Cathode Ray Tube Experiment

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Grade: ____________

Contents

1 Part A

1.1 Introduction and theoretical background

1.1.1 experiment goals

For part A, we aim to verify the theory behind how electric and magnetic fields affect a moving electron.

1.1.2 theory

In this experiment, we study the effects of the electric and magnetic fields on a ray of electrons, more precisely on the direction of the beam.

A cathode ray tube is used to generate the electron beam. It works by heating a cathode to the temperature at which it starts to emit electrons. The electrons emitted are filtered by energy level and accelerated by two meshes, and then the beam is focused using two anodes.

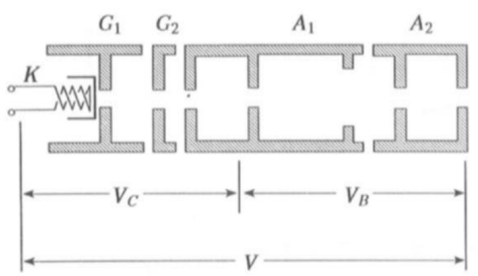


Figure 1: ray tube diagram

Where:

- K is the heated cathode.
- G_1 is the first mesh.
- G_2 is the second mesh.
- A_1 is the first anode.
- A_2 is the second anode.

From the diagram, we see that the first mesh acts as a physical barrier that only allows electrons emitted in a certain direction to go through; in addition, it also acts as a potential barrier also only allowing electrons with enough energy to pass, thus filtering the emitted electrons by direction of movement and energy.

The second mesh receives the electrons that were able to pass through the first mesh and accelerates them using a high voltage gate, thus, we get a group of electrons moving approximately in the right direction at high speeds. All we need to do now is concentrate them into a focused beam.

That is what the anodes are for, both anodes are charged tubes, the first anode slows down the electrons, and the second one speeds them up. The repetitive acceleration and deceleration of the electron beam focus it according to the electrostatic lens notion depicted in A.1

To measure the effect of an electric field on the concentrated electron beam, a uniform electric field perpendicular to the beam is needed, which is done using a simple two-plate capacitor. The equation describing the electric field inside a two-plate capacitor is as follows:

$$\vec{E} = -\frac{V_d}{d}$$

Where d is the distance between the capacitors' plates

From electrodynamics, we know that under a constant electric field $\vec{E}$, an electron has an electric force working on it, equivalent in size to the electric field times the charge of the electron and opposite in direction to the electric field, and using newtons second law, we get:

$$\vec{F} = -q_e E = \frac{eV_d}{d} \Rightarrow \vec{a} = \frac{eV_d}{dm_e} \tag{1}$$

Assuming that the electron is moving in the $\hat{x}$ direction and the acceleration is only in the $\hat{y}$ direction, it is possible to describe the deviation angle of the beam by the ratio of the initial velocity $\vec{v}_x$ and the acquired velocity $\vec{v}_y$.

$$\tan\theta = \frac{v_y}{v_x} \tag{2}$$

where θ is the scattering angle of the electron beam. See 2

And if the time the electrons spend under that acceleration is known and marked by t, we can find the velocity $\vec{v}_y$ using simple kinematics.

$$\vec{v}_y = \vec{a}_y \cdot t$$

Note that t can be calculated from $l = v_x t$, considering the electron is spending little time in the capacitor, where l is the length of the capacitor.

thus we can substitute $\vec{v}_y$ and $\vec{a}_y$ in (2) and get:

$$\tan\theta = \frac{v_y}{v_x} = \frac{a_y l}{v_x^2} = \frac{eV_d l}{mv_x^2 d} \tag{3}$$

The term apparent in the denominator, mv_x^2, is twice the kinetic energy of the electrons. From energy conservation we can say that the kinetic energy of the electron is equal to the potential energy that accelerated it at the beginning eV, not to be confused with V_d, which is the potential difference in the capacitor. Thus we get:

$$2eV = mv_x^2$$

$$\Rightarrow \tan\theta = \frac{V_d l}{2Vd} \tag{4}$$

The scattering angle of the electrons is determined only by the voltages applied in the CRT!

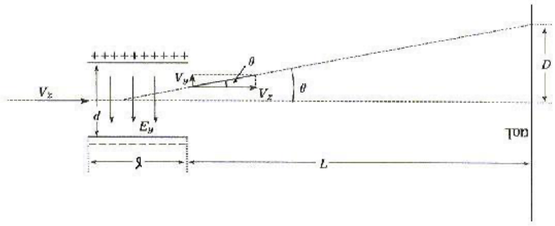


Figure 2: electron velocity in a cappacitor

Now, from simple geometry, we can see that:

$$\tan\theta = \frac{D}{L + \frac{l}{2}} = \frac{V_d l}{2Vd} \tag{5}$$

Note that we consider that the scattering point is in the middle of the capacitor, which arises from the assumption that the force acted on the electrons is uniform throughout the capacitor and is always pointing in the $\hat{y}$ direction.

