1.- $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2 + 1,\ y_1 + y_2 + 1)$

Sean $X, Y, Z \in H$ y $\alpha, \beta \in \mathbb{R}$

S1) $\bar{X} + \bar{Y} \in H$ Cerradura de la Suma

si $\bar{X} + \bar{Y} \in H$: $\bar{X} + \bar{Y} = (x_1 + x_2 + 1,\ y_1 + y_2 + 1)$

Verificación

$$\bar{X} = (x_1, y_1)$$
$$\bar{Y} = (x_2, y_2)$$
$$\bar{X} + \bar{Y} = (x_1 + x_2,\ y_1 + y_2) : (x_1 + x_2 + 1,\ y_1 + y_2 + 1)$$
$$\bar{X} + \bar{Y} \in H$$

S2 $\bar{X} + (\bar{Y} + \bar{Z}) = (\bar{X} + \bar{Y}) + \bar{Z}$ Asociativa

$$\bar{X} + (\bar{Y} + \bar{Z}) = (x_1, y_1) + (x_2 + x_3 + 1,\ y_2 + y_3 + 1)$$
$$= (x_1 + (x_2 + x_3 + 1) + 1,\ y_1 + (y_2 + y_3 + 1) + 1)$$
$$= (x_1 + x_2 + x_3 + 1 + 1,\ y_1 + y_2 + y_3 + 1 + 1)$$
$$= (x_1 + x_2 + 1) + x_3 + 1,\ (y_1 + y_2 + 1) + y_3 + 1$$
$$[(x_1 + x_2 + 1), (y_1 + y_2 + 1)] + (x_3 + 1,\ y_3 + 1)$$
$$= (\bar{X} + \bar{Y}) + \bar{Z}$$

S3) $\exists\, \bar{0} \in H \mid \bar{X} + \bar{0} = \bar{X}$ Elemento neutro

$$\bar{0} \in H = (-1, -1)$$
$$\bar{X} + \bar{0} = \bar{X}$$
$$(x_1, y_1) + (-1, -1) = (x_1 - 1 + 1,\ y_1 - 1 + 1) = (x_1, y_1)$$

S4) $\exists\, -\bar{X} \in H \mid \bar{X} + (-\bar{X}) = 0$ Inverso aditivo

$$-\bar{X} \in H = (-x_1 - 2, -y_1 - 2)$$
$$\bar{X} + (-\bar{X}) = (x_1, y_1) + (-x_1 - 2, -y_1 - 2) =$$
$$= (x_1 - x_1 - 2 + 1,\ y_1 - y_1 - 2 + 1) = (-1, -1)$$

S5) $\bar{X} + \bar{Y} = \bar{Y} + X$ Conmutativa

$$\bar{X} + \bar{Y} = (x_1, y_1) + (x_2, y_2)$$
$$= (x_1 + x_2 + 1,\ y_1 + y_2 + 1$$
$$= (x_2 + x_1 + 1,\ y_2 + y_1 + 1)$$
$$= (x_2, y_2) + (x_1, y_1)$$

M1) $\alpha \bar{X} \in H$ cerradura producto

si $\alpha \bar{X} \in H$, $\alpha \bar{X} = \alpha (x_1, y_1)$
$$= (\alpha x_1, \alpha y_1)$$
$$\alpha \bar{X} \in H$$

*M2) $\alpha (\bar{X} + \bar{Y}) = \alpha \bar{X} + \alpha \bar{Y}$ 1ª Distribución

$$\alpha (\bar{X} + \bar{Y}) = \alpha\, [(x_1, y_1) + (x_2, y_2)]$$
$$= \alpha\, (x_1 + x_2 + 1,\ y_1 + y_2 + 1)$$
$$= (\alpha x_1 + \alpha x_2 + \alpha,\ \alpha y_1 + \alpha y_2 + \alpha)$$

$$\alpha \bar{X} = (\alpha x_1, \alpha y_1)$$
$$\alpha \bar{Y} = (\alpha x_2, \alpha y_2)$$
$$\alpha \bar{X} + \alpha \bar{Y} = (\alpha x_1 + \alpha x_2 + 1,\ \alpha y_1 + \alpha y_2 + 1)$$
$$\therefore\ \alpha (\bar{X} + \bar{Y}) \neq \alpha \bar{X} + \alpha \bar{Y}$$

