Second year review

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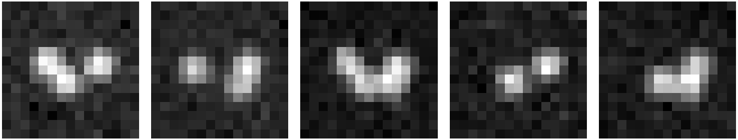
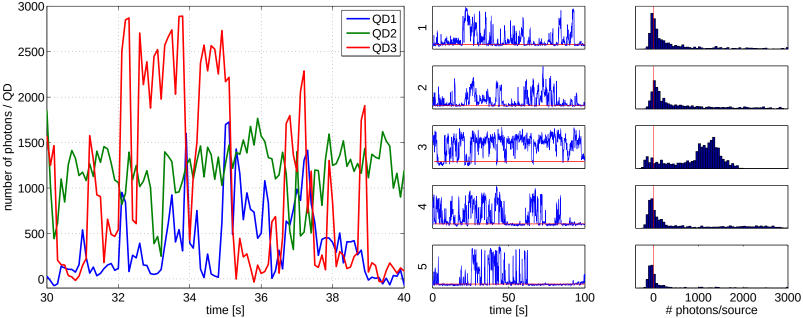
Overview

1. Introduction
2. Model comparison
3. Theoretical limits of the LM
4. Out of focus PSF
5. Future work

Introduction: Quantum Dots and LM





Model

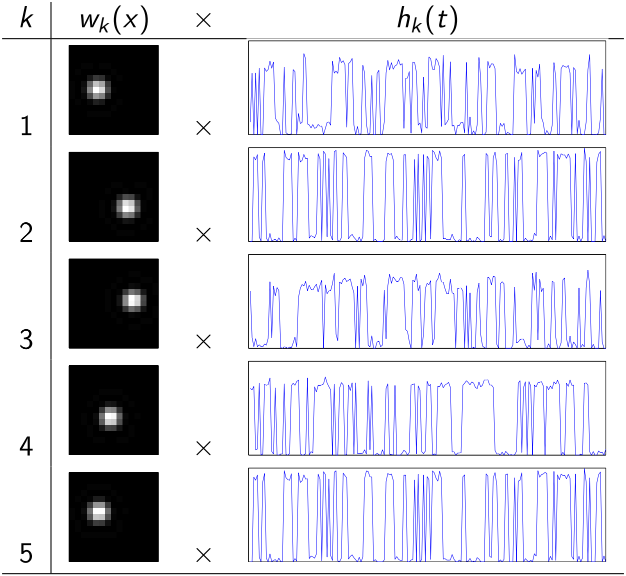
- model of one source

$$s_k(x,t) = w_k(x)h_k(t)$$

- model of an image

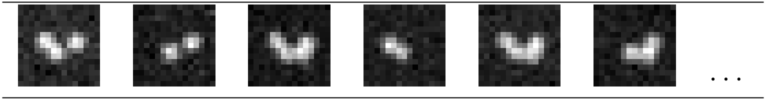
$$\begin{aligned} I(x,t) &= \sum_{k=1}^{N} s_k(x,t) \\ &= \sum_{k=1}^{N} w_k(x)h_k(t) \end{aligned}$$

Model



Model

$$I(x, t)$$



time $\rightarrow$

Model

- matrix form - factorisation problem

$$I(x,t) = W(x)H(t)$$

- relaxed model

$$E[I(x,t)] = W(x)H(t)$$

- non-negativity constraints

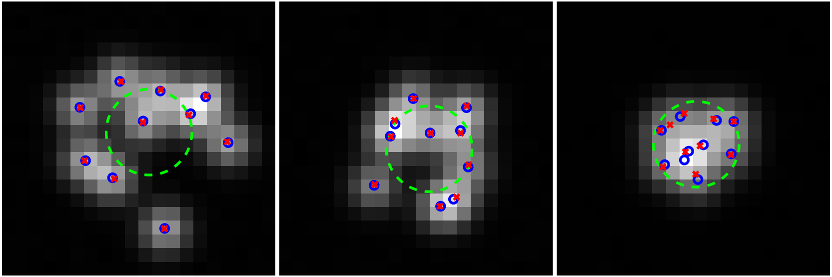
$$I \geq 0,\; W \geq 0,\; H \geq 0$$

Algorithms

- Maximum a posteriori fitting (MAP)
  [Harrington et al., 2008]
- Non-negative matrix factorisation (NMF)
  [Lee & Seung, 2001]
- Gamma - Poisson model (GaP)
  [Canny, 2004]