For the second part of the experiment, we want tot test the effect of a constqnt magnetic field on the electron beam.

In this expirement, a magnetic field is created using two spools of copper wire each with n turns, and a current I flowing through them, they are placed on either side of the CRT, and the magnetic field is perpendicular to the electron beam, see the figure A.2.

If the spools are running through the CRT, it is calculated using the following equation:

$$B_0 = \mu_0 n I$$

Where μ_0 is the magnetic permeability of free space.
But it doesn't, so the magnetic field inside the CRT would be slighttly less than that, and we will define it as being $B = K_1 B_0$

In the presence of a magnetic field, the force acting on the electrons is given by the Lorentz force equation:

$$\vec{F}_e = e^-(\vec{E} + \vec{v} \times \vec{B}); F = \frac{m_e v^2}{R} = m\omega^2 R \tag{6}$$

Where $\vec{B}$ is the magnetic field, $\vec{v}$ is the velocity of the electrons, ω is the angular velocity of the electrons, and R is the radius of the circular motion, see A.3.

From the equation it is clear that the force acting on the electrons is perpendicular to both the velocity and the magnetic field, thus no work would be done on the electrons, and the law of conservation of energy applies.

When the electron beam is under the effect of the magnetic field it starts shifting in a circular manner, and the angle of deviation θ is given by the following equation:

$$\tan\theta = \frac{D - y_0}{\frac{L}{2} - a} \tag{7}$$

And we can use the small angle approximation to get:

$$\tan\theta \approx \sin\theta = \frac{2a}{r} \approx \theta | \theta \approx \tan\theta = \frac{D - y_0}{\frac{L}{2} - a} \tag{8}$$

And using trigonometry in figure A.3 we can see that the distance y_0 is given by:

$$y_0 = r - AB = r(1 - \cos\theta) \approx \frac{r\theta^2}{2} \tag{9}$$

combining equations(8) and (9) we get:

$$\begin{aligned} D &= y_0 + \left(\tfrac{L}{2} - a\right)\tan\theta \approx \frac{r\,\theta^2}{2} + \left(\tfrac{L}{2} - a\right)\tan\theta \\ &\approx \frac{r}{2}\left(\frac{2a}{r}\right)^2 + \left(\tfrac{L}{2} - a\right)\frac{2a}{r} = \frac{2a^2}{r} + \frac{L\,a}{r} - \frac{2a^2}{r} = \frac{L\,a}{r}\,. \end{aligned}$$

And plugging in equation (6) we get:

$$D = LaB\frac{e^-}{mv} = \sqrt{\frac{e^-}{m}}La\frac{B}{\sqrt{2V}} \tag{10}$$

Where we got the expretion $v = \sqrt{\frac{2eV}{m}}$ from the previus part.

And similarly to the effective magnetic field we cannot consider a to be exactly the width of the effevtive magnetic field, so again we consider it is equaal to $K_2 \cdot L/2$, where K_2 is a constant that depends on the system.
we plug a, B and $K = K_1K_2$ into equation (10) and we get:

$$D = K\mu_0\sqrt{\frac{e}{m}}\frac{L^2nI}{2\sqrt{2}\sqrt{V}} \tag{11}$$

1.2 Materials and Methodology

1.2.1 equipment

The equipment used was:

1. variable power supply.
2. voltage control box.
3. voltmeters.
4. Cathode Ray Tube

1.2.2 instructions

1. Connect the CRT to the power supply and manipulate the voltage until a small green dot appears on the screen. Set the voltage of the capacitor to 0V.
2. Record the position of the green dot as the origin point of the measurements.
3. Increase the voltage of the capacitor in 2V step increments and record the position of the green dot relative to the origin point for each step.
4. Repeat the process three times for different voltages of the CRT.

For the second part of the experiment,

1. Similerly to the first part, the CRT is connected to a power supply and the voltage is manipulated until a small green dot appears on the screen.
2. Two spools of wire are placed on either side of the CRT,making sure both north sides are pointing in the same direction, they are connected in series to an ampre meter and a power supply.
3. The voltage is set to zero and the innitial position of the green dot is recorded.
4. The voltage is increased in even steps and the current in the spools and the position of the green ot are recorded.
5. The process is repeated for 2 different voltages of the CRT.

1.3 data analysis process

After collecting the position data of the green dot, a fit of $D(\frac{V_d}{V}) = \frac{V_d}{V} \cdot \frac{(L+\frac{l}{2})l}{2d}$ is plotted and the linearity of the data according to equation (5) is checked.

The errors expected are as follows:

- The error in the position of the green dot is a resolution error of the screen, which is $\Delta D = \frac{0.01}{\sqrt{12}}$.

