Quantizing neural networks

Optimizing AI - Session 2

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Course organisation

Sessions

1. Deep Learning and Transfer Learning,
2. Quantization,
3. Pruning,
4. Factorization,
5. Distillation,
6. Operators and Architectures,
7. Embedded Software and Hardware for DL.
8. Presentations for challenge.

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Today's Summary

1. Objectives

2. Quantization : Basics
   - Floating Point
   - Integers, Fixed Point
   - Quantization

3. Quantization : Neural Networks
   - Quantization Post Training
   - Quantization Aware Training

Plan

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Motivation

1. Reduce model size
   - Fewer bits → Reduced memory footprint
2. Decrease memory access
   - GPU & CPU : reduce Cache usage
3. Computational complexity

Table: Performance on the ImageNet dataset and complexities

Network	Alexnet	Inceptionv1	ResNet50	ResNet152
Top-5 error	16.4%	6.7%	5.25%	4.49%
Num. Weights	61M	7M	25.5M	63.75M
Num. MAC	724M	1.43G	3.9G	11.31G

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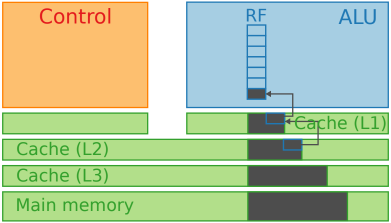

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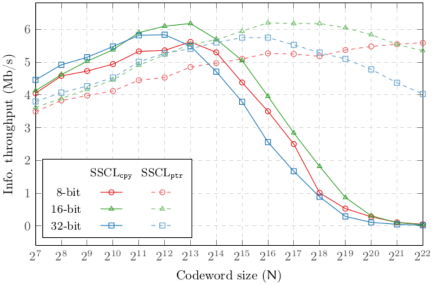

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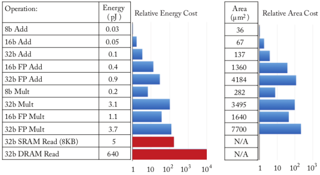

Operation:	Energy (pJ)	Area (µm²)
8b Add	0.03	36
16b Add	0.05	67
32b Add	0.1	137
16b FP Add	0.4	1360
32b FP Add	0.9	4184
8b Mult	0.2	282
32b Mult	3.1	3495
16b FP Mult	1.1	1640
32b FP Mult	3.7	7700
32b SRAM Read (8KB)	5	N/A
32b DRAM Read	640	N/A

Figure 7.1: The area and energy cost for additions and multiplications at different precision, and memory accesses in a 45 nm process. The area and energy scale different for multiplication and addition. The energy consumption of data movement (red) is significantly higher than arithmetic operations (blue). (Figure adapted from [121].)

From : Sze, Vivienne, et al. "Efficient processing of deep neural networks." Synthesis Lectures on Computer Architecture

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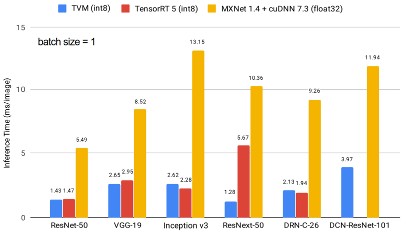

From : https://github.com/vinx13/tvm-cuda-int8-benchmark/

Plan

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3 Quantization : Neural Networks