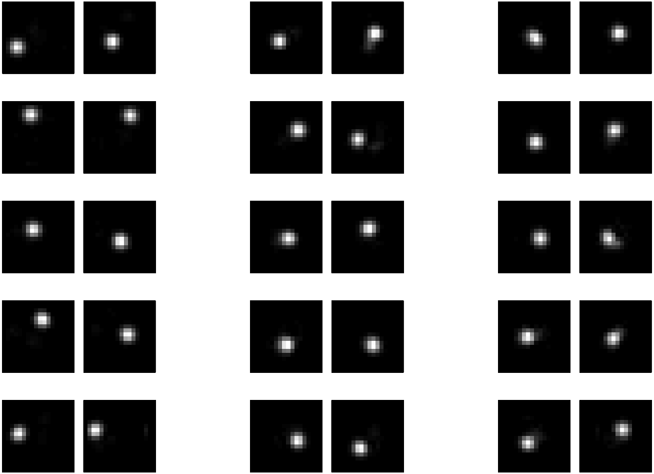
Simulated Data

- Localised $W(x)$



Simulated Data

- Separated $W(x)$, N $= 10$



Model comparison (estimation of K)

1. Principal component analysis (PCA)
2. Variational lower bound [Buntine & Jakulin, 2006]

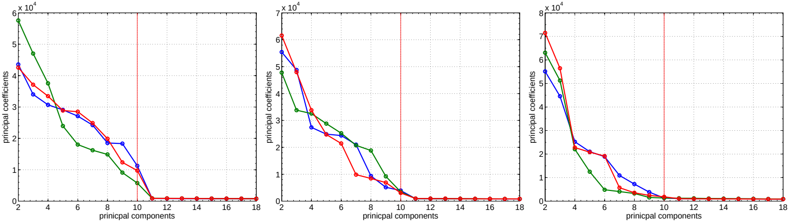
$$p(K|D) \propto p(D|K)p(K)$$

3. Analysis of the correlations in residuals

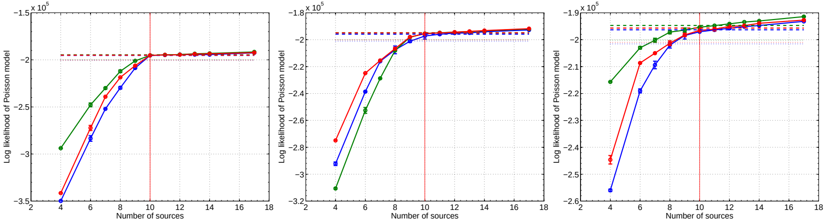
$$s_{xt} = \frac{d_{xt} - \sum_{k=1}^{K} w_{xk} h_{kt}}{\sqrt{\sum_{k=1}^{K} w_{xk} h_{kt}}}$$

Model comparison (estimation of K)

- PCA

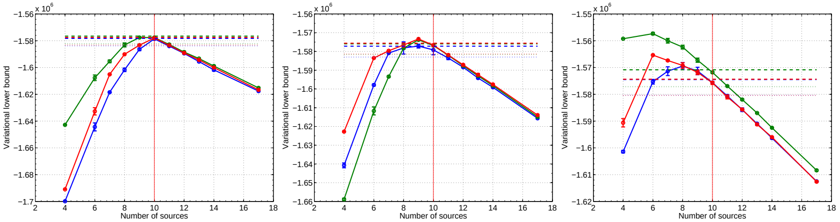


- Log likelihood

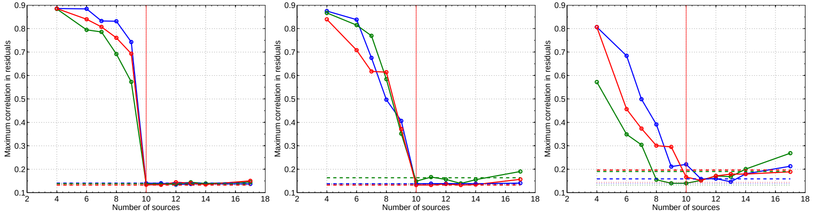


Model comparison (estimation of K)

- Variational lower bound [Buntine & Jakulin, 2006]



- Correlations in residuals (data - model)



