Clase 30-10-20

viernes, 30 de octubre de 2020 10:51 a. m.

Ejemplo Obtener el o los valores de $z \in \mathbb{C}$ que satisfacen la siguiente ecuación:

$$\left(\frac{z_1 z_2}{z^{2/5}}\right) z_3 - z_4 = z_5$$

donde:

$$z_1 = 5(\cos 300° + i \operatorname{sen} 300°)$$

$$z_2 = e^{\frac{4}{3}\pi i}$$

$$z_3 = 4(\cos 30° + i \operatorname{sen} 30°)$$

$$z_4 = 4\left(\cos \frac{\pi}{2} + i \operatorname{sen} \frac{\pi}{2}\right)$$

$$z_5 = -2\sqrt{3} - 2i$$

Obtener el resultado en forma polar y de Euler y trazar el Diagrama de Argand.

Solución:

Para despejar $z^{2/5}$

$$\frac{z_1 z_2 z_3}{z^{2/5}} - z_4 = z_5$$

$$\frac{z_1 z_2 z_3}{z^{2/5}} = z_5 + z_4$$

$$\frac{z_1 z_2 z_3}{z_4 + z_5} = z^{\frac{2}{5}}$$

Entonces:

$$z = \left(z^{2/5}\right)^{5/2}$$

$$z = \left(\frac{z_1 z_2 z_3}{z_4 + z_5}\right)^{5/2}.$$

Por partes:

$$z_1 = 5 \operatorname{cis} 300^\circ$$

$$z_2 = e^{\frac{4}{3}\pi i} = \operatorname{cis} 240^\circ$$

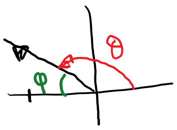
$$z_3 = 4 \operatorname{cis} 30^\circ$$

$$\therefore\ z_1 z_2 z_3 = 20 \operatorname{cis}(300^\circ + 240^\circ + 30^\circ) = 20 \operatorname{cis} 210^\circ$$

$$z_4 = 4 \operatorname{cis} 90^\circ = 4i$$

$$z_5 = -2\sqrt{3} - 2i$$

$$\therefore\ z_4 + z_5 = -2\sqrt{3} + 2i \xrightarrow{\text{polar}} r = \sqrt{(-2\sqrt{3})^2 + (2)^2}$$

II 

$$r = \sqrt{12+4} = \sqrt{16} = 4$$

$$\theta = 180^\circ - \varphi$$

$$\varphi = \tan^{-1}\left(\frac{2}{2\sqrt{3}}\right) = 30^\circ$$

$$\therefore\ \theta = 150^\circ$$

$$z_4 + z_5 = 4 \operatorname{cis} 150^\circ$$

Ahora:

$$z = \left(\frac{z_1 z_2 z_3}{z_4 + z_5}\right)^{5/2} = \sqrt{\left(\frac{20 \operatorname{cis} 210^\circ}{4 \operatorname{cis} 150^\circ}\right)^5} = \sqrt{(5 \operatorname{cis} 60^\circ)^5} = \sqrt{3125 \operatorname{cis} 300}$$

$$z = 25\sqrt{5} \operatorname{cis} \frac{300 + k(360^\circ)}{2}, \quad k = 0, 1$$

Para $k=0$, $w_0 = 25\sqrt{5} \operatorname{cis} 150^\circ = 25\sqrt{5}\, e^{\frac{5}{6}\pi i}$

Para $k=1$, $w_1 = 25\sqrt{5} \operatorname{cis} 330^\circ = 25\sqrt{5}\, e^{\frac{11}{6}\pi i}$