- The error in setting the voltage of the acppasitor is a resolution error, which is $\Delta V_d = \frac{0.001}{\sqrt{12}}$. (after rounding)
- the error of the length of the CRT is a resolution error of the ruler, which is $\Delta L = \frac{0.01}{\sqrt{12}}$.
- the error of the distance between the plates of the capacitor is a little more complicated, as it is an indirect error after taking the average between the max and min distance between the plates where each one has an error of $\Delta = 0.0127$ according to the manual. so we get that the overall error of d is $\Delta d = \frac{1}{2}\sqrt{\Delta d_{min}^2 + \Delta d_{max}^2}$.
- Similarly, the error of the length of the cappasitor is as given in the manual: $\Delta l = 0.0127$

we expect a linear graph of the form:

$$y(x) = a_1 x + a_0 \tag{12}$$

where we expect $a_0 \approx 0$, and the error of a_1 to be:

$$\Delta a_1 = \sqrt{\left(\frac{\Delta L l}{2d}\right)^2 + \left(\frac{\Delta l}{2d}(L+l)\right)^2 + \left(\frac{\Delta d}{d}\frac{(L+l)\,l}{2}\right)^2} \tag{13}$$

and the error of $\frac{V_d}{V}$ is to be:

$$\Delta\frac{V_d}{V} = \left(\frac{\Delta V_d}{V}\right) \tag{14}$$

and from that the value of $\frac{l}{d}$ is extracted and cmpared to the theoretical value range. Thee epeccted error of $\frac{l}{d}$ is :

$$\Delta\frac{l}{d} = \sqrt{\left(\frac{\Delta l}{d}\right)^2 + \left(\frac{l}{d^2}\Delta d\right)^2} \tag{15}$$

We get from (13) that $\Delta a_1 = 0.23$

The values extracted fronm the fits are tested against the thier theoretical value using the N_σ test.

For the second part: Two linear plots will be fitted to the data, the firat for the function $D(I)$ and the second for the function $D(\frac{I}{\sqrt{V}})$.
For both fits the expectted values of a_0 is 0, and the expected value of $a_1 = K\mu_0\sqrt{\frac{e}{m}}\frac{L^2 n}{2\sqrt{2V}}$ for the first fit, and $a_1 = K\mu_0\sqrt{\frac{e}{m}}\frac{L^2 n}{2\sqrt{2}}$ and value we wish to extract is K. for the second fit the value we wish to extract is $\sqrt{\frac{e}{m}}$.

The errors are as follows:

- The error in the position of the green dot is a resolution error of the screen, which is $\Delta D = \frac{0.01}{\sqrt{12}}$.
- The error in setting the voltage of the acppasitor is a resolution error, which is $\Delta V_d = \frac{0.001}{\sqrt{12}}$. (after rounding)
- The error in reading the current in the spools is a resolution error, which is $\Delta I = \frac{0.001}{\sqrt{12}}$.
- The error of the length of the CRT is a resolution error of the ruler, which is $\Delta L = \frac{0.01}{\sqrt{12}}$.

for the second graph:

$$\Delta \frac{I}{\sqrt{V}} = \sqrt{\left(\frac{\Delta I}{\sqrt{V}}\right)^2 + \left(\frac{I \Delta V}{2V^{\frac{3}{2}}}\right)^2} \tag{16}$$

$$\sqrt{\frac{e}{m}} \pm \Delta \sqrt{\frac{e}{m}} = \frac{\sqrt{8}\,|a_1|}{\mu_0 L^2 n} \pm \sqrt{\frac{e}{m}} \sqrt{\left(\frac{\Delta a_1}{a_1}\right)^2 + \left(\frac{2\Delta L}{L}\right)^2} \tag{17}$$

$$\kappa \pm \Delta\kappa = \frac{\sqrt{8}\,|a_1|}{\mu_0\, L^2\, n} \left(\sqrt{\tfrac{e}{m}}\right)^{-1}_{\text{theo}} \pm \kappa \sqrt{\left(\frac{\Delta a_1}{a_1}\right)^2 + \left(\frac{2\,\Delta L}{L}\right)^2} \tag{18}$$

1.4 Results

First part
First we compute the values we want to verify:
$a_1 = 17.485 \pm 0.23$ and the ratio $(\ell/d)_{max} = 4.39 \pm 0.13$, $(\ell/d)_{min} = 0.757 \pm 0.080$.
Below are the linear fits, residuals, and the numerical results for each data set.