Theoretical limits for LM - Cramer Rao bound

- Covariance matrix of an unbiased estimator θ

$$\mathbf{Q}(\theta) \geq \mathbf{I}^{-\mathbf{1}}(\theta)$$

- Fisher information as a curvature of the log-likelihood function

$$\mathbf{I}_{ij}(\theta) = -\mathbb{E}\left[\frac{\partial^2 \mathcal{L}}{\partial\theta_i \partial\theta_j}\right]$$

Two sources

- centres of the sources

$$\theta = (c_1,\, c_2)$$

- likelihood

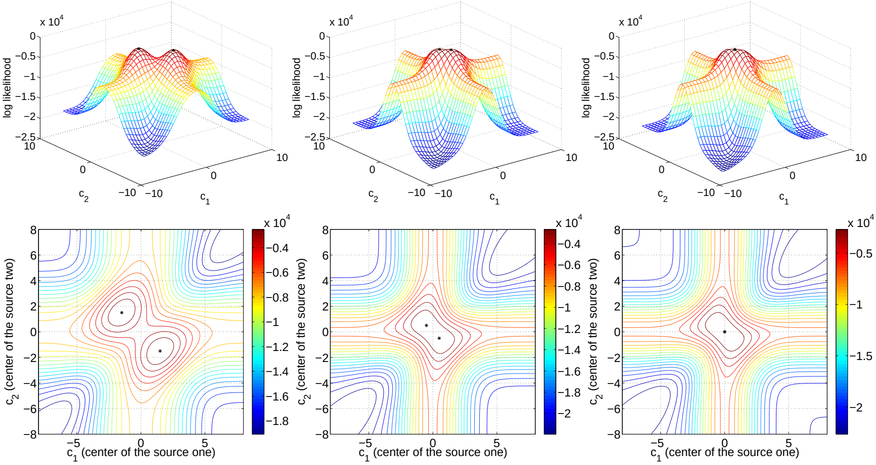
$$l(\theta) = \prod_{p=1}^{N} \mathrm{Po}(n_p|\lambda_p(\theta))$$

- mean intensity image

$$\lambda(\theta) = \Lambda_1 f_1(c_1) + \Lambda_2 f_2(c_2) + b$$

Fisher information as a curvature of the log-likelihood

$$\mathbb{E}_{true}\left[\log l(\theta)\right] = \sum_{p=1}^{N} \left(\lambda_p^{true} \log \lambda_p(\theta) - \lambda_p(\theta)\right) + A$$



Blinking sources

- blinking situation - intensity $\mathbf{\Lambda} = (\Lambda_1, \Lambda_2)$ is a random variable distributed over four states:

$$\left\{\mathbf{\Lambda}^1 = (\Lambda_1, 0),\ \mathbf{\Lambda}^2 = (0, \Lambda_2),\ \mathbf{\Lambda}^3 = (\Lambda_1, \Lambda_2),\ \mathbf{\Lambda}^4 = (0, 0)\right\}$$

- likelihood

$$l(\theta) = \prod_{p=1}^{N} p(n_p|\theta) = \prod_{p=1}^{N} \sum_{i=1}^{4} p(n_p|\theta, \mathbf{\Lambda}^i) p(\mathbf{\Lambda}^i)$$

Localisation precision - source intensity

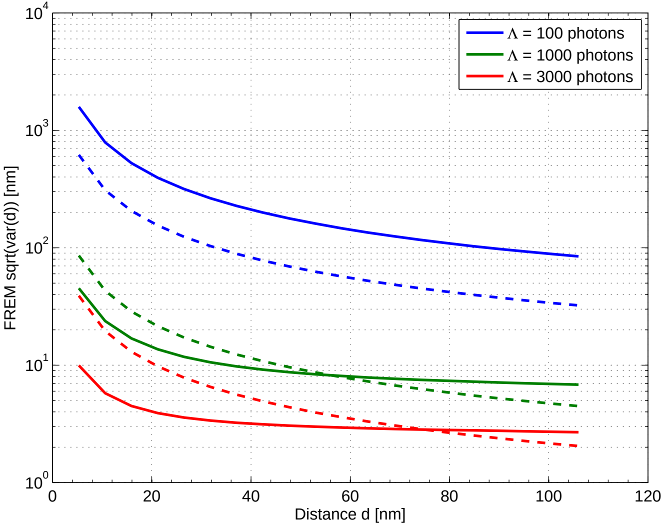


Figure: FREM (fixed background 100 photons)

