

Logistic Regression

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Binary classification

Classification : discrete target (output vector).

Binary classification: $\{0, 1\}$ target

Example of binary classification task: Spam/not spam emails.

Idea: consider hypothesis $h_{\mathbf{w}}$ such that

$$0 \leq h_{\mathbf{w}} \leq 1.$$

- ▶ if $h_{\mathbf{w}}(\mathbf{x}) \geq 0.5$, predict 1;
- ▶ if $h_{\mathbf{w}}(\mathbf{x}) < 0.5$, predict 0.

Logistic Regression

Hypothesis: $h_{\mathbf{w}}(\mathbf{x}) = \sigma(\mathbf{w}^T \mathbf{x})$, where

$$\sigma(t) = \frac{1}{1 + e^{-t}}$$

is the **sigmoid function**.

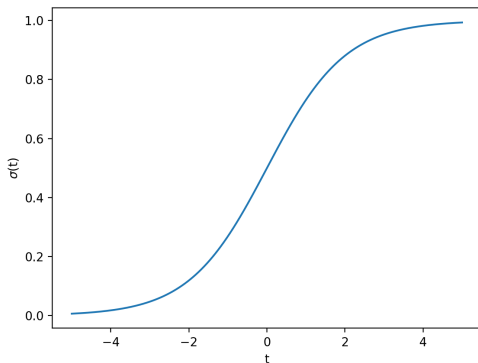


Figure: Sigmoid function

Linear decision boundary

Model: $h_{\mathbf{w}}(x_1, x_2) = \sigma(w_0 + w_1x_1 + w_2x_2)$

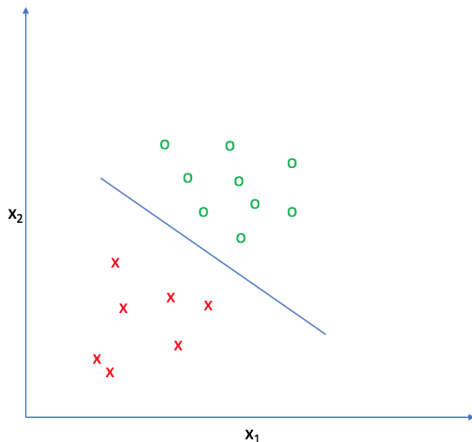


Figure: An example of linear decision boundary

Non-linear decision boundary

Model: $h_{\mathbf{w}}(x_1, x_2) = \sigma(w_0 + w_1x_1 + w_2x_2 + w_3x_1^2 + w_4x_2^2)$

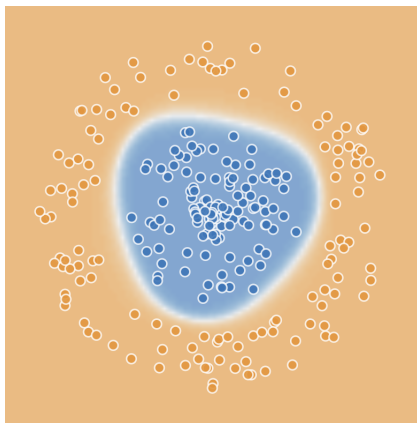


Figure: An example of non-linear decision boundary

Cost function