*M3) $(\alpha + \beta)\bar{X} = \alpha \bar{X} + \beta \bar{X}$ 2ª Ley de Distribución

$$(\alpha + \beta)\bar{X} = (\alpha + \beta)(x_1, y_1)$$
$$= ((\alpha + \beta)x_1, (\alpha + \beta)y_1)$$
$$= (\alpha x_1 + \beta x_1,\ \alpha y_1 + \beta y_1)$$

$$\alpha \bar{X} = \alpha (x_1, y_1) = (\alpha x_1, \alpha y_1)$$
$$\beta \bar{X} = \beta (x_1, y_1) = (\beta x_1, \beta y_1)$$
$$\alpha \bar{X} + \beta \bar{X} = (\alpha x_1 + \beta x_1 + 1,\ \alpha y_1 + \beta y_1 + 1)$$
$$= ((\alpha + \beta)x_1 + 1, (\alpha + \beta)y_1 + 1)$$

$$\therefore\ (\alpha + \beta)\bar{X} \neq \alpha \bar{X} + \beta \bar{X}$$

H no es espacio vectorial

$$\bar{X} + \bar{X}^{-1} = (-1, -1)$$
$$(x_1, y_1) + (x_1^{-1}, y_1^{-1}) = (x_1 + x_1^{-1} + 1,\ y_1 + y_1^{-1} + 1)$$

donde: $x_1 + x_1^{-1} + 1 = -1 \Rightarrow x_1^{-1} = -x_1 - 2$

$y_1 + y_1^{-1} + 1 = -1 \Rightarrow y_1^{-1} = -y_1 - 2$

$$\therefore\ \bar{X}^{-1} = (-x_1 - 2, -y_1 - 2)$$

2) $\bar{u} = (2,-3,1)$ $\bar{v} = (x,y,z)$ ②

a) $\bar{u} \cdot \bar{v} = (2,-3,1) \cdot (x,y,z) = 0$

$= 2x - 3y + z = 0$ plano que pasa por el origen en $\mathbb{R}^2$

Sean $\bar{u}, \bar{w} \in H$ y $\alpha \in \mathbb{R}$

S1) $\bar{x} + \bar{y} \in H$

si $\bar{x} + \bar{y} \in H$: $2x - 3y + z = 0$

Verificación

$\bar{x} = (x_1, y_1, z_1) : 2x_1 - 3y_1 + z_1 = 0$

$\bar{y} = (x_2, y_2, z_2) : 2x_2 - 3y_2 + z_2 = 0$

$\bar{x} + \bar{y} = (x_1 + x_2, y_1 + y_2, z_1 + z_2) : 2(x_1 + x_2) - 3(y_1 + y_2) + z_1 + z_2 = 0$?

$2x_1 + 2x_2 - 3y_1 - 3y_2 + z_1 + z_2 = 0$?

$\underbrace{(2x_1 - 3y_1 + z_1)}_{0} + \underbrace{(2x_2 - 3y_2 + z_2)}_{0} = 0$

$= 0$ ✓

$\therefore \bar{x} + \bar{y} \in H$

S2) $\alpha \bar{x} \in H$

si $\alpha \bar{x} \in H$: $2x - 3y + z = 0$

Verificación:

$\alpha \bar{x} = \alpha (x_1, y_1, z_1)$

$= (\alpha x_1, \alpha y_1, \alpha z_1) : 2\alpha x_1 - 3\alpha y_1 + \alpha z_1 = 0$?

$\alpha \underbrace{(2x_1 - 3y_1 + z_1)}_{0} = 0$

$\therefore \alpha \bar{x} \in H$

y H es un subespacio vectorial

b) $2x - 3y + z = 0$

$z = -2x + 3y$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} +1 \\ 0 \\ -2 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix} y$$

Los vectores $\bar{x} = (1,0,-2)$ y $\bar{y} = (0,1,3)$ son linealmente independientes

(3)

3) $\text{gen}\{\bar{v}_1, \bar{v}_2, \bar{v}_3\} = \{\bar{v} \in \mathbb{R}^3,\ \bar{v} = a_1\bar{v}_1 + a_2\bar{v}_2 + a_3\bar{v}_3\}$

$(x,y,z) = c_1\bar{v}_1 + c_2\bar{v}_2 + c_3\bar{v}_3$ CL

$(x,y,z) = c_1(2,0,1) + c_2(4,1,1) + c_3(1,-1,1)$

$= (2c_1 + 4c_2 + c_3,\ c_2 - c_3,\ c_1 + c_2 + c_3)$

$$\left.\begin{aligned} x &= 2c_1 + 4c_2 + c_3 \\ y &= c_2 - c_3 \\ z &= c_1 + c_2 + c_3 \end{aligned}\right\} \text{SE}$$

$$\begin{bmatrix} 2 & 4 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{2H}$$

