1) $p(x) = 2x^6 + 3x^5 + ax^4 + 15x^3 - 32x^2 + 12x$

1	2	3	a	15	-32	12
		2	5	5+a	20+a	-12+a
	2	5	5+a	20+a	-12+a	a

$a = 0$

rn	1	1
rrp	2	0
rrn	1	1
rc	6	4
TOTAL	6	6

$\therefore p(x) = 2x^6 + 3x^5 + 15x^3 - 32x^2 + 12x$

$p(x) = x\,q_1(x)$ $\quad \boxed{\alpha_1 = 0}$

$q_1(x) = 2x^5 + 3x^4 + 15x^2 - 32x + 12$ 2 ó 0 rrp

$q_1(-x) = -2x^5 + 3x^4 + 15x^2 + 32x + 12$ 1 rrn

factores de $a_0(p)$: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 12$

factores de $a_1(q)$: $\pm 1, \pm 2$

prr $\left(\frac{p}{q}\right)$: $\pm 1, \pm 2, \pm 3, \pm 6, \pm 12, \pm \frac{1}{2}, \pm \frac{3}{2}$

$\alpha_2 = 1$

1	2	3	0	15	-32	12
		2	5	5	20	-12
	2	5	5	20	-12	0

$\boxed{\alpha_2 = 1}$

$q_1(x) = (x-1)\,q_2(x)$

$q_2(x) = 2x^4 + 5x^3 + 5x^2 + 20x - 12$

con $\alpha = -3$: $2x^4 - 5x^3 + 5x^2 - 20x - 12$

-3	2	5	5	20	-12
		-6	3	-24	12
	2	-1	8	-4	0

$\boxed{\alpha_3 = -3}$

$q_2 = (x+3)\,q_3(x)$

$q_3(x) = 2x^3 - x^2 + 8x - 4$

$\alpha = \frac{1}{2}$:

$\frac{1}{2}$	2	-1	8	-4
		1	0	4
	2	0	8	0

$\boxed{\alpha_4 = \frac{1}{2}}$

$q_3(x) = (x - \frac{1}{2})\,q_4(x)$

$q_4(x) = 2x^2 + 8$

$= 2(x^2 + 4)$; $x^2 = -4$

$\boxed{\alpha_5 = 2i}$

$\boxed{\alpha_6 = -2i}$

$\therefore p(x) = 2x(x-1)(x+3)(x-\frac{1}{2})(x-2i)(x+2i)$

5
16
12
3

2) $f(y) = 4y^9 - 20y^8 + 49y^7 - 100y^6 + 136y^5 - 80y^4 + 16y^3$

$\alpha_1 = 0$
$\alpha_2 = 0$
$\alpha_3 = 0$

$f(y) = y^3(4y^6 - 20y^5 + 49y^4 - 100y^3 + 136y^2 - 80y + 16)$

$q_1(y) = 4y^6 - 20y^5 + 49y^4 - 100y^3 + 136y^2 - 80y + 16$ 6 ó 4 ó 2 ó 0 rrp

$q_1(-y) = 4y^6 + 20y^5 + 49y^4 + 100y^3 + 136y^2 + 80y + 16$ 0 rrn

rn	3	3	3	3
rrp	6	4	2	0
rc	0	2	4	6
TOT	9	9	9	9

factores de a_0: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

factores de a_n: $\pm 1, \pm 2, \pm 4$

prr $\left(\frac{p}{q}\right)$: $1, \frac{1}{2}, \frac{1}{4}, 2, 4, 8, 16$

con $\alpha = 1$:

1	4	-20	49	-100	136	-80	16
		4	-16	33	-77	59	-21
	4	-16	33	-77	59	-21	|-5

$\frac{1}{2}$	4	-20	49	-100	136	-80	16
		2	-9	20	-40	48	-16
	4	-18	40	-80	96	-32	|0