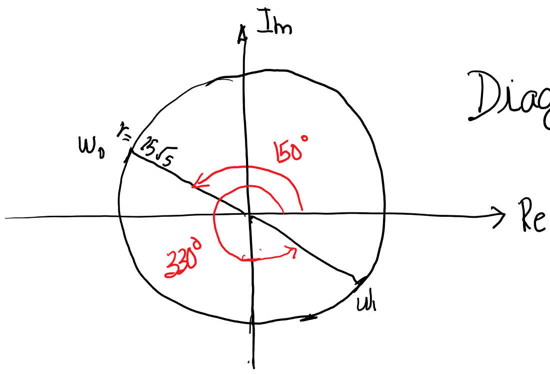


Diagrama de Argand

Ejercicio

Efectuar las siguientes operaciones:

a) $1-e^{-\pi i}$, b) $\dfrac{1-e^{\frac{\pi}{2}i}}{1+e^{\frac{\pi}{2}i}}$, c) $i+e^{2\pi i}$

$$1-e^{\pi i} = 1-(-1) = 2$$

a) $1-e^{-\pi i} = 1-\dfrac{1}{e^{\pi i}} = 1-\dfrac{1}{\cos\pi + i\,\text{sen}\,\pi} = 1-\dfrac{1}{-1} = 1+1 = 2$

b) $\dfrac{1-(\cos\frac{\pi}{2}+i\,\text{sen}\,\frac{\pi}{2})}{1+(\cos\frac{\pi}{2}+i\,\text{sen}\,\frac{\pi}{2})} = \dfrac{1-(0+i)}{1+(0+i)} = \dfrac{1-i}{1+i}\cdot\dfrac{1-i}{1-i} = \dfrac{1-2i-1}{1+1} = \dfrac{-2i}{2} = -i$

c) $i+e^{2\pi i} = i+(\cos 2\pi + i\,\text{sen}\,2\pi) = i+1 = 1+i$

Ejercicio

Representar en el Diagrama de Argand las soluciones de la ecuación:

$$\frac{4-4i}{z^{3/4}} = 2e^{\pi i}$$

$$z^{3/4} = \frac{4-4i}{2e^{\pi i}}$$

$4-4i \xrightarrow{\text{polar}} r = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

$\theta = 360° - \varphi$ IV Cuadrante

$\varphi = \tan^{-1}\left(\frac{4}{4}\right) = 45° \Rightarrow \theta = 315°$

$\therefore\ 4-4i = 4\sqrt{2}\ \text{cis}\ 315°$

$2e^{\pi i} = 2\ \text{cis}\ 180°$

$\therefore\ z^{3/4} = \dfrac{4\sqrt{2}\ \text{cis}\ 315°}{2\ \text{cis}\ 180°} = 2\sqrt{2}\ \text{cis}\ 135°$

y $z = \left(z^{3/4}\right)^{4/3} = \left(2\sqrt{2}\ \text{cis}\ 135°\right)^{4/3} = \sqrt[3]{\left(2\sqrt{2}\ \text{cis}\ 135°\right)^4}$

$= \sqrt[3]{64\ \text{cis}\ 540°} = \sqrt[3]{64\ \text{cis}\ 180°}$

$= 4\ \text{cis}\left(\dfrac{180°+k(360°)}{3}\right)$, $k=0,1,2$

Para $K=0$: $w_0 = 4 \text{ cis } \frac{180°}{3} = 4 \text{ cis } 60° = 4e^{\frac{\pi}{3}i}$

Para $K=1$, $w_1 = 4 \text{ cis } 180° = 4e^{\pi i}$

Para $K=2$, $w_2 = 4 \text{ cis } 300° = 4e^{\frac{5}{3}\pi i}$

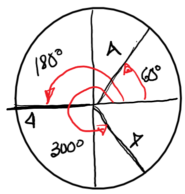


Diagrama de Argand

Tarea

Dados: $z_1 = 1+i$, $z_2 = \sqrt{2}\, e^{\frac{\pi}{4}i}$, $z_3 = e^{2\pi i}$, $z_4 = 8 \text{ cis } 30°$,

obtener $z \in \mathbb{C}$ tal que:

$$z_1 + z_2 = \frac{z_3 \cdot z_4}{z^{3/2}}$$ Trabajar Euler

Trazar el Diagrama de Argand.