Logistic Regression

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Binary classification

Classification : discrete target (output vector).
Binary classification: $\{0, 1\}$ target

Example of binary classification task: Spam/not spam emails.

Idea: consider hypothesis h_w such that

$$0 \leq h_{\boldsymbol{w}} \leq 1.$$

- if $h_{\boldsymbol{w}}(\boldsymbol{x}) \geq 0.5$, predict 1;
- if $h_{\boldsymbol{w}}(\boldsymbol{x}) < 0.5$, predict 0.

Logistic Regression

Hypothesis: $h_{\boldsymbol{w}}(\boldsymbol{x}) = \sigma(\boldsymbol{w}^T \boldsymbol{x})$, where

$$\sigma(t) = \frac{1}{1 + e^{-t}}$$

is the **sigmoid function**.

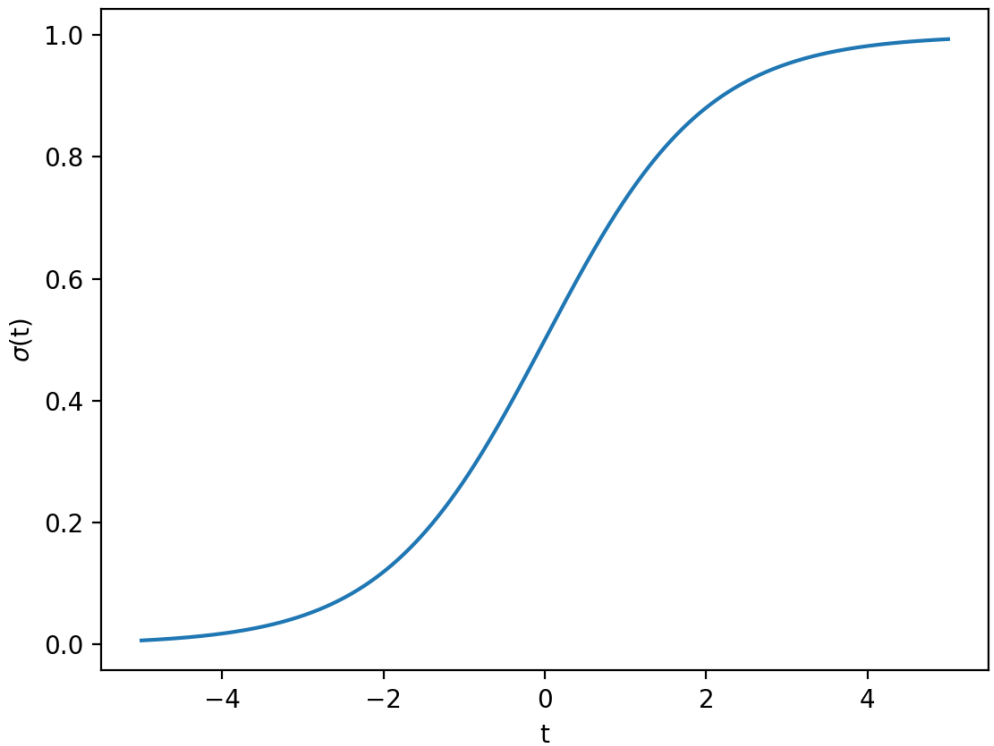

Figure: Sigmoid function

Linear decision boundary

Model: $h_{\boldsymbol{w}}(x_1, x_2) = \sigma(w_0 + w_1 x_1 + w_2 x_2)$

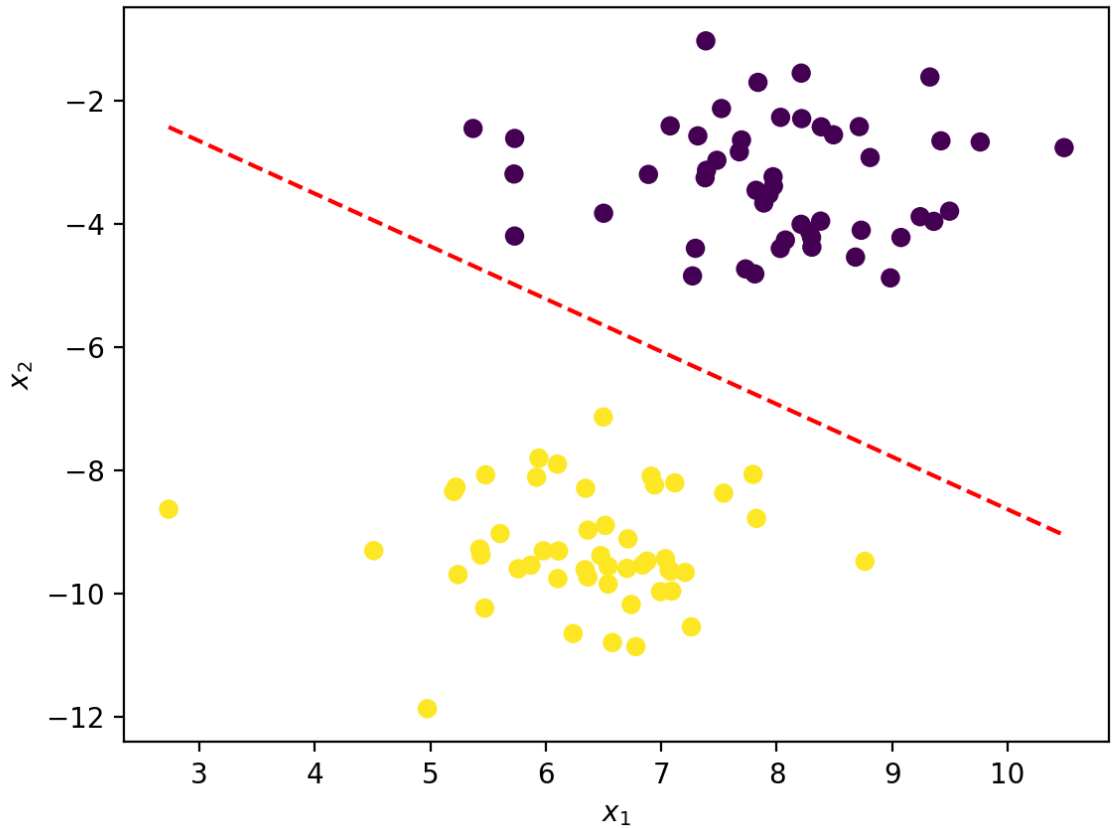

Figure: An example of linear decision boundary

Non-linear decision boundary

Model: $h_{\boldsymbol{w}}(x_1, x_2) = \sigma(w_0 + w_1 x_1 + w_2 x_2 + w_3 x_1^2 + w_4 x_2^2)$

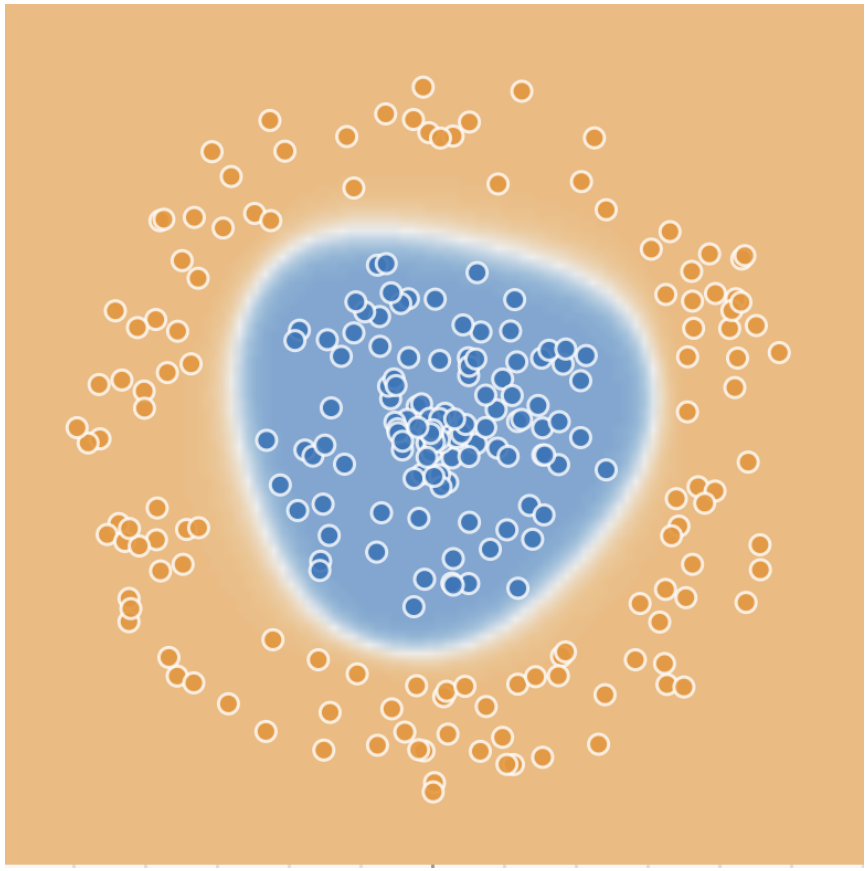

Figure: An example of non-linear decision boundary

Cost function