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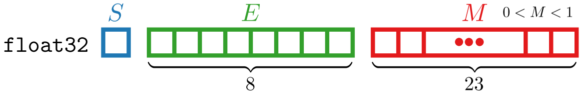
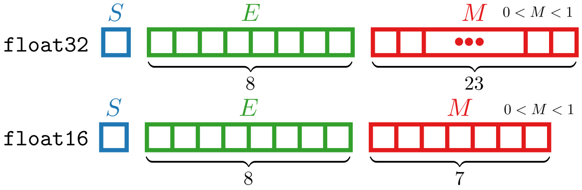
Floating Point



precision $M: 2^{-23} = 10^{-7}$

$$(-1)^S \times 2^{E-127} \times (1 + M)$$

range: 1.17×10^{-38} to $3.4 \times 10^{+38}$



$0 < M < 1$

precision $M: 2^{-23} = 10^{-7}$

$$(-1)^S \times 2^{E-127} \times (1+M)$$

range: 1.17×10^{-38} to $3.4 \times 10^{+38}$

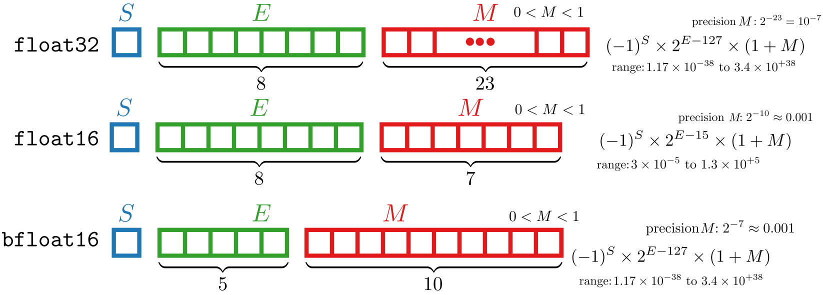
$0 < M < 1$

precision M: $2^{-10} \approx 0.001$

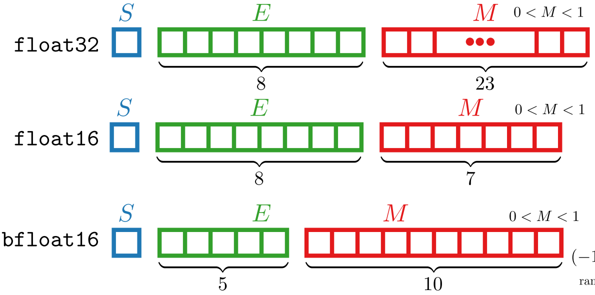
$$(-1)^S \times 2^{E-15} \times (1+M)$$

range: 3×10^{-5} to $1.3 \times 10^{+5}$

Floating Point



Floating Point



float32: $0 < M < 1$

precision $M: 2^{-23} = 10^{-7}$

$$(-1)^S \times 2^{E-127} \times (1+M)$$

range: 1.17×10^{-38} to $3.4 \times 10^{+38}$

float16: $0 < M < 1$

precision M: $2^{-10} \approx 0.001$

$$(-1)^S \times 2^{E-15} \times (1+M)$$

range: 3×10^{-5} to $1.3 \times 10^{+5}$

bfloat16: $0 < M < 1$

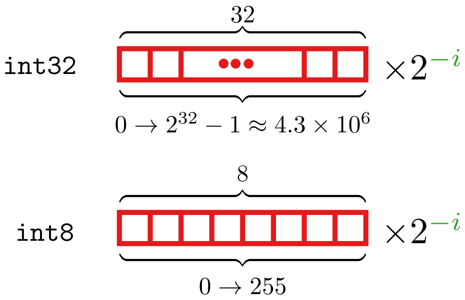
precision M: $2^{-7} \approx 0.001$

$$(-1)^S \times 2^{E-127} \times (1+M)$$

range: 1.17×10^{-38} to $3.4 \times 10^{+38}$

- To add to FP numbers:
  - Shift M according to E (`int shift` n_E `bits`)
  - Add M (`int add` n_M `bits`)
  - Normalize $(0 < M < 1)$
- To multiply to FP numbers:
  - Multiply M (`int mult` n_M `bits`)
  - Add E (`int mult` n_E `bits`)
  - Normalize $(0 < M < 1)$

Integers, fixed point



- Fixed point ($-i$)
- Short range
- Simple computation

Quantizing

In practice, data/weights are often random and may have high variation ranges (ex: Gaussian distribution). Quantizing with full precision requires too many bits $\rightarrow$ need approximations to reduce the number of bits:

