

Individual HW05

Your Name Here

```
library(bayesrules) # R package for our textbook
library(tidyverse) # Collection of packages for tidying and plotting data
library(janitor) # Helper functions like tidy and tabyl
library(rstan) # for MCMC
library(bayesplot) # for plotting
```

1 BR Exercise 8.7

For each situation, find the appropriate credible interval using the “middle” approach

- (a) A 99% credible interval of λ where $\lambda|y \sim \text{Gamma}(1,5)$
- (b) A 95% credible interval of μ where $\mu|y \sim N(10, 2^2)$
- (c) An 80% credible interval of μ where $\mu|y \sim N(-3, 1^2)$

2 BR Exercise 8.8

There's more than one approach to constructing a 95% credible interval. The “middle 95%” approach reports the range of the middle 95% of the posterior density, from the 2.5th to the 97.5th percentile. The “highest posterior density” approach reports the 95% of posterior values with the highest posterior densities.

- (a) Let $\lambda|y \sim \text{Gamma}(1,5)$. Construct the 95% highest posterior density credible interval for λ . Represent this interval on a sketch of the posterior pdf. *Hint:* The sketch itself will help you identify the appropriate CI. Do not try to find a solution that will generalize to any posterior density, just focus on this specific density.
- (b) Repeat part a using the middle 95% approach.
- (c) Compare the two intervals from parts a and b. Are they the same? If not, how do they differ and which is more appropriate here?
- (d) Let $\mu|y \sim N(-13, 2^2)$. Construct the 95% highest posterior density credible interval for μ .
- (e) Repeat part d using the middle 95% approach.

- (f) Compare the two intervals from parts d and e. Are they the same? If not, why not?

3 BR Exercise 8.21

The loon is a species of bird common to the Ontario region of Canada. Let λ denote the typical number of loons observed by a birdwatcher across a 100-hour observation period. To learn about λ , we'll utilize bird counts (Y_1, \dots, Y_n) collected in n different outings.

- (a) Explain which Bayesian model is appropriate for this analysis: Beta-Binomial, Gamma-Poisson, or Normal-Normal.
- (b) Your prior understanding is that the typical rate of loon sightings is 2 per 100 hours with a standard deviation of 1 per 100-hours. Specify an appropriate prior model for λ and explain your reasoning.
- (c) The `loons` data in the `{bayesrules}` package contains loon counts in different 100-hour observation periods. How many data points do we have and what's the average loon count per 100 hours?
- (d) In light of your prior and data, calculate and interpret a (middle) 95% posterior credible interval for λ . NOTE: You'll first need to specify your posterior model of λ

4 BR Exercise 8.22

Let's continue our analysis of λ , the typical rate of loon sightings in a 100-hour observation period. You hypothesize that birdwatchers should anticipate a rate of less than 1 loon per observation period.

- (a) State this as a formal hypothesis test (using H_0 , H_a , and λ notation)
- (b) What decision might you make about these hypotheses utilizing the credible interval from the previous exercise?
- (c) Calculate and interpret the posterior probability that your hypothesis is true.
- (d) Putting this together, explain your conclusion about λ

5 8.24: Posterior predictive distribution of Loon sightings (modified slightly)

- (a) Simulate the posterior model of λ , the typical rate of loon sightings per observation period, with `{rstan}` using 4 chains and 10000 iterations per chain. Perform some MCMC diagnostics to confirm that your simulation has stabilized.

- (b) Use your MCMC simulation to approximate the posterior predictive model of Y' , the number of loons that a birdwatcher will spy in their *next* observation period. Construct a visualization of this model.
- (c) Summarize your observations/conclusions of the posterior predictive model of Y'
- (d) Approximate the probability that the birdwatcher observes 0 loons in their next observation period.

6 8.13: Posterior predictive Normal-Normal with calculus

Let $Y = y$ be an observed data point from a $N(\mu, \sigma^2)$ model. Further, suppose σ is known and that μ is unknown with a $N(\theta, \tau^2)$ prior.

- (a) Identify the posterior pdf of μ given the observed data, $f(\mu|y)$
- (b) Let $Y' = y'$ be the value of a *new* data point. Identify the conditional PDF of Y' given μ , $f(y'|\mu)$
- (c) Identify the posterior predictive pdf of Y' , $f(y'|y)$
- (d) Suppose $y = -10$, $\sigma = 3$, $\theta = 0$ and $\tau = 1$. Specify and sketch the posterior predictive pdf of Y'