See Figures A.5–A.7 in Appendix A, and Tables B.1–B.3 in Appendix B.
after performing the N_σ test for each set we get:

$N_\sigma^{(1)}$	$N_\sigma^{(2)}$	$N_\sigma^{(3)}$
23.99	18.70	9.27

Table 1: Results part A (slope)

We can see that none of the results are within the 3σ range, and thus we can conclude that the results are not consistent with the theory.
furthermore the results in of the third set is closest tot the theoretical value by a big margin and we can also see that the value of a_0 was closes to 0 in that voltage too, so we can conclude that the third voltage set was the most accurate.
after extracting the value of $\frac{\ell}{d}$ from the fits and again performing the N_σ test we get:

$\frac{\ell}{d}_1$	$\frac{\ell}{d}_2$	$\frac{\ell}{d}_3$
3.848 ± 0.065	4.058 ± 0.092	3.725 ± 0.143

Table 2: Results part A ($\frac{\ell}{d}$)

and it is clear that the results are consistent with the theory and fall in the desired range

Second part
In this part we worked with two different sets and preformed two different fits for each set. For results of the first fit see figures A.8 and A.9 and the corresponding tables B.4 and B.5 in the appendix.

the values of K that we get are:

K_{set1}	K_{set2}
0.0664 ± 0.00006	0.1076 ± 0.00125

Table 3: Results part A (K)

The values have low relative error and are smaller than 1, which is expected as the magnetic field is not uniform and the spools are not perfectly aligned.

For the results of the second fit see figures A.10 and A.11 and the corresponding tables B.6 and B.7 in the appendix.

after extracting the values of $\sqrt{\frac{e}{m}}$ we get:

$\sqrt{\frac{e}{m}}_{set1}$	$\sqrt{\frac{e}{m}}_{set2}$
58169.2 ± 5683	44109.5 ± 7814

Table 4: Results part A ($\sqrt{\frac{e}{m}}$)

The theoretical value of $\sqrt{\frac{e}{m}}$ is , after performing the N_σ test we get:

$N_\sigma^{(1)}$	$N_\sigma^{(2)}$
63.6	48.0

Table 5: Results part A (N_σ)

The results are not consistent with the theory, and we can see that the results were an order of magnitude less than the expected result.

2 Part B

Again lets compute the values we will be testing:

2.1 Introduction and theoretical background

2.1.1 experiment goals

What is the energy distribution of electrons in a cathode ray tube determined by varying a retarding potential, and how does this distribution compare to the theoretical Maxwell-Boltzmann distribution?

2.1.2 theory

Electrons are emitted from a heated cathode via thermionic emission. Due to the thermal energy of the cathode, the electrons are not emitted with a single energy but rather follow a statistical distribution. For an idealized case, the Maxwell-Boltzmann distribution can describe the energy distribution of the electrons emitted. In terms of energy, this distribution is given by:

$$f(E) = \frac{1}{K_B T} \cdot e^{\frac{-E}{K_B T}} \tag{19}$$

where

- E is the kinetic anergy.
- K is boltzmann's constant.
- T is the temperature.

In this experiment, a retarding potential V_r is applied between the electron gun and the screen. This creates an energy barrier such that only electrons with kinetic energy E greater than the potential energy eV_r (with e being the electron charge) can overcome the barrier and reach the screen(or have a much better chance of doing so).

Thus, the measured current (or brightness) $I(V_r)$ is related to the number of electrons with energies above eV_r by the equation

$$I(V_r) = I_0 \int_{eV_r}^{\infty} f(E)\, dE, \tag{20}$$

where

- I_0 is the total current when $V_r = 0$ (i.e., when no energy filtering is applied), and
- $f(E)$ is the electron energy distribution function.

This expression shows that $I(V_r)$ represents the *cumulative distribution function (CDF)* for the electron energies, with the integration starting at the energy threshold eV_r.

Energy Distribution Since $I(V_r)$ is the integrated form of $f(E)$, we can recover the energy distribution by differentiating $I(V_r)$ with respect to the retarding potential V_r, we get

$$\frac{d}{dV_r}\left(\int_{eV_r}^{\infty} f(E)\,dE\right) = -e\,f(eV_r). \tag{21}$$

Differentiating the measured current gives

$$\frac{dI}{dV_r} = -I_0 e\,f(eV_r). \tag{22}$$

Rearranging, we obtain the energy distribution function:

$$f(eV_r) = -\frac{1}{I_0 e}\frac{dI}{dV_r}. \tag{23}$$

Here, the substitution $E = eV_r$ connects the retarding potential directly to the electron kinetic energy.

2.2 equipment and Methodology

2.2.1 Equipment

The equipment used was:

1. variable power supply.
2. voltage control box.
3. voltmeters.
4. Cathode Ray Tube
5. cammera
6. construction paper and tape to cover the camera and CRT.

2.2.2 instructions

The retarding voltage V_r in the CRT was controlled, and the electron current, represented as "brightness" on the screen, was measured using a digital camera with constant ISO (exposure) settings. This brightness was then converted to current using numerical methods. If available, a light sensor or a photodiode was preferred.