- **Remove l the least significant bits** $\rightarrow$ residual errors, requires **round** operations to improve the performance.
  $X_{\text{round}} = 2^l \text{round}(2^{-l} \times X)$
- **Remove q the most significant bits** $\rightarrow$ risk of overflow, requires **saturation** mechanism.
  $X_{saturate} = \min(X, 2^{n-q} - 1)$ (for n bits unsigned)

Exemple: $\mathbf{D} = \{-47, 64, 3, -26\}$ requires 9 bits for full precision. But the value 64 is closed to the value 63.
Therefore $\hat{\mathbf{D}} = \{-47, 63, 3, -26\}$ can be used instead $\rightarrow$ 1 MSB removed + saturation at $2^6 - 1$, slight loss of precision.

Estimating the impact of quantization

Impact on weights

Signal-to-Quantization Noise Ratio metric.
W_k: weight number index k in the set.
$\hat{W}_k$: quantized weight index k in the set.
L: number of element in the set.

$$\mathsf{SQNR}(\hat{W}) = \frac{\sum_{k=0}^{L-1} |W_k|^2}{\sum_{k=0}^{L-1} \underbrace{|W_k - \hat{W}_k|^2}_{\text{quantization error}}}$$

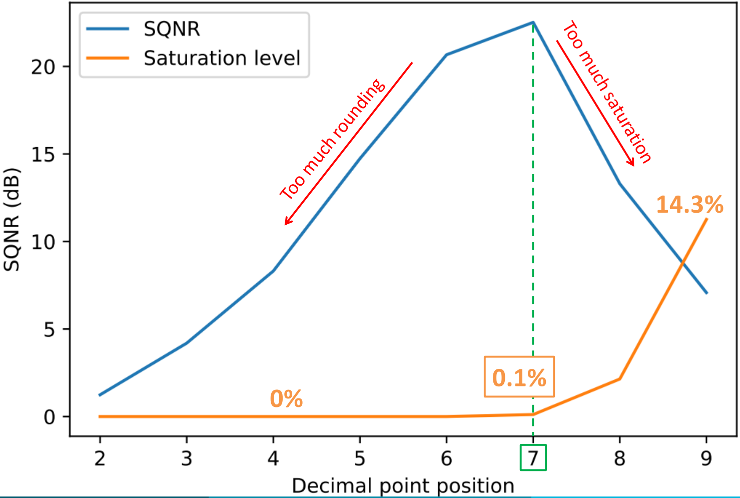
Generally expressed in dB: $\mathsf{SQNR}_{\mathsf{dB}} = 10\log_{10}(\mathsf{SQNR})$

Impact on network performance

Directly measure the accuracy of the network. For instance: Top-1 or Top-5 errors.

Quantizing trained networks: example

Fully connected network with 1 hidden layer, trained on MNIST.
6 bits weights (hidden layer) $\rightarrow$ SQNR optimal at $\times 2^{-7}$ (see figure).



Plan

Quantization Post Training : Weights

Start by considering weights with a few number of bits n.
Find the point position (2^{-v}) which maximize the SQNR for each trained weight sets $\hat{W}(v)$:

$$\hat{W}_k(v) = 2^v \underbrace{\text{max}(\text{min}}_{\text{saturation}}(\text{round}(2^{-v} \times W_k), 2^{n-1} - 1), -2^{n-1})$$

Quantize $\rightarrow$ measure accuracy $\rightarrow$ increase the number of bits and repeat.

Different weight sets can be considered

- Whole network,
- per layer,
- per neuron.

Finer sets segmentation $\rightarrow$ better accuracy.
Depends on how weights are stored in hardware (parallel accesses).