Localisation precision - source intensity

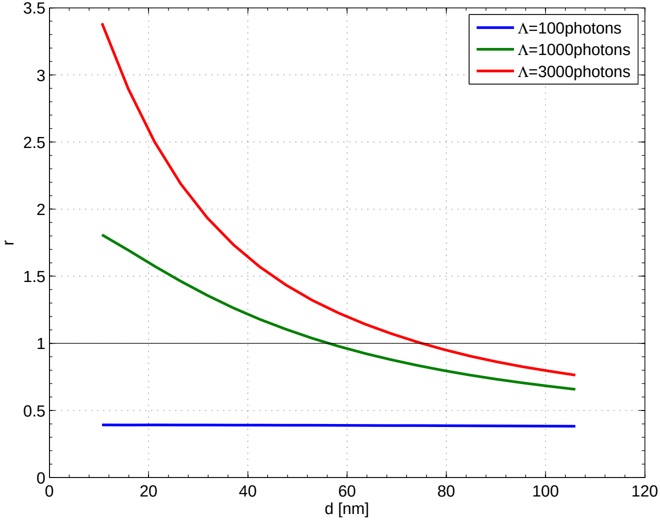


Figure: Comparison blinking vs static: $r = \sqrt{\text{var}^{\text{static}}(d)/\text{var}(d)}$

Localisation precision - background

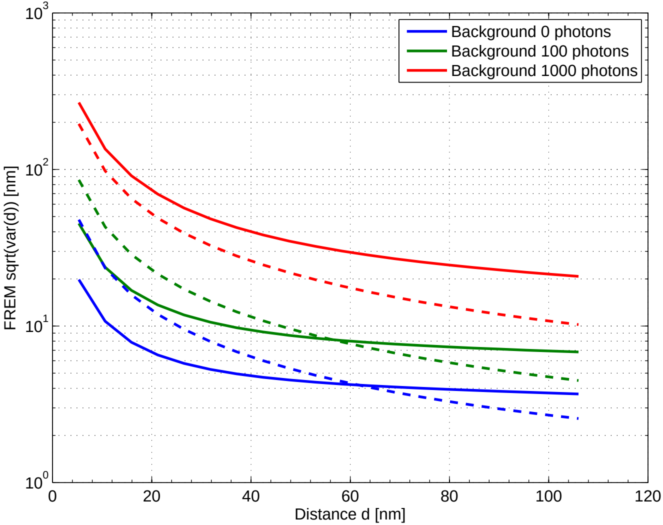


Figure: FREM (fixed $\Lambda = 10^3$ photons)

Localisation precision - background

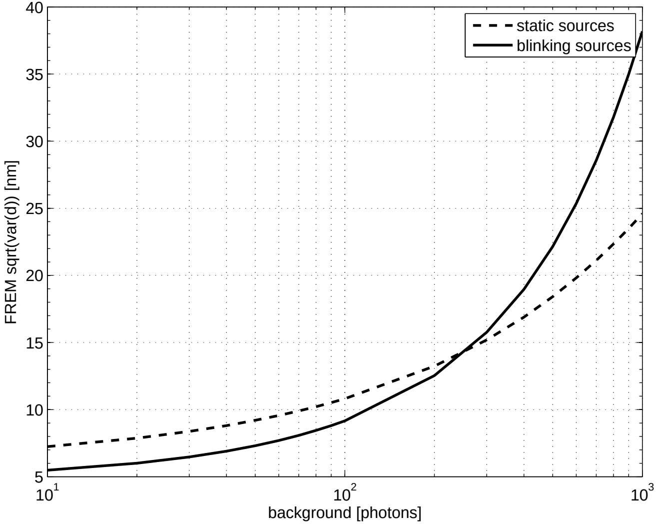
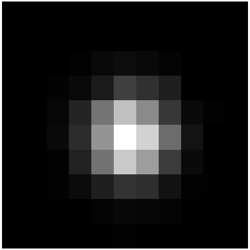
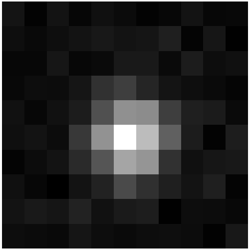


Figure: Effect of the background $\Lambda = 10^3$ photons, $d = 40$ nm.