Experiment Steps

1. The retarding voltage was set to zero, and the brightness of the screen was measured to obtain I_0, the maximum electron current.
2. V_r was gradually increased in small increments, with brightness measurements taken at each increment.
3. The images captured were converted into grayscale, and the electron current was evaluated numerically.

Regressions Planned The results of the energy density distribution were fitted to the Maxwell-Boltzmann distribution.

Physical Constants Recovered From this experiment, the average energy (temperature) was extracted, although the primary interest was matching the Maxwell-Boltzmann fit.

2.3 data analysis process

The data was analyzed using a jupyter notebook the was build from the ground up with the help of Chatgpt. the code converted the images to grayscale and found the midpoint of the brightest spot in the image and its radius, that was partially done manually, the numpy library was then used to calculate the average brightness of each area. The results were plotted and fitted to the Maxwell-Boltzmann distribution.
and $\chi^2_{reduces}$ and p-probability were calculated for the plot.

2.4 Results

the plot of the probability density of the position of the electrons (which corresponds to the number of electrons hitting a spot on the screen) as a function of the retartding voltage:

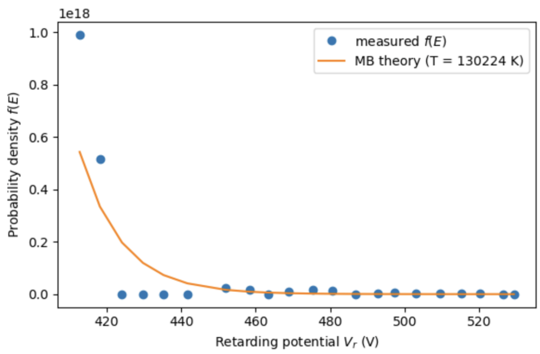


Figure 3: Maxwell-Boltzmann fit

We can see that the energy levels of the electron fall on the high side of the Maxwell-Boltzmann distribution, which is expected as the electrons are emitted from a heated cathode and filtered according to energy level and then accelerated (the Theoretical MB distribution is shown in A.4).

On the otherhand the $\chi^2_{reduces}$ and p-probability are as follows:

χ^2_{red}	p-probability
1.238×10^{18}	0.005

Table 6: Results part B

In addition we got an average "Tempreture" of $130224K$ and i couldnt find any source reccording an energy level of that magnitude, this sugests that there was an incorrect convertion of pixels to cm or a light leak. further detail in the discution.

3 Discussion

3.1 Part A

The results of Part A reveal significant deviations from the theoretical predictions. For all three voltage sets, the measured slopes of $D(V_d/V)$ yielded N_σ values far exceeding 3σ, indicating that systematic errors dominate over statistical uncertainties. Possible sources include:

- **Non-uniform electric field:** Fringe effects and inaccuracies in plate alignment may have caused E to differ from the ideal V_d/d, biasing the deflection.
- **Measurement resolution:** The digitized dot position on the CRT screen was extracted from photographs, and the pixel-to-distance conversion likely underestimates true uncertainties, leading to underestimated error bars.
- **Neglected lensing effects:** Electrostatic focusing elements in the CRT can alter the beam trajectory in ways not accounted for by the simple capacitor model.

I suspect That the most, since after looking at the residuals we ses that nearly in all graphs we undercalculated the errors. On the other hand, the inferred ratio ℓ/d agreed roughly with theory but carried large uncertainties. In the magnetic deflection measurements, the extracted constant K was significantly different between both sets of measurments, as expected for a non-ideal coil geometry, but their relative uncertainties remained below 2%, suggesting reasonable internal consistency. However, the values of $\sqrt{e/m}$ derived from the $D(I/\sqrt{V})$ fits were on the order of magnitude below the theoretical value

3.2 Part B

The Maxwell–Boltzmann fit to the energy distribution (Part B) produced a reduced chi-squared $\chi^2_{\text{red}} \sim 10^{18}$ and a p-value ≈ 0.005, indicating a poor fit despite the qualitative shape matching expectations at high energies. The deduced electron "temperature" of 1.3×10^5 K is far above realistic cathode temperatures.