Quantization Post Training : Activation

Start by considering activations with a few number of bits n.
Find the point position (2^{-v}) which maximize the SQNR for each activation sample set $\hat{W}(v)$:

$$\hat{W}_k(v) = 2^v \underbrace{\text{max}(\text{min}}_{\text{saturation}}(\text{round}(2^{-v} \times W_k), 2^{n-1} - 1), -2^{n-1})$$

Quantize $\rightarrow$ measure accuracy $\rightarrow$ increase the number of bits and repeat.

Also different strategies

- Whole network,
- per layer,
- per neuron.

Finer sets segmentation $\rightarrow$ better accuracy.
Depends on how activations are stored (parallel accesses).

Quantization Aware Training

- Quantize Forward
- Quantize Backward & Forward
- Weights
- Weights & Activations

Quantization Aware Techniques yield way better accuracy

Quantization while Learning - Binary Connect

Algorithm 1 SGD training with BinaryConnect. C is the cost function for minibatch and the functions binarize(w) and clip(w) specify how to binarize and clip weights. L is the number of layers.

Require: a minibatch of (inputs, targets), previous parameters w_{t-1} (weights) and b_{t-1} (biases), and learning rate η.
Ensure: updated parameters w_t and b_t.

1. Forward propagation:
$w_b \leftarrow \text{binarize}(w_{t-1})$
For $k = 1$ to L, compute a_k knowing a_{k-1}, w_b and b_{t-1}

2. Backward propagation:
Initialize output layer's activations gradient $\frac{\partial C}{\partial a_L}$
For $k = L$ to 2, compute $\frac{\partial C}{\partial a_{k-1}}$ knowing $\frac{\partial C}{\partial a_k}$ and w_b

3. Parameter update:
Compute $\frac{\partial C}{\partial w_b}$ and $\frac{\partial C}{d b_{t-1}}$ knowing $\frac{\partial C}{\partial a_k}$ and a_{k-1}
$w_t \leftarrow \text{clip}(w_{t-1} - \eta \frac{\partial C}{\partial w_b})$
$b_t \leftarrow b_{t-1} - \eta \frac{\partial C}{\partial b_{t-1}}$

Courbariaux, Matthieu, Yoshua Bengio, and Jean-Pierre David. "Binaryconnect: Training deep neural networks with binary weights during propagations." Advances in neural information processing systems. 2015.
https://arxiv.org/pdf/1511.00363.pdf

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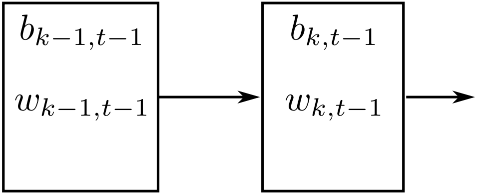

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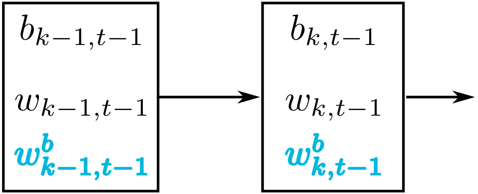

$$w^b = \begin{cases} +1, & \text{if } w \geq 0 \\ -1, & \text{otherwise} \end{cases}$$

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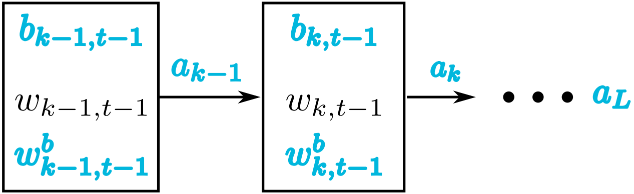

Quantization while Learning - Binary Connect

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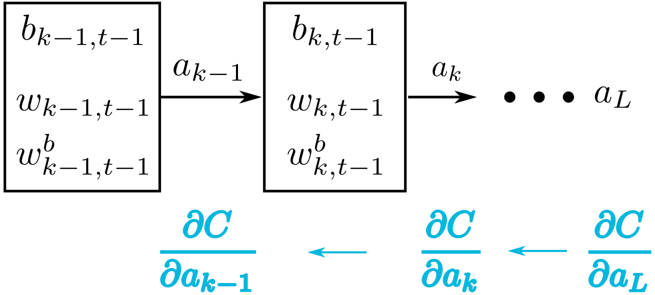

