a formal hypothesis test (using $H_0,\,H_a,\,$ and λ notation)
- (b) What decision might you make about these hypotheses utilizing the credible interval from the previous exercise?
- (c) Calculate and interpret the posterior probability that your hypothesis is true.
- (d) Putting this together, explain your conclusion about λ

5 Q5 (TBA Wed)

6 Q6 (TBA Wed)