Solución por Gauss:

$$\left[\begin{array}{ccc|c} 2 & 4 & 1 & x \\ 0 & 1 & -1 & y \\ 1 & 1 & 1 & z \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 1 & z \\ 0 & 1 & -1 & y \\ 2 & 4 & 1 & x \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 1 & z \\ 0 & 1 & -1 & y \\ 0 & 2 & -1 & x-2z \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & -y+z \\ 0 & 1 & -1 & y \\ 0 & 0 & 1 & x-2y-2z \end{array}\right] \to$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -2x+3y+5z \\ 0 & 1 & 0 & x-y-2z \\ 0 & 0 & 1 & x-2y-2z \end{array}\right]$$

El espacio generado por $\{(2,0,1), (4,1,1), (1,-1,1)\}$ es $\mathbb{R}^3$

4) $\begin{vmatrix} 1 & 1 & 1 & 0 \\ -1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1 \begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \end{vmatrix} = 1(1) - 1(-1) + 0 = 2 \neq 0 \quad \therefore$ sí es una base

$c_1(1-x) + c_2(1+x) + c_3(1+x+x^2) + c_4(x+x^3) = 0 + 0x + 0x^2 + 0x^3$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ -1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$c_1 + c_2 + c_3 = 0$

$-c_1 + c_2 + c_3 + c_4 = 0$

$c_3 = 0$

$c_4 = 0$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \to \cdots \to \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

$c_1 = c_2 = c_3 = c_4 = 0$

son 4 polinomios l. indep en P^3

$\therefore B$ es una base para P_3

5) $B_1 = \{(1,4,-5), (0,3,2), (1,-5,-2)\}$, $B_2 = \{(-2,-3,-2), (-1,-4,-5), (-3,-5,-4)\}$

$(1,4,5) = c_1(-2,-3,-2) + c_2(-1,-4,-5) + c_3(-3,-5,-4)$

$(0,3,2) = (-2c_1 - c_2 - 3c_3,\ -3c_1 - 4c_2 - 5c_3,\ -2c_1 - 5c_2 - 4c_3)$

$(1,-5,-2) =$

$$\begin{array}{lcc}
1 = -2c_1 - c_2 - 3c_3 & 0 & 1 \\
4 = -3c_1 - 4c_2 - 5c_3 & 3 & -5 \\
-5 = -2c_1 - 5c_2 - 4c_3 & 2 & -2
\end{array}$$

$$\begin{bmatrix} -2 & -1 & -3 \\ -3 & -4 & -5 \\ -2 & -5 & -4 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ -5 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ -2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} -2 & -1 & -3 & 1 & 0 & 1 \\ -3 & -4 & -5 & 4 & 3 & -5 \\ -2 & -5 & -4 & -5 & 2 & -2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1/2 & 3/2 & -1/2 & 0 & -1/2 \\ -3 & -4 & -5 & 4 & 3 & -5 \\ -2 & -5 & -4 & -5 & 2 & -2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 1/2 & 3/2 & -1/2 & 0 & -1/2 \\ 0 & -5/2 & -1/2 & 5/2 & 3 & -13/2 \\ 0 & -4 & -1 & -6 & 2 & -3 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1/2 & 3/2 & -1/2 & 0 & -1/2 \\ 0 & 1 & 1/5 & -1 & -6/5 & 13/5 \\ 0 & -4 & -1 & -6 & 2 & -3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 7/5 & 0 & 3/5 & -9/5 \\ 0 & 1 & 1/5 & -1 & -6/5 & 13/5 \\ 0 & 0 & -1/5 & -10 & -14/5 & 37/5 \end{array}\right] \rightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 7/5 & 0 & 3/5 & -9/5 \\ 0 & 1 & 1/5 & -1 & -6/5 & 13/5 \\ 0 & 0 & 1 & 50 & 14 & -37 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -70 & -19 & 50 \\ 0 & 1 & 0 & -11 & -4 & 8 \\ 0 & 0 & 1 & 50 & 14 & -37 \end{array}\right] \qquad M_T = \begin{bmatrix} -70 & -19 & 50 \\ -11 & -4 & 8 \\ 50 & 14 & -37 \end{bmatrix}$$

$$(X)_{B_2} = \begin{bmatrix} -70 & -19 & 50 \\ -11 & -4 & 8 \\ 50 & 14 & -37 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -20 \\ -3 \\ 13 \end{bmatrix}$$