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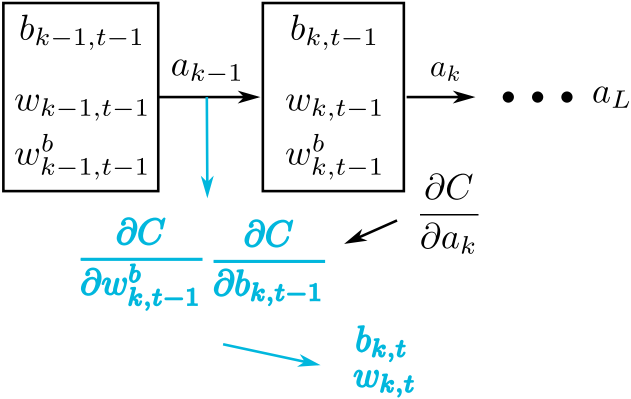

Binarization : Stochastic vs Deterministic

- Deterministic

$$w_b = \begin{cases} +1, & \text{if } w \geq 0 \\ -1, & \text{otherwise} \end{cases}$$

- Stochastic

$$w_b = \begin{cases} +1, & \text{with probability } p = \sigma(w) \\ -1, & \text{with probability } 1 - p \end{cases}$$

avec

$$\sigma(x) = \text{clip}(\frac{x+1}{2}, 0, 1) = \max(0, \min(1, \frac{x+1}{2}))$$

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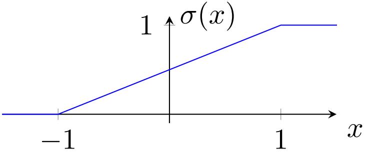
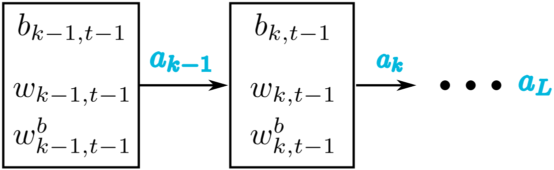


Quantization while Learning - Binary Weighted network (XNOR-NET)



$$a_k = \begin{cases} +\alpha, \\ -\alpha. \end{cases}$$

Rastegari, Mohammad, et al. "Xnor-net: Imagenet classification using binary convolutional neural networks." European conference on computer vision. Springer, Cham, 2016. https://arxiv.org/pdf/1603.05279.pdf

Quantization while Learning - Binary Weighted network (XNOR-NET)

	Network Variations	Operations used in Convolution	Memory Saving (Inference)	Computation Saving (Inference)	Accuracy on ImageNet (AlexNet)
Standard Convolution	Real-Value Inputs: 0.11 -0.21 ... -0.34 / -0.25 0.61 ... 0.52; Real-Value Weights: 0.12 -1.2 ... 0.41 / -0.2 0.5 ... 0.68	+ , − , ×	1x	1x	%56.7
Binary Weight	Real-Value Inputs: 0.11 -0.21 ... -0.34 / -0.25 0.61 ... 0.52; Binary Weights: 1 -1 ... 1 / -1 1 ... 1	+ , −	~32x	~2x	%56.8
BinaryWeight Binary Input (**XNOR-Net**)	Binary Inputs: 1 -1 ... -1 / -1 1 ... 1; Binary Weights: 1 -1 ... 1 / -1 1 ... 1	XNOR , bitcount	~32x	~58x	%44.2

Rastegari, Mohammad, et al. "Xnor-net: Imagenet classification using binary convolutional neural networks." European conference on computer vision. Springer, Cham, 2016. `https://arxiv.org/pdf/1603.05279.pdf`