$$H=\{(x,y,z,w)\in\mathbb{R}^4:\ x=z,\ y=w\}\qquad \bar{v}=(1,-1,-1,1)$$

$$H=\{(x,y,x,y),\ x,y\in\mathbb{R}\}$$

$$\begin{pmatrix}x\\y\\z\\w\end{pmatrix}=\begin{pmatrix}1\\0\\1\\0\end{pmatrix}x+\begin{pmatrix}0\\1\\0\\1\end{pmatrix}y \qquad B_H=\{\underbrace{(1,0,1,0)}_{\bar{v}_1},\ \underbrace{(0,1,0,1)}_{\bar{v}_2}\}$$

$$\bar{u}_1=\frac{(1,0,1,0)}{\sqrt{2}}=\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}},0\right)$$

$$\bar{v}_2'=\bar{v}_2-(\bar{v}_2\cdot\bar{u}_1)\bar{u}_1=(0,1,0,1)-\left[(0,1,0,1)\cdot\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}},0\right)\right]\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}},0\right)$$

$$=(0,1,0,1)-0$$

$$\bar{u}_2=\frac{\bar{v}_2'}{|\bar{v}_2|}=\frac{(0,1,0,1)}{\sqrt{2}}=\left(0,\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)\qquad B''_H=\left\{\left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}},0\right),\left(0,\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)\right\}$$

$$\dim\mathbb{R}^4=4$$

b) $\dim H=2 \implies \dim H^{\perp}=2$

$$\therefore (x,y,z,w)\cdot(1,0,1,0)=0 \implies x+z=0$$
$$x=-z,\ z\in\mathbb{R}$$

$$(x,y,z,w)\cdot(0,1,0,1)=0 \implies y+w=0$$
$$y=-w,\ w\in\mathbb{R}$$

$$\therefore \begin{pmatrix}x\\y\\z\\w\end{pmatrix}=\begin{pmatrix}-1\\0\\1\\0\end{pmatrix}z+\begin{pmatrix}0\\-1\\0\\1\end{pmatrix}w \qquad B_{H^{\perp}}=\{\underbrace{(-1,0,1,0)}_{\bar{v}_1^{\perp}},\ \underbrace{(0,-1,0,1)}_{\bar{v}_2^{\perp}}\}$$

0 -1 0 1

$$\bar{u}_1^{\perp}=\left(-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}},0\right)\ ;$$

$$\bar{v}_2^{\perp\prime}=\bar{v}_2^{\perp}-(\bar{v}_2^{\perp}\cdot\bar{u}_1^{\perp})\bar{u}_1^{\perp}=(0,-1,0,1)-\left[(0,-1,0,1)\cdot\left(-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}},0\right)\right]\left(-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}},0\right)$$

$$\bar{v}_2^{\perp\prime}=(0,-1,0,1)-0 \qquad \bar{u}_2^{\perp}=\left(0,-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)\qquad \boxed{B''_{H^{\perp}}=\left\{\left(-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}},0\right),\left(0,-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)\right\}}$$

$$\bar{p}=\text{proy}_{H^{\perp}}\bar{v}=(\bar{v}\cdot\bar{u}_1^{\perp})\bar{u}_1^{\perp}+(\bar{v}\cdot\bar{u}_2^{\perp})\bar{u}_2^{\perp}$$

$$\bar{p}=\left[(1,-1,-1,1)\cdot\left(-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}},0\right)\right]\left(-\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}},0\right)+\left[(1,-1,-1,1)\cdot\left(0,-\frac{1}{\sqrt{2}},0,+\frac{1}{\sqrt{2}}\right)\right]\left(0,-\frac{1}{\sqrt{2}},0,+\frac{1}{\sqrt{2}}\right)$$

$$\bar{p} = -\frac{2}{\sqrt{2}}\left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0\right) + \frac{2}{\sqrt{2}}\left(0, -\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) = \left(\frac{2}{2}, 0, -\frac{2}{2}, 0\right) + \left(0, -\frac{2}{2}, 0, \frac{2}{2}\right) = (1,-1,-1,1)$$

b)

$$\bar{h} = \text{proy}_H \bar{v} = (\bar{v}\cdot\bar{u}_1)\bar{u}_1 + (\bar{v}\cdot\bar{u}_2)\bar{u}_2$$

$$= \left[(1,-1,-1,1)\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0\right)\right]\left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0\right) + \left[(1,-1,-1,1)\left(0, \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)\right]\left(0, \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

$$\boxed{\bar{h} = \text{proy}_H \bar{v} = \bar{0} = (0,0,0,0)}$$

c) $\bar{v} = \bar{h} + \bar{p} = (0,0,0,0) + (1,-1,-1,1) = (1,-1,-1,1) = \bar{v}$