$\alpha_4 = \frac{1}{2}$

$q_1(y) = (y - \frac{1}{2})\, q_2(y)$

$q_2(y) = 4y^5 - 18y^4 + 40y^3 - 80y^2 + 96y - 32$

$\frac{1}{2}$	4	-18	40	-80	96	-32
		2	-8	16	-32	32
	4	-16	32	-64	64	|0

$\alpha_5 = \frac{1}{2}$

$q_2(y) = (y - \frac{1}{2})\, q_3(y)$

$q_3(y) = 4y^4 - 16y^3 + 32y^2 - 64y + 64$

2	4	-16	32	-64	64
		8	-16	+32	-64
	4	-8	16	-32	|0

$\alpha_6 = 2$

$q_3(y) = (y-2)\, q_4(y)$

$q_4(y) = 4y^3 - 8y^2 + 16y - 32$

2	4	-8	16	-32
		8	0	32
	4	0	16	|0

$\alpha_7 = 2$

$q_4(y) = (y-2)\, q_5(y)$

$q_5(y) = 4y^2 + 16 = 4(y^2 + 4)$

$y^2 = \sqrt{-4}$ — $\alpha_8 = 2i$, $\alpha_9 = -2i$

$f(y) = 4y^3 (y - \frac{1}{2})^2 (y-2)^2 (y-2i)(y+2i)$

3) $x + y - z = 1$
$2x + 3y + az = 3$
$x + ay + 3z = 2$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 2 & 3 & a & 3 \\ 1 & a & 3 & 2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & a+2 & 1 \\ 0 & a-1 & 4 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 1 \\ 0 & 1 & a+2 & 1 \\ 0 & 0 & (a+3)(a-2) & -a+2 \end{array}\right]$$

$R_2 \rightarrow R_2 - 2R_1$ $\quad$ $R_3 \rightarrow R_3 - (a-1)R_2$
$R_3 \rightarrow R_3 - R_1$

Para que el sistema no tenga solución: $a = -3$
tenga infinidad de sol's: $a = 2$
tenga sol. única: $a \neq -3$ y $a \neq 2$

$$\left[\begin{array}{ccc|c} 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & m \end{array}\right]$$

4) $x + y + z + 2t = 0$
$-2x - 3y + 3z + 2t = -6$
$-y + 2t = -1$
$2x + 3y - z + t = 5$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 0 \\ -2 & -3 & 3 & 2 & -6 \\ 0 & -1 & 0 & 2 & -1 \\ 2 & 3 & -1 & 1 & 5 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 0 \\ 0 & -1 & 5 & 6 & -6 \\ 0 & -1 & 0 & 2 & -1 \\ 0 & 1 & -3 & -3 & 5 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 1 & 2 & 0 \\ 0 & 1 & -5 & -6 & 6 \\ 0 & -1 & 0 & 2 & -1 \\ 0 & 1 & -3 & -3 & 5 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 6 & 8 & -6 \\ 0 & 1 & -5 & -6 & 6 \\ 0 & 0 & -5 & -4 & 5 \\ 0 & 0 & 2 & 3 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 6 & 8 & -6 \\ 0 & 1 & -5 & -6 & 6 \\ 0 & 0 & 1 & 4/5 & -1 \\ 0 & 0 & 2 & 3 & -1 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 16/5 & 0 \\ 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & 4/5 & -1 \\ 0 & 0 & 0 & 7/5 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 16/5 & 0 \\ 0 & 1 & 0 & -2 & 1 \\ 0 & 0 & 1 & 4/5 & -1 \\ 0 & 0 & 0 & 1 & 5/7 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -16/7 \\ 0 & 1 & 0 & 0 & 17/7 \\ 0 & 0 & 1 & 0 & -11/7 \\ 0 & 0 & 0 & 1 & 5/7 \end{array}\right]$$

$x = -16/7$
$y = 17/7$
$z = -11/7$
$t = 5/7$