A Figures Appendix

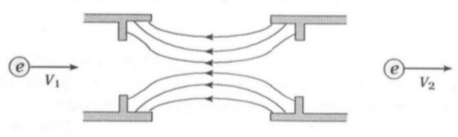


Figure A.1: Electrostatic lens

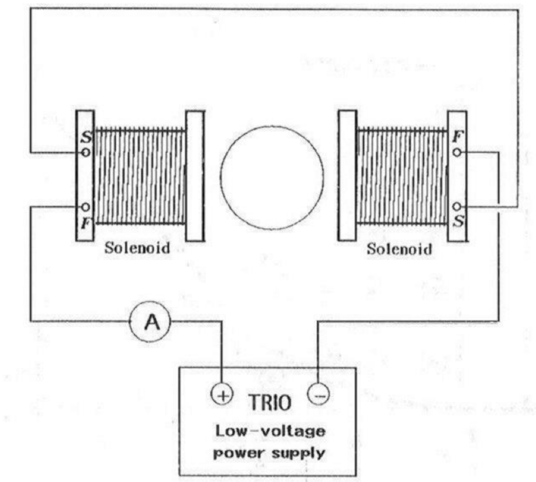


Figure A.2: Magnetic system

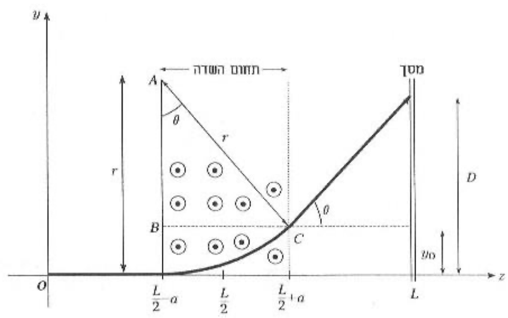


Figure A.3: Electron path in a magnetic field

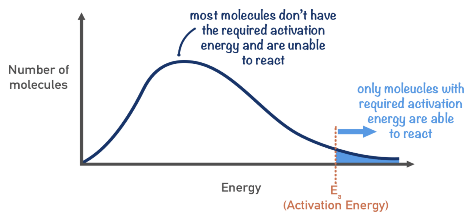
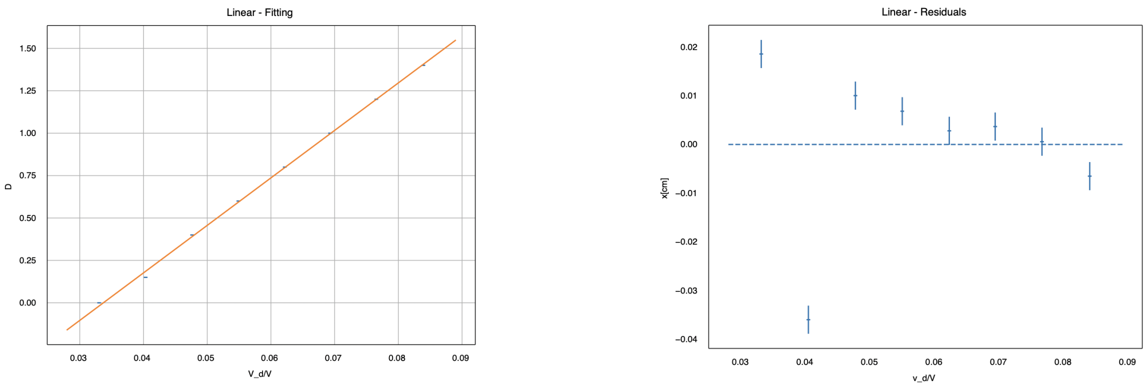


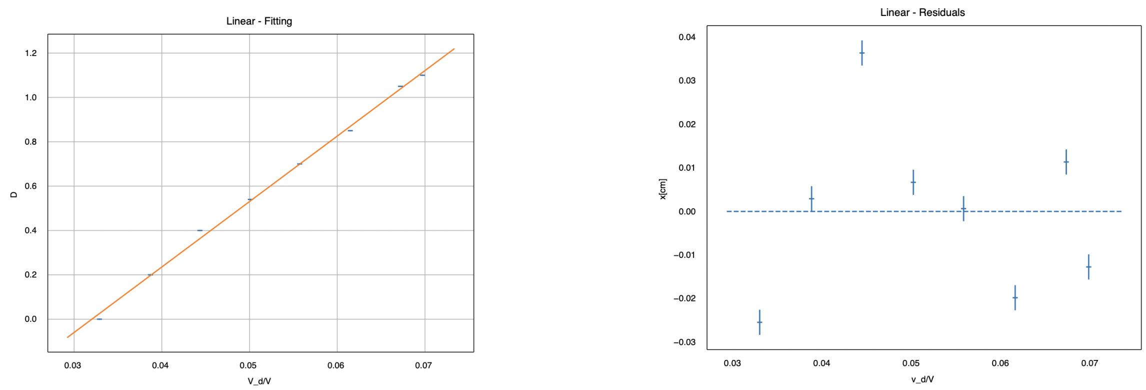
Figure A.4: theoretical Maxwell-Boltzmann distribution