2) $A = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix}$ $\quad \bar{A}^t = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix} = A \Rightarrow$ A es hermitiana, por lo tanto es unitariamente diagonalizable

$$p(\lambda) = (2-\lambda)(5-\lambda) - (3-3i)(3+3i) = 10-2\lambda-5\lambda+\lambda^2-(9+9i-9i+9) = \lambda^2-7\lambda-8$$

$|A-\lambda I| = (\lambda-8)(\lambda+1) = 0 \qquad \lambda_1 = 8, \lambda_2 = -1$ valores reales diferentes

para $\lambda_1 = 8$: $(A-\lambda I)\bar{v} = \bar{0}$

$$\begin{bmatrix} -6 & 3-3i \\ 3+3i & -3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Haciendo ceros 1er renglón: $(3+3i)x_1 - 3x_2 = 0$

$$(1+i)x_1 = x_2 \Rightarrow \bar{v}_1 = \begin{bmatrix} 1 \\ 1+i \end{bmatrix}$$

$$|\bar{v}_1| = \sqrt{3} \Rightarrow \bar{u}_1 = \left(\frac{1}{\sqrt{3}}, \frac{1+i}{\sqrt{3}}\right)$$

para $\lambda_2 = -1$, $(A-\lambda I)\bar{v} = \bar{0}$

$$\begin{bmatrix} 3 & 3-3i \\ 3+3i & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Haciendo ceros el 2º renglón: $3x_1 + (3-3i)x_2 = 0$

$$x_1 = (-1+i)x_2 \Rightarrow \bar{v}_2 = \begin{bmatrix} -1+i \\ 1 \end{bmatrix}$$

$|\bar{v}_2| = \sqrt{3}$, $\quad \bar{u}_2 = \left(\frac{-1+i}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right)$

$$\therefore \; U = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1+i \\ 1+i & 1 \end{bmatrix}$$

$$U^* A U = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1-i \\ -1-i & 1 \end{bmatrix} \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix} \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1+i \\ 1+i & 1 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 2+3-3i+3i+3 & 3-3i+5-5i \\ -2-2i+3+3i & -3-3i+3i-3+5 \end{bmatrix} \begin{bmatrix} 1 & -1+i \\ 1+i & 1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 8 & 8-8i \\ 1+i & 1 \end{bmatrix} \begin{bmatrix} 1 & -1+i \\ 1+i & 1 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 8+8+8 & -8+8i+8-8i \\ 1+i-1-i & -1+i-i-1-1 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 24 & 0 \\ 0 & -3 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 0 & -1 \end{bmatrix}$$

$- \; 4x^2 - 2xy + 4y^2 = 25$

a) $\bar{X}^t A \bar{X} = [x, y] \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 25$

b) $p(\lambda) = (4-\lambda)(4-\lambda) - 1 = 16 - 4\lambda - 4\lambda + \lambda^2 - 1 = \lambda^2 - 8\lambda + 15 = (\lambda-3)(\lambda-5)$

$\lambda_1 = 3, \; \lambda_2 = 5$

para $\lambda_1 = 3$: $(A - \lambda I)\bar{v} = 0$

$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad x - y = 0, \; x = y \quad \bar{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad |\bar{v}_1| = \sqrt{2} \quad \bar{u}_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$

para $\lambda_2 = 5$, $(A - \lambda I)\bar{v} = \bar{0}$

$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad -x_1 - x_2 = 0, \quad x_1 = -x_2 \quad \bar{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad |\bar{v}_2| = \sqrt{2}, \quad \bar{u}_2 = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$

$Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$, $\quad |Q| = \frac{1}{2} + \frac{1}{2} = 1 \quad \cos\theta = \frac{1}{\sqrt{2}}, \; \theta = 45°$

$\operatorname{sen}\theta = \frac{1}{\sqrt{2}}$ I Cuad

$$D = Q^t A Q = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 3 & 3 \\ -5 & 5 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 6 & 0 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}$$

$$\therefore \bar{X}'^t D \bar{X}' = [x', y'] \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix} = 3x'^2 + 5y'^2$$

$3x'^2 + 5y'^2 = 25$ elipse con $\theta = 45°$

$\frac{x'^2}{\frac{25}{3}} + \frac{y'^2}{5} = 1$ elipse con eje mayor el eje x'

$a^2 = 25, \; a = 5$

$b^2 = \frac{25}{3}, \; b = \frac{5}{\sqrt{3}} = 2.88$