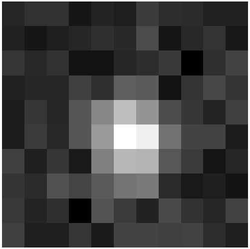
Simulated data: Two points, $d = 40$ nm $\Lambda = 10^3$ photons



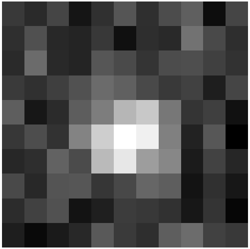
bg=0



$bg = 10^2$



$bg = 5 \cdot 10^2$



$bg = 10^3$

"Resolution" of the correlation residual analysis

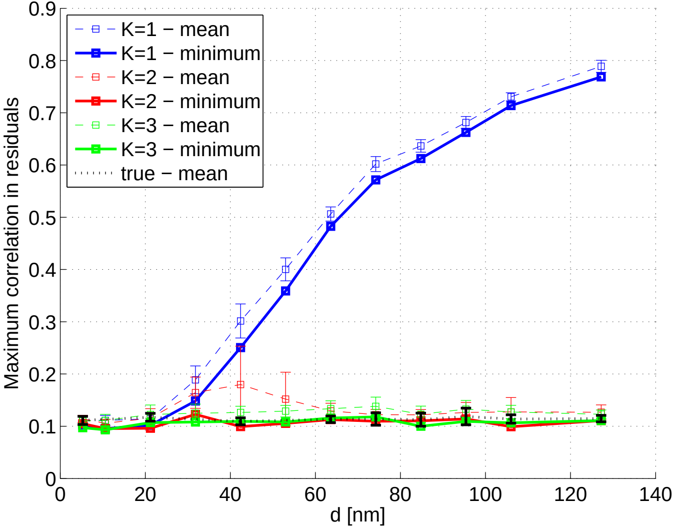
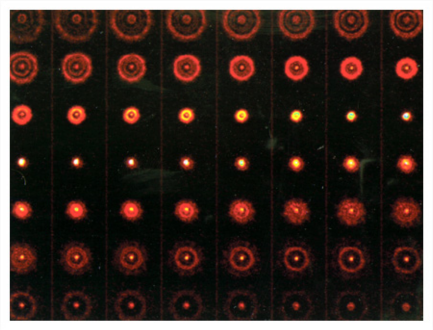


Figure: $K_{true} = 2$

Point Spread Function 3D



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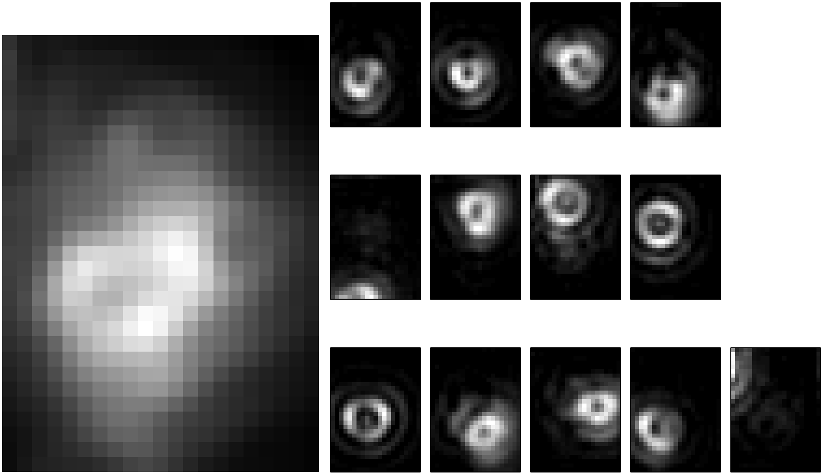
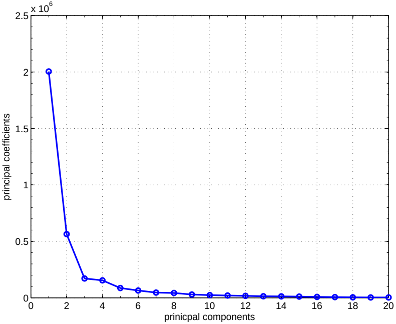
Real data

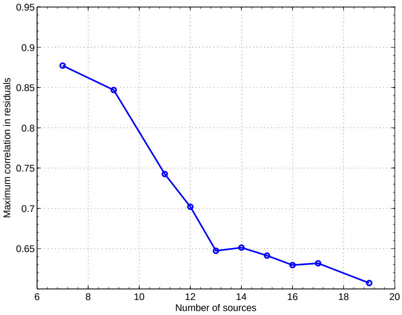
- Separated $W(x)$, N $= 13$



Real data

- PCA vs Correlations in Residuals





Future plans

- Write up: thesis chapters, paper (which journal?)
- More experimental work (visit of the Rainer's lab)
  - QD data, real sample
  - local illumination of the sample using spatial light modulator (SLM)
  - LM with photo - switchable proteins