(a) Linear fit

(b) Residuals

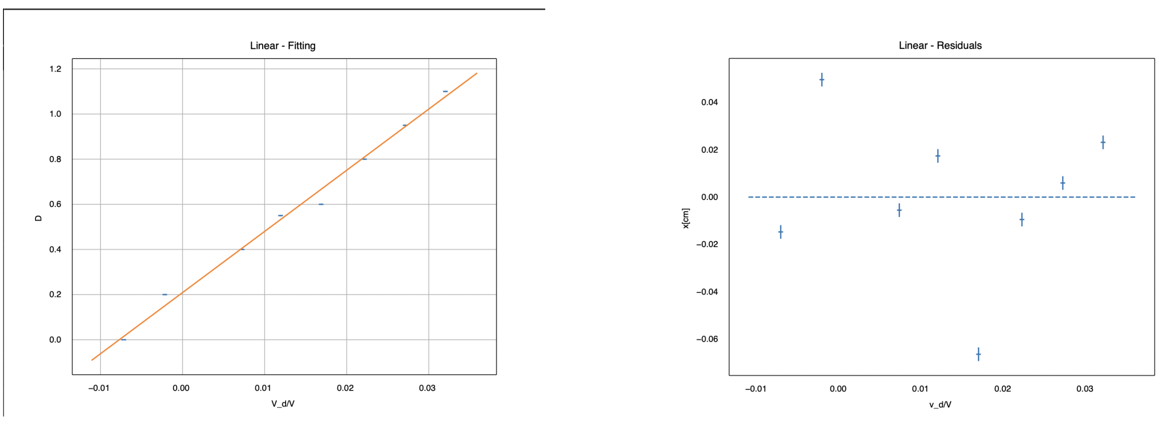
Figure A.5: First set of $D(\frac{V_d}{v})$ measurements



(a) Linear fit

(b) Residuals

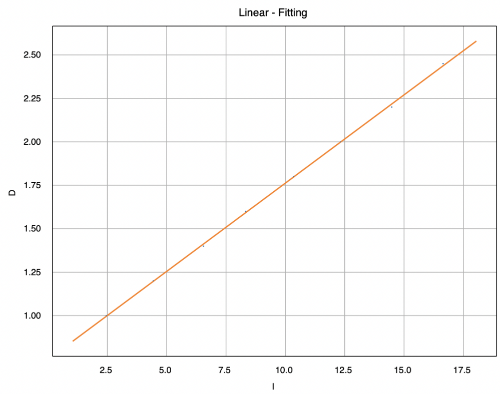
Figure A.6: Second set of $D(\frac{V_d}{v})$ measurements



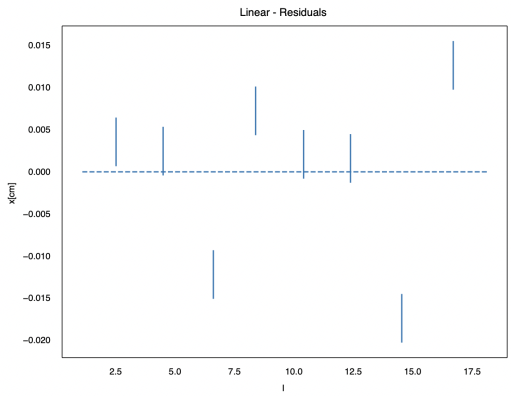
(a) Linear fit

(b) Residuals

Figure A.7: Third set of $D(\frac{V_d}{v})$ measurements

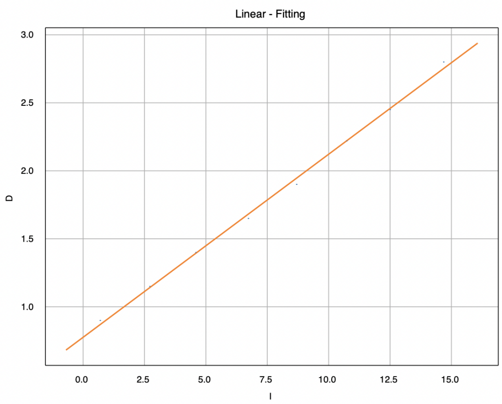


(a) First set D(I) fit

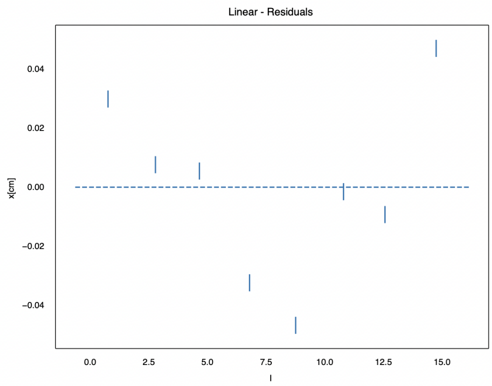


(b) First set D(I) residual

Figure A.8: First set D(I) measurements

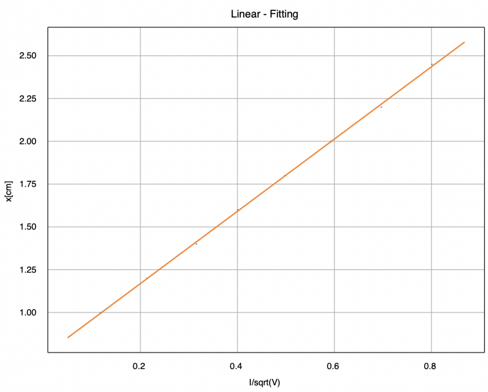


(a) Second set D(I) fit

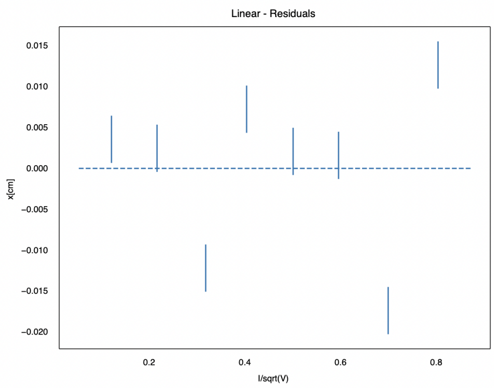


(b) Second set D(I) residual

Figure A.9: Second set D(I) measurements

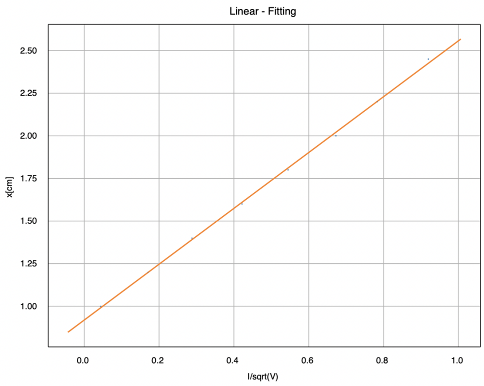


(a) First set $D(\frac{I}{\sqrt{V}})$ fit

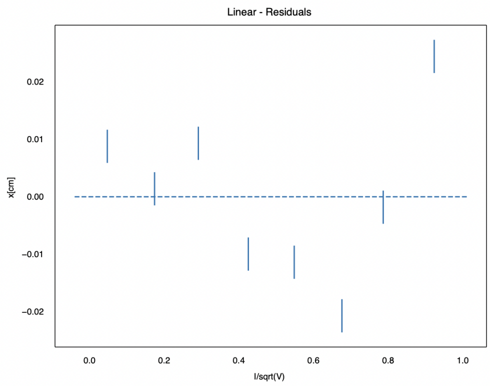


(b) First set $D(\frac{I}{\sqrt{V}})$ residual

Figure A.10: First set $D(\frac{I}{\sqrt{V}})$ measurements



(a) Second set $D\left(\frac{I}{\sqrt{V}}\right)$ fit



(b) Second set $D\left(\frac{I}{\sqrt{V}}\right)$ residual

Figure A.11: Second set $D\left(\frac{I}{\sqrt{V}}\right)$ measurements

B Tables Appendix

χ^2_{red}	p-probability	a_0	a_1
4.184	3.27×10^{-4}	-0.943 ± 0.0227	27.996 ± 0.373

Table B.1: Results from the first set of $D(\frac{V_d}{v})$ measurements

χ^2_{red}	p-probability	a_0	a_1
5.574	8.613×10^{-6}	-0.945 ± 0.0324	29.521 ± 0.601

Table B.2: Results from the second set of $D(\frac{V_d}{v})$ measurements

χ^2_{red}	p-probability	a_0	a_1
19.40	9.218×10^{-23}	0.208 ± 0.0188	27.097 ± 1.011

Table B.3: Results from the third set of $D(\frac{V_d}{v})$ measurements

χ^2_{red}	p-probability	a_0	a_1
13.76	1.06×10^{-15}	$7.467 \pm 8.63 \times 10^{-3}$	$1.01 \pm 8.22 \times 10^{-4}$

Table B.4: Results from first set D(I) measurements

χ^2_{red}	p-probability	a_0	a_1
130.1	2.14×10^{-15}	0.775 ± 0.022	$0.134 \pm 2.553 \times 10^{-3}$

Table B.5: Results from second set D(I) measurements

χ^2_{red}	p-probability	a_0	a_1
13.76	1.059×10^{-15}	$0.746 \pm 0.863 \times 10^{-3}$	2.110 ± 0.017

Table B.6: Results from first set $D\left(\frac{I}{\sqrt{V}}\right)$ measurements

χ^2_{red}	p-probability	a_0	a_1
28.44	3.33×10^{-34}	0.919 ± 0.010	1.637 ± 0.019

Table B.7: Results from second set $D\left(\frac{I}{\sqrt{V}}